THE REAL OPTIONS EFFECT OF UNCERTAINTY ON INVESTMENT AND LABOUR DEMAND

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Abstract

This paper shows that, contrary to common beliefs, the real options effect of uncertainty plays no role in the *long run* rate of investment. This is proven for both the standard investment model with Cobb-Douglas production and Brownian motion demand, and also for a broader class of models with multiple lines of capital, labor and general demand stochastics. Real options and irreversibility, however, are shown to play an important role in the *short run* dynamics of investment and labor demand. Specifically, they reduce the short run response of investment and hiring to current demand shocks, and lead to a lagged response to past demand shocks.

Keywords: Investment, labor demand, uncertainty, real options.

JEL Classification: D92, E22, D8

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1. Introduction

It is commonly understood that the real options effect of uncertainty reduces investment[1]. To be precise, when investment is irreversible so that capital cannot be resold for its full purchase price, a firm's optimal investment rule takes on a threshold form. Investment will only occur when demand rises to some upper threshold while disinvestment will occur only when demand falls to some lower threshold. It has often been assumed that because uncertainty raises the upper threshold for investment[2], it reduces the long run rate of investment.

This appears to be confirmed by a variety of papers which find an inverse relationship between uncertainty and investment. Caballero (1991) and Lee and Shin (2000) demonstrate that in a two period model in which firms start off with no capital stock the *real options effect* of uncertainty unambiguously acts to reduce investment in the first period. Pindyck (1993) and Sakellaris (1994) report that in a competitive multi-period (three or more periods) model of investment in which firms again start off with no initial capital stock the *real options effect* of uncertainty also unambiguously reduces first period investment. And Dixit and Pindyck's (1994) survey on the investment under uncertainty literature implies a negative impact of real options on investment when they report that

> "a larger σ [the standard deviation of demand] means a lower long-run average growth rate of the capital stock, and thus less investment on average" [page 373]

[1] The real options effect of uncertainty is defined as "the effect of uncertainty that arises from a firm's option to choose the timing of its investment" when this investment is irreversible.

[2] The effect of uncertainty in raising the investment threshold is demonstrated, for example, by Bertola (1988), Pindyck (1988), Dixit (1989), Bentolila and Bertola (1990), Bertola and Caballero (1994), Dixit and Pindyck (1994) and Abel and Eberly (1996).

In this paper we argue that these existing papers cannot be used to draw inferences about the *real options effect* of uncertainty on long run investment since they either amalgamate differing short and long run impacts, or introduce Jensen's inequality effects in to demand growth. This paper distinguishes the real options effect in the short run from its effect in the long run, and controls for these Jensen's inequality effects. In section 2 we show that *real options play no role in determining the long run rate of investment.*

The intuition for this result is that while the real option effect of uncertainty increases the investment threshold, reducing the rate of *investment* in times of strong demand, it also lowers the disinvestment threshold, reducing the rate of *disinvestment* in times of weak demand. These effects on the rate of investment and the rate of disinvestment exactly cancel out in the long run. This result is proven for both the standard Cobb-Douglas production function and Brownian motion demand model (proposition 1) and also a more general class of multi-capital production functions and stochastic demand processes (proposition 2). As a corollary to proposition 1 we also show that the negative effect of uncertainty on long run investment quoted above from Dixit and Pindyck (1994) derives from a Jensen's inequality effect and is unrelated to real options.

Although real options do not affect long run investment, it is shown in section 3 that they play an important role in the *short run* dynamics of investment and labor demand. The separation of the investment/disinvestment thresholds and the hiring/firing thresholds is proven to reduce the short run *responsiveness* to demand shocks at any level of aggregation (proposition 3). This explains the findings of Caballero (1991), Pindyck (1993), Sakellaris (1994) and Lee and Shin (2000) whose assumption that firms start off with no capital is critical since it ensures that first period investment will necessarily be positive. Because higher uncertainty reduces the investment *response* it reduces this first period investment in their model and generates a negative real options effect of uncertainty on investment. We show that the investment and hiring response to demand shocks is convex with larger shocks leading to proportionally larger responses (proposition 4). In addition to reducing the short run responsiveness, real options and irreversibilities are also shown to induce strong dynamics into the investment and hiring process, leading to lagged responses to past demand shocks (proposition 5).

These predictions from real options and irreversibility - in particular the low short run responsiveness and the longer lagged response - match the stylized facts from estimating firm, industry and macro level investment and labor demand models[3]. Estimates of the short run elasticities of investment and hiring to demand shocks are generally moderate in size, with point estimates on annual data often around one quarter and one half respectively. Additional lagged responses, however, lead to a much higher long run response, with some suggestion that the long run elasticities of both factors is about unity.

Related literature to this paper includes Abel and Eberly (1995) who examine the influence of uncertainty on the *level* of the capital stock under complete irreversibility with no depreciation for a range of parameter values, and find an ambiguous response. Our work complements these results by examining the real options effect of uncertainty and irreversibility on long run investment - that is on the *growth* of the capital stock. Since we find no impact on the long run *growth* of the capital stock this suggests that any real options effects of uncertainty on the

[3]See for example Chirinko (1993) on investment and Hamermesh (1993) on labor demand.

level of the capital stock are stationary and independent of long run investment. Our results are also fully analytical, unambiguous, and obtained for a more general model which allows for partial irreversibility, depreciation, multiple-lines of capital, and broader demand stochastics.

The impact of real options and irreversibilities on short run dynamics has previously been noted by Bertola and Caballero (1994), who model aggregate investment in an economy composed of a continuum of homogenous firms, each operating with a single line of irreversible capital. We extend these results in a number of directions by generalizing to firms with multiple-lines of partially irreversible labor and capital, allowing for a less restrictive demand processes, allowing for heterogeneity across firms, and dispensing with the need for aggregation across a large numbers of units. *This allows us to make predictions on the short run dynamic effects of real options on investment and hiring in data-sets at all levels of aggregation*, from the plant and firm level up to the industry and macro level, with these being robust to a variety of production and demand processes.

Finally, a number of papers cited above[4] also examine a result, demonstrated in Hartman (1972) and Abel (1983), that if a firm can freely adjust its labor force used in production after investment has been undertaken, this can lead the marginal revenue product of capital to be convex in price, so that greater uncertainty may increase the *level* of the capital stock. The result, however, depends on particular modelling assumptions over both the revenue function and the form of the demand shock so that, for example, if the demand shock is modelled as a quantity rather than a price shock this uncertainty effect disappears or can even be reversed. This Hartman-Abel effect of uncertainty also requires firms to undertake

[4] Caballero (1991), Pindyck (1993), Sakellaris (1994) and Lee and Shin (2000).

frequent adjustment of its labor force, which if labor is believed to be subject to adjustment costs, may not be optimal. Since our results are valid both with and without this uncertainty effect we do not discuss this issue any further.

