the plasma are strongly coupled to the transverse modes [4-6]. This will happen, for large enough $\boldsymbol{B}_0$, when the propagation vector $\boldsymbol{k}$ does not lie along $\boldsymbol{B}_0$ [4-6]. For example, an approximate solution of $|\boldsymbol{R}| = 0$ is given at low temperatures, by [4 eq. (76)]

$$-s^2 = \omega_p^2 + \Omega_e^2 \sin^2\theta + \omega_p^2\Omega_e^2 \sin^2\theta/c^2k^2 \quad (6A)$$

where ω_p and Ω_e denote the electron plasma and gyro frequencies. On the other hand, the corresponding solution of $\epsilon_L = 0$ (which we denote by $s^2{}_{app}$) is given by

$$-s^2{}_{app} = \omega_p^2 + \Omega_e^2 \sin^2\theta. \quad (6B)$$

Eq. (6B) is the square of the frequency at which the "side peaks" in the spectrum of scattered light is usually assumed to occur. Eq. (6A), however, predicts that, due to the transverse motion of the plasma, the square frequency of the "side peaks" will be shifted by an amount equal to

$$\omega_p^2\Omega_e^2 \sin^2\theta/c^2k^2. \quad (7)$$

Clearly, this shift is significant when

$$c^2k^2 \gtrsim \Omega_e^2 \sin^2\theta. \quad (8)$$

If $n_e = 10^{12}$ cm^{-3} one can obtain a value for k as small as 2 cm^{-1} so that eq. (8) will be satisfied when $\Omega_e \gtrsim 6 \times 10^{10}$ sec^{-1} ($B_0 \gtrsim 3500$ gauss). Under these circumstances the frequency shifts of the "side peaks" which are due to the transverse motion of the plasma will be quite significant.

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WAVELENGTH EFFECTS IN MICROWAVE ACOUSTIC RESONANCE

A. M. STONEHAM
Theoretical Physics Division, Atomic Energy Research Establishment, Harwell, Didcot, Berks

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Stevens and Tucker [1] showed recently that the finiteness of the phonon wavelength in microwave acoustic resonance altered the moments of the absorption line. Their approach, based on the moment method of Van Vleck [2], shares its fundamental weakness that changes in the observed line shape are not completely described by the changes in a small number of its moments. These may be due to some combination of line broadening, a line shift or the appearance of satellite lines. Further, in [1] the moments are related to the phonon wave vector $\boldsymbol{q}$, which is replaced by $\boldsymbol{q}_0$, the wave vector appropriate to the peak of the resonance line. In general $\boldsymbol{q}$ varies if the phonon frequency ω is varied to measure the line shape, i.e., $\boldsymbol{q} = \boldsymbol{q}(\omega)$. But the moments are integrals over ω - if $\gamma(\omega)$ is the line shape, the Nth moment is $\langle\omega^N\rangle = \int_{-\infty}^{\infty} \mathrm{d}\omega\, \omega^N\gamma(\omega)$ - and it is not clear what meaning can be given to an equation which relates this moment to $\boldsymbol{q}$, which is a function of the integration variable ω.

A different approach is practical in certain simple cases, and gives some insight into the results of [1]. It indicates that the moments are altered by the appearance of additional weak resonance lines, with intensity quadratic in $\boldsymbol{q}$ and position independent of $\boldsymbol{q}$. These will be difficult to detect, even though there may be large changes in the moments. The first moment of the absorption line is changed by the variation of wavelength with ω.

In the simplest model showing the effects discussed in [1] spins with effective spin $\sigma = \frac{1}{2}$ interact with a static magnetic field and are coupled in pairs by the isotropic exchange interaction $\hbar j \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, with only weak interactions between the pairs. The eigenstates of one such pair can be written in terms of the states $|\sigma_{1Z}, \sigma_{2Z}\rangle$. They are $|\pm\frac{1}{2}, \pm\frac{1}{2}\rangle$, with energy $\pm\hbar\omega_0$, the symmetric state $|s\rangle = \{|\frac{1}{2}, -\frac{1}{2}\rangle + |-\frac{1}{2}, \frac{1}{2}\rangle\}/\sqrt{2}$ with energy zero, and the antisymmetric state $|a\rangle = \{|\frac{1}{2}, -\frac{1}{2}\rangle + - |-\frac{1}{2}, \frac{1}{2}\rangle\}/\sqrt{2}$ with energy $-\hbar j$. Transitions are induced between the levels of this pair by the microwave phonon interaction: $h = h_1 + h_2 = A\{\sigma_{x1} \sin(\omega t + \varphi + \boldsymbol{q}\cdot\boldsymbol{r}_1) + \sigma_{x2} \sin(\omega t + \varphi + \boldsymbol{q}\cdot\boldsymbol{r}_2)\}$. Any dependence of A on $\boldsymbol{q}$ or ω will be neglected. We write $\boldsymbol{a} = \boldsymbol{r}_2 - \boldsymbol{r}_1$. When $\boldsymbol{q}\cdot\boldsymbol{a}$ is zero the only allowed transitions are between $|s\rangle$ and $|\pm\frac{1}{2}, \pm\frac{1}{2}\rangle$, both transitions involving phonons of energy $\hbar\omega_0$. If $\boldsymbol{q}\cdot\boldsymbol{a}$ is finite transitions between $|a\rangle$ and $|\pm\frac{1}{2}, \pm\frac{1}{2}\rangle$ become slightly allowed, for $\langle a|h|\frac{1}{2}, \frac{1}{2}\rangle = \{\langle\frac{1}{2}, -\frac{1}{2}|h_2|\frac{1}{2}, \frac{1}{2}\rangle - \langle -\frac{1}{2}, \frac{1}{2}|h_1|\frac{1}{2}, \frac{1}{2}\rangle\}/\sqrt{2}$ and $\langle a|h|-\frac{1}{2}, -\frac{1}{2}\rangle$ do not vanish, giving satellite lines at frequencies $\omega_{\pm} = |(\omega_0 \pm j)|$. In the long wave limit $\boldsymbol{q}\cdot\boldsymbol{a} \ll 1$ we obtain, after averaging over the phase φ:

