Shearing Interferometer for Quantifying the Coherence of Hard X-Ray Beams

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We report a quantitative measurement of the full transverse coherence function of the 14.4 keV x-ray radiation produced by an undulator at the Swiss Light Source. An x-ray grating interferometer consisting of a beam splitter phase grating and an analyzer amplitude grating has been used to measure the degree of coherence as a function of the beam separation out to 30 μm. Importantly, the technique provides a model-free and spatially resolved measurement of the complex coherence function and is not restricted to high resolution detectors and small fields of view. The spatial characterization of the wave front has important applications in discovering localized defects in beam line optics.

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Coherence is the common denominator of many of the most exciting x-ray research techniques that exploit the radiation produced by third generation synchrotron sources. X-ray photon correlation spectroscopy [1], coherent x-ray diffraction imaging [2], lensless imaging [3], and phase contrast radiography or tomography [4] are only a few examples of recently developed techniques. Furthermore, the virtues of coherent beams of x rays have motivated the construction of soft x-ray lasers and the strong effort to build fourth generation hard x-ray free electron laser sources. While it is widely recognized that coherence-based experiments demand characterization tools to monitor the stability, shape, and size of the x-ray beam, the intrinsic coherence properties are equally important. Since coherence is readily modified by the beam line optics, this can have a dramatic impact on the quality of measured data [5,6].

Traditionally, the coherence properties of x rays are characterized by a single number, the so-called (transverse) coherence length. In classical optics this number is related to the visibility of interference fringes in a Young's double slit experiment [7]. Although this can be quite a challenging experiment for hard x rays, it has been demonstrated successfully [8]. Measurements of the coherence length were also reported by using precisely defined objects, such as polished slits or fibers [9].

A more general description should quantify the coherence on multiple length scales. In visible light optics, this is achieved by the so-called mutual coherence function (MCF). The MCF is defined as the correlation between two wave fields separated in space and time [7]. In the case of a quasimonochromatic experiment, the time average of the MCF over a time interval larger than the typical fluctuations present in the source, T, is called the mutual intensity function (MIF) [10],

$$J(\vec{r}_1, \vec{r}_2) = \langle E(\vec{r}_1, t)E^*(\vec{r}_2, t)\rangle_T, \tag{1}$$

where $\vec{r}_1$, $\vec{r}_2$ are points in a plane perpendicular to the optical axis and $E(\vec{r}_i, t)$ are the corresponding field values. It is usual to normalize the MIF as

$$\gamma(\vec{r}_1, \vec{r}_2) = \frac{J(\vec{r}_1, \vec{r}_2)}{\sqrt{J(\vec{r}_1, \vec{r}_1)J(\vec{r}_2, \vec{r}_2)}}, \tag{2}$$

which is known as the complex coherence factor (CCF).

Whereas most classical x-ray applications do not require it, some recently developed coherent x-ray techniques [2] explicitly rely on precise knowledge of the CCF. Consequently, several attempts have been made to measure this CCF for specific experimental arrangements at synchrotron x-ray sources. Using a cleverly designed mask comprising a whole range of different Young's slit apertures in a uniformly redundant array (URA), J. Lin *et al.* recently demonstrated that the full coherence function of an x-ray beam can be obtained [11]. However, several constraints restrict the use of URAs. For URA apertures of up to a hundred microns, the far-field diffraction pattern contains extremely fine details, which cannot be resolved with the required accuracy for hard x rays. This, in turn, limits the spatial range over which the degree of coherence can be measured to a few microns for hard x rays. Another severe drawback of this approach is that detailed knowledge about the fine structure of the URA enters the numerical data processing from which the CCF is extracted.

In this Letter we present a new approach, which overcomes these limitations and leads to a quantitative and model-free method for measuring the CCF. Instead of measuring the far-field diffraction pattern, we measure the near-field interference fringes produced by a simple periodic grating. It has been shown [12] that the CCF can be obtained by recording the near-field pattern at several distances behind such a grating. Most simply, the grating can be understood as a beam splitter, diffracting the incoming x rays by an angle of $\pm\lambda/p$ (first order), where λ is the wavelength of the radiation used and p the periodicity of the grating. Accordingly, at a distance d behind the grating, the two diffracted beams become separated by a distance $2d\lambda/p$. The fringe visibility as a function of d directly corresponds to a measurement of the CCF, which is equivalent to the degree of coherence [7], on the length scale of $2d\lambda/p$ (see Fig. 1). However, if a periodic object with a periodicity of a few microns is used, the problem of resolving the interference fringes, of periodicity $p/2$, is still difficult because it requires special detectors with micrometer resolution for hard x rays.

