Monetary models and technology shocks

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Abstract

Adding variable capital utilisation to a dynamic new Keynesian framework gives a model which can produce realistic responses to both technology and monetary shocks. This requires the assumption of a much lower level of nominal rigidity than is usual.

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1 Introduction

Dynamic new Keynesian (DNK) models using Calvo (1983) pricing are common in the literature (e.g. Bernanke, Gertler and Gilchrist (1999)). Such models generally assume that prices are fixed on average for a year. As noted in Casares and McCallum (2000), models with this level of nominal rigidity give an unrealistically large response to monetary shocks unless capital adjustment costs are included. This addition is unsatisfactory for a number of reasons. Firstly, a standard critique of real business cycle models is that they require large technology shocks to match the data. By weakening the response to technology shocks, capital adjustment costs make DNK models equally susceptible to this criticism. Although much of the recent literature ignores technology shocks (a notable exception is Kim (2000)), few would deny that real factors have some role to play in driving the business cycle and a framework capable of modelling both sorts of shock is theoretically appealing. Secondly, the empirical evidence on capital adjustment costs is far too limited to do anything but confine the calibration within the broadest range. Thirdly, it is methodologically unsatisfactory to introduce costs merely to fix another problem in the model. While adjustment costs have important roles to play, few would argue that capital costs are more important than, say, labour costs.

This paper presents a model which can be calibrated to produce a response to monetary shocks similar to DNK models but also a strong response to technology shocks. I investigate the tradeoffs involved in the calibration of this model.

2 The Model

I combine the features of two models. The first is the real business cycle model of King and Rebelo (1999) which generates strong amplification of technology shocks from variable capital utilization and indivisible labour. The second is a standard DNK model taken from Bernanke, Gertler and Gilchrist (1999). The model consists of six types of agent. Households with rational expectations supply labour, consume final goods and hold money balances and bonds. Intermediate goods firms are imperfect competitors producing differentiated output from labour and capital. Changing the intensity of utilization of capital brings a benefit of extra output, but incurs a cost of faster depreciation. Final goods firms aggregate intermediate goods à la Dixit-Stiglitz to produce a homogenous final good. To motivate costly capital adjustment, capital goods producing firms take final goods and turn them into capital. A monetary authority sets nominal interest rates and a government rebates seigniorage proceeds as a lump sum to households keeping the budget always balanced.

2.1 Households

The representative household solves the problem[1]

$$\max_{\{c_{t+j},n_{t+j},m_{t+j},k_{t+j}\}_{j=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta^i u(c_{t+i}, n_{t+i}, m_{t+i}) \tag{1}$$

subject to a series of budget constraints

$$w_t n_t + r_t^k (z_t) k_t + \Pi_t + \frac{m_{t-1}}{1+\pi_t} + b_{t-1} = c_t + q_t x_t + m_t + \frac{b_t}{1+r_t} \tag{2}$$

where w_t is the wage, n_t labour, r_t^k return to capital, z_t rate of capital utilization, k_t level of capital stock, c_t consumption, q_t price of capital in terms of final goods, x_t investment, π_t inflation m_t holdings of money and b_t holdings of real zero-coupon bonds paying an interest rate r_t.

The linearized first-order conditions[2] with respect to consumption, labour, capital and bonds are

$$-\sigma\hat{c}_t + (1-\sigma)\kappa\hat{n}_t = \hat{\lambda}_t \tag{3}$$

$$(1-\sigma)\hat{c}_t + \frac{(1-\sigma)^2}{\sigma}\kappa\hat{n}_t = \hat{\lambda}_t + \hat{w}_t \tag{4}$$

$$E_t\left(r\hat{r}_{t+1} - r^k\hat{r}_{t+1}^k\right) = [1-\delta(z)]E_t\hat{q}_{t+1} - \hat{q}_t - \delta_z(z)\hat{z}_t \tag{5}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{r}_t \tag{6}$$

σ is the coefficient of relative risk aversion and $\delta(z_t)$ is depreciation, an increasing function of the intensity of capital utilization.

The linearized law of motion of capital is

$$\hat{k}_{t+1} = \delta\hat{x}_t + [1-\delta(z)]\hat{k}_t - z\delta_z(z)\hat{z}_t \tag{7}$$

[1] If a_t is the level of a variable, a is the steady states and $\hat{a}_t$ the linearization.

[2] A full statement of the problem and derivation of the linearized system is available on request from the author.

