Self-Similar Propagation and Amplification of Parabolic Pulses in Optical Fibers

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Ultrashort pulse propagation in high gain optical fiber amplifiers with normal dispersion is studied by self-similarity analysis of the nonlinear Schrödinger equation with gain. An exact asymptotic solution is found, corresponding to a linearly chirped parabolic pulse which propagates self-similarly subject to simple scaling rules. The solution has been confirmed by numerical simulations and experiments studying propagation in a Yb-doped fiber amplifier. Additional experiments show that the pulses remain parabolic after propagation through standard single mode fiber with normal dispersion.

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The establishment of self-similarity is a key element in the understanding of many widely differing nonlinear physical phenomena, including the propagation of thermal waves in nuclear explosions, the formation of fractures in elastic solids, and the scaling properties of turbulent flow [1]. In particular, the presence of self-similarity can be exploited to obtain asymptotic solutions to partial differential equations describing a physical system by using the mathematical technique of symmetry reduction to reduce the number of degrees of freedom [2]. Although the powerful mathematical techniques associated with the analysis of self-similar phenomena have been extensively applied in certain areas of physics such as hydrodynamics, their application in optics has not been widespread. However, some important results have been obtained, with previous theoretical studies considering asymptotic self-similar behavior in radial pattern formation [3], stimulated Raman scattering [4], and in the nonlinear propagation of ultrashort pulses with parabolic intensity profiles in optical fibers with normal dispersion [5]. This latter case has also been studied using numerical simulations, with results suggesting that parabolic pulses are generated in the amplification of ultrashort pulses in nonlinear optical fiber amplifiers with normal dispersion [6]. To date, however, there has been no experimental demonstration of self-similar parabolic pulse propagation either in optical fibers or in nonlinear optical amplifiers.

In this Letter we present results of calculations using self-similarity methods [1–4] to analyze pulse propagation in an optical fiber amplifier described by the nonlinear Schrödinger equation (NLSE) with gain and normal dispersion. These calculations show that parabolic pulses are, in fact, exact asymptotic solutions of the NLSE with gain, and propagate in the amplifier self-similarly subject to exponential scaling of amplitude and temporal width. In addition, the pulses possess a strictly linear chirp. Our theoretical results are confirmed both by numerical simulations and by experiments which have taken advantage of the current availability of high gain optical fiber amplifiers and of recent developments in methods of ultrashort pulse characterization. In particular, the use of the pulse characterization technique of frequency-resolved optical gating (FROG) [7] has allowed us to measure the intensity and chirp of parabolic pulses generated in a Yb-doped fiber amplifier, and to compare these experimental results directly with theoretical predictions. Additional experiments have demonstrated that the pulses remain parabolic in profile during propagation in normally dispersive fiber, confirming the self-similar nature of propagation in this regime.

These asymptotic self-similar parabolic pulses are of fundamental interest since they represent a new class of solution to the NLSE with gain and, from a practical point of view, their linear chirp facilitates efficient pulse compression. In particular, the asymptotic pulse characteristics are found to be determined only by the incident pulse energy and the amplifier parameters, with the initial pulse shape determining only the map toward this asymptotic solution. In addition, all of the incident pulse energy contributes to the output parabolic pulse. This is in contrast to the better known soliton solutions of the NLSE in the absence of gain [8], which require accurate control of the input pulse energy and where a given input pulse develops into a soliton of fixed amplitude shedding the remaining energy into a continuum. "Parabolic fiber amplifiers" therefore have potential wide-ranging applications in many areas of current optical technology, allowing the generation of well-defined linearly chirped output pulses from an optical amplifier, even in the presence of input pulse distortions. High power linearly chirped parabolic pulses can be efficiently compressed and indeed, after compression of the parabolic pulses generated in our experiments, we have generated pulses of 80 kW peak power having 70 fs duration. Parabolic amplifiers thus allow access to a convenient fiber-based method of generating and transmitting high-power optical pulses, rivaling soliton propagation, stretched-pulse Gaussian pulse propagation [9], as well as existing chirped pulse amplification systems.

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Our theoretical analysis considers the evolution of pulses in an optical amplifier in the absence of gain saturation and for pulses with spectral bandwidths less than the amplifier bandwidth. In this case, propagation can be described by the NLSE with gain [10]:

$$i\,\frac{\partial A}{\partial z} = \frac{1}{2}\,\beta_2\,\frac{\partial^2 A}{\partial T^2} - \gamma|A|^2 A + i\,\frac{g}{2}\,A\,. \qquad (1)$$

Here, $A(z,T)$ is the slowly varying pulse envelope in a comoving frame, β_2 is the group velocity dispersion (GVD) parameter, γ is the nonlinearity parameter, and g is the distributed gain coefficient. In the absence of gain ($g = 0$), it is possible to solve the NLSE exactly using the inverse scattering method to obtain the well-known soliton solutions [7], but, in the presence of gain, solutions usually require numerical simulations. However, the NLSE with gain can also be analyzed using symmetry reduction, with the solutions obtained in this way representing exact self-similar solutions which appear in the asymptotic limit $z \to \infty$ [1–4].