We solved this problem by inserting a second analyzer grating (amplitude grating, period $p/2$) at a distance d behind the beam splitter grating as shown in Fig. 1. Such a *shearing interferometer* [13,14] can effectively circumvent the problem of having to measure the interference fringes directly. In our case we used a beam splitter phase grating (G1) with a periodicity of $p = 4$ μm and a height of the Si structures of 18 μm. The height was chosen to render this grating a perfect π-phase shifting grating for an x-ray energy of $E = 14.4$ keV ($\lambda = 0.0861$ nm), at which the experiments were carried out. The analyzer amplitude grating (G2) had a periodicity of $p/2 = 2$ μm and the empty spaces between the Si bars were filled with a highly absorbing material, gold. Both gratings [Figs. 1(b) and 1(c)] were fabricated using a process involving electron-beam lithography, deep etching into silicon, and, for the absorption grating, subsequent electroplating of gold [15,16].

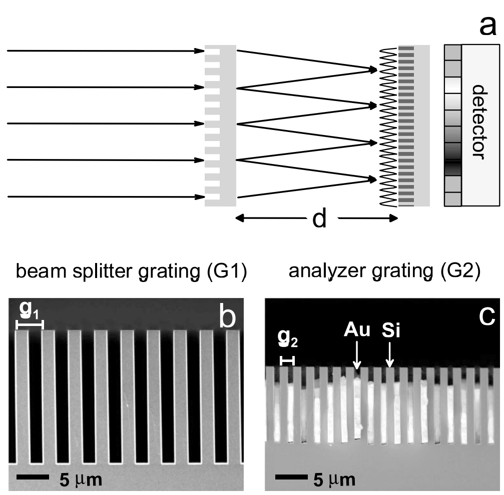

FIG. 1. (a) Schematic layout of the experiment based on a hard x-ray shearing interferometer. The beam splitter grating (G1) splits the incident beam into two diffraction orders, which form a periodic interference pattern in the plane of the analyzer grating (G2). (b) Electron micrograph of G1 fabricated in silicon. (c) G2 made by filling the grooves of a silicon grating with electroplated gold.

By rotating G2 by an angle θ, typically 0.5°, with respect to G1, around the axis of the x-ray beam, a Moiré fringe pattern is formed [17] as shown in Fig. 2. Importantly, the periodicity of the Moiré pattern is larger than the one formed directly behind G1 by a factor of $1/\sin\theta$. These magnified fringes, ranging from 50–500 μm, can therefore be conveniently and efficiently recorded with standard x-ray detectors with large pixel sizes and correspondingly large fields of view.

Our x-ray experiments have been carried out at the beam line X06SA of the Swiss Light Source (SLS, Switzerland). A liquid nitrogen cooled Si(111) double reflection monochromator was used and adjusted to a wavelength of $\lambda = 0.0861$ nm [18]. G1 was placed at a distance $D = 22.7$ m from the undulator source. A fiber coupled CCD detector (Photonic Science Hystar) with an effective pixel size of 17.9 μm has been used. Apart from a 170 μm thick chemically vapor deposited (CVD) diamond filter (at 12.0 m from the source), a 250 μm thick polished Be window (at 13.8 m from the source), and a 50 μm thick Kapton foil (at 22.0 m from the source), no further optical element was situated in the x-ray beam path. No attempt was made to collimate using slits or apertures in order to improve the raw coherence properties.

Figure 2 shows a series of detector images for different intergrating distances d in the form of linear two-dimensional contour plots of the detected intensity. The exposure time was 1.0 s. Flat- and dark-field corrections have been applied to the raw data. Clearly visible are the

