Competition and Innovation: An inverted U relationship?

by

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COMPETITION AND INNOVATION: AN INVERTED U RELATIONSHIP*

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Abstract

This paper investigates the relationship between product market competition and innovation. It uses the radical policy reforms in the UK as instruments for changes in product market competition, and finds a robust inverted-U relationship between competition and patenting. It then develops an endogenous growth model with step-by-step innovation that can deliver this inverted-U pattern. In this model, competition has an ambiguous effect on innovation. On the one hand, it discourages laggard firms from innovating, as it reduces their rents from catching up with the leaders in the same industry. On the other hand, it encourages neck-and-neck firms to innovate in order to escape competition with their rival. The inverted-U pattern results from the interplay between these two effects, together with the effect of competition on the equilibrium industry structure. The model generates two additional predictions: on the relationship between competition and the average technological distance between leaders and followers across industries; and on the relationship between the distance of an industry to its technological frontier and the steepness of the inverted-U. Both predictions are supported by the data.

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1. Introduction

Economists have long been interested in the relationship between product market competition and innovation. Both the theoretical IO and the more recent endogenous growth literatures tackle the issue. IO theory often predicts that innovation should decline with competition, as more competition reduces the monopoly rents that reward successful innovators.\(^1\) However, empirical work such as Geroski (1995), Nickell (1996) and Blundell, Griffith and Van Reenen (1999) has pointed to a positive correlation between product market competition and innovative output. Understanding the relationship between competition and innovation is central for policy makers in the US and Europe, where a common belief that more competition is good for innovation has driven a series of reforms to toughen up competition policy. But what is the evidence on the relationship between competition and innovation?

This paper examines the relationship using firm panel data and finds clear non-linearities in the form of an inverted U-shape. This is confirmed using both flexible non-parametric estimators like kernels and splines, and also using more parsimonious parametric estimators including time and industry controls. A major issue in this line of research is identifying the direction of cause and effect between competition and innovation. To tackle this we use UK data and exploit the major policy reforms undertaken over the 1970s and 1980s, which dramatically changed the nature and extent of competition across industries and overtime. The radical policies of the Thatcher administration, the introduction of the European Single Market Program (SMP) and the reforms imposed by the Monopolies and Mergers Commission together provide a number of natural experiments across time and industries to identify the causal impact of competition on innovation. Recent advances in non-parametric instrumental

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\(^1\)See, inter alia, Dasgupta-Stiglitz (1980) and also the first generation of Schumpeterian growth models Aghion-Howitt (1992), Caballero-Jaffe (1993). However, the replacement effect in Arrow (1962) and the efficiency effects in Gilbert and Newbury (1982) and Reingenum (1983) go in the opposite direction.
variable estimation enable us to use these natural experiments to identify a non-linear functional form. Controlling for endogeneity by using the policy instruments shifts the relationship towards the competitive direction, as we would expect, but the strong inverted-U shape is maintained, with its Schumpeterian regime at high levels of competition. This inverted U relationship proves to be robust to a number of controls and experiments.

The possibility of an inverted-U relationship between competition and innovation was already hinted at by Scherer (1967), who showed a positive relationship between patenting activity and firm size in the cross section, but interestingly, a diminishing impact at larger sizes when allowing for nonlinearities.

As it turns out, however, to our knowledge most existing models of product market competition do not predict an inverted-U relationship between competition and innovation or between competition and productivity growth. Basic endogenous growth models (see Aghion and Howitt (1992)) and their precursors in the IO or trade literatures predict a negative effect of competition on innovation and productivity growth (this we refer to as the "Schumpeterian effect" of product market competition), whereas agency models of competition as an incentive scheme (e.g. Hart (1983), Schmidt (1997) and Aghion-Dewatripont-Rey (1999)) predict a positive relationship, as does the recent paper by Boldrin-Levine (2003) on perfectly competitive innovations.

In order to interpret our empirical findings we develop a simple extension of Aghion-Harris-Vickers (1997) in which both current technological leaders and their followers in any industry can innovate, and where innovations by leaders and followers all occur step-by-step. In this model, innovation incentives depend not so much upon post-innovation rents as in previous endogenous growth models where all innovations are made by outsiders, but upon the difference between post-innovation and

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2 See our discussion in Section 3 below.
3 See also Aghion-Harris-Howitt-Vickers (2001).
pre-innovation rents of incumbent firms. In this case, more competition may foster innovation and growth, because it may reduce a firm’s pre-innovation rents by more than it reduces its post-innovation rents. In other words, competition may increase the incremental profits from innovating, and thereby encourage R&D investments aimed at “escaping competition”. This should be particularly true in sectors where incumbent firms are operating at similar technological levels; in these “neck-and-neck” sectors, pre-innovation rents should be especially reduced by product market competition. On the other hand, in sectors where innovations are made by laggard firms with already very low initial profits, product market competition will mainly affect post innovation rents and therefore the Schumpeterian effect of competition should dominate.

The essence of the inverted-U relationship between competition and innovation is that the fraction of sectors with neck-and-neck competitors is itself endogenous, and depends upon equilibrium innovation intensities in the different types of sectors. More specifically, when competition is low a larger equilibrium fraction of sectors involve neck-and-neck competing incumbents, so that overall the escape competition effect is more likely to dominate the Schumpeterian effect. On the other hand, when competition is high, the Schumpeterian effect is more likely to dominate because a larger fraction of sectors in equilibrium have innovation being performed by laggard firms with low initial profits. The model is consistent with the raw data both in terms of the shape of the relationship and also the distribution of firms across the regions. Moreover it provides additional testable predictions on the relationship between competition and the composition of industries and more specifically between competition and the average degree of "neck-and-neckness" in the economy.

The rest of the paper is structured as follows. Section 2 displays the empirical evidence on the existence of an inverted-U relationship between competition and innovation. Section 3 argues that existing models of competition and innovation
cannot account for the inverted-U pattern; it develops a theoretical rationale for this relationship and derives some additional empirical predictions. Section 4 tests the additional predictions. Finally Section 5 concludes.

2. The impact of competition on innovation

The early empirical literature inspired by Schumpeter (1943) estimated linear cross-sectional relationships and typically found a negative relationship between competition and innovation, confirming the theoretical prejudices of the era. Scherer (1967) developed this research by allowing for additional nonlinearities, and in a cross-sectional analysis on Fortune 500 firms discovered a significant inverted-U shape, with higher competition initially increasing then decreasing the rate of innovation. But research since then has returned to estimating linear specifications, spearheaded by Nickell (1996) and Blundell, Griffith and Van Reenen (1999), who estimate specifications on panel data finding a positive effect of competition on innovation. Is this linear restriction valid? Is there really no role for the Schumpeterian effect emphasized in the theoretical literature? We take a fully flexible non-linear approach and estimate the relationship between product market competition and innovation on a panel of UK firms, exploiting the radical policy reforms of the Thatcher era to identify the impact of competition.