For the NLSE with gain in Eq. (1), this technique yields an asymptotic self-similar solution in the limit $z \to \infty$, provided that $g \neq 0$ and that $\gamma\beta_2 > 0$. The solution is

$$A(z,T) = A_0(z)\{1 - [T/T_0(z)]^2\}^{1/2}\exp[i\varphi(z,T)], \quad |T| \leq T_0(z)\,, \qquad (2)$$

with $A(z,T) = 0$ for $|T| > T_0(z)$. This corresponds to a compactly supported pulse with a parabolic intensity profile, and a quadratic phase given by

$$\varphi(z,T) = \varphi_0 + 3\gamma(2g)^{-1}A_0^2(z) - g(6\beta_2)^{-1}T^2, \qquad (3)$$

where φ_0 is an arbitrary constant. The corresponding constant linear chirp is given by $\delta\omega(T) = -\partial\varphi(z,T)/\partial T = g(3\beta_2)^{-1}T$. In the asymptotic regime, this pulse propagates self-similarly, maintaining its parabolic shape subject to the exponential scaling of its amplitude $A_0(z) = |A(z,0)|$ and effective width parameter $T_0(z)$ according to

$$A_0(z) = 0.5(gE_{\rm IN})^{1/3}(\gamma\beta_2/2)^{-1/6}\exp(gz/3)\,, \qquad (4)$$

$$T_0(z) = 3g^{-2/3}(\gamma\beta_2/2)^{1/3}E_{\rm IN}^{1/3}\exp(gz/3)\,, \qquad (5)$$

where $E_{\rm IN}$ is the energy of the input pulse to the amplifier. Significantly, these results imply that it is only the energy of the initial pulse (and not its specific shape) which determines the amplitude and width of the asymptotic parabolic pulse.

These theoretical predictions have been confirmed by numerical simulation of the NLSE using the standard split-step Fourier method [8]. Gaussian input pulses having a range of pulse durations (FWHM) from 100 fs–5 ps, but fixed energy $E_{\rm IN} = 12$ pJ, were propagated in a 6 m long fiber amplifier with realistic parameters corresponding to Yb-doped fiber: $\gamma = 5.8 \times 10^{-3}\ {\rm W^{-1}\,m^{-1}}$, $\beta_2 = 25 \times 10^{-3}\ {\rm ps^2\,m^{-1}}$, $g = 1.9\ {\rm m^{-1}}$ [11]. Figure 1(a) compares the evolution of the amplitude of the propagating pulse

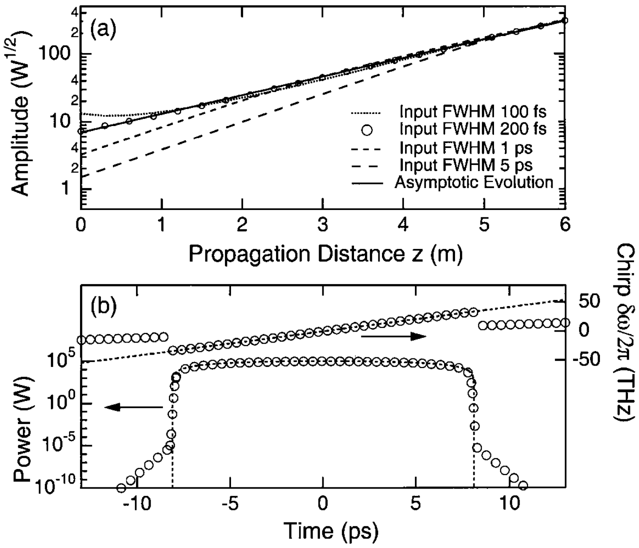

FIG. 1. (a) Simulation results showing the evolution of pulse amplitude as a function of propagation distance for Gaussian pulses of duration 100 fs–5 ps, compared with calculated asymptotic result (see legend). (b) Simulated output intensity (circles, left axis) and chirp (circles, right axis) corresponding to the 200 fs input pulse, compared with the asymptotic parabolic pulse results (dotted lines).

obtained from simulations with the analytic prediction for $A_0(z)$ given by Eq. (4). The evolution of the pulse in the amplifier approaches the asymptotic limit in all cases. Indeed, Fig. 1(b) shows the output pulse characteristics for the input 200 fs pulse, illustrating the excellent agreement (over 10 orders of magnitude) between the intensity and chirp of the simulation output (circles) and the expected asymptotic pulse profile from Eq. (2) (dashed line).