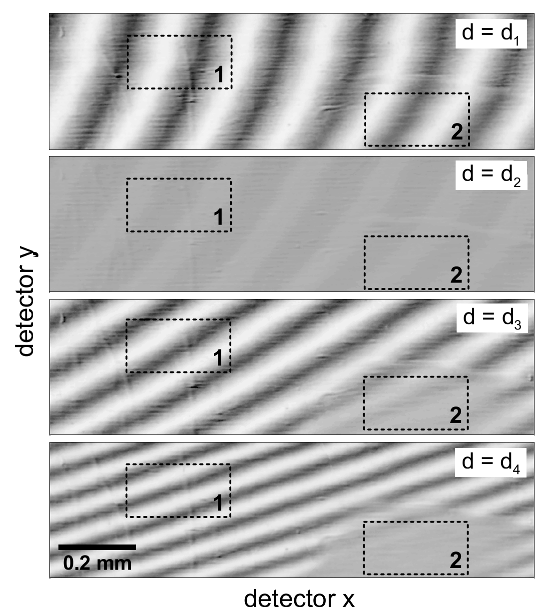

FIG. 2. Linear contour plots of the intensity detected on the detector (black corresponds to a high intensity) for different intergrating distances d ($d_1 = 23.2$ mm, $d_2 = 46.5$ mm, $d_3 = 116.1$ mm, and $d_4 = 209.1$ mm). The areas 1 and 2 indicate the areas in which the visibility has been extracted. Note the almost complete loss of fringe visibility at $d = d_4$ in area 2.

Moiré fringes whose periodicity (and tilt angle) changes from approximately 195 μm (at $d = d_1$) to 72 μm (at $d = d_4$). This effect is due to the divergence of the beam and can be efficiently used to determine the wave front curvature of the x-ray beam [19]. Important for our goal here, the measurement of the CCF, is not the periodicity or the inclination angle of the fringes, but the change of the visibility of the fringe pattern as a function of distance d. The visibility $|\gamma| = (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$ is plotted in Fig. 3 as a function of the distance d and the beam separation $x = 2d\lambda/p$ for two different areas in the field of view [20]. The envelope of the fringe visibility, governed by a rather slow decrease with d, is the relevant quantity for the purpose of measuring the CCF. The rapid oscillations, with maxima at distances $d = (2m + 1)p^2/8\lambda$ and minima at $d = 2mp^2/8\lambda$ ($m = 0, 1, 2, \ldots$), can be attributed to the Talbot effect of classical optics theory and are illustrated further in Fig. 4 [21].

In order to interpret the results, we assumed a Gaussian intensity distribution $I(s_x, s_y) = I_0 \exp(-s_x^2/2\sigma_x^2 - s_y^2/2\sigma_y^2)$ in the source plane (s_x, s_y denote the coordinates perpendicular to the optical axis in the source plane) [22]. According to the propagation laws of the MIF (Fourier transformation), the CCF is also a Gaussian of the form

$$|\gamma(x, y)| = \gamma_0 \exp(-x^2/2\xi_x^2 - y^2/2\xi_y^2), \qquad (3)$$

where $\xi_x = \lambda D/2\pi\sigma_x$ and $\xi_y = \lambda D/2\pi\sigma_y$ denote the sigma values for the width of the CCF in the vertical and horizontal directions. Fitting a one-dimensional version of this Gaussian CCF to the decaying maxima values of the fringe visibility extracted in area 1 (see Fig. 2) leads to a good fit with a value of $\xi_x = 18.4$ μm for the width of the CCF (coherence length) in the vertical direction [23]. Using Eq. (3) we obtain an effective source size value of $\sigma_x = 16.9$ μm, which is approximately a factor of 1.9 larger than the minimum sigma width of the electron beam in the undulator source. Similar deviations from the ideal result for the sigma value of the CCF have been obtained in [21,24]. Although this small disagreement was attributed in the latter references to the beam line optics, we attribute this rather small deviation from the ideal theoretical value to the fact that a slightly enlarged effective source could be caused by the limited depth of focus of the electron beam in the storage ring at the position of the undulator [25].

In area 2, $|\gamma|$ decays much faster than in the remaining field of view. This clearly indicates a coherence degradation of the original wave front in that part of the field of view. Several attempts to fit this reduced CCF again with a single Gaussian [Eq. (3)] failed. However, a double Gaussian CCF of the form $|\gamma(x)| = f \exp(-x^2/2\xi_1^2) + (1 - f)\exp(-x^2/2\xi_2^2)$ [26] nicely fitted the results with values of $\xi_1 = 16.9$ μm, $\xi_2 = 1.6$ μm, and $f = 0.17$ [see Fig. 3(b)].

This result can be explained as follows [6]. A single Gaussian CCF takes into consideration only the original intensity distribution in the undulator source plane. If, however, the wave front of the original beam is scattered, refracted, or distorted due to imperfections present in a component of the beam line optics (e.g., filters, windows, crystals, etc.), a second, virtual, source is created, and the MCF is modified correspondingly.