The plan of the paper is as follows. Section 2 starts by examining the standard model with a single line of capital and flexible labor and demonstrates that real options have no impact on the long run rate of investment. We then consider a broad class of models which allow for multiple lines of capital and inflexible labor under any degree of aggregation, and show that real options still play no role in determining the long run rate of investment. Section 3 then uses this general framework to consider the short run effects of real options and irreversibilities, and proves that these will retard the investment and hiring response to demand shocks, and lead to a dynamic lagged response to demand shocks. Some concluding remarks are then made in section 4.

2. Long Run Impact of Real Options on Investment

We start off by examining a stylized investment model with one line of capital and flexible labor which is commonly used in the irreversible investment literature[5].

2.1. The Standard Model with a Single Line of Capital and Flexible Labor

The firm's revenue function, $R(K, P)$, in terms of its capital stock (K) and its demand conditions (P) is modelled as having the following form

$$R(K, P) = \frac{1}{a} P^{1-a} K^{a} \tag{2.1}$$

where this can be shown to nest a Cobb-Douglas production function and an iso-elastic demand curve. In this setup labor is assumed to be a completely flexible factor of production and has been optimized out of the revenue function. We assume that the firm's demand conditions follow a Brownian motion process with drift μ and variance σ^2. The firm is assumed to maximize the expected present value of sales revenues, minus the cost of buying capital at a price B, plus the proceeds received from selling capital at a price S, where $B > S$. The optimal investment strategy then maximizes its total discounted profits

$$\max_{\{I(s)\}} E_t \left\{ \int_t^{\infty} \exp^{-r(s-t)} (\frac{1}{a} P^{1-a}(s) K(s)^{a} ds - B dI^{+}(s) + S dI^{-}(s)) \right\} \tag{2.2}$$

$$\text{subject to} \quad dK(t) = -\delta K dt + I(t)$$

where r is the discount rate I^{+} denotes positive investment and I^{-} denotes negative investment, and δ is the rate of capital depreciation.

[5] This model of partial irreversibility is taken directly from Abel and Eberly (1996) and is a generalisation of the complete irreversibility case in Bertola (1988) and Dixit and Pindyck (1994) Chapter 11.

Abel and Eberly (1996) demonstrate that the profit maximizing investment behavior can be described in terms of the firm's current marginal revenue product of capital $P^{1-a}K^{a-1}$, and an investment and disinvestment threshold, which is summarized in Table 1 below. These thresholds are represented by the investment and disinvestment user costs of capital, C_I and C_D respectively[6], and two real options terms $\Phi_I > 1$ and $\Phi_D > 1$.

Table 1: The threshold behavior of investment with real options

Invest if:	$P^{1-a}K^{a-1} = C_I\Phi_I$
Do Nothing if:	$C_D/\Phi_D < P^{1-a}K^{a-1} < C_I\Phi_I$
Dis–Invest if:	$P^{1-a}K^{a-1} = C_D/\Phi_D$

After taking logs, these thresholds provide bounds for the logged capital stock under partial irreversibility, which using lower case to denote logged variables, can be stated as follows

$$p - \left(\frac{c_I + \phi_I}{1-a}\right) \leq k \leq p - \left(\frac{c_D - \phi_D}{1-a}\right) \tag{2.3}$$

To pinpoint the impact of real options we make a ceteris paribus comparison to the situation in which the firm acts *as if* it has no option to delay its investment. By modelling the firm *as if* it has a now or never investment choice we can turn off the real options effect but keep all other parameters and the evolution of demand constant[7]. Under this hypothetical alternative we know from Jorgenson (1963)

[6] Jorgenson (1963) demonstrates that when capital costs a constant price B to buy, the investment user cost of capital can be expressed as $C_I = (r+\delta)B$. This can be generalised for capital resale at a constant price S, to denote the disinvestment user cost of capital as $C_D = (r+\delta)S$.

[7] This 'no real options' scenario can be equivalently described as the situation in which the firm always acts *as if* the current level of demand will extend into the future with complete certainty.

that the firm will only invest when its marginal revenue product of capital is equal to its investment user cost of capital C_I, and only disinvest when its marginal revenue product of capital is equal to its disinvestment user cost of capital C_D. In the absence of real options the investment rule and capital stock, K_{no}, would satisfy the threshold optimality conditions expressed in table 2 below.

Table 2: The threshold behavior of investment without real options

Invest if:	$P^{1-a}K_{no}^{a-1} = C_I$
Do Nothing if:	$C_D < P^{1-a}K_{no}^{a-1} < C_I$
Dis–Invest if:	$P^{1-a}K_{no}^{a-1} = C_D$

After taking logs, these thresholds provide bounds for the logged capital stock with no real options under partial irreversibility, and can be stated as follows,

$$p - \frac{c_I}{1-a} \leq k_{no} \leq p - \frac{c_D}{1-a} \tag{2.4}$$

To evaluate the dynamics of investment we use the result that for continuous changes in the capital stock the instantaneous rate of investment is equal to the change in the log of the capital stock plus depreciation[8]

$$\frac{I_t}{K_t} = d\log K_t + \delta \tag{2.5}$$

This allows us to use results on the long run growth rate of the capital stock to determine the long run rate of investment. Proposition 1 below demonstrates, perhaps surprisingly, that even though real options play an important role in the

[8]It should be noted that since the evolution of these capital stocks is undertaking according to the threshold rules in Tables 1 and 2, they are 'variation finite' processes. Hence, they have zero quadratic variation and so do not require any Ito's Lemma adjustments to their drift rates when converted from logs to levels (see Harrison, 1990).

determination of the investment thresholds, they play no limiting role in long run investment.

Proposition 1: Real Options have no limiting effect on long run investment.