2.1. Measuring innovation

There is a large literature on measuring innovation intensity, with the most commonly used measures at the firm level being research and development and patenting activity. In the UK R&D expenditure was not a mandatory reporting item prior to 1990 so it is not available for the majority of firms. Our main measure of innovation is therefore a citation weighted patent counts. The patents are those taken out by UK firms in the US patent office. We also estimate the model using firms’ reported R&D expenditure (on a much smaller sample of firms) as a robustness checks. The
US patenting office data was chosen because it provides comprehensive and detailed electronic patenting data from 1971 onwards, and it is where all significant innovations are patented internationally. We also match in information on all citations to and from these patents and use this to weigh patents by citation counts (see Hall, Jaffe and Trajtenberg et al. (2001) and Bloom and Van Reenen (2002)).

2.2. Measuring Competition

Our main indicator of product market competition is the Lerner Index, or price cost margin, following Nickell (1996). This measure has several advantages over indicators such as market share or the Herfindahl concentration index, which rely on precise definitions of geographic and product markets. This is particularly true in our application as many UK firms operate in international markets so that traditional market concentration measures will be extremely misleading. We use accounting data to construct a firm Lerner Index measure of competition based on rents over value-added (see further details in Appendix 1).

2.3. Using Policy Instruments

The major obstacle to empirical research in this area is that competition and innovation are mutually endogenous. Without addressing this any results we find are likely to be biased towards finding a more-negative relationship between competition and innovation if higher levels of innovation act, for example, to reduce competition. We address this problem in a sequence of steps. First, we allow for industry effects, thus removing bias that results from correlation between permanent levels of innovative activity and product market competition. Second, we use a set of policy instruments that provide exogenous variation in the degree of industry-wide competition. Since we are including industry and time effects this approach identifies the competition effect through the differential timing of the introduction of changes across industries in the policies. The three sets of policy instrument used are the Thatcher era privatiza-
tions, the EU Single Market Programme, and the Monopoly and Merger Commission investigations that resulted in structural or behavior remedies being imposed on the industry. These have changed competition over a range of industries across the last two decades, enabling identification across time and industries.\footnote{See Aghion, Bloom, Blundell, Howitt and Griffith (2002), henceforth ABBGH (2002), and Griffith (2001) for further details on the instruments.}

2.4. A non-linear relationship

We use a flexible non-parametric kernel density estimator to investigate the basic shape of the relationship between competition and innovation. In figure 1 this kernel is plotted for innovation (proxied by citation weighted patents) against competition (proxied by the Lerner index) for the UK from 1971-1994. From this raw data graph we can see a clear non-linearity in the relationship, with this appearing to take the form of an inverted U-shape. There is an initial region where higher competition raises average innovation, followed by a relatively flat region where innovation is unresponsive to competition, with finally a region where yet higher competition reduces innovation. Firms are empirically distributed across all three regions so that any linear relationship will provide a poor approximation to the aggregate relationship between competition and innovation.

[Figure 1 here]

2.4.1. A Method of Moments Estimator

While a Kernel estimator is useful for estimating fully flexible relationships it is difficult to allow for the inclusion of any other conditioning variables, like time or industry effects. To do this we take a more structural approach. Denoting $n$ as the hazard rate and $c$ as the measure of competition, we start by defining the competition innovation relationship as

$$n = e^{g(c)} \tag{2.1}$$
Suppose the patent process has a Poisson distribution with hazard rate (2.1). The resulting count of patents in any time interval has the probability distribution

$$Pr[p = k|c] = e^{g(c)k} e^{-e^{g(c)}} / k!$$

(2.2)

and the expected number of patents satisfies

$$E[p|c] = e^{g(c)}$$

(2.3)

Parametric models that study count data processes typically base their specification on this Poisson model with a parametric (linear) form for $g(c)$, but they relax the strong assumptions on higher moments implicit in (2.2).\(^5\) We follow this approach in our empirical analysis, basing our estimator on the first moment (2.3). Because we are particularly interested in allowing the data to determine the shape of the relationship between innovation and product market competition, we adopt a flexible specification for $g(c)$.

In our data firms $i = 1, ..., N_t$ are grouped into $J$ mutually exclusive industries with $j = 1, ..., J$. We observe firms for $t = 1, ..., T_i$ periods. Our principle competition measure, the industry average Lerner index, (B.1) is measured at the industry level while patents are measured for each firm. Following from the specification of the conditional mean (2.3) we write

$$E[p_{it}|c_{jt}] = e^{g(c_{jt})},$$

(2.4)

where $g(c)$ is nonparametric. This directly identifies the innovation hazard (2.1).

Note also that (2.4) is fully nonparametric but will be extended into a semiparametric specification as we introduce more conditioning variables into the mean specification.

It is very likely that firms in different industries will have observed levels of patenting activity that have no direct causal relationship with product market competition but reflect other institutional features of the industry. Consequently industry fixed

\(^5\)See Hausman, Hall and Griliches (1984), for example.
effects are essential to remove any spurious correlation or ‘endogeneity’ of this type. Time effects are also included to remove common macroeconomic shocks. Conditional on industry and time, average patent behavior is related to industry competition according to

\[ E[p_{it} | c_{jt}, x_{jt}] = e^{g(c_{jt}) + x_{it}' \beta}, \]  
(2.5)

where \( x_{it} \) represent a complete set of time and industry dummy variables. We use moment condition (2.5) to define a semiparametric moment estimator and approximate \( g(c) \) with a spline following Ai and Chen (2001). The results are plotted in figures 2. This confirms the nonlinearities found in the Kernel estimator, again displaying an inverted U-shape relationship. Plotted alongside the spline in figure 2 is a more parsimonious exponential quadratic specification which includes controls for year and industry effects, with the dotted lines displaying the 95% confidence intervals. It can be seen that the exponential quadratic specification provides a very reasonable approximation to the non-parametric spline. The estimated coefficients for the exponential quadratic model are presented in the first column of results in Table 1.

[Figure 2 here]

[Table 1 here]

The underlying distribution of the data is shown by the intensity of the points on the estimated curves. These indicate that the peak of the inverted U lies near the median of the distribution (the median is 0.95) so that firms are well spread across the U-shape. We can also see that a simple linear relationship would yield a positive slope, confirming the results presented in Nickell (1996).

Before moving to the results using the policy instruments we also consider three robustness checks. The first is shown in Figure 3. We present the relationship fitted separately for each of the top four innovating industries in our sample. In each case there is an apparent inverted U shape. The second and third are reported here with
full details and figures in ABBGH (2002). The second uses a lagged measure of the Lerner index as an alternative approach to minimize the feedback from innovation to competition. This yields very similar results. The third uses firm level R&D expenditure as an alternative innovation measure, where we have a substantially smaller sample, but again the inverted-U shape distinctly appears.

[Figure 3 here]

2.4.2. Endogeneity

The inclusion of industry and time dummies may not be sufficient to remove all spurious correlation between the competition measure and the patent count. In particular, relative changes in the competition measure across industries in the UK may be indirectly caused to some extent by shocks to UK patents. Now, recall that our main measure of product market competition was constructed as an average of the firm level measure using data from firms both within our sample and outside our sample. This already alleviates the endogeneity problems that arise due to time varying, firm specific shocks or measurement errors. However, our main approach to remove such temporal correlations is to use the policy variables described in section 2.3. as excluded instruments that determine the competition structure of the industry but have no direct effect on the level of patenting.