According to [27], a virtual source at a distance D_1 from the original source contributes a second Gaussian component to the CCF with an effective, but reduced, coherence length of $\xi_2 = (\lambda D/2\pi\sigma_x)(1 - D_1/D)/(1 + \alpha_x D_1/\sigma_x)$, where α_x is the sigma value of the source divergence angle

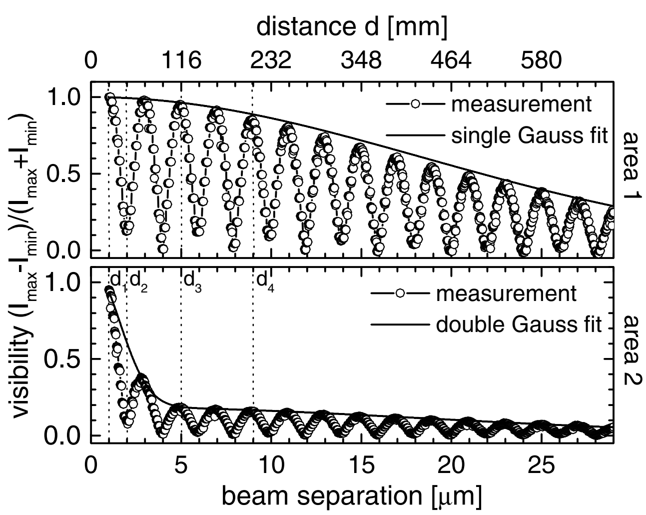

FIG. 3. Measured fringe visibility of the resulting Moiré interference pattern as a function of the distance d and the beam separation $x = 2d\lambda/p$ in area 1 and area 2.

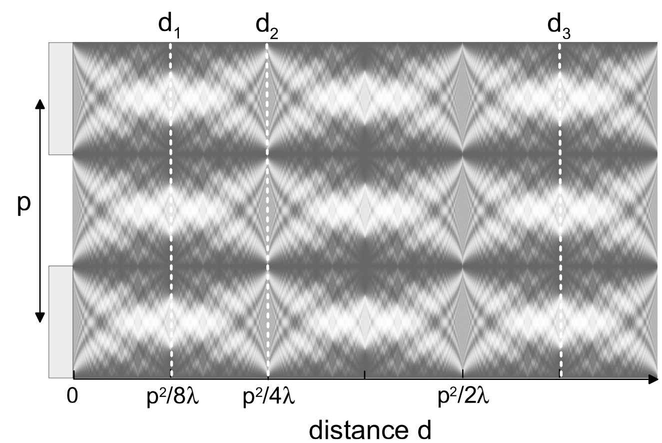

FIG. 4. Linear contour plot of the calculated near-field distribution of the electric field amplitude behind a beam splitter phase grating G1 with a periodicity p. The dashed lines at $d = d_1, \ldots, d_3$ (d_4 is outside the plotting range) indicate the positions of G2 for the corresponding detector images displayed in Fig. 2. We have assumed perfect lateral coherence and a ratio of 0.5 between the perfectly π phase shifting and undisturbed zones.

of the synchrotron beam. Evaluating this equation for given $\alpha_x = 10\ \mu$rad and $\sigma_x = 16.9\ \mu$m, we find $D_1 = 11.3$ m. Thus we conclude that the degradation of a fraction $(1 - f) = 83\%$ of the beam coherence is caused by an imperfect part of the CVD diamond filter, which is located at $D_1 = 12.0$ m from the source.

In conclusion, we have shown that a shearing interferometer can be used to measure the CCF of a hard x-ray beam with unprecedented accuracy. The method presented in this Letter exhibits major advantages over already existing methods. One is that the CCF can be extracted from the measured data without any prior assumptions of its shape. Another advantage is that the method can use detectors with large pixels, and a correspondingly large field of view, which is particularly important for imaging applications. Since the length scales over which the CCF can be measured are very large, the method will be particularly useful for future, highly coherent, hard x-ray sources where coherence lengths of the order of a few hundred microns are expected. Finally, by using a so-called "phase-stepping" technique [28] the CCF can even be measured in each detector pixel, resulting in a 2D mapping of the CCF over the whole field of view with a spatial resolution of a few microns. We expect that this technique will find widespread applications for the characterization and optimization of single optical components or whole experimental setups at present synchrotron and future free electron x-ray laser sources.

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