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THE GEOMETRY OF SLUMS: BOUNDARIES, PACKING & DIVERSITY

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ABSTRACT

The geometry of squatter settlements on the northeastern coast of Brazil is examined and compared to settlements in the central region of Kenya. In particular, fragmented structures, often squatter settlements composed of islands of dwelling or 'habitations' in these settlements, and void areas which are unoccupied regions between dwellings, are studied. We find that such settlements, when constrained by urban and natural boundaries, present robust configurational patterns, which we can express best by statistical distributions with the scaling properties. Such scaling properties, when considered in the context of urban planning, can be useful for describing and predicting the spatial and social parameters of such squatter settlements. Several scaling functions and other mathematical formulae, which are of interest in planning, are also derived and discussed.

1. INTRODUCTION

There are two major issues that have been ever present in discussions concerning the form of cities within urban planning: scale and regularity. Throughout history, cities have usually been described and modeled as linear aggregations of islands – groups of connected buildings – that are regular and have the same pattern in different parts of the form and across different scales. This principle of regularity can be seen in large cities with very different cultures such as Barcelona, Manhattan, Brasilia … and so on. However, the history of urbanization and planning intervention has shown that controlling of the evolution of a city is not a simple task. Many forms of city deviate from the idealized patterns and rules which are desired by planners of the formal city, and the norm for city development is usually evolution with a decentralized dynamic. Amongst the best examples of such structures, informal developments such as squatter settlements stand out as urban structures which evolve outside the frame of general urban restrictions on urban growth with little or no public intervention. These structures are very different from those at the other end of the spectrum – cities which are highly structured geometrically and formally planned. Informal structures usually manifest themselves as the result of an irregular distribution of different islands or groups of dwellings and because of this, they have usually been considered disordered, non-logical and hence difficult to model.

In this work, we will show that such settlements are not disordered. We will argue that they can be defined as complex structures, and such complexity can be quantified through the spatial patterns which describe the irregularity of their configuration. By analyzing the frequency of occurrence of the basic units which comprise such settlements which we define as clusters of built forms based on dwellings and clusters of voids or empty spaces between dwellings, we will invoke scaling laws which relate their size to their frequency.

In recent years a great deal of effort in pure and applied science has been devoted to the study of nontrivial spatial and temporal scaling laws which are robust, i.e. independent of the details of particular systems [1]. These studies at present involve a multitude of complex systems which are formed from a large number of small units which communicate through short-range interactions, conditioned by both deterministic rules and random influences. As examples of particular interest, we can cite the study of the geometry of railway networks [2], studies of the dynamics of traffic jams [3], and the modeling of urban growth patterns [4], amongst many others, all them exhibiting different but related types of scaling laws or power-law behavior. On the other hand, in the last two decades, the idea of disorder associated with the spatio-temporal configurations of cities has been informed by the concept of complexity [5]. In the next section, we report some results of our study of the geometry of squatter settlements. Our analysis focuses and quantifies in particular, the fragmentation occurring in both the voids and built space comprising these urban structures. The data used in the first part of this present work comes from an ensemble of nine squatter settlements or *"favelas"* distributed in different areas of the metropolitan region of Recife, on the northeastern coast of Brazil[1]. In the last section of the paper, we present a brief comparison between these data and those from the configurational analysis of two squatter settlements situated in the Mathare Valley in Nairobi, in the central region of Kenya[2].

2. RESULTS AND DISCUSSION

To give a general view of the kind of urban structure with which we are dealing, we show in Figure 1 images of the nine settlements examined in this first part of this study. Each small cell of irregular shape in this figure represents a single habitation or dwelling which is actually the precise space defined by the roofs of these buildings. From these images, the settlements exhibit a seemingly disordered or spontaneous but fragmented structure,

[1] Source of Data: URB–Recife (Recife Urbanization Enterprise), Brazil
[2] Source of Data: Housing Research and Development Unit - University of Nairobi – MATHARE VALLEY: A Case Study of Uncontrolled Settlement – 1971 – Nairobi., Kenya.

characterized by a diversity of size of islands of habitation with irregularity of distribution and shape. These islands define a variable number s of dwellings where $s = 1$ implies an isolated habitation, $s = 2$ a pair of contiguous dwellings, and so on. A careful examination of Figure 1 reveals that s varies in the interval from 1 to 19. We can observe that there are a large number of small islands in each settlement, and as well a small number of large ones, a typical feature of scaling in complex systems. Another important characteristic of the settlements studied is that all them are embedded in urban networks and most of them adhere to very rigid boundary conditions. The development of these settlements has occurred not through a spontaneous process of spreading, but more as a kind of packing process. Consequently, as the spatial limit of the settlement is previously defined, the diversity of size of these islands seems to be some sort of response of a system optimizing its occupation of space.