To accomplish this we specify a reduced form model for the competition measure

\[ c_{jt} = \pi(z_{jt}) + x'_{it}\gamma + v_{jt}, \]  

(2.6)

with

\[ E[v_{jt}|z_{jt}, x_{it}] = 0 \]  

(2.7)

where \( z_{it} \) denote the policy instruments. The idea is then to use functions of the \( v_{jt} \) as controls in an extended version of the moment condition (2.5). The control function assumption can be expressed as

\[ E[e^{u_{it}}|c_{jt}, x_{it}, v_{jt}] = 1, \]  

(2.8)
so that controlling for \( v_{it} \) in the conditional moment condition is sufficient to retrieve the conditional moment assumption (see Newey, Powell and Vella (1999) for a discussion of the additive nonparametric model). In estimation we use the extended moment condition (2.4)

\[
E[p_{it}|c_{jt}, x_{it}, v_{jt}] = e^{g(c_{jt}) + x_{it}^T \beta + \rho(v_{jt})}.
\]

(2.9)

To recover the parameters of interest we can integrate over the distribution of \( v \) and recover the ‘average structural function’ (see Blundell and Powell (2003))

\[
E[p_{it}|c_{jt}, x_{it}] = \int e^{g(c_{jt}) + x_{it}^T \beta + \rho(v_{jt})} dF_v.
\]

(2.10)

This is achieved using the empirical distribution for \( v \).6

The second column in Table 1 presents the estimates for our exponential quadratic specification that control for endogeneity using our set of instruments. The coefficient estimates are similar to the first column. In the bottom part of the table we present some diagnostic statistics. They show that the instruments are significant in the reduced form, that the policy instruments in particular are significant, and that they have explanatory power.

[Figure 4 here]

Figure 4 plots the relationship between innovation and product market competition and displays a similar inverted U relationship to that found in our baseline specification, but with the peak shifted to the right. As we would expect, instrumenting for competition, which removes the feedback from innovation to (lower) competition, leads to a more positive effect of competition. So while the position of the U-shape moves under instrumental variables the inverted-U shape relationship is still preserved.

6 The nonlinear form of the regression specification (both quadratic and exponential) implies that the control function approach will differ from standard instrumental variables and GMM. In such models the control function approach is likely to be much better behaved (see Blundell and Powell (2003)).
3. Explaining the inverted-U

3.1. Main existing theories of competition and innovation

In this subsection we briefly summarize what existing theories have to say about the relationship between competition and innovation or competition and productivity growth. As it turns out, none of them can account for the inverted-U pattern described in the previous section.

3.1.1. The IO models of product differentiation and price competition

The two leading models of price competition and product differentiation in theoretical IO are the Hotelling linear model (and the circular version of that model by Salop (1977)) and the symmetric model of monopolistic competition by Dixit and Stiglitz (1977). This latter model has been the template for Romer’s (1990) model of endogenous growth with increasing product variety. Both models are described in details in Chapter 7 of Tirole (1988) and they both deliver the same prediction: more intense product market competition, modeled as a reduction in unit transport cost in Salop (1977) or as an increase in the substitutability between differentiated products in Dixit-Stiglitz (1977), reduces the rents of those firms that successfully enters the market, and therefore it discourages firms to enter the market in the first place. Entry in these models is what captures the notion of innovation. Thus these models can only account for the decreasing part of the inverted-U curve: increased product market competition discourages innovation by reducing post-entry rents. As Dasgupta-Stiglitz (1980) put it, “ex post competition drives out ex ante competition”.

3.1.2. The endogenous growth paradigm and the Schumpeterian effect of product market competition

The prediction that product market competition has an unambiguously negative effect on “entry” or innovation is shared by most existing models of endogenous growth (e.g Romer (1990), Aghion-Howitt (1992), Grossman-Helpman (1991)). In all of
these models, an increase in product market competition, or in the rate of imitation, has a negative effect on productivity growth by reducing the monopoly rents that reward new innovation. This discourages firms from engaging in R&D activities, thereby lowering the innovation rate, and also lowering the rate of long-run growth which in these models is proportional to the innovation rate. In the product variety framework of Romer (1990) this property is directly inherited from the Dixit-Stiglitz model upon which that model is built. But the same effect is also at work in the Schumpeterian (or quality-ladder) models of Aghion-Howitt (1992) and Grossman-Helpman (1991), which also share the prediction that property-rights protection is growth-enhancing. In fact, in these models the reason why competition policy is unambiguously detrimental to growth is the same as the reason why patent protection is unambiguously good for growth: patent protection raises monopoly rents from innovation whereas increased product market competition destroys these rents. Thus, if we were to take these models at face value when making policy prescriptions, patent policy and anti-trust should never be pursued at the same time.

3.1.3. Circular model with vertical differentiation

Consider again the circular model, but now suppose that some firms have higher unit costs than others. Thus, firms are not only horizontally differentiated along the circle, but they are also vertically differentiated by their costs. Then, as shown in Aghion-Schankerman (2003), more intense product market competition, modeled again as a reduction in the unit transport cost \( t \), can enhance “innovations” through several channels that counteract the negative effect pointed out above. First, by increasing the market share of low-cost firms at the expense of high-cost firms (we refer to this as the “selection effect” of product market competition), more intense competition may end up encouraging entry by low-cost firms (especially if potential low-cost entrants are far less numerous than high-cost entrants). Second, and again because it increases the market share of low-cost firms relative to high-cost firms,
more intense competition will induce high-cost firms to invest in “restructuring” in order to become low-cost firms themselves. Note that such an investment amounts to a quality-improving innovation that allows the high-cost firm to suffer less from more intense competition. Thus, introducing vertical differentiation in the Salop model might well reverse the effect of product market competition on entry, in which case the previously negative relationship between competition and innovation will turn into an upward-sloping curve, not an inverted-U.


Consider an economic environment in which firms are subject to “agency problems” due to the non-observability of both managerial effort and managerial performance (for example managers can manipulate profits over time). In standard models of moral hazard (e.g. Holmstrom (1979)), agents’ efforts are not observable but output performance is observable, so that the firm’s owner (the principal) can design a wage schedule contingent upon the agent’s performance so as to provide effort incentives to the agent (under the assumption that effort and performance are positively correlated). When profits or other measures of performance are not observable, monetary incentive schemes are no longer feasible and one has to look for alternatives. One alternative, analyzed by Holmstrom (1982), is to rely on career concerns and the market for managers. Another alternative, analyzed by Hart (1983) and Schmidt (1997), is to rely on product market competition together with managers’ fear of losing their jobs as a result of the firm going bankrupt. For example, suppose some exogenous change, say in demand conditions, occurs, which induces profit-maximizing firms to innovate in order to reduce costs. Then, to avoid losing control the resulting private benefits, non-profit maximizing managers will be forced to also reduce costs so as to preserve their market share and thereby their profit flow above the bankruptcy threshold. Thus, even though relative performance schemes may not be feasible due
to the non-observability of firms’ revenues, “the market mechanism makes actions and utilities of different managers interdependent via prices”. Competition then acts as an incentive scheme to induce even non-profit maximizing firms to reduce slack and improve their management methods. Thus *competition increases productive efficiency in firms subject to agency problems.*

Similar considerations lead to a positive relationship between competition and productivity growth in the dynamic version of this model by Aghion et al (1999). Either the economy is dominated by profit maximizing firms, in which case, as in previous endogenous growth models, increased product market competition reduces monetary rents and thereby discourages growth-enhancing innovations. Or the economy is dominated by firms with "satisficing" managers as in Hart (1983), in which case product market competition fosters growth by managers concerned with keeping their private benefits from remaining on the job. The relationship between competition and innovation is negative in the former case, positive in the latter, but never does it show the inverted-U pattern identified in the previous section.