Proof: For partially irreversible investment combining the conditions (2.3) and (2.4) we find that the difference between the capital stock with real options and without real options is bounded by a finite constant,

$$\frac{-\log \phi_I}{1-a} \leq k - k_{no} \leq \frac{\log \phi_D}{1-a} \tag{2.6}$$

Hence, K and K_{no} have the same long run limiting[9] growth rate since

$$\begin{aligned} \lim_{T\to\infty} \frac{1}{T} |\log(K_{T+t}/K_t) - \log(K_{no,T+t}/K_{no,t})| &= \lim_{T\to\infty} \frac{1}{T} |(\log(K_{T+t}/K_{no,T+t}) - \log(K_t/K_{no,t})| \\ &\leq \lim_{T\to\infty} \frac{1}{T} |\frac{\log \phi_I}{1-a} + \frac{\log \phi_D}{1-a}| \\ &= 0 \end{aligned} \tag{2.7}$$

By (2.5) they also have the same long run rate of investment. For completely irreversible investment the distance between the capital stock with real options and without real options is, after any initial common investment episode, a fixed distance apart[10],

$$k - k_{no} = \frac{-\log \phi_I}{1-a} \tag{2.8}$$

Hence, by the same logic as above they also have the same long run rate of investment.

[9] This and all other limits in the paper are the standard deterministic limits.

[10] For complete irreversibility the (positive investment) rules from Tables 1 and 2 are still optimal, so that during any common investment episode the levels of the capital stock with options and without options will be $\frac{\log \phi_U}{1-a}$ apart. Since subsequent depreciation imparts a linear trend to both capital stocks this $\frac{\log \phi_U}{1-a}$ gap between the two levels will persist from then onwards.

These results are independent of any assumptions on the rate of capital depreciation, the rate of demand growth, the degree of uncertainty or the degree of irreversibility.

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The intuition for this proof is that the capital stocks with and without real options are both contained within the real options investment thresholds. These thresholds are a fixed and bounded distance apart so that the gap between the two capital stocks is also fixed and bounded. Over time the importance of this fixed gap for long run investment tends to zero, as the evolution of demand becomes the first order determinant of investment. This is illustrated in figure 1, which plots in bold the investment profile for the capital stock with real options and without real options for a random 10 year realization of logged demand[11]. Also plotted in figure 1 in feint are the investment and disinvestment thresholds for the capital stock with real options.

FIGURE 1 ABOUT HERE

In figure 1 both the capital stocks with and without real options have been normalized to start off at one unit so that total cumulative investment can be measured from the current level of the capital stock. It can be seen that these two capital stocks evolve in a similar manner to each other. Figure 2 plots the evolution of these two capital stocks for the same demand process continued over a

[11] Logged demand is drawn from a Brownian process with 5% drift and 20% standard deviation. Returns to capital are assumed homogenous of degree 0.75, consistent with constant returns to scale production and a price elasticity of 4. Capital depreciates at 10% per year, costs $1 per unit to buy and can be resold for $0.75 per unit. The firm's annual discount rate is 10%.

fifty year period from which the equivalence between the long run rates of growth is much clearer[12].

FIGURE 2 ABOUT HERE

While the real options effect of uncertainty plays no role in long run investment, there is a specific case in which uncertainty does play a long run role by directly affecting the growth rate of logged demand. This effect is independent from real options.

Corollary to Proposition 1: Uncertainty can decrease (increase) expected long run investment, independently of real options and irreversibility, if logged demand is concave (convex) in the Brownian motion term.

Proof: From the definition of the thresholds for the level of capital stock with and without real options, equations (2.3) and equations (2.4), we can see that the long run growth rate of both levels of capital will be equal to the long run growth rate of logged demand,

$$\begin{aligned} \lim_{T\to\infty} \frac{1}{T} |\log(K_{T+t}/K_t)| &= \lim_{T\to\infty} \frac{1}{T} |\log(K_{no,T+t}/K_{no,t})| \\ &= \lim_{T\to\infty} \frac{1}{T} |(\log(P_{T+t}/P_t)| \end{aligned}$$

Jensen's inequality states that long run growth of demand will be decreasing (increasing) in the variance of demand if $\log P$ is concave (convex) in the underlying

[12] For Ss models with *fixed costs* of adjustment, for example as analyzed by Grossman and Laroque (1990), the long run rate of investment is also independent from any option value effects of uncertainty, since the investment rule takes on a similar threshold form.

Brownian motion process. But, since this affects the level of the capital stock both with and without real options, this Jensen's effect of uncertainty is independent of real options.

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This explains Dixit and Pindyck's (1994) result that the expected long run rate of investment is equal to $\mu - \frac{\sigma^2}{2}$, and so reduced by higher uncertainty, since *logged demand* is concave in the *level of demand.* But this Jensen's effect is not a robust theoretical prediction since it relies on the initial assumptions on the functional form of the demand process. For example, if we assume that *logged demand,* rather than the *level of demand,* is a Brownian motion process with mean μ and standard deviation σ, then this effect of uncertainty would disappear entirely.

2.2. A Model With Multiple Lines of Capital and Labor Adjustment Costs

The investment model outlined above, as is common in the literature, treats all types of capital within the firm as homogenous and labor as completely flexible because this makes the firm's optimization problem analytically tractable. But even a cursory glance at establishment and firm level data will reveal that it is common for firms to operate at several different production locations, across industry classifications, and with different capital mixes and vintages. Furthermore, since at least the work of Oi (1962) it has been recognized that hiring and firing workers involves recruitment, training, reorganization and compensation costs, which makes labor a costly factor to adjust. We generalize the firm level production function to allow for N separate lines of capital and M types of labor and a more flexible demand process. This is done in a general way by considering the class of models which satisfy the following three assumptions[13]:

1. The sales function is jointly concave and homogenous of degree λ in all N lines of capital and M types of labor, where $\lambda < 1$. Individual lines of capital and types of labor within each plant are also complementary in production[14].

2. Lines of capital cost $\mathbf{B} = \{B_1, B_2, ...B_N\}$ to buy and can be resold for $\mathbf{S} = \{S_1, S_2, ...S_N\}$ where $0 < \mathbf{S} < \mathbf{B}$. Labor can be hired at a cost $\mathbf{H} = \{H_1, H_2, ...H_M\}$ and fired at a cost $\mathbf{F} = \{F_1, F_2, ...F_M\}$, where these costs include the present discounted value of all future wage payments, and $0 < \mathbf{F} < \mathbf{H}$.

3. The firm level demand shock has a multiplicative impact upon revenue and is generated by a stationary Markov process[15].

Since firms may operate using several lines of capital and types of labor we have to generalize our threshold investment rule. Eberly and Van Mieghem (1997) demonstrate that the investment policy of a production plant with N lines of capital and M types of labor satisfying conditions (1) to (3) will be of a multi dimensional threshold form as characterized in Table 3.

[13] This class of models includes the Cobb-Douglas production and Brownian demand model we discussed previously in section (2.1).