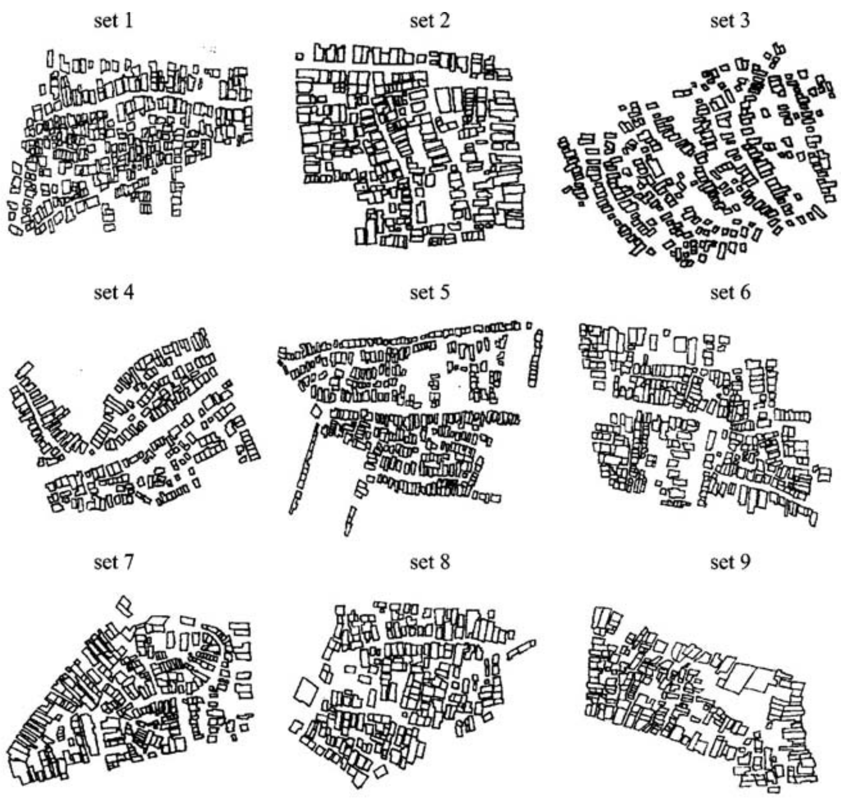

Figure 1: The Nine Spontaneously Developing Settlements in the Recife Region

2.1 Geometry and Functions

To make clear to the general reader who is not familiar with the kind of mathematics used here, in the following paragraphs, we will present a geometrical example of the statistical method suggested in this work. As example, it provides a comparison between two hypothetical geometrical structures representing, respectively, regular and irregular settlements. The geometrical form of these two settlements is shown in Figure 2.

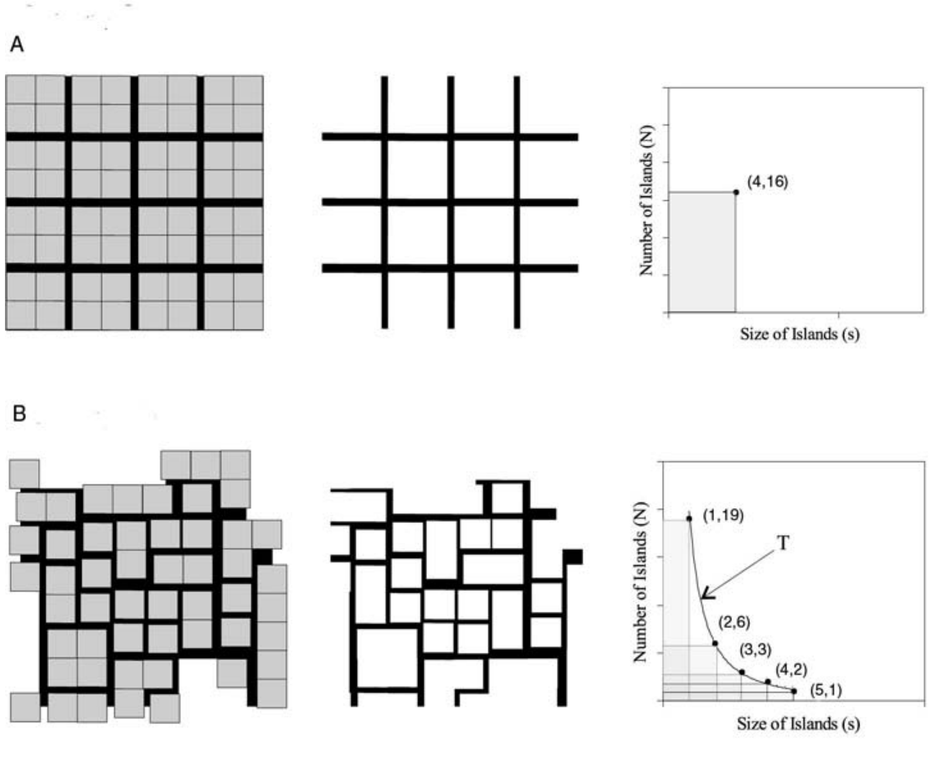

Figure 2: Two Hypothetical Geometrical Structures:
2A: A Regular Settlement, and 2B: An Irregular Settlement

Each one is represented by the map of islands, the grid of void spaces, and the frequency graph which relates the number of islands *N* to their sizes *s*.