Interestingly, Schmidt (1997) points out that effect of product market competition on managerial incentives, becomes ambiguous when managers respond to monetary incentives. In this case an increase in product market competition has two opposite effects on managerial incentives. On the one hand it increases the threat of bankruptcy for firms that do not reduce costs in response to cost cutting by competitors. On the other hand it reduces equilibrium profits and therefore the extent to which high-powered incentive schemes can be used to reward good performance by managers. But this ambiguous effect does not generate an inverted-U relationship between competition and managerial incentives.


In a provocative paper entitled "Perfectly Competitive Innovation", Boldrin and Levine (2002) argue that an increase in the rate of diffusion of a technological in-
novation, may in fact increase the stream of future consumption services generated (after costly copying) by the innovation. This should in turn increase the amount of quasi-rents that accrue to the initial innovator, and thereby encourage innovation in the first place. This story generates a positive relationship between innovation and "imitation", but again not an inverted-U.

3.2. A theoretical framework

Thus, none of the main existing models of competition and entry/innovation/productivity growth, can explain the inverted-U pattern uncovered in the previous section, and which had been already pointed out by Scherer (1967). In this section we develop a simple extension of the quality-ladder model of endogenous growth, which generates the desired inverted-U shape relationship between competition and innovation from the combination between three basic effects: (a) the Schumpeterian effect pointed out above; (b) an "escape competition effect" whereby incumbent firms innovate in order to escape competition with a neck-and-neck rival; (c) a composition effect whereby the fraction of industries dominated by neck-and-neck competing incumbents, depends itself upon the degree of product market competition through its effects on innovation incentives in the various types of sectors.

3.2.1. Consumers

Suppose that a final good is produced at any time $t$ using input services from a continuum of intermediate sectors, according to the production function:

$$
\ln y_t = \int_0^1 \ln x_j d_j,
$$

(3.1)

where each $x_j$ is an aggregate of two intermediate goods produced by duopolists in sector $j$, defined by the subutility function:

$$
x_j = v(x_{A_j}, x_{B_j})
$$
where \( v \) is homogeneous of degree one and symmetric in its two arguments. A special case is CES:

\[
x_j = \left( x_{A_j}^{\alpha_j} + x_{B_j}^{\alpha_j} \right)^{\frac{1}{1-\alpha_j}}
\]

(3.2)

where a higher \( \alpha_j \in (0, 1] \) reflects a higher degree of substitutability between the two goods in industry \( j \).

The log-preference assumption made in (3.1) implies that in equilibrium individuals spend the same amount on each basket \( x_j \). We normalize this common amount to unity by using current expenditure as the numeraire for the prices \( p_{A_j} \) and \( p_{B_j} \) at each date. Thus the representative household chooses each \( x_{A_j} \) and \( x_{B_j} \) to maximize \( v(x_{A_j}, x_{B_j}) \) subject to the budget constraint: \( p_{A_j} x_{A_j} + p_{B_j} x_{B_j} = 1 \).

### 3.2.2. Technology levels, R&D and innovations

Each firm produces using labor as the only input, according to a constant-returns production function, and takes the wage rate as given. Thus the unit costs of production \( c_A \) and \( c_B \) of the two firms in an industry are independent of the quantities produced. Now, let \( k \) denote the technology level of duopoly firm \( i \) in some industry \( j \); that is, one unit of labor currently employed by firm \( i \) generates an output flow equal to:

\[
A_i = \gamma^{k_i}, \quad i = A, B,
\]

(3.3)

where \( \gamma > 1 \) is a parameter that measures the size of a leading-edge innovation; (equivalently, it takes \( \gamma^{-k_i} \) units of labor for firm \( i \) to produce one unit of output). The state of an industry is then fully characterized by a pair of integers \((l, m)\), where \( l \) is the leader’s technology and \( m \) is the technology gap of the leader over the follower.

We define \( \pi_m \) (respectively \( \pi_{-m} \)) to be the equilibrium profit flow of a firm \( m \) steps ahead of (respectively behind) its rival.\(^7\)

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7 The above logarithmic technology along with the cost structure \( c(x) = x.\gamma^{-k} \) implies that the profit in the industry depends only on the gap \( m \) between the two firms, and not on absolute levels of technology.
For expositional simplicity, we assume that knowledge spillovers between leader and follower in any intermediate industry are such that the maximum sustainable gap is $m = 1$. That is, if a firm already one step ahead innovates, the lagging firm will automatically learn to copy the leader’s previous technology and thereby remain only one step behind. Thus, at any point in time, there will be two kinds of intermediate sectors in the economy: (i) leveled or neck-and-neck sectors where both firms are at technological par with one another, so that $m = 0$; (ii) unleveled sectors where one firm (the leader) lies one step ahead of its competitor (the laggard or follower) in the same industry, so that $m = 1$.\(^8\)

By spending the R&D cost $\psi(n) = n^2/2$ in units of labor, a leader (or frontier) firm moves one technological step ahead with a Poisson hazard rate of $n$. We call $n$ the “innovation rate” or “R&D intensity” of the firm. We assume that a follower firm can move one step ahead with hazard rate $h$ even if it spends nothing on R&D, by copying the leader’s technology. Thus $n^2/2$ is the R&D cost of a follower firm moving ahead with a hazard rate $n + h$. Let $n_0$ denote the R&D intensity put up by each firm in a neck-and-neck industry; and let $n_{-1}$ denote the R&D intensity by a follower firm in an unleveled industry; if $n_1$ denotes the R&D intensity of the leader in an unleveled industry, note that $n_1 = 0$, since our assumption of automatic catch-up means that a leader cannot gain any further advantage by innovating.