[14] This complementarity is defined such that for a production function $F(K_1, K_2...K_N; L_1, L_2...L_M)$ the marginal product of any individual factor of production is increasing in every other factor of production. For example, $\partial F(K_1, K_2...K_N)/\partial K_i$ would be increasing in all K_j, $j \neq i$. This condition is technically known as supermodularity and is described in more detail in Dixit (1997).

[15] Stationarity implies that this process does not depend on time, while the Markov property implies that the future behaviour of the process depends on its present position but not on how it got there.

Table 3: N+M dimensional investment and hiring threshold behavior

For Lines of Capital $i = 1, 2, ...N$		For Types of Labor $j = 1, 2, ...M$	
Invest if:	$K_i = K_i^I$	Hire if:	$L_j = L_j^H$
Do Nothing if:	$K_i^I < K_i < K_i^D$	Do Nothing if:	$L_j^H < L_j < L_j^F$
Dis–Invest if:	$K_i = K_i^D$	Fire if:	$L_j = L_j^F$

For this more general set up we demonstrate in proposition 2 below that once again real options play no role in determining long run investment.

Proposition 2: For models satisfying assumptions (1) to (3) real options have no effect on the long run rate of investment.

Proof: In Appendix A

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This suggests that even in quite general models of production and investment, after conditioning on demand growth to remove any Jensen's inequality effects, uncertainty plays no role in determining the long run rate of investment.

3. The Short Run Impact of Real Options and Irreversibility on Investment and Labor Demand

The combination of real options and irreversibility do play an important role, however, in shaping the short run response to demand shocks. To explore this issue we first develop a methodology for characterizing these responses which is robust to any degree of aggregation. This is important because it enables us to make predictions on the dynamics of investment and hiring at the establishment, firm, industry, and macro level - thereby developing results that apply to data at the much lower levels of aggregation that is commonly used in empirical work. To ensure as wide a generality as possible we also maintain the multiple line of capital and inflexible labor model outlined by assumptions (1) to (3) in section (2.2). While our results will be stated for investment and hiring, to avoid repetition we will prove them only for investment, with the proof for hiring following by symmetry[16].

First, we define $F(-x)$ to be the cumulative distribution of log capital within each firm according to the size of the demand shock x required to move the capital up to its investment threshold. This implies, for example, that $F(0) = 1$ because all lines of capital will be either on or below their investment demand threshold and so require a shock of zero or greater to start investing. In contrast, if $F(-\Delta \log P) = 0$, this would imply that all lines of capital would start investing after a (presumably large) demand shock of size $\Delta \log P$. Figure 3 plots an example density function, $f(x) = dF(x)$, of capital below its investment threshold with the shaded area representing $F(-d \log P)$, the lines of capital which would invest after a demand shock of size $d \log P$.

FIGURE 3 ABOUT HERE

Second, we define the investment function for capital at each point x of this cumulative distribution $F(-x)$ as follows[17]

$$d \log K(-x) = I(-x, \Delta \log P) \quad \text{where} \quad I(-x, \Delta \log P) \geq 0 \quad \text{if} \quad \Delta \log P \geq x$$
$$\text{and} \quad I(-x, \Delta \log P) = 0 \quad \text{if} \quad \Delta \log P \leq x$$

[16] Note that we define the rate of labor hiring as the change in the *logged* labor force.

[17] This investment function actually also depends on the whole distribution of capital below the threholds $F(.)$, so could be fully written out for a position x and shock $\Delta \log P$ as $I(-x, \Delta \log P, F(.))$. However, since this does not effect the discussion of the main results this is compressed to the shorter form $I(-x, \Delta \log P)$ in the main text.

The right hand side conditions follow by the definition of $-x$ as the smallest demand shock required to move capital at that position up to the investment threshold. This investment function will be increasing in the size of the demand shock so that $\frac{\partial I(-x,\Delta \log P)}{\partial \Delta \log P} \geq 0$. For firms with multiple lines of capital this investment function will also be convex (increasing at an increasing rate) due to the assumed complementarity of capital in production, so that $\frac{\partial^2 I(-x,\Delta \log P)}{(\partial \Delta \log P)^2} \geq 0$. Combining these two definitions we can characterize total firm level investment to a demand shock of size $\Delta \log P$ as

$$\Delta \log(K) = \int_{-\Delta \log P}^{0} I(x, \Delta \log P) dF(x) \tag{3.1}$$

One important effect of real options and irreversibility is to reduce the investment and hiring response to a demand shock. This arises because firms will act more cautiously when capital and labor is partially irreversible and their market conditions are uncertain - any investment and hiring represents a gamble from which the firm can not easily extricate itself if conditions turn bad. This is noted in proposition 3 below

Proposition 3: Real options and irreversibility will reduce the responsiveness of investment and hiring to demand shocks.

Proof: In response to a positive demand shock of size $\Delta \log P$ the investment response across all lines of capital will be

$$\begin{aligned} \Delta \log(K) &= \int_{-\Delta \log P}^{0} I(x, \Delta \log P) dF(x) \\ &\leq \int_{-\Delta \log P}^{0} \frac{1}{1-\lambda}(\Delta \log P + x) dF(x) \\ &= \frac{1}{1-\lambda}\left(\Delta \log P - \int_{-\Delta \log P}^{0} F(x) dx\right) \end{aligned} \tag{3.2}$$

$$\leq \frac{1}{1-\lambda}\Delta \log P$$

where the second line follows because $I(x, \Delta \log P) \leq \frac{1}{1-\lambda}(\Delta \log P + x)$ by the complementarity of capital in production (see appendix B for details), and the third line follows by integration by parts. If there is any capital lying below the investment threshold, so that $F(-x) > 0$ for some $x > 0$, we obtain the strict inequality that $\Delta \log(K) < \frac{1}{1-\lambda}\Delta \log P$. To isolate the impact of real options and irreversibility we make the comparison to the hypothetical completely reversible level of capital, K_R. For this reversible capital we show in appendix B that

$$\Delta \log(K_R) = \frac{1}{1-\lambda}\Delta \log P \tag{3.3}$$

Combining (3.2) and (3.3) demonstrates that the investment response is lower under partial irreversibility than under complete reversibility.

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The intuition for this result is that the region of inaction between the investment/disinvestment thresholds and the hiring/firing thresholds acts as a buffer against demand shocks. Within this region the response to shocks will be zero unless they are large enough to ensure that capital and labor are moved up against their investment and hiring thresholds. And even when the demand shock is large enough to ensure this happens the investment and hiring response will still be reduced by the zone of inaction.