Figure 2A shows a regular grid, composed by a group of identical islands, distributed within a square area. It is really very simple to obtain global parameters from the local units of this regular structure. For example, as we observe, each island contains four cells (dwellings/habitations) and all the islands are identical to one another. Each cell of habitation has area a_h of 10m^2. We can easily conclude that the area of each island A_i is 40m^2 (where $A_i = 4a_h$, and the total built area A is 640m^2 ($A_i = 16.4.a_h$). Dividing the total area by the area of each dwelling A/a_h, we get the number of dwellings (64 units). If the average population per dwelling is 3, one can easily calculate the total population of the settlement as $P = 3.4.16 = 192$. Another way to calculate general parameters of these forms is through the graphs which are shown in Figure 2A. The graph in Figure 2A plots the number of islands N_i according their sizes s, with the shaded area representing the total number of dwellings $N_h = N_i s = 16.4 = 64$. This is a simple procedure, for the linear function generates a regular shape. Thus, it is not hard to understand why planners have preferred regular shapes when designing urban structures. This kind of geometrical scheme, despite the great range of social constraints that it forces upon us, has been very common through the history of urban planning, not simply due to aesthetic considerations, but mainly because such shapes can be easily reproduced, controlled, analyzed, measured, and planned.

When analyzing Figure 2B which refers to an irregular settlement, one can note that the overall structure is not forced into a regular square, and thus the local units (islands) are different in size to each other. It might therefore seem impossible to obtain global parameters from just knowing information about a simple local unit, which was the case in the first example. But it is in fact possible to obtain global properties by describing each one of the units in the system. For example, to calculate the total built area A for the irregular settlement in Figure 2B, we have to multiply the size s of each class of fragment (island) by the total number of fragments in that class, $n(s)$, and by the average area a_h of each dwelling or habitation and sum over all classes of size.

As we have argued, there is a difference between disordered systems and complex ones. When they are complex, they usually follow rules that can be described by non-linear functions, and we suggest that such irregular structures (as those presented in Figure 2b, which represent the squatter settlements of figure 1) have the statistical properties of complex systems. For example, in the graph of Figure 2B which shows number of islands N_i according to their sizes *s*, the sum of area of the shaded rectangles defined by the coordinates x, y on the graph (groups of islands according to their sizes) represents the total number of dwellings N_h. But the peculiarity of this new graph is that the coordinates x, y follow a curve that defines a power law because there is a logical distribution between the large number of small islands and the small number of large ones. So, to calculate the total shaded area which is equivalent to the total number of dwellings of the respective structure, we simply need to know the curve of this function, and this amounts to knowing its exponent *T*. In this work, we have found that all the settlements studied present similar distribution of sizes, i.e., they follow non-linear functions with a similar value of exponent *T*, whether these functions pertain to built forms as islands or to void spaces.

2.2 Statistical Analysis of the Convex Spaces

It is important to verify that the fragmentation observed on the maps of squatter settlements is not only characterized by different sizes of island, but also by the irregularity of the shape of those islands and their informal distribution over the space. Such irregularity is clearly observed when the void space is analyzed. The linear distribution of regular islands usually generate grids that can be easily modeled geometrically, while the irregular ones form a kind of deformed structure that seems to be fairly disordered (see Figure 2). But as will be shown in the first part of this analysis, such structure of void spaces can be described by non-linear geometrical patterns which directly relate to the informality of the respective built structures.

So to quantify the geometry of the settlements, we initially investigated their void spaces, formed by alleys, courts and other interstitial regions surrounding the islands of habitation. To do this, we performed an informal covering of this void space by a set of contiguous convex polygonal spaces as first suggested by Hillier and Hanson [6]. According to these authors, a convex map for a structure defined on the plane is "the least set of fattest spaces" that covers the system. To adapt the convex space concept to the fragmentation approach applied here, we considered as void areas all the spaces which were not covered by roofs in terms of their plan. Thus, these convex spaces, in our approach, are not only common area where those who use the space can gather, but also the areas behind fences such as gardens, courts, etc. Figure 3 shows the convex maps of the nine settlements studied.

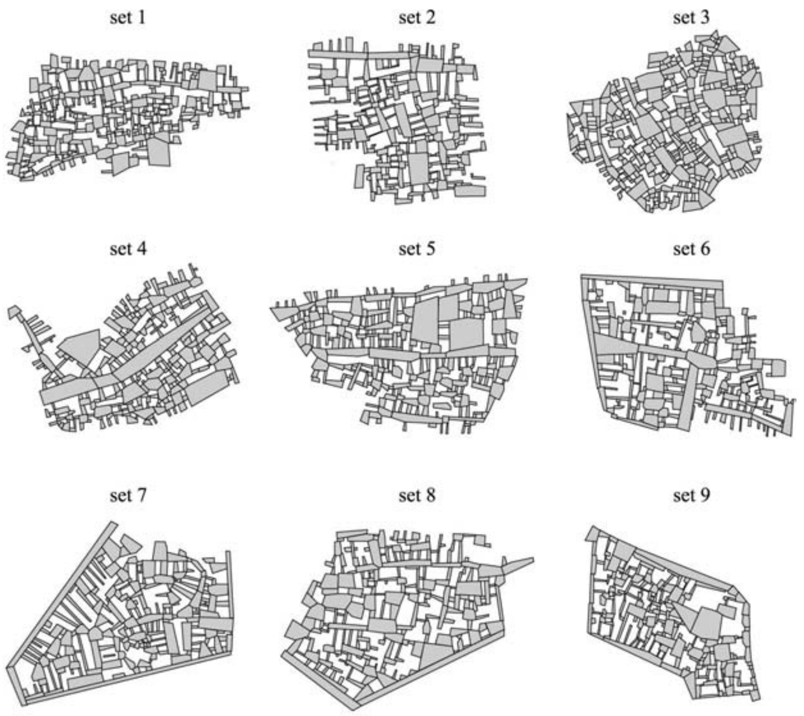

Figure 3: The Covering of the Void Spaces of the Nine Settlements by Convex (Polygonal) Spaces which have a Distribution of Areas f(a)