### 3.2.3. Bellman equations

Let $V_m$ (resp. $V_{-m}$) denote the steady state value of being currently a leader (resp. a follower) in an industry with technology gap $m$, and let $r$ denote the individual rate of time preference and let $w$ denote the wage rate, which we take as given and

\(^8\)Aghion et al (2001) analyze the more general case where $m$ is unbounded. However, unlike in this section, that paper provides no close form solution for the equilibrium R&D levels and the steady-state industry structure, and therefore cannot formally establish qualitative results such as the existence of an inverted-U relationship between competition and innovation or characterize the relationship between competition and the distribution of technological gaps.
equal to one assuming an infinitely elastic supply of labor.\textsuperscript{9,10} We have the following Bellman equations:

\begin{align*}
    rV_1 &= \pi_1 + (n_{-1} + h)(V_0 - V_1); \\
    rV_{-1} &= \pi_{-1} + (n_{-1} + h)(V_0 - V_{-1}) - (n_{-1})^2/2; \\
    rV_0 &= \pi_0 + \overline{n}_0(V_1 - V_0) + n_0(V_{-1} - V_0) - (n_0)^2/2.
\end{align*}

(3.4)  
(3.5)  
(3.6)

In words, the annuity value \( rV_1 \) of currently being a technological leader in an industry with gap \( m = 1 \) at date \( t \) equals the current profit flow \( \pi_1 \), minus the expected capital loss \( (n_{-1} + h)(V_0 - V_1) \) from having the follower catch up by one step with the leader. The annuity value \( rV_{-1} \) of currently being a laggard, is equal to the current profit flow \( \pi_{-1} \) plus the expected capital gain \( (n_{-1} + h)(V_0 - V_{-1}) \) from catching-up with the leader minus the R&D cost \( (n_{-1})^2/2 \). Finally, in the Bellman equation for a neck-and-neck firm, there is no help factor \( h \) because there is no leader, and \( \overline{n}_0 \) denotes the R&D intensity by the other firm in the same sector; in a symmetric Nash equilibrium both firms’ R&D intensities are equal, that is:

\[ \overline{n}_0 = n_0. \]

Now, using the fact that each firm chooses its own R&D intensity to maximize its current value, that is, to maximize the right-hand side of the corresponding Bellman equation, we obtain the first order conditions:

\begin{align*}
    n_{-1} &= V_0 - V_{-1}; \\
    n_0 &= V_1 - V_0.
\end{align*}

(3.7)  
(3.8)

\textbf{3.2.4. Product-market competition}

Boone (2000) makes the convincing argument that any parameter increase that would result in increasing the relative profit shares of more technologically advanced firms,
would be a suitable measure of product market competition. For example, Aghion-Howitt-Harris-Vickers (2001) measure competition using the elasticity of substitution parameter $\alpha$ in the CES case:\footnote{Although $\alpha$ is ostensibly a taste parameter, it can be shown to satisfy Boone’s requirement.}

$$v(x_A, x_B) = (x_A^\alpha + x_B^\alpha)^{\frac{1}{\alpha}}.$$  

Here, we simply assume undifferentiated Bertrand competition except for the possibility of collusion if both firms are leveled. Thus a laggard makes zero profit, that is $\pi_{-1} = 0$, whereas the profit flow of a neck-and-neck firm ranges from $\pi_0 = 0$ if there is perfect competition to $\pi_0 = \frac{\pi_1}{2}$ if there is maximum collusion. Product market competition is then simply parametrized inversely by $\pi_0$, or directly by the incremental profit $\Delta \equiv \pi_1 - \pi_0$.

### 3.2.5. Individual innovation intensities: the Schumpeterian and "escape competition" effects

Eliminating the $V$’s between the Bellman equations and first-order conditions (3.4) to (3.8), yields the reduced form R&D equations:

\begin{align*}
\frac{(n_0)^2}{2} + (r + h)n_0 - (\pi_1 - \pi_0) &= 0 \quad (3.9) \\
\frac{(n_{-1})^2}{2} + (r + h + n_0)n_{-1} - \pi_0 - \frac{(n_0)^2}{2} &= 0. \quad (3.10)
\end{align*}

This system is recursive, as the first equation solves for $n_0$, and then given $n_0$ the second equation solves for $n_{-1}$. We obtain:

\begin{align*}
n_0 &= -(r + h) + \sqrt{(r + h)^2 + 2(\pi_1 - \pi_0)} \quad (3.11) \\
n_{-1} &= -(r + h + n_0) + \sqrt{(r + h)^2 + n_0^2 + 2\pi_1}. \quad (3.12)
\end{align*}

We immediately see that $n_0$ increases whereas $n_{-1}$ decreases with higher product market competition.\footnote{From (3.11):}

$$\frac{\partial n_0}{\partial \pi_0} = -\frac{1}{\sqrt{(r + h)^2 + 2\Delta}} < 0$$
that results from reducing the rents that can be captured by a follower who succeeds in catching-up with its rival by innovating. The former effect (on $n_0$) we refer to as an “escape-competition effect”, namely that more competition induces neck-and-neck firms to innovate in order to escape competition, as the incremental value of getting ahead is increased with higher PMC. The latter effect is the basic Schumpeterian effect that results from reducing the rents that are captured by a follower who succeeds in catching up with the leader by innovating. On average, an increase in product market competition will thus have an ambiguous effect on growth as it induces faster productivity growth in currently neck-an-neck sectors and slower growth in currently unleveled sectors. The overall effect on growth will thus depend on the (steady-state) fraction of leveled versus unleveled sectors. But this steady-state fraction is itself endogenous as it depends upon equilibrium R&D intensities in both types of sectors. Now, we shall proceed to show under which condition this overall effect is an inverted-U, and at the same time derive additional predictions for further empirical testing.

Let

$$x = n_0.$$ 

Since by equation (3.11), $x$ is an increasing function of $\Delta$, we can use $x$ as our proximate measure of product market competition, which in turn will prove convenient when deriving our main predictions below. Note that $x$ takes values on the interval $[x = \sqrt{h^2 + \pi_1} - h, \bar{x} = \sqrt{h^2 + 2\pi_1} - h]$, with $x = x$ corresponding to maximum collusion ($\pi_0 = \pi_1/2$) and $x = \bar{x}$ corresponding to maximum competition ($\pi_0 = 0$).

From this and (3.12):

$$\frac{\partial n_{-1}}{\partial \pi_0} = \frac{\partial n_0}{\partial \pi_0} \left[ -1 + \frac{n_0}{\sqrt{(r+h)^2 + (n_0)^2 + 2\pi_1}} \right] > 0$$
3.2.6. Average innovation rate

Let $\mu_1$ (resp. $\mu_0$) denote the steady-state probability of being an unleveled (resp. neck-and-neck) industry. During any unit time interval, the steady-state probability that a sector moves from being unleveled to leveled is $\mu_1 (n_{-1} + h)$, and the probability that it moves in the opposite direction is $2\mu_0 n_0$. In steady state, these two probabilities must be equal:

$$\mu_1 (n_{-1} + h) = 2\mu_0 n_0.$$ 

This, together with the fact that:

$$\mu_1 + \mu_0 = 1,$$

implies that the average flow of innovations is:

$$I = 2\mu_0 n_0 + \mu_1 (n_{-1} + h) = 2\mu_1 (n_{-1} + h) = \frac{4n_0 (n_{-1} + h)}{2n_0 + n_{-1} + h}. \quad (3.13)$$

3.2.7. The inverted-U pattern

We shall now establish the possibility of an inverted-U pattern analytically, and also provide the main intuition underlying it. For this purpose it will be convenient to reexpress the aggregate innovation rate as being proportional to

$$\nu(x) = x \sqrt{\frac{x^2 + B - r - x}{x^2 + B - r + x}},$$

where

$$B = (r + h)^2 + 2\pi_1.$$

The following propositions are proved for the case where $r$ is small, however the results can be shown by simulations to hold more generally.