FIGURE 4 ABOUT HERE

Figure 4 plots an investment response, as an example, for a an economy comprised of a set of firms operating with a single line of capital and flexible labor, and where these firms are uniformly distributed between their investment and disinvestment thresholds. We can see that the investment response to a demand shock under partial irreversibility (the curved darker line) is always smaller than the investment response under complete reversibility (the straight lighter line). It can also be seen that while the demand response is always lower under partial irreversibility, this response is *proportionally* larger for larger shocks than for smaller shocks. This leads to a low but increasing and convex investment and hiring response to demand shocks, as noted in proposition 4 below.

Proposition 4: Real options and irreversibility will lead to an increasing convex investment and hiring response to demand shocks.

Proof: Taking the first derivative of the investment response in (3.1) with respect to the demand shock yields a positive result, simply indicating that larger shocks lead to more investment

$$\frac{\partial \Delta \log(K)}{\partial \Delta \log P} = \int_{-\Delta \log P}^{0} \frac{\partial I(x, \Delta \log P)}{\partial \Delta \log P} dF(x) \geq 0 \tag{3.4}$$

Taking the second derivative we see that the investment response is also increasing in the size of the shock

$$\begin{aligned} \frac{\partial^2 \Delta \log(K)}{\partial (\Delta \log P)^2} &= \int_{-\Delta \log P}^{0} \frac{\partial^2 I(x, \Delta \log P)}{(\partial \Delta \log P)^2} dF(x) + \frac{\partial I(-x, \Delta \log P)}{\partial \Delta \log P}|_{x=-\Delta \log P} dF(-\Delta \log P) \\ &\geq 0 \end{aligned}$$

where the first term is non-negative by the assumed complementarity of different lines of capital in production and the second term is non-negative by the non-decreasing nature of cumulative distribution functions.

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This prediction matches the results of Caballero, Engel and Haltiwanger (1997) and Cooper and Haltiwanger (2000), who estimate employment and investment functions respectively on a panel of US establishment level data, and find a convex and increasing response.

Finally, in addition to the reduction in the *short run* response of investment and hiring to demand shocks, real options and irreversibility also lead this response to be spread out over time, imparting rich and persistent dynamics to these processes. This is noted in proposition 5 below.

Proposition 5: Real Options and irreversibility will lead investment and hiring to be increasing in all past demand shocks.

Proof: In Appendix C

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This can be interpreted in terms of the basic concept of pent up demand. Firms and industries with a history of strong recent demand growth will have a distribution of capital and labor lying close to their investment threshold and will display a strong investment and hiring response, while firms with a recent history of bad demand shocks will be less disposed to hire or invest. Empirically this will lead investment and labor demand to appear to respond to both current and lagged demand shocks. This matches the stylized facts from estimating firm and macro investment and labor demand equations. These display lagged responses to demand shocks usually spread over several quarters and years[18].

[18] See, for example, Chirinko (1993) and Hammermesh (1993).

4. Conclusion

In this paper we have shown that real options play no role in determining the long run rate of investment. This is demonstrated for both the standard model with Cobb-Douglas production and Brownian demand shocks, and also for a broader class of models with multiple lines of capital, inflexible labor and a generalized demand process. However, real options and irreversibilities are shown to play an important role in shaping the short run dynamics of investment and hiring. They reduce the short run response of investment and hiring to current demand shocks and create lags in the response to past demand shocks.

The predictions of this model are consistent with the stylized facts from estimating firm, industry and macro level investment and labor demand equations. Hence, using real options to build a structural framework for estimating investment and labour demand should help to bridge the gap between what is often empirically preferable (reduced form models) and what is often theoretically preferable (their structural counterparts). And from the policy perspective, the time varying response elasticity of investment and employment over the business cycle can be explained by variations in the degree of macro uncertainty. Thus using measures of the current degree of macro uncertainty could improve the predictions of the response elasticities of investment and employment, helping policymakers to better model the effects of tax and interest rate changes.

Appendix A

PROOF OF PROPOSITION 2:

For a plant with N+M lines of capital and types of labor satisfying assumptions (1) to (3) we define $V(\mathbf{K}, \mathbf{L}, P)$ to be its value function given its current capital stock $\mathbf{K} = \{K_1, K_2, ...K_N\}$, labor force $\mathbf{L} = \{L_1, L_2, ...L_M\}$, and demand condition P. By theorems 9.6, exercise 9.9 and theorem 9.10 respectively of Stokey et al. (1983) this value function will inherit the concavity and homogeneity properties of plant level sales and will be once continuously differentiable. We define $U = \{B_1, ...B_N, H_1...H_M\}$ and $D = \{S_1, ...S_N, F_1...F_M\}$ to be the combined N+M dimensional {buy,hire} and {sell,fire} prices of capital and labor. Following Eberly and Van Mieghem (1997) we can define the plants investment and hiring thresholds by the vector of first differential of $V(\mathbf{K}, \mathbf{L}, P)$ with respect to each line of capital and type of labor,

$$D \leq \nabla V(\mathbf{K}, \mathbf{L}, P) \leq U \tag{4.1}$$

Since this value function is homogenous of degree one[19] in $(\mathbf{K}, \mathbf{L}, P^{\frac{1}{1-\lambda}})$ its vector of first derivatives will be homogeneous of degree zero in $(\mathbf{K}, \mathbf{L}, P^{\frac{1}{1-\lambda}})$ so that this condition can be re-written as

$$D \leq \nabla V(\mathbf{K}P^{-\frac{1}{1-\lambda}}, \mathbf{L}P^{-\frac{1}{1-\lambda}}, 1) \leq U \tag{4.2}$$

By the concavity of $\nabla V(\mathbf{K}, \mathbf{L}, P)$ this defines a bounded continuation region such that

$$P^{\frac{1}{1-\lambda}} \mathbf{\Gamma}_K^I \quad \leq \quad \mathbf{K} \leq P^{\frac{1}{1-\lambda}} \mathbf{\Gamma}_K^D \tag{4.3}$$

[19] By assumptions (1) to (3) sales is homogeneous of degree λ in $\mathbf{K}$ and $\mathbf{L}$, and homogeneous of degree one in P (due to the multiplicative nature of the shock), and so sales is jointly homogenous of degree one in $(\mathbf{K}, \mathbf{L}, P^{\frac{1}{1-\lambda}})$.