Each covering here is a family of fragments with a wide diversity of sizes, which are defined in this analysis by the variable area a_c. It is observed that such fragmentation, in any particular settlement, is characterized by a statistical distribution $f(a_c)$ which gives the frequency[3] of convex spaces within each interval[4] of area a_c. In spite of the non-uniqueness of these coverings, we have observed that the distribution $f(a_c)$ is robust. Moreover $f(a_c)$ has the scaling property and satisfies

$$f(a_c) \sim a_c^{-\beta}, \quad \beta = 1.6 \pm 0.2 , \qquad (1)$$

with the exponent β independent of the settlement within the indicated statistical limits. This scaling property suggest that the diversity of sizes of the resulting convex spaces follows a robust distribution of areas and indicates a large number of small convex spaces, and a small number of large ones.

Scaling or power-law behavior with nontrivial exponents is a characteristic of complex systems and means that the systems exhibiting such distributions have no characteristic scale or size [7]. The explanation of these scaling laws with nontrivial exponents is one of the great challenges in the study of complex systems. Figure 4 shows a log-log plot of the measured distribution $< f(a_c) >$ of areas of convex spaces after dividing these spaces by classes of areas or bins, and averaging over the entire ensemble of settlements given in Figure 3. On the horizontal axis of Figure 4, the values a_c for areas were normalized with respect to the area of the largest convex space for each settlement. To help the reader in developing insights into the distribution of areas of convex spaces, we exhibit in the inset of Figure 4 a non-averaged variable $f(a_c)$ – in this case for the settlement 1 shown in Figure 3. As can be seen from

[3] Number of convex spaces divided by the total number of convex spaces to cover the entire void space.

[4] Since the variable a_c is in principle continuous, to arrive in equation 1 some sort of discretization is needed as usual.

these figures, both $< f(a_c) >$ and $f(a_c)$ are essentially equal and have the same scaling exponent β, within the statistical fluctuations.

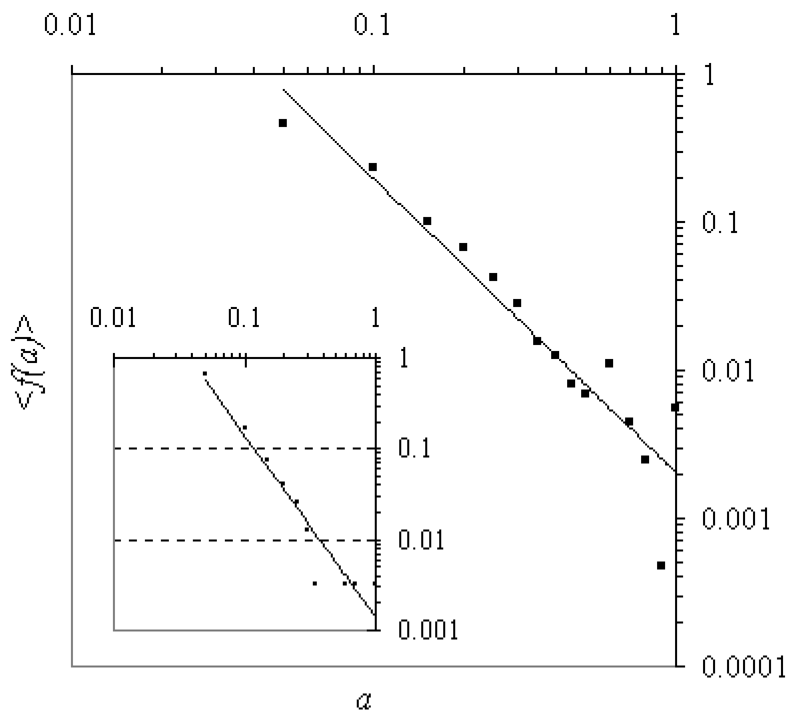

Figure 4: Log-log Plot of the Measured, Averaged, and Normalized Distribution of Areas of the Convex Spaces <f(a)> for the Ensemble of Nine Settlements Shown in Figure 1.

$<f(a)> \sim a^{-\beta}$ with $\beta = 1.6 \pm 0.2$ is represented by the continuous line in the main plot. The inset gives *f(a)* for settlement 2 shown in Figure 3

2.3 Statistical Analysis for the Islands of Habitation

Inspired by statistical studies of the fragmentation dynamics in physical systems and in order to obtain a more complete description of the geometrical properties of the urban structures studied here, we have measured another distribution function, namely $f(s)$, the frequency of islands with *s* contiguous habitations or dwellings for each settlement. The discrete variable *s* gives a measure of the size or area of an island. This distribution is consequently defined for a region that is complementary to the void space defined in the previous paragraph.