**Proposition 3.1.** Take $r = 0$. Whenever the value

$$\tilde{x} = \sqrt{B/3}$$
is interior to the interval \([x, \bar{x}]\), the aggregate innovation rate \(\nu(x)\) follows an inverted-U pattern, with \(\nu'(x) > 0\) for all \(x \in [x, \bar{x}]\) and \(\nu'(x) < 0\) for all \(x \in (\bar{x}, \|x\|]\). If \(\bar{x} > \|x\|\), then \(\nu'(x) > 0\) for all \(x \in [x, \bar{x}]\) so that the escape competition effect always dominates. If \(\bar{x} < x\), then \(\nu'(x) < 0\) for all \(x \in [x, \bar{x}]\) so that the Schumpeterian effect always dominates. Moreover, each of these cases arises for a non-empty subset of parameter values.

**Proof:** See Appendix.

### 3.2.8. Composition effect and the logic of the inverted-U

The inverted-U shape can be simply explained as follows. When there is not much product market competition, with \(\pi_0\) close to \(\pi_1/2\) (\(x\) close to \(\bar{x}\)), there is hardly any incentive for neck-and-neck firms to innovate, and therefore the overall innovation rate will be highest when the sector is unleveled. Thus the industry will be quick to leave the unleveled state (which it does as soon as the laggard innovates) and slow to leave the leveled state (which will not happen until one of the neck-and-neck firms innovates). As a result the industry will spend most of the time in the leveled state, where the escape-competition effect dominates (\(n_0\) is decreasing in \(\pi_0\)). In other words, if the degree of competition is very low to begin with, an increase in competition should result in a faster average innovation rate.

On the other hand, when competition is initially very high, with \(\pi_0\) close to 0 (\(x\) close to \(\bar{x}\)), there is relatively little incentive for the laggard in an unleveled state to innovate. Thus the industry will be relatively slow to leave the unleveled state. Meanwhile the large incremental profit \(\pi_1 - \pi_0\) gives firms in the leveled state a relatively large incentive to innovate, so that the industry will be relatively quick to leave the leveled state. As a result, the industry will spend most of the time in the unleveled state where the Schumpeterian effect is at work on the laggard, while the leader never innovates. In other words, if the degree of competition is very high to
begin with, an increase in competition should result in a slower average innovation rate.

Hence the possibility of an inverse-U relationship between competition and innovation. When competition is low, an increase will raise innovation through the escape-competition effect, but when it becomes intense enough it may lower innovation through the Schumpeterian effect on laggards. The reason why one effect dominates when competition is low and the other when competition is intense is the “composition effect” of competition on the steady-state distribution of technology gaps across sectors.

3.2.9. Additional predictions

In addition to providing a rationale for the inverted-U pattern uncovered in the previous section, the model delivers two additional predictions, which we test on our UK sample in the following section.

1. First, the expected technological gap in an industry increases as product market competition increases, that is the distribution shifts towards a lower probability of being neck-and-neck. To see this, note that the expected technological gap is given by:

\[ G = \mu_0 \cdot 0 + \mu_1 \cdot 1 = \mu_1 = \frac{2n_0}{2n_0 + n_{-1} + h}, \]

which can be reexpressed as

\[ G = \left[ 1 + \frac{\sqrt{x^2 + B - r - x}}{2x} \right]^{-1}. \]

This latter expression is clearly increasing in \( x \) and therefore with product market competition.

2. Second, there tends to be a positive interaction between the escape competition effect and the average distance of the industry to its frontier, in the sense that in industries where firms are closer to their technological frontier over time, the
escape competition effect tends to be stronger (that is, the increasing part of the inverted-U tends to be steeper). To see this in the context of our model, suppose there are industries with large spillover parameter $h$ and industries with small $h$. Those with large $h$ will tend to be more neck-and-neck on average over time, as the expected technological gap

$$G = [1 + \frac{n-1 + h}{2n_0}]^{-1}$$

is decreasing in $h$.\(^{13}\) Now, one can compare the magnitude of the escape competition effect across industries with different values of $h$. Then one can establish:

**Proposition 3.2.** Take $r = 0$. The peak of the inverted-U is larger and occurs at a higher degree of competition, in more neck-and-neck industries. More formally, let $\Delta$ be the incremental profit at which $x = \bar{x} = \sqrt{B/3}$; then both $\Delta$ and $\nu(\bar{x})$ are increasing in $h$.

**Proof:** See Appendix.

4. Empirical support for the additional predictions

To assess the second and third theoretical predictions a measure of the size of the technology gap between firms within an industry is required.

\(^{13}\)This stems from the fact that $n_0$ is decreasing in $h$ whereas $n_{-1} + h$ is increasing in $h$. To see the former, note that:

$$\frac{\partial n_0}{\partial h} = -\frac{n_0}{n_0 + r + h} \in (-1, 0),$$

whereas the latter follows from:

$$\frac{\partial n_{-1}}{\partial h} = \frac{r + h}{\sqrt{(r + h)^2 + n_0^2 + 2\pi_1}} - 1 + \left(\frac{n_0}{\sqrt{(r + h)^2 + n_0^2 + 2\pi_1}} - 1\right) \frac{\partial n_0}{\partial h} > -1$$

since

$$n_0 < \sqrt{(r + h)^2 + n_0^2 + 2\pi_1}$$

and

$$\frac{\partial n_0}{\partial h} < 0.$$
4.1. Measuring the technology gap

We capture the gap by the proportional distance a firm is from the technological frontier, as measured by total factor productivity. More formally, we let:

\[ m_{it} = \frac{TFP_{Ft} - TFP_{it}}{TFP_{Ft}}, \]  

(4.1)

where \( F \) denotes the frontier firm (with the highest TFP) and \( i \) denotes non-frontier firms. For the frontier firm our measure is

\[ m_{Ft} = \frac{TFP_{Ft} - TFP_{F-1t}}{TFP_{Ft}}, \]  

(4.2)

where \( F - 1 \) denotes the firm just behind the frontier. In the empirical application below we use an industry level measure \( m_j \) that is the average across firms in the industry. A lower value of \( m_j \) indicates that firms in industry \( j \) are technologically closer to the frontier (and therefore more like the neck and neck firms in our theoretical section) while a high value of \( m_j \) indicated a large technological gap with the frontier (so that firms in that industry are more like laggard firms in an unleveled industry).

4.2. Composition effect

The empirical analysis in section 2 studied the impact of product market competition at the industry level on the level of patenting activity. We now look at the importance of similarities in technology across firms in the same industry - defined by the size of the technology gap or the degree of neck-and-neckness. The second key prediction derived from the theoretical discussion is that, in equilibrium, the average technology gap between leaders and followers should be an increasing function of the overall level of industry-wide competition (so that average neck-and-neckness should be a decreasing function of competition).

Figure 5 presents a kernel smoothed plot of our measure of the average technological distance from the frontier, \( m \), for each industry time observation against the industry competition index. This shows a strongly positive relationship as predicted
by the theory. In particular, more competitive industries display a lower degree of neck-and-neckness.