$$P^{\frac{1}{1-\lambda}}\mathbf{\Gamma}_L^H \quad \leq \quad \mathbf{L} \leq P^{\frac{1}{1-\lambda}}\mathbf{\Gamma}_L^F \tag{4.4}$$

where $\mathbf{\Gamma}_K^I$, $\mathbf{\Gamma}_K^D \in R_N$ and $\mathbf{\Gamma}_L^H$, $\mathbf{\Gamma}_L^F \in R_M$, with these being functions of the curvature of the value function and the cost of reversibility. Hence for capital we can write

$$\frac{1}{1-\lambda}\log P + \log \mathbf{\Gamma}_K^I \leq \log \mathbf{K} \leq \frac{1}{1-\lambda}\log P + \log \mathbf{\Gamma}_K^D \tag{4.5}$$

By revealed preference these optimal investment and disinvestment thresholds contain the investment and disinvestment thresholds for a firm that ignores its option to delay the investment decisions so that

$$\log \mathbf{\Gamma}_K^I - \log \mathbf{\Gamma}_L^D \leq \log \mathbf{K} - \log \mathbf{K_{no}} \leq \log \mathbf{\Gamma}_K^D - \log \mathbf{\Gamma}_K^I \tag{4.6}$$

where $\log \mathbf{K_{no}}$ is the vector of no real options capital stocks.

Following the same logic as in proposition 1 we can show that $\log \mathbf{K}$ and $\log \mathbf{K_{no}}$ have the same long run limiting growth rate since

$$\begin{aligned} \lim_{T\to\infty}\frac{1}{T}|\log(\mathbf{K}_{T+t}/\mathbf{K}_t) - \log(\mathbf{K}_{\mathbf{no},T+t}/\mathbf{K}_{\mathbf{no},t})| &= \lim_{T\to\infty}\frac{1}{T}|(\log(\mathbf{K}_{T+t}/\mathbf{K}_{\mathbf{no},T+t}) - \log(\mathbf{K}_t/\mathbf{K}_{\mathbf{no},t})| \\ &\leq \lim_{T\to\infty}\frac{1}{T}|\log \mathbf{\Gamma}^I + \log \mathbf{\Gamma}^D| \qquad (4.7) \\ &= 0 \end{aligned}$$

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Appendix B

NOTES FOR THE PROOF OF PROPOSITION 3:

From equation (4.1) above it can be seen that for completely reversible capital and labor we have

$$\nabla_{\mathbf{K}}V(\mathbf{K}P^{\frac{-1}{1-\lambda}}, 1) = \mathbf{r} \tag{4.8}$$

where $\mathbf{r}$ is the reversible cost of capital. Thus $\log \mathbf{K} = \frac{1}{1-\lambda} \log P + \log \mathbf{\Gamma}_K^r$. The investment response to a demand shock can then be written as, $\Delta \log K_i = \frac{1}{1-\lambda} \Delta \log P$, which, since this holds for the log of each line of capital, holds for the log of the total capital stock,

$$\Delta \sum_{i=1}^{N} \log K_i = \frac{\sum_{i=1}^{N} dK_i}{\sum_{i=1}^{N} K_i} \tag{4.9}$$

$$= \frac{1}{1-\lambda} \Delta \log P. \tag{4.10}$$

To show that

$$I(x, \Delta \log P) \leq \frac{1}{1-\lambda} (\Delta \log P + x). \tag{4.11}$$

it is sufficient to note that under irreversibility every line of capital and type of labor may not be adjusting, and because these are supermodular (complementary) in production, the investment response to a demand shock at the investment threshold will be less than the reversibility case where all factors of production would adjust. This inequality will be strict if production is strictly supermodular and some factors do not adjust fully.

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Appendix C

PROOF OF PROPOSITION 5:

The approach of this proof is to demonstrate that for any demand shock in period t a lagged positive demand shock in any period $t-s$, $s>0$, will increase investment in period t. And the larger the demand shock in period $t-s$ the larger the increase in investment in period t. Thus current investment will be an increasing function of past demand shocks.

Suppose in the absence of a some past demand shock the cumulative density of capital below the investment thresholds has the form $F(x)$. The investment function at each point x in response to a demand shock $\Delta \log P_t$ takes the value $I(-x, \Delta \log P_t, F(.))$, where we use the fuller notation which accounts for the impact of the distribution of capital on the investment response of each line of capital. Since all lines of capital are (weakly) complimentary in production the investment function will be weakly increasing in all weakly decreasing transformations of $F(.)$ across its support.

Now consider a counter factual in which the firm experienced a shock in period $t - s$ of magnitude $\Delta \log P_{t-s} > 0$ so that the distribution below the investment threshold is now characterised by $\widetilde{F(.)}$. We can then define the new investment function at each point x on the *old* cumulative density function by $I(-x, \Delta \log P_t, \widetilde{F(.)})$. For each point x on the old cumulative density function this new investment function must be greater or equal to than the old investment function since $\widetilde{F(.)} \leq F(.)$ across the support of x, because the lagged demand shock will have moved all lines of capital closer to their investment thresholds. Hence, using our characterization for investment we can write

$$\begin{aligned}
\widetilde{\Delta(\log K)} &= \int_{-\Delta \log P_t - g(F(.),\widetilde{F(.)})}^{0} I(-x, \Delta \log P_t, \widetilde{F(.)}) dF(x) \\
&\geq \int_{-\Delta \log P_t}^{0} I(-x, \Delta \log P_t, \widetilde{F(.)}) dF(x) \\
&\geq \int_{-\Delta \log P_t}^{0} I(-x, \Delta \log P_t, F(.)) dF(x) \\
&= \Delta \log(K)
\end{aligned}$$

where $g(F(.), \widetilde{F(.)}) \geq 0$ is a positive function reflecting the impact of past demand shocks on the *density* of capital which will invest today. Hence, current investment is increasing in lagged demand shocks. Since ,$I(-x, \Delta \log P_t, \widetilde{F(.)})$ is increasing in negative transformations of $\widetilde{F(.)}$, and $\widetilde{F(.)}$ is decreasing in the size of the lagged demand shock, current investment will display a larger response to larger lagged demand shocks. Since this result holds by symmetry for negative demand shocks the proof is complete.