We have found that the distribution $f(s)$ also obeys a scaling relation; it is given by $f(s) \sim s^{-\tau}$, with $\tau = 1.6 \pm 0.2$, independent of the settlement. Thus, the value of the exponent τ is equal to the value of β appearing in equation (1) within the statistical fluctuations. This means that, as an open structure, all the settlements analyzed present a similar distribution pattern for their built elements, and this suggests that the diversity of built spaces generate an equally diverse structure of void spaces. It is interesting to observe that such similarity of distributions between the convex spaces and the islands (defined by the similar values of the exponents) suggests a strong relation between the built and void structures. Such correlation can also be supported by the similar degree of break-up verified on the settlements that can be measured by the relation between the number of convex spaces and the number of buildings [6] in each settlement. We note that practically all the settlements manifest a high and similar level of breakup, i.e., the number of cells (houses) is close to the number of convex spaces. In a regular grid, the number of convex spaces is considerably reduced when compared to the number of cells of the system.

Scaling distributions of fragments with an exponent similar to this are commonly found in statistical models and experiments of fragmentation dynamics in various physical, chemical and ecological systems [8]. As an illustration we can cite the distribution of sizes of fragments for different types of collapse of two-dimensional brittle solids as cement plates (which also presents a hyperbolic dependence with the size or area of the fragments and has an exponent τ varying in the interval 1.5 to 1.8 [9]). The origin of this numerical value however is not yet clear. Another important illustration is the distribution of areas of the urban settlements around Berlin and London recently discussed by Makse et al. which generate exponents near to 1.9 [4]. However, Makse et al. deal with scaling distributions at a very large scale when compared to the "microscopic" scale of lengths within the settlements examined in the present work. We will comment on these peculiarities below.

The distribution of islands of dwellings averaged over the ensemble of settlements shown in Figure 1 and normalized to the total number N of islands in each settlement $< f(s) >$ is depicted in Figure 5.

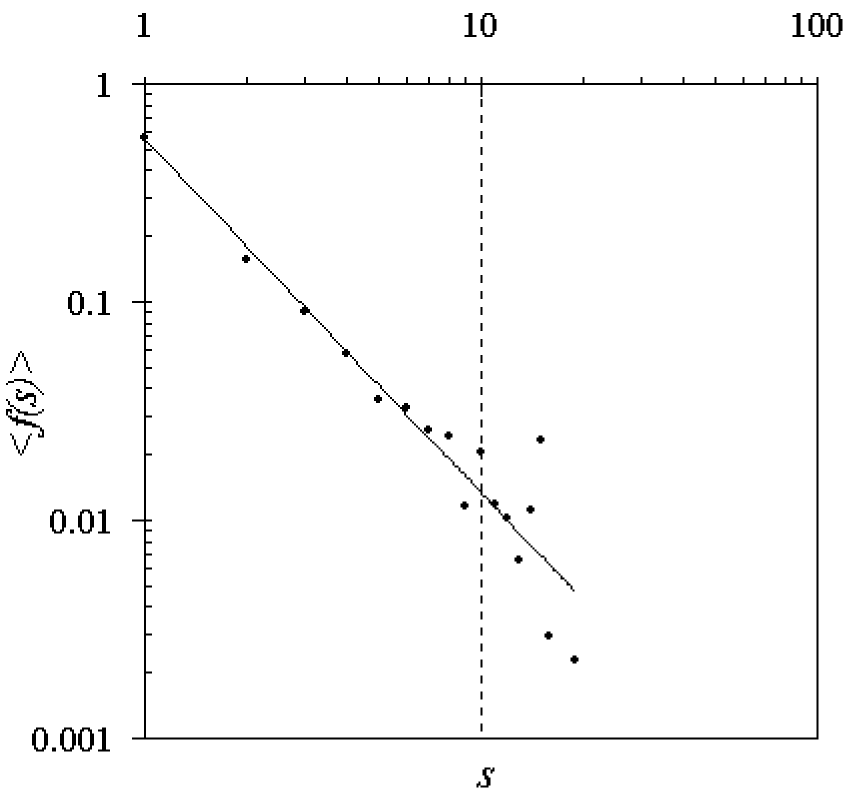

Figure 5: Log-log Plot of the Measured, Averaged, and Normalized Distribution of Sizes of Islands with s Dwellings <f(s)> for the Ensembles of Figure 1.

The continuous line represents the best adjust given by the power-law $<f(s)> = <f(1)> . s^{-\tau}$, with $<f(1)> = 0.49$, and $\tau = 1.6$

The continuous line in Figure 5 represents the best fit given by

$$<f(s)> = <f(1)> s^{-\tau} \ , <f(1)> = 0.49 \pm 0.08 \ , \tau = 1.6 \pm 0.2 \ ; \ 1 < s < s_{max} \ , \qquad (2)$$

where $< f(1) >$ is the average number of islands with a single habitation, divided by the total number N of islands, that is $< n(1)/N >$. The largest size of islands $s_{\max}$ present in the settlement studied is 19. The largest size $s_{\max}$ can be a measure of the diversity of size of the islands D (i.e. D can represent the number of different classes of size for the islands). In particular, $s_{\max}$ converges to D when the level of complexity increases. From now on, we will identify $s_{\max}$ with D in all equations.