$$\overline{|\langle a|h|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle|^2} = \overline{|\langle s|h|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}\rangle|^2}\,(\tfrac{1}{2}\boldsymbol{q}\cdot\boldsymbol{a})^2. \quad (1)$$

These transitions are weaker by $(\frac{1}{2}\boldsymbol{q}\cdot\boldsymbol{a})^2$ than the main pair line; for X band phonons and spins on nearest neighbour sites this factor is typically 10^{-5} or 10^{-6}. The resultant resonance spectrum is independent of the magnetic field orientation when the spin-spin interaction is isotropic exchange alone. If a more general form of interaction between the members of the pair is used the resonance spectrum is anisotropic, although the overall picture is not changed. Both the allowed and slightly allowed transitions are broadened by dipolar interactions with spins in different pairs.

The moments of the spectrum can be calculated from

$$\langle\omega^N\rangle = \left\{\sum_{ij} \omega_{ij}{}^N W_{ij}\right\} \Big/ \left\{\sum_{ij} W_{ij}\right\} \quad (2)$$

where W_{ij} is the transition probability between levels i and j and ω_{ij} the corresponding frequency. There is no difficulty in treating the case $\boldsymbol{q} = \boldsymbol{q}(\omega)$; for the main line we use $\boldsymbol{q} = \boldsymbol{q}(\omega_0)$ and for the satellites $\boldsymbol{q}_{\pm} = \boldsymbol{q}(\omega_{\pm})$. To order $(\boldsymbol{q}\cdot\boldsymbol{a})^2$ (1) and (2) yield

$$\langle\omega^N\rangle = \omega_0^N + \tfrac{1}{8}\{(\omega_+^N - \omega_0^N)(\boldsymbol{q}_+\cdot\boldsymbol{a})^2 + (\omega_-^N - \omega_0^N)(\boldsymbol{q}_-\cdot\boldsymbol{a})^2\}.$$

The denominator of (2) is independent of $\boldsymbol{q}\cdot\boldsymbol{a}$, as the appearance of transitions involving $|a\rangle$ slightly diminishes the intensity of those involving $|s\rangle$. If the phonon wavelength does not change with frequency, i.e., $\boldsymbol{q}_+ = \boldsymbol{q}_-$, then the first moment $\langle\omega\rangle$ is simply ω_0 and $\langle\omega^2\rangle = \omega_0{}^2 + \frac{1}{4} j^2(\boldsymbol{q}\cdot\boldsymbol{a})^2$, in agreement with the result of [1]. Thus in this simple case the changes correspond solely to the appearance of satellite lines. If $\omega = v|\boldsymbol{q}|$ the lines at ω_+ and ω_- have different intensities; $\langle\omega\rangle$ is raised from ω_0 by $\frac{1}{8}(\boldsymbol{q}_0\cdot\boldsymbol{a})^2 j$ when $\omega_0 < j$ and $\omega_{\pm} = j \pm \omega_0$, and by $\frac{1}{8}(\boldsymbol{q}_0\cdot\boldsymbol{a})^2 j^2/\omega_0$ when $\omega_0 > j$ and $\omega_{\pm} = \omega_0 \pm j$.

In practice the satellites will be hard to detect, as they are much weaker than the main resonance line. If j is small the satellites will lie under the main resonance line, with negligible effect on both the line shape and its moments. When j is large (as in the example given in [1] they will lie outside the frequency range in which the line shape is measured, and the shape actually measured will probably be the same as that of the photon resonance line given by ordinary spin resonance experiments [3]. The changes in the observed line shape are unlikely to be observable, despite the large increases in the moments, in analogy with other situations in both paramagnetic and ferromagnetic resonance [2, 4]. In exceptional circumstances the satellites will be in a region where their intensity is sufficient for detection, j being several times the linewidth of the main line. As their intensity is quadratic in $\boldsymbol{a}$ this favours cases where j has such a value for widely separated spins. For a given ω_0 the dependence of intensity on $\boldsymbol{q}$ favours crystals with a low sound velocity. In this case the positions of the satellites give the magnitude of the exchange interaction directly; these must be measured for several magnetic field orientations if anisotropic exchange or dipolar interactions are appreciable.

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