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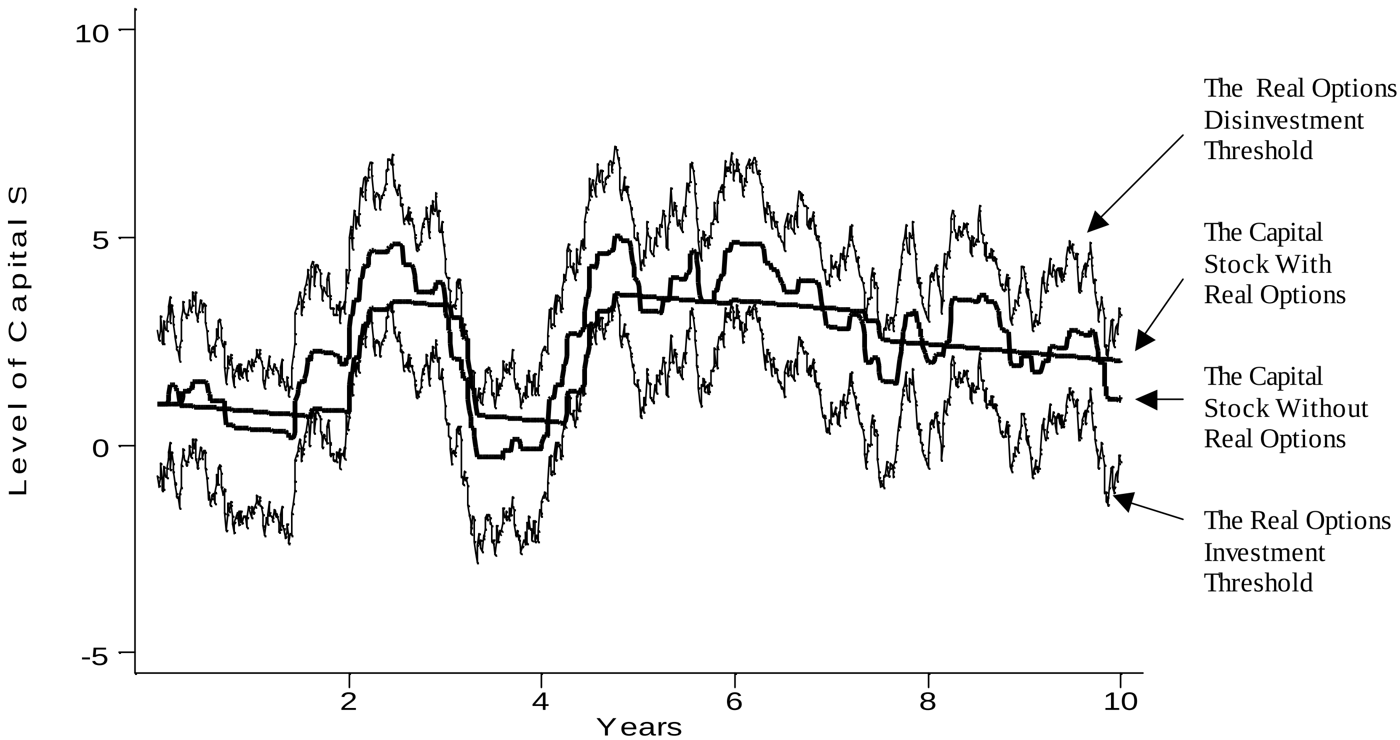
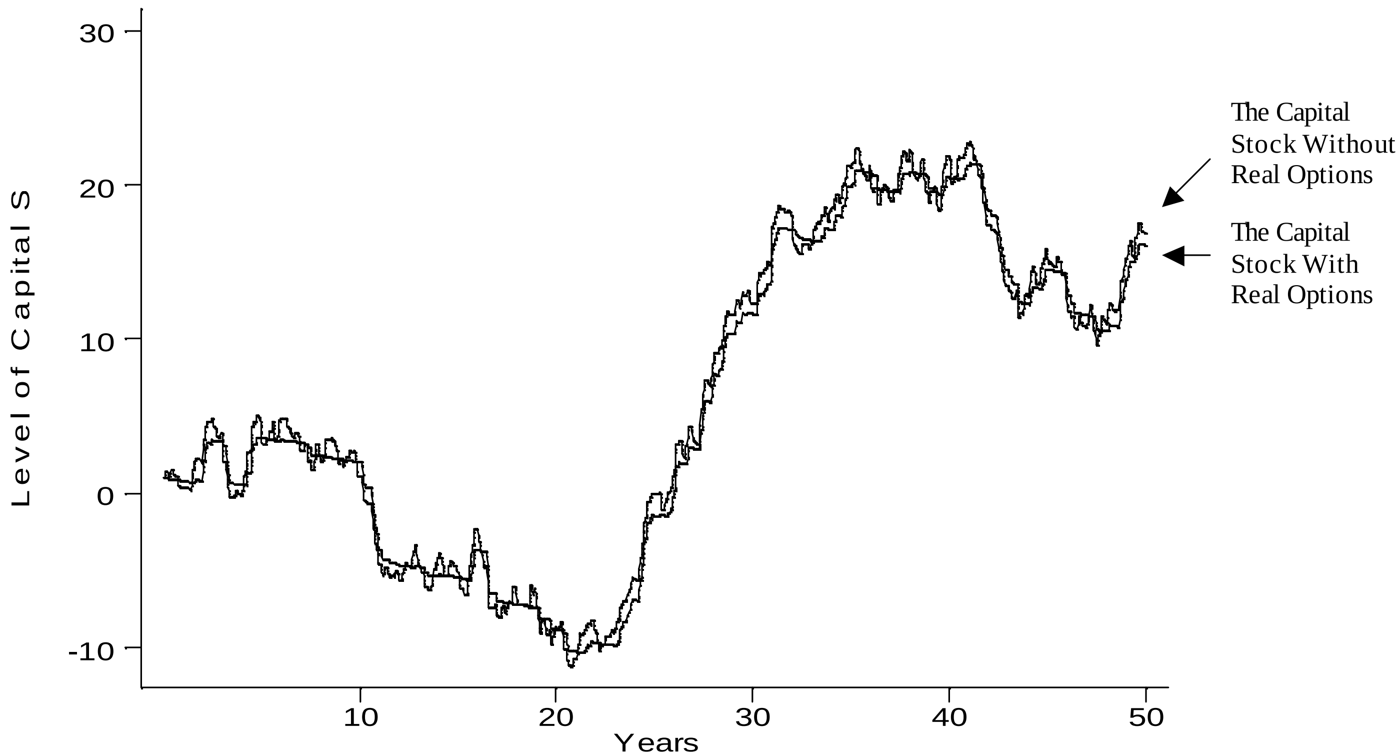
Figure 1. The Level of the Capital Stock with Real Options and Without Real Options, and the Real Options Investment and Disinvestment Thresholds.