The non-normalized distribution $< n(s) >$ satisfies in general the constraint $< n(s = D) >= 1$, that is, there is on average, a single island with the maximum size $s = D$ in the scaling. This result leads to $1 =< n(1) > D^{-\tau}$ or

$$< n(1) >= D^{\tau}, \qquad (3)$$

i.e., there is a simple relationship connecting the number of smallest islands (s=1), n(1), with the observed diversity of size of islands D. When the first quantity increases (or decreases), the second also increases (or decreases), and the degree of coupling between $n(1)$ and D is controlled by the nontrivial exponent τ of the distribution. We can say, as a consequence, that $n(1)$ and D are two sides of a same coin. To increase the diversity of size of islands in a spontaneous settlement, we need to increase the total number of islands, and this last quantity is mainly controlled by the number of isolated habitations $n(1)$. The dependence of $n(1)$ on D given in equation (3) allows equation (2) to be written as

$$<n(s)> = <n(1)> s^{-\tau} = (s/D)^{\tau} \;, \tau = 1.6 \pm 0.2 \;. \qquad (4)$$

This means that if one knows the diversity (generally equivalent to the size of the largest island) of a squatter settlement, it is possible to estimate the number of islands with any size *s*, and vice versa.

The diversity of sizes as represented by the variable *D* here (see the definition in section 2.3) has been identified in studies of fragmentation dynamics with the overall complexity of the process or structure [7]. It is of interest to know how this diversity of size increases with the total number of islands N_i. To examine this relationship, for each settlement represented in Figure 1, we define a family of arbitrary balls with irregular contours in a such way that the family of balls contains successively $N_i = 1, 5, 10, 15, 20, ..., 100$ islands. The dependence of *D* with N_i is shown in Figure 6 in a log-log plot for all settlements; the diamond signs in this figure refer to the ensemble averages. The continuous line in Figure 6 represents the best fit $D = 1.1\, N_i^{0.49}$ for $1 < N_i < 80$. This relation says that the diversity of the size of islands in a spontaneous settlement increase as the square root of the total number of islands. This last scaling law is in agreement with several recent experiments [9] and computer simulations [8] that have studied the evolution of the diversity in a fragmented system when its global size (e.g. the number of fragments) increases.

3. OBTAINING PARAMETERS OF INTEREST

We have suggested that irregular settlements within the spatial constraints and statistical fluctuations described here present the same pattern of fragmentation. These patterns occur in both the built and void spaces, and are defined by their size distribution. We also argue that it is possible to estimate global properties from general or local information pertaining to these settlements. Consequently, we can estimate the global parameters as functions of total area, number of houses and population. Obviously the more developed the settlement (the higher the level of diversity, greater the number of habitations, etc.), the more precise will be the estimate.

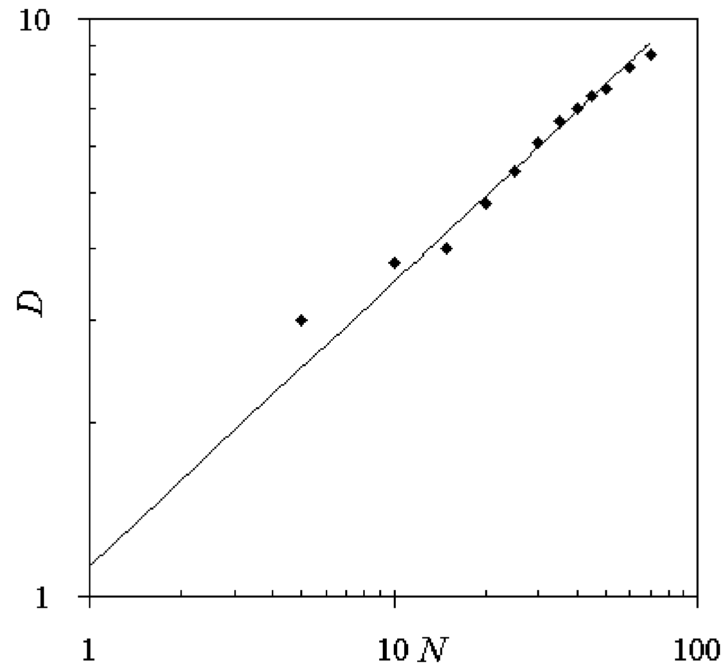

Figure 6: The Diversity of Sizes of Islands of Dwellings, D, as a Function of the Number of Islands N_i.

The continuous line represents the best fit $D = 1.1\ Ni^{0.49}$

The reader can observe that the distribution in equation (4) is extremely economical: it depends only on a single parameter, namely *D* – the diversity of size of the settlement, or on $< n(1) >$ – the number of islands with just one habitation, since all settlements have a common exponent τ. With equation (4) valid, on the average, for all the settlements studied, we can calculate several statistical parameters of interest after some simple integrations as we will show below.

The total area *A* occupied by the islands in a certain settlement is given by

$$A = \int_{s=1}^{s=D} n(s)\ s\ a_h\ ds = \int_{s=1}^{s=D} \left[\frac{s}{D}\right]^{-\tau} s\ a_h\ ds = a_h \frac{[D^2 - D^\tau]}{2-\tau}$$
$$= a_h \frac{\langle n(1)\rangle^{2/\tau} - \langle n(1)\rangle}{2-\tau} \quad , \qquad (5)$$

where a_h is the average area per dwelling or habitation. Since a_h can be easily guessed, equation (5) shows that the total built area of a settlement is controlled by a single parameter, the diversity of size of island D *or* $n(1)$. Moreover, the total number of habitations in a settlement $N_h = A / a_h$ takes the simple form

$$N_h = \frac{[D^2 - D^\tau]}{2-\tau}$$
$$= \frac{\langle n(1)\rangle^{2/\tau} - \langle n(1)\rangle}{2-\tau} \quad . \qquad (6)$$