[Figure 5 here]

In some senses this is a surprising effect as one might expect that higher competition would reduce the spread within industries because low productivity firms would exit, compressing the distribution. What this results demonstrates is that this static compression effect is dominated by a more powerful dynamic effect whereby higher competition increases innovation among similar (neck-and-neck) firms expanding the spread.

4.3. Technological Gap and Product Market Competition.

The third theoretical prediction is that the inverted U shaped relationship between competition and growth should be steeper in more neck and neck industries. To assess this prediction, we consider a subsample of our data - firms in industries with below median technological gap - these are more neck and neck industries. Figure 6 presents a picture of the baseline exponential quadratic specification, as well as the same specification estimated on the sample of firms in high neck and neck industries. Two features stand out clearly. First, more neck and neck industries show a higher level of innovation activity for any level of product market competition.14 Second, the inverted U curve is steeper for the more neck and neck industries which accords well our theoretical predictions. The estimated exponential quadratic specification for the high neck and neck firms are shown in the first column of Table 2.

[Figure 6 here]

[Table 2 here]

As a check on these results, in addition to the full set of time and industry effects we control for endogeneity in the neck and neck split, as well as in the degree of com-

14 The unconditional mean of citation weighted patents for high neck and neck firms is 8.58, compared to 6.68 for the entire sample.
petition. As additional instruments we use information on the state of technology and costs in France and the US.\textsuperscript{15} The instruments have explanatory power, as evidenced by the high $R^2$ on the reduced form shown in Table 2 and the significance of both the policy instruments and the other instruments. The impact of controlling for the endogeneity of the sample separation between high and low neck and neck industries in the exponential quadratic specification is presented in Table 2. The strong quadratic pattern remains, and one can also show that the impact of controlling for endogeneity is to reinforce the escape-competition effect for more neck and neck firms.\textsuperscript{16}

5. Conclusions

This paper investigates the relationship between product market competition (PMC) and innovation using a flexible non-linear estimator. To identify the causal impact of competition we exploit a series of major policy reforms in the UK undertaken under the Thatcher government, the European Single Market, and Competition Policy reforms. We find evidence that the competition-innovation relationship takes the form of a balanced inverted-U shape, with firms distributed across both the increasing and decreasing sections of the U-shape. This inverted-U was robust to a number of different estimation strategies including fully-flexible kernel regressions and semi-parametric instrumental variables controlling for year and industry effects.

To understand what is driving this inverted-U shape we extend the current theoretical literature on step-by-step innovation to produce a model delivering a balanced inverted-U prediction. In this model competition may increase the incremental profit from innovating, labelled the "escape competition effect"; but on the other hand competition may reduce innovation incentives for laggards, labelled the "competition

\textsuperscript{15}We use additional instruments that vary at the industry-year level for the same industries in France and US. The instruments include: imports penetration; output minus costs over output; estimate of markup from Martins et al (1996) interacted with a time trend; TFP; R&D intensity. All instruments vary over industry and time and are included in levels and squared terms.

\textsuperscript{16}The results are available from the authors.
effect". The balance between these two effects changes between low and high levels of competition generating a balanced inverted-U relationship. In addition, this extension of the theory provides two new predictions. First, the equilibrium degree of technological ‘neck-and-neckness’ among firms should decrease with PMC, and second, that the higher the average degree of ‘neck-and-neckness’ in an industry, the steeper the inverted-U relationship between PMC and innovation in that industry. We take these two extra predictions to the data and find the data to be consistent with these. This dual empirical and theoretical approach provides useful results on the impact of competition and closeness in technology space on innovation, and also a model to understand this and experiment with potential policy reforms.
A. Appendix A

A.1. Proof of Proposition 3.1:

Let

\[ \nu(x) = \frac{x\sqrt{x^2 + B} - r - x}{\sqrt{x^2 + B} - r + x} \]

measure the innovation rate as a function of competition measured by \( x = n_0 \), where \( x \in [\underline{x} = \sqrt{h^2 + \pi_1 - h}, \overline{x} = \sqrt{h^2 + 2\pi_1 - h}] \). For \( r = 0 \), we have:

\[ \nu'(x) = B\left(\frac{1}{\sqrt{x^2 + B} - r + x}\right)^2(1 - \frac{2x}{\sqrt{x^2 + B}}). \]

The expression

\[ f(x) = 1 - \frac{2x}{\sqrt{x^2 + B}} \]

is decreasing in \( x \). Thus \( \nu(x) \) is concave, thus an interior zero of \( f \) corresponds to a maximum of \( \nu \). Thus, if \( \bar{x} \) denotes such a value, we have:

\[ \bar{x}^2 + B = 4\bar{x}^2 \]

or equivalently:

\[ \bar{x} = \sqrt{B/3}. \]

The inverted-U pattern will obtain whenever \( \bar{x} \in (\underline{x}, \overline{x}) \). Now let \( \eta = h/\sqrt{\pi_1} \). One can easily establish that:

\[ \frac{\bar{x}}{\underline{x}} = \frac{\sqrt{\eta^2 + 2 - \eta}}{\sqrt{(\eta^2 + 2)/3}}, \quad \frac{\bar{x}}{\overline{x}} = \frac{\sqrt{\eta^2 + 1 - \eta}}{\sqrt{(\eta^2 + 2)/3}}. \]

Thus the inverted-U pattern will obtain whenever

\[ \sqrt{\eta^2 + 1} < \sqrt{(\eta^2 + 2)/3} + \eta < \sqrt{\eta^2 + 2}; \]

the escape competition effect will strictly dominate over the whole interval \( [\underline{x}, \overline{x}] \) whenever

\[ \sqrt{(\eta^2 + 2)/3} + \eta \geq \sqrt{\eta^2 + 2}; \]

finally the Schumpeterian effect will dominate over the whole interval \( [\underline{x}, \overline{x}] \) whenever

\[ \sqrt{(\eta^2 + 2)/3} + \eta < \sqrt{\eta^2 + 1}. \]

Each of the corresponding three regions is non-empty, which establishes the Proposition.
A.2. Proof of Proposition 3.2:

For \( r \) equal to zero, we have:

\[ B = h^2 + 2\pi_1. \]

Thus \( h \) will affect \( \tilde{x} = \sqrt{B/3} \) and \( \nu(\tilde{x}) \) via its positive effect on \( B \). Assume that \( \tilde{x} \) is interior to the interval \((\underline{x}, \bar{x})\). From the envelope theorem, the marginal effect of \( B \) on

\[ \nu(\tilde{x}) = \max_{x \in (\underline{x}, \bar{x})} \nu(x) \]

is just equal to the direct effect

\[ E = \frac{\partial}{\partial B} \left\{ x \sqrt{x^2 + B - r - x} \right\} \]

which is unambiguously positive. The marginal effect of \( h \) is \( E \frac{\partial B}{\partial h} \) which is therefore also positive. Therefore more neck-and-neck industries (those with larger \( h \)) have a higher peak in the inverted-U. Moreover, the peak occurs at the value of \( \pi_0 \) such that

\[ x = n_0 = \sqrt{B/3}, \] or equivalently:

\[ 0 = -\sqrt{h^2 + 2(\pi_1 - \pi_0)} + h + \sqrt{(h^2 + 2\pi_1)/3}. \quad (A.1) \]