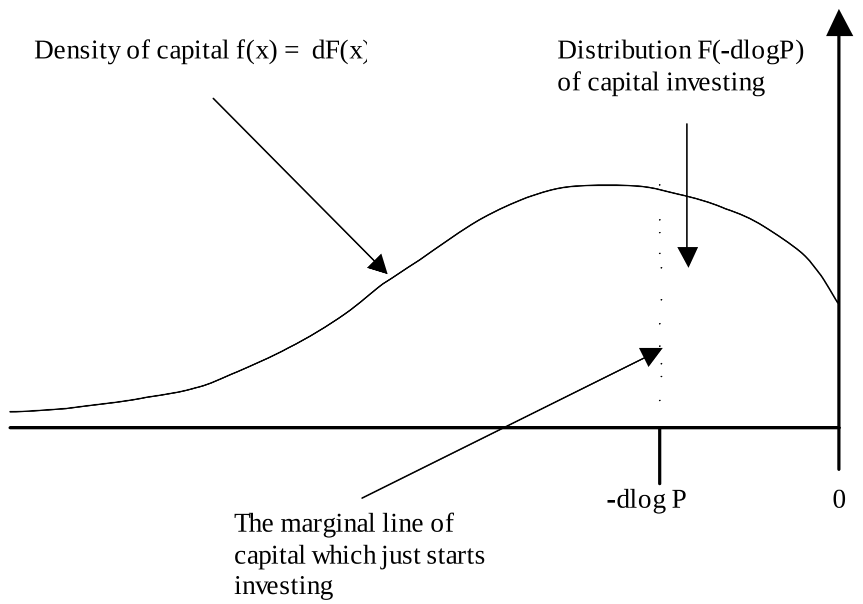
Notes: The graph plots the evolution of the capital stock with real options (in bold) between its lower investment threshold (in feint) and its upper disinvestment threshold (in feint) in response to a randomly drawn demand process (see footnote 11 for details). Also plotted in bold is the evolution of the no real options capital stock in response to the same demand process.

Figure 2. The Level of the Capital Stock With and Without Real Options.



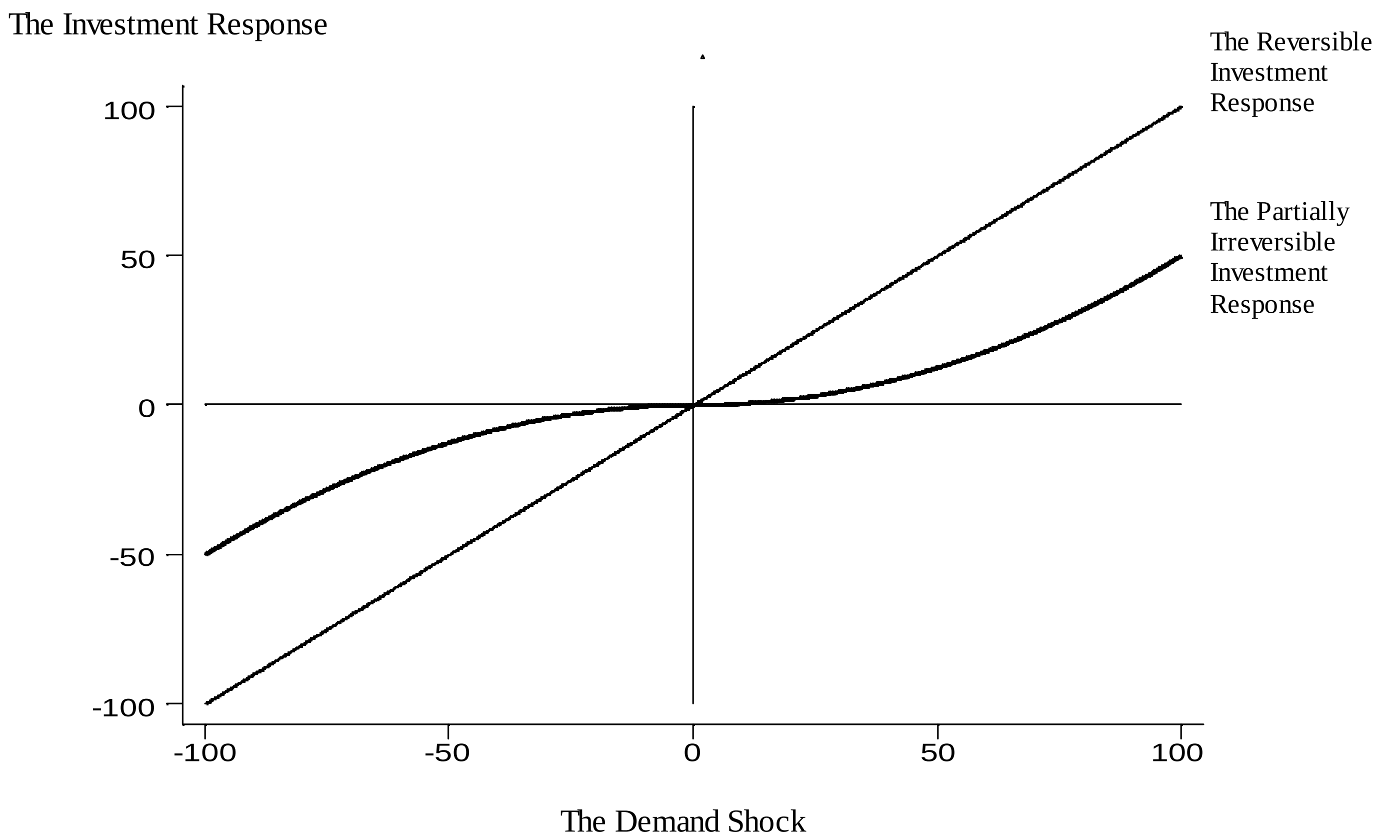
Notes: The graph plots the evolution of the capital stock with real options (the less variable line) in response to a randomly drawn demand process (see footnote 11 for details). Also plotted is the evolution of the n real options capital stock (the more variable line) in response to the same demand process.

Figure 3. The Distribution of Capital Below its Investment Threshold



Notes: The figure plots an example distribution of capital according to the size of the shock required t initiate investment. Thus, capital at point 0 will invest in response to a positive shock of any size, while the shaded area of capital up to point -dlog P would invest in response to a shock of size dlog P (or greater).

Figure 4. The Investment Response to Demand Shocks When Capital is Uniformly Distributed below the Investment Threshold



Notes: The figure plots the investment response (in bold) of an economy of firms with partially irreversible capital which are uniformly distributed between their investment and disinvestment thresholds. Also plotted (in feint) is the investment response for this economy if capital were to be completely reversible.