Why frequencies are natural

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Abstract: Research in mathematical cognition has shown that *rates*, and other interpretations of x/y, are hard to learn and understand. On the other hand, there is extensive evidence that the brain is endowed with a specialized mechanism for representing and manipulating the numerosities of sets – that is, frequencies. Hence, base-rates are neglected precisely because they are rates, whereas frequencies are indeed natural.

Barbey & Sloman (B&S) are to be congratulated for laying out the explanations for base-rate neglect so clearly and systematically. However, to a researcher not from the field of normative rationality research, but from the field of mathematical cognition, it is surprising that none of the explanations make reference to what is known about how we process numerical quantities (Butterworth 2001). From this perspective, another type of explanation can be proposed for base-rate neglect. It is in the word "rate." Rates can be expressed formally as x/y, and it is well known from research in mathematical cognition and education that humans are very bad at understanding x/y however it is interpreted – as a fraction, as a proportion, or as a rate. For example, it is well known that children find it hard to learn and understand fractions and simple operations on them (Bright et al. 1988; Hartnett & Gelman 1998; Mack 1995; Smith et al. 2005). It has also been found that most third and fourth graders cannot order fractions by size and cannot explain why there are two numbers in a given fraction (Smith et al. 2005). In particular, they seem to have trouble getting away from whole numbers – for example, when they say that 1/56 is smaller than 1/75 because 56 is smaller than 75

(Stafylidou & Vosniadou 2004). This has been called "whole number bias" (Ni & Zhou 2005) and can be found in adults as well as children (Bonato et al., in press). Whole number bias is not simply a function of the symbolic form of the rate, for example, 3/5, because it appears also in non-symbolic formats such as arrays of dots (Fabbri et al., submitted).

The advantage of presentations in terms of frequencies, and therefore of whole numbers, rather than rates, again is well supported by research in mathematical cognition. This has nothing to do with the relative computational simplicity of representing the problem in terms of frequencies as compared with rate-based Bayesian formulations; rather, it has to do with the fact that the human brain is configured from birth to represent sets and their numerosities. Infants can discriminate small sets on the basis of their numerosity (Antell & Keating 1983; Starkey & Cooper 1980; Wynn et al. 2002). This seems to be an inherited capacity since other primates can do the same in the wild (Hauser et al. 1996), and can learn to do it relatively easily (Brannon & Terrace 2000). Indeed, monkeys readily learn to select the larger of two numerosities (Brannon & Terrace 1998; Matsuzawa 1985).

These primate capacities are not merely analogous to those of humans, but appear to have been inherited from a common ancestral system. Evidence for this comes from recent research showing that the primate brain areas for numerosity processing are homologous to human brain areas. Studies have demonstrated that the intraparietal sulcus (IPS) in humans processes the numerosities of sets (Piazza et al. 2002). It has recently been demonstrated that when monkeys are required to remember the numerosity of a set before matching to sample, the homologous IPS brain area is active (Nieder 2005). This is evidence that we have inherited the core of our system from the common ancestor of humans and macaques.

The concept of the numerosity of a set is abstract, because sets logically contain any type of member that can be individuated. Members need not be visible objects, and they need not be simultaneously present. It turns out that the human numerosity system in the IPS responds when members of the set are distributed as a sequence in time or simultaneously distributed in a spatial array (Castelli et al. 2006) and for auditory as well as visual sets (Piazza et al. 2006). Indeed, the neural process of extracting numerosity from sets of visible objects appears to be entirely automatic, since repeated presentation of different sets with same numerosity produces a reduction in neural firing in the IPS, called "adaptation," even when numerosity is task-irrelevant (Cantlon et al. 2006; Piazza et al. 2004; 2007).

"Frequency" is just a way of referring to this numerosity property of a set, and so it too is natural. "Natural sampling" can be interpreted to be a way of making an estimate of numerosity when the set is distributed in time or in space. Humans and other species are born with the capacity to make these estimates of the approximate size of a set, using a specialized brain system probably related to the system for exact numerosities. This system also responds to environmental stimuli in rapid and automatic manner (Cantlon et al. 2006; Dehaene et al. 1999; Lemer et al. 2003; Piazza et al. 2004). So natural sampling too is natural, in the sense that it depends on an innate system.

B&S note that accounts involving specialized modules (Cosmides & Tooby 1996), specialized frequency algorithms (Gigerenzer & Hoffrage 1995), or specialized frequency heuristics (Gigerenzer & Hoffrage 1995; Tversky & Kahneman 1974) appeal to evolution. However, these claims depend on general arguments about ecological rationality rather than on specific facts about the evolution of dedicated neural system. On the other hand, there is a clear account, well supported by a range of evidence, as I have indicated, for the evolution of numerosity processing. Indeed, the evidence suggests that numerosity processing is a classic Fodorian cognitive module: domain-specific, automatic, with a dedicated brain system, and innate (though Fodor himself cites the number domain as the responsibility of classic central processes; cf. Fodor 1983). Therefore, the critical difference between normative Bayesian reasoning and actual human preferences for sets and their frequencies appears to be rooted in the evolution of a specialized "number module" for processing numerosities (Butterworth 1999). As far as I know, there is no comparable evolutionary account of a specialized brain system for x/y.

Base-rate is neglected because rates are neglected.