If the average population per dwelling p is estimated by any mean, the total population P can be expressed in terms of the diversity D *or* $n(1)$ as

$$P = p\ N_h = p \frac{[D^2 - D^\tau]}{2-\tau}$$
$$= p.N_h = p. \frac{\langle n(1)\rangle^{2/\tau} - \langle n(1)\rangle}{2-\tau} \quad . \qquad (7)$$

Thus, the total population P is known if D *or* $n(1)$ is given and vice-versa. Another quantity of interest is the total number of islands in a given settlement N_i which is expressed as

$$N_i = \int_{s=1}^{s=D} n(s)\ ds = \frac{[D^\tau - D]}{\tau - 1}$$
$$= \frac{\langle n(1)\rangle - \langle n(1)\rangle^{1/\tau}}{\tau - 1} \quad . \qquad (8)$$

An interesting result concerning the density of islands of size $s=1$ comes from equation (8): in leading order, this equation reads $N_i =< n(1) > /(\tau-1)$ (since $1/\tau = 0.625$, and the term $< n(1) >^{1/\tau}$ can be neglected in respect to the linear term $< n(1) >$, that is

$$\frac{< n(1) >}{N_i} = \tau - 1 = 0.6 \quad . \tag{9}$$

The density $n(1)/N_i$ varies in the nine settlements studied from 0.46 to 0.66 with an average of 0.57 in agreement with equation (9). Furthermore, if we substitute (9) in (6), we obtain the total number of dwellings N_h as a function of the total number of islands N_i

$$N_h = \frac{(N_i(\tau-1))^{2/\tau} - (N_i(\tau-1))}{2-\tau} . \tag{10}$$

From equation (10), with $\tau = 1.6$, we get $N_h = 305$ for settlement 6 ($N = 110$), and $N_h = 245$ for settlement 8 ($N = 94$). The actual values of N_h for settlements 6 and 8 are respectively 323 and 215, i.e. equation (10) gives N_h values with a typical uncertainty of 6%-10%.

In summary, if the distribution of islands or fragments for a particular fragmented system is known, all the statistical quantities of interest can be obtained. Hyperbolic distributions of the type given in equations (2) or (4) are specially robust and universal when they appear in a certain class of problems [1]. We conjecture that all spontaneous settlements that present the same kind of spatial constraints, are described by a single hyperbolic distribution of the type given in equation (4) with the scaling exponent τ assuming the robust value $\tau = 1.6 \pm 0.2$.

4. BOUNDARIES, PACKING & DIVERSITY

It is important to remember that the evolution of the settlements analyzed in this work is not related to spreading through diffusion, as the boundaries around the settlements limit the extent of their growth. Actually, growth here is more like a process of packing within a limited space for development. So when the number of islands increases, the global extent of the settlement remains the same and the density of the system begins to rise, as each new building added to the system has to adhere to constraints posed by spatial availability.

If a settlement were composed only by islands of size 1 (isolated dwellings), the resulting density would be considerably lower, due to the large number of void spaces, and we could conclude that in this case, occupancy would be not optimized. On the other hand, if a structure were composed of a small number of really large islands, the density would be really high. However, the resulting occupancy would not be appropriate due to problems of access, privacy and salubrity caused by the lack of void spaces connecting houses. This would also be a kind of 'non-optimized' development. So we can conclude that the best response to optimize the spatial layout of a decentralized system is through meeting the diversity of size of its islands as reflected in what actually happens in practice.

We can verify this relation about boundaries, packing and diversity by analyzing an actual example: squatter settlements along the Mathare Valley in Nairobi, Kenya. In this case, we have compared the structure of two settlements (A and B) of similar age but with distinctly different configurational patterns. In Figure 7, a general view of the valley and the area around it is shown. The settlements A and B are highlighted (the shaded and circled areas) in the general map and their built structure is detailed in the specific plans, in which the shaded objects represent the islands of dwellings or habitations. Settlement A is completely constrained by rigid boundaries: it is surrounded on the left and below by high-density urban structures, above by the Gitathuru river, below by the Juja road and on the right by property limits. On the other hand, settlement B is completely free from such constraining boundaries, whether natural, urban or property related.