Additional simulations have been carried out to investigate the dependence on fiber parameters and pulse initial conditions in more detail. As the fiber gain is increased for a given input pulse, the exponential growth of the pulse amplitude and width is correspondingly increased in agreement with Eqs. (4) and (5), and the parabolic asymptotic limit is reached in a shorter propagation distance. Simulations also show that for a fiber of fixed gain, while the effect of intensity or phase modulation on an input pulse modifies the length scale over which the evolution to the asymptotic limit occurs, the asymptotic parabolic pulse solution is nonetheless reached in all cases after sufficient propagation distance. In this context, we also note that, although at $|T| = T_0(z)$ the solution in Eq. (2) has infinite slope, this is the case only in the asymptotic limit. At intermediate propagation distances, simulations and analysis predict low amplitude wings on the parabolic pulse which decay exponentially as a function of T, and which vanish in the limit $z \to \infty$. Indeed, these wings can be seen in the simulation results in Fig. 1(b) at instantaneous power levels less than 10^{-5} W.

To experimentally verify that parabolic pulses are indeed generated in fiber amplifiers, we injected femtosecond pulses into a high gain Yb-doped fiber amplifier, and carried out FROG characterization of the amplified pulses. Figure 2 shows the experimental setup. Here, a fiber-based pulsed seed source was used to generate Gaussian input pulses of 200 fs FWHM at a wavelength of 1.06 μm and at a repetition rate of 63 MHz [11]. These pulses were then injected into a 3.6 m length of Yb-doped fiber co-directionally pumped at 976 nm, with a gain of 30 dB in this geometry. The input pulse energy in the fiber was estimated at 12 pJ. Complete pulse characterization of the output pulses was carried out using FROG based on second-harmonic generation (SHG) in a KDP (potassium dihydrogen phosphate) crystal, with the experimental configuration used being similar to that described in [7]. FROG measurements were carried out on the pulses directly after the Yb-doped fiber amplifier, as well as after subsequent propagation in 2 m of standard undoped single mode fiber (SMF). Intensity and chirp retrieval from the measured FROG traces was carried out using the standard FROG retrieval algorithm, with the root-mean-squared error between the measured FROG trace and that associated with the retrieved pulse being acceptably low ($G < 0.007$) in all cases [7].

We first discuss the characterization of the pulses directly from the amplifier. The solid lines in Fig. 3 show the measured intensity and chirp for an amplifier gain of 30 dB, corresponding to a distributed gain coefficient of $g = 1.9\ \mathrm{m}^{-1}$. In this case, the output pulse energy was 12 nJ, the temporal FWHM was $\Delta\tau = 2.6$ ps, the spectral FWHM was $\Delta\lambda = 32$ nm, and the corresponding duration bandwidth product was $\Delta\tau\Delta\nu \approx 22$. Note that, in the figure, the arbitrary intensity profile obtained from the FROG retrieval algorithm has been scaled to show the instantaneous power in kilowatts. The experimental intensity and chirp are compared with the results of NLSE simulations (circles) and the predicted asymptotic parabolic pulse characteristics (short dashes) for this length of fiber. Both the measured intensity and chirp are in good agreement with the results of NLSE simulations. The experimentally observed weak oscillations in the wings are attributed to higher order dispersion and resonant effects not included in Eq. (1). More significantly, however, the measured intensity profile is also in agreement (over 2 orders of magnitude) with the asymptotic parabolic pulse predicted by Eq. (2), using the experimental fiber parameters given above. To emphasize the parabolic nature of these pulses, the figure also includes a sech^2 fit to the measured intensity profile (long dashes). These parabolic pulse characteristics are consistent with the results in Fig. 1 for a 200 fs input pulse, where asymptotic behavior would be expected after 3.6 m of propagation. We therefore interpret these results as the first direct experimental characterization of a parabolic pulse from an optical fiber amplifier, and as a confirmation of our theory of asymptotic pulse evolution described above.