2.2 Firms

The representative intermediate goods firm minimizes costs

$$\min_{\{n_t, k_t, z_t\}} w_t n_t + r_t^k (z_t) k_t \tag{8}$$

subject to a production function

$$y_t = a_t (z_t k_t)^{1-\alpha} n_t^{\alpha} \tag{9}$$

where y_t is output and a_t technology. The linearized first-order conditions with respect to labour, capital and capital utilization are

$$\hat{w}_t = \hat{y}_t - \hat{n}_t + \widehat{mc}_t \tag{10}$$

$$\hat{r}_t^k = \hat{y}_t - \hat{k}_t + \widehat{mc}_t \tag{11}$$

$$\hat{z}_t = \frac{1}{1+\xi}\left(\hat{y}_t - \hat{k}_t - q_t + \widehat{mc}_t\right) \tag{12}$$

where mc_t is the firm's marginal cost and ξ the elasticity of the marginal rate of depreciation to utilization. The linearized production function is

$$\hat{y}_t = \hat{a}_t + (1-\alpha)\left(\hat{z}_t + \hat{k}_t\right) + \alpha \hat{n}_t \tag{13}$$

Firms' pricing decisions follow a Calvo (1983) type process with a probability of changing prices in any period of $1-\phi$. The linearized first-order condition is

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_\pi \widehat{mc}_t \tag{14}$$

where κ_π is the new Keynesian Phillips curve constant.

The profit maximizing behaviour of capital producing firms with a production function $g(x)$ gives a linearized relation for the price of capital

$$\hat{q}_t = \psi^{-1}\left(\hat{x}_t - \hat{k}_t\right) \tag{15}$$

where ψ is the elasticity of the cost of capital with respect to investment.

2.3 General equilibrium

The economy's resource constraint is

$$\hat{y}_t = \frac{c}{y}\hat{c}_t + \frac{x}{y}\hat{x}_t \tag{16}$$

The nominal interest rate is defined as

$$\hat{R}_t = E_t\hat{\pi}_{t+1} + \hat{r}_t \tag{17}$$

and the monetary authority follows a Taylor rule (after Casares and McCallum (2000)):

$$\hat{R}_t = (1-\mu_3)\left[\mu_1\hat{\pi}_t + \mu_2\widehat{mc}_t\right] + \mu_3\hat{R}_{t-1} + \varepsilon_t^m \tag{18}$$

where μ_1, μ_2 and μ_3 are constants and ε_t^m is a white noise shock. Technology follows an autoregressive process:

$$\hat{a}_t = \rho\hat{a}_{t-1} + \varepsilon_t^a \tag{19}$$

where ε_t^a is a white noise shock.

An equilibrium for this economy is a set of six allocations $\left\{\hat{c}_t, \hat{n}_t, \hat{y}_t, \hat{k}_t, \hat{z}_t, \hat{x}_t\right\}$, six prices $\left\{\hat{w}_t, \hat{r}_t^k, \hat{\pi}_t, \hat{r}_t, \hat{q}_t, \hat{R}_t\right\}$ and two shadow prices $\left\{\hat{\lambda}_t, \widehat{mc}_t\right\}$ which satisfy the fourteen equations (3) to (7) and (10) to (18). I simulate the system using standard methods[3].

2.4 Calibration

To calibrate the model on quarterly data I use the values shown in table 1. The results of the paper depend on the interaction between three key parameters. ϕ measures the stickiness of prices. The average length that prices are fixed is given by $\frac{1}{1-\phi}$. ξ is a measure of the costs of increasing utilization. As ξ becomes large, the model approaches one with constant utilization. ψ measures the cost of capital adjustment. Large values mean the price of capital hardly changes with respect to investment so capital can be adjusted costlessly.

I consider three calibrations of the model. Case 1, with $\phi = 0.75, \psi = \xi = \infty$, is a DNK model without capital adjustment costs, with prices fixed on average for one year. Case 2, with $\phi = 0.75, \psi = 4, \xi = \infty$, adds capital adjustment costs. Case 3, with $\phi = 0.25, \psi = \infty, \xi = 0.5$, has no capital adjustment costs, prices fixed on average for only four months, and variable capital utilization.

3 Results

Figure 1 and 2 show the responses of output and investment to a 1% persistent technology shock. The response of case 1 is similar to a baseline RBC

[3] MATLAB programs available on request from the author

model so, to match investment volatility in the data, would need a technology shock with a standard deviation of around 1% (see King and Rebelo (1999)). Such a technology shock is widely considered to be improbably large. Capital adjustment costs mean the response of case 2 is much weaker, with the impact effect on investment less than half that in case 1. So to produce a realistic volatility of investment from real shocks alone, the model would need to be driven by a technology shock with a standard deviation of around 2%. Or, to put it another way, real factors have very little cyclical role in such a model. Case 3 shows a strong response of both output and investment as a result of the amplification provided by variable capital utilization. To produce similar investment volatilities to case 2, case 3 would need to be driven by a technology shock with a standard deviation of a much more credible 0.3%.

Figure 3 shows the response of output to a 1% white noise shock in the monetary policy rule. Case 1 shows the excessive response characteristic of DNK models without capital adjustment costs (see Casares and McCallum (2000) for a discussion). Output responds by 6% response to a 1% innovation which is the stuff of central bankers' dreams but unsupported by empirical work. In case 2 the introduction of capital adjustment costs reduces this response to that common in DNK models. Case 3 produces a response of comparable magnitude, but from a much reduced level of nominal rigidity. Note that case 2 generates the hump-shaped response characteristic of models with adjustment costs.