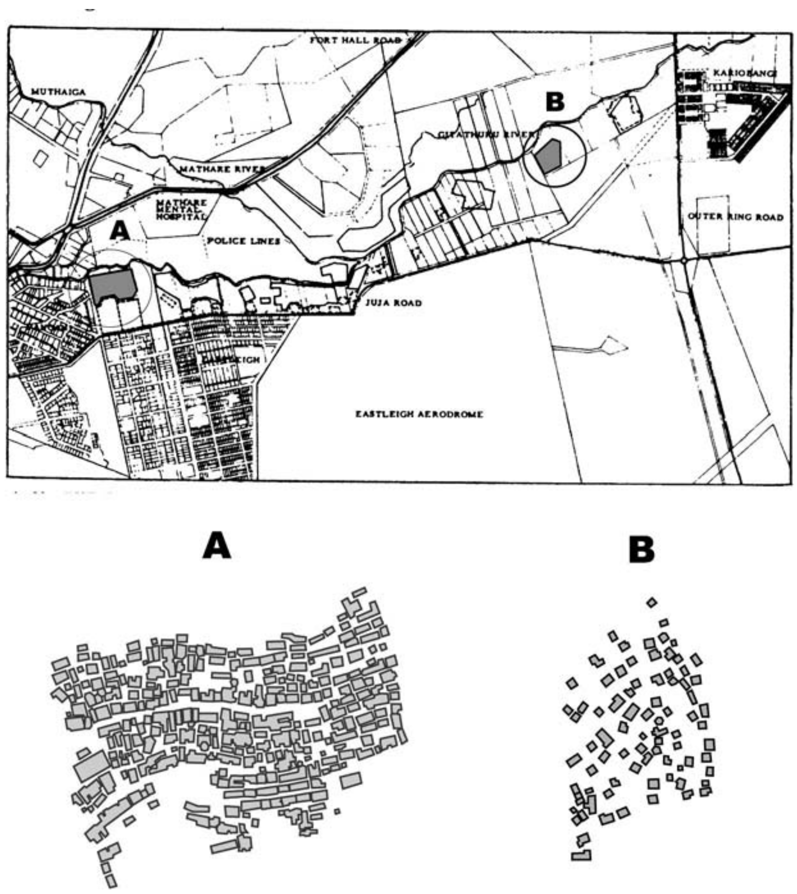

Figure 7: General View of the Mathare Valley, Nairobi in Kenya, and the Area Around It.

The settlements A and B are highlighted as the shaded and circled areas in the general map and have their built structure detailed in specific plans, in which the shaded objects represent the islands of dwellings

By analyzing the detailed maps of built structures, we clearly observe great differences in the configurational structure of these settlements. Settlement B is practically composed of islands of isolated habitation $(s = 1)$ spread in a low-density structure while settlement A is highly packed and dense and presents islands with a size from $s = 1$ to $s = 8$ (which show a high level of diversity compared with B). What is more important, the frequency of islands $f(s)$ according to the sizes follows a power law (see Figure 8) which is defined by an exponent with a value of 1.4. This is similar to the exponent found for the settlements in Brazil. These results seem to indicate a deterministic order, where boundary is the cause, packing is the effect, and diversity is the route through which the settlement patterns evolves.

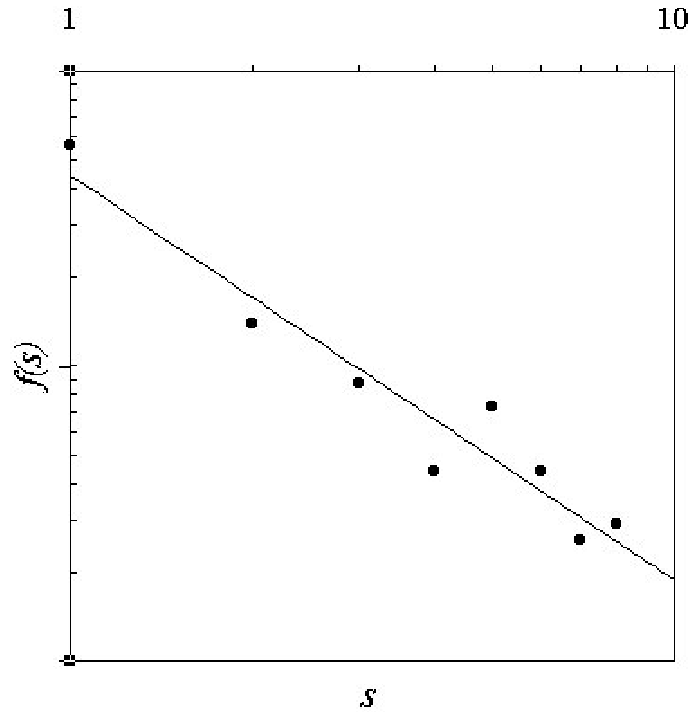

Figure 8: Log-log plot of the Measured, and Normalized Distribution of Sizes of Islands with s Dwellings <f(s)> for the Settlement A Ensemble of Figure 7.

The continuous line represents the best adjust given by the power-law $<f(s)> = <f(1)>.s^{-\tau}$, with $<f(1)> = 0.44$, and $\tau = 1.4$

5. CONCLUSIONS

The fragmentation in squatter settlements in northeastern Brazil has been investigated and compared to settlements in the central region of Kenya, and several nontrivial scaling functions describing these urban structures have been found. The statistical analysis used here has been inspired by recent studies of fragmentation dynamics mainly in physical systems. We have shown that there are robust, possibly universal distribution functions associated with the fragmented structures of these spontaneous settlements. Using these distribution functions, many statistical quantities of interest can be obtained. In particular, we have stressed the importance of the diversity of size in obtaining these quantities of interest. We conjecture that the hyperbolic distribution functions given in equations (1) and (4) are robust and universal; that is, they control the statistical aspects discussed here in all squatter settlements, irrespective of different cultural particularities.

An important characteristic of the settlements studied is that all them are embedded in urban networks and most of them adhere to very rigid boundary conditions. The development of these settlements occurs not as a spreading diffusion-like process, but as a kind of packing process. Consequently as the spatial limits of these settlement are previously defined, the diversity of size is a response of the system to optimize its spatial occupancy. If there are no rigid boundaries, the response is trivial: houses are distributed in a disordered way, isolated into islands of size $s = 1$. Although conclusions are clear, further work is required and detailed analysis of other spontaneous settlement growth in different regions is needed to test the robustness of the present conjectures.

6. REFERENCES

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