As previously predicted in Ref. [5], an attractive feature of high power parabolic pulses is that they propagate self-similarly in normally dispersive fiber, allowing for highly nonlinear propagation over substantial fiber lengths without optical wave breaking. We have been able to verify this prediction experimentally by launching the amplified pulses shown in Fig. 3(a) into a 2 m length of undoped fiber (SMF) and using FROG to characterize the output pulses. The output pulses after propagation had broadened both temporally and spectrally with $\Delta\tau = 4.4$ ps, $\Delta\lambda = 50.5$ nm, and $\Delta\tau\Delta\nu \approx 60$. Figure 3(b) shows the measured intensity and chirp (solid lines), together with parabolic (short dashes) and sech^2 (long dashes) fits. The pulse intensity profile was found to remain parabolic, confirming the self-similar nature of pulse propagation, although we note that the dynamic range of the parabolic profile is reduced due to the presence of a low energy background having its origin in the weak oscillations in

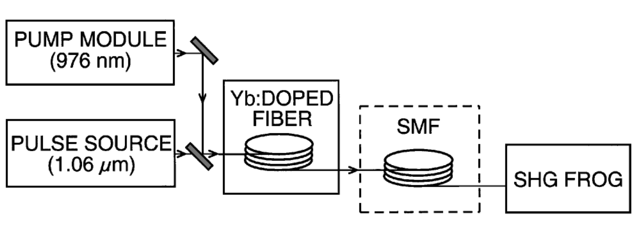

FIG. 2. Experimental setup used for parabolic pulse generation and measurement. Pulse characterization via FROG was carried out for the pulses directly from the 3.6 m Yb-doped fiber amplifier as well as after propagation in 2 m of undoped fiber (enclosed by dashed lines).

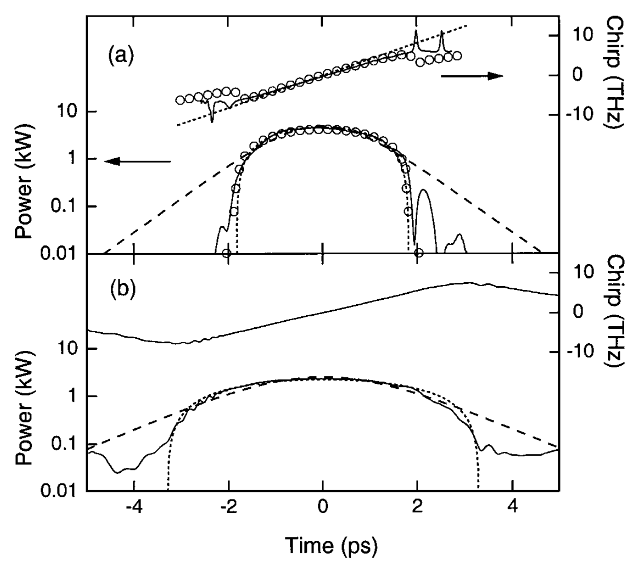

FIG. 3. (a): Intensity (left axis) and chirp (right axis) for pulses directly from Yb-doped amplifier for a gain of 30 dB. The solid lines are the experimental results, compared with NLSE simulation (circles), asymptotic parabolic pulse profile (short dashes), and sech^2 fit (long dashes). (b) The solid lines show measured intensity and chirp after propagation through 2 m of SMF, compared with parabolic (short dashes) and sech^2 (long dashes) fits.

the wings of the amplified pulses. Importantly, despite the significant temporal and spectral broadening in this nonlinear regime, the chirp remains linear, a characteristic feature of parabolic pulse propagation [5]. To demonstrate the potential of high power parabolic pulses in ultrafast optics, we used a simple dispersive grating pair to compress these parabolic pulses, obtaining a minimum pulse duration of $\Delta\tau = 68$ fs with a corresponding peak power of 80 kW. The pulses do not compress to the expected transform limited pulse duration of around 30 fs because of third order dispersion in the bulk grating compressor, but we note that this can easily be eliminated with an improved compressor design [12].

In conclusion, we have developed a theoretical treatment of the amplification of pulses in high gain fiber amplifiers which predicts the formation of high power parabolic pulses from any input pulse. We have also demonstrated experimentally that a Yb-doped fiber amplifier with 30 dB gain does indeed yield parabolic pulses, whose intensity and chirp characteristics are in quantitative agreement with our theoretical predictions. This is the first experimental demonstration of the existence of parabolic pulses. In view of their self-similar propagation and the ease with which they can be compressed, we expect that these parabolic pulses will find wide application. Indeed, we anticipate that parabolic pulse propagation in optical fibers may well become as important and as widely studied as the propagation of optical fiber solitons.

Note added in proof.—After acceptance of this manuscript, we learned of recent results studying the generation of self-similarity and Cantor set fractals in nonlinear soliton-supporting systems [13,14]. We also note that self-similarity techniques have been used to analyze the evolution of self-written waveguides in photosensitive materials [15]. It is likely that the study of self-similarity phenomena in nonlinear optics will become an increasingly important field of research in the near future.

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