The trade off between the three parameters is investigated further in table 1 which shows combinations of ϕ, ψ and ξ which give an approximately 1% response of output to a monetary shock[4]. For any given value of capital adjustment costs, a lower level of nominal rigidity requires capital utilization to be easier to vary i.e. low ξ. As capital adjustment costs increase, the depreciation cost of increasing utilization also increases so the amplification effect of variable utilization is dampened. With $\psi = 4$, more elastic utilization has almost no effect as the benefits in terms of extra output are fully offset by the depreciation costs.

For this model to match the response to technology shocks of the simplest RBC model we need to assume that prices are fixed on average for at most seven months ($\phi = 0.60$). The model can give realistic responses to both monetary and technology shocks for much lower levels of nominal rigidity - in case 3 prices are fixed on average for only four months ($\phi = 0.25$). This compares unfavorably with the value of a year ($\phi = 0.75$) common in the literature. Such high levels of nominal rigidity require high capital

[4] I follow King and Rebelo (1999) in taking $\xi \epsilon [0.1, \infty]$

adjustment costs to reduce their excessive response to monetary shocks. If a model is also to produce realistic responses to technology shocks, and in the continued absence of conclusive empirical evidence on the other parameters, we need to reconsider what constitutes an acceptable level of nominal rigidity.

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Figure 1 : Response of output to a 1% persistent technology shock

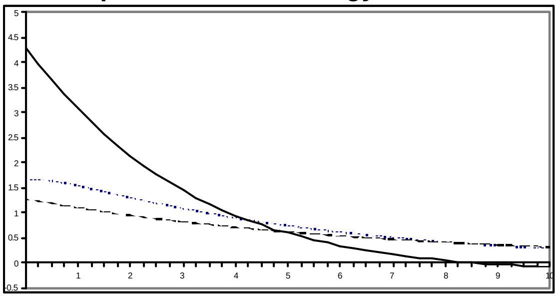


Figure 2 : Response of investment to a 1% persistent technology shock

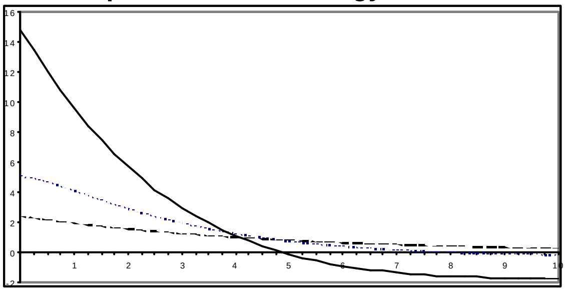
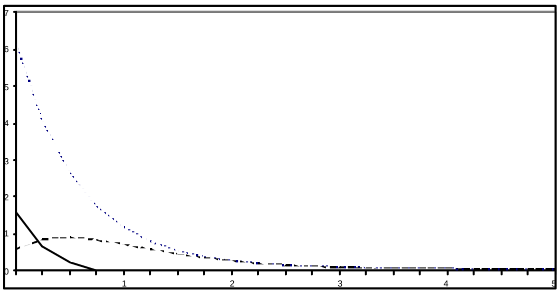


Figure 3 : Response of output to a 1% white noise monetary policy rule shock



x-axis : years after shock
(note x range is different in figure 3)
y-axis : percent deviation from the steady state

Case 1 :
Case 2 :
Case 3 :

Table 1 : Calibration

	Value	**Description**
β	0.99	Quarterly discount factor
σ	1	Coefficient of relative risk aversion
θ	3	Preference parameter on labour
α	0.65	Factor share of labour
δ	2.5%	Steady state depreciation per quarter
ρ	0.95	Autoregressive parameter of technology shock
μ_1	1.5	Monetary policy rule parameter
μ_2	0.1	Monetary policy rule parameter
μ_3	0.8	Monetary policy rule parameter

Table 2 : Parameter combinations giving a 1% impact response of output on a 1% white noise shock to the monetary policy rule

j =4			j =8			j =20			j =¥		
ϕ	ξ	Tech	ϕ	ξ	Tech	ϕ	ξ	Tech	ϕ	ξ	Tech
0.75 (4.0)	∞	2.0	0.72 (3.6)	∞	1.4	0.62 (2.6)	∞	1.0	0.42 (1.7)	∞	0.7
0.72 (3.6)	0.4	2.0	0.65 (2.9)	0.7	1.1	0.60 (2.5)	3.0	0.9	0.35 (1.5)	3.0	0.6
0.70 (3.3)	0.1	2.0	0.60 (2.3)	0.1	0.9	0.50 (2.0)	0.7	0.7	0.25 (1.3)	0.8	0.4
						0.42 (1.7)	0.1	0.4	0.10 (1.1)	0.1	0.1

The numbers in brackets are the average number of quarters for which prices are fixed
"Tech" is the standard deviation of the technology shock required to generate realistic volatility of investment

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