The peak lies further to the right on the \( x \) line in more neck-and-neck industries if \( \frac{d\pi_0}{dh} < 0 \), where \( \pi_0 \) is implicitly defined by (A.1). But precisely, applying the implicit functions theorem to equation (A.1), we get:

\[ \frac{d\pi_0}{dh} = -\sqrt{h^2 + 2(\pi_1 - \pi_0)}.F, \]

where

\[ F = -\frac{h}{\sqrt{h^2 + 2(\pi_1 - \pi_0)}} + 1 + \frac{h/3}{\sqrt{(h^2 + 2\pi_1)/3}} \]

\[ > -h + \sqrt{h^2 + 2(\pi_1 - \pi_0)} > 0. \]

This establishes the proposition.
B. Appendix B

B.1. Data

The Lerner Index is price minus marginal cost over price. One difficulty we face is that we do not observe marginal cost. For the numerator we use operating profits net of depreciation and provisions. We deduct an estimate of the financial cost of capital (cost of capital*capital stock) from our measure of profits. This is more like price minus average cost. We divided this by sales.

\[ l_{it} = \frac{\text{operating profit - financial cost}}{\text{sales}}. \]

At the firm level the Lerner Index varies from 0 to 0.38, has a mean of 0.09 and a median value of 0.08.

In our econometric analysis below we present results with this individual firm level index. However, we also find that our results are robust to using the industry level aggregate. This robustness to the use of the industry level product competition measure is important since it is the industry level variation in the policy instruments which we exploit to purge the endogeneity in the competition measure. Consequently for our central specification we relate firm level innovation activity to the industry level competition index. Identification will come from variation across industries over time. The industry level index, denoted \( c_{jt} \), is an unweighted average across all firms in the industry,

\[ c_{jt} = 1 - \frac{1}{N_{jt}} \sum_{i \in j} l_{it}, \]  

where \( i \) indexes firms, \( j \) indexes industry, \( t \) indexes time and \( N_{jt} \) is the number of firms in industry \( j \) in year \( t \). A value of 1 indicates perfect competition (price equals marginal cost) while values below 1 indicate some degree of market power. We classify firms by the 2-digit SIC code in which the firm had the largest proportion of its sales in 1995, where the median share of sales accounted for by this industry is 90\%.\(^{17}\) As an alternative competition measure we derive an estimate of the industry substitution parameter \( \alpha \) as described in (3.2), and we find our specifications robust to this alternative measure.

B.1.1. Summary Statistics

Table 1 shows the sample of firms where we have only accounting data and the sample where we have both accounting and patents data. The table shows that the firms we have in our sample are similar in terms of their Lerner Index to those not in our sample - both are used to construct our industry measure of the Lerner. At the industry level the Lerner averages 4\% and ranges from 13\% in Office & Computing Machinery in 1973 to less than 1\% in Motor Vehicles in 1982. Patenting levels also vary strongly across industries, in part due to different patenting intensities, which the industry dummies will control for.

Table 2 presents the descriptive statistics on our sample of 330 firms which remain after matching the accounting and innovation data and cleaning the data (removing firms with missing observations, firms involved in major mergers, or firms with less

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\(^{17}\)For firms operating in more than one market the Lerner Index will represent a weighted average of the degree of product market competition across these markets. This could lead to measurement error and attenuation bias, tending to flatten our estimated shape making it harder to find a non-monotonic relationship. We discuss this further in the empirical section below.
than three years of consecutive data). From this we can see that the patent count is highly skewed, with most firms taking out no patent in any given year, but one firm (ICI in 1974) taking out 409 patents. The employment figures reflect firm size, with about 1,200 employees in the median firm.

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Figure 1: Kernel Regression of Innovation on Competition

Kernel regression, bw = .025, k = 6
x=(1-Lerner), y=CW Patents

Note: Bandwidth of 0.025 in Epanechnikov Kernel

Figure 2: Innovation and Product Market Competition: Exponential quadratic and the semiparametric specifications: with year and industry effects
Figure 3: Innovation and Product Market Competition: Four highest patenting industries

Motor vehicles

Chemicals

Electrical and electronics

Food, beverages and tobacco

Figure 4: Innovation and Product Market Competition: Controlling for Endogeneity
Figure 5: Technology Gap and Competition: Kernel Regression of The Composition effect

Figure 6: Innovation and Product Market Competition: The neck and neck split
## Table 1: Exponential Quadratic: Basic specification

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**Notes:** All specifications include year and industry effects. Column (1) shows Poisson estimates. Significance test show likelihood ratio test statistic and P-value from F test of joint significance. Column (2) includes control function, excluded variables are: policy instruments, imports over value-added in same industry-year, TFP in same industry-year, output minus variable costs over output in same industry-year and estimate of mark-up from industry-country regression (Martins et al 1996) interacted with time trend, all for USA and France.
### Table 2: Exponential Quadratic: Neck and Neck specification

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<tbody>
<tr>
<td><strong>Dependent variable:</strong> Citation weighted patents</td>
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<tr>
<td><strong>Observations</strong></td>
<td>1197</td>
<td>1184</td>
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<tr>
<td><strong>Constant</strong></td>
<td>-137.3</td>
<td>-73.65</td>
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<tr>
<td></td>
<td>(18.57)</td>
<td>(19.65)</td>
</tr>
<tr>
<td>jtc</td>
<td>292.8</td>
<td>156.0</td>
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<tr>
<td></td>
<td>(39.8)</td>
<td>(42.0)</td>
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<tr>
<td>jtc²</td>
<td>-153.5</td>
<td>-80.46</td>
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<td></td>
<td>(21.3)</td>
<td>(22.49)</td>
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</table>

Significance of \(jtc\), \(jtc²\)

- **Significance of policy instruments in reduced form:** - 12.17 (0.00)
- **Significance of other instruments in reduced form:** - 24.91 (0.00)
- **Control functions in regression:** - 5.99 (2.69)
- **R² of reduced form:** - 0.84

**Notes:** All specifications include year and industry effects. Column (1) shows Poisson estimates. Significance test show likelihood ratio test statistic and P-value from F test of joint significance. Column (2) includes control function, excluded variables are: policy instruments, imports over value-added in same industry-year, TFP in same industry-year, output minus variable costs over output in same industry-year and estimate of mark-up from industry-country regression (Martins et al 1996) interacted with time trend, all for USA and France.
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<th>Mean (s.d)</th>
<th>Median</th>
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<th>Max</th>
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<td>Patents</td>
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<td>6.68 (26.53)</td>
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<td>0.95 (0.020)</td>
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<td>11 (31.3)</td>
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<td>Observations per firm</td>
<td>17.2 (5.06)</td>
<td>19</td>
<td>5</td>
<td>22</td>
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<tr>
<td>Technology gap (m)</td>
<td>0.56 (0.127)</td>
<td>0.59</td>
<td>0.085</td>
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        A clarification of the Goodwin model of the growth cycle
04-05  G L Albano  C Leaver
        Transparency, recruitment and retention in the public sector
04-06  P Aghion  N Bloom  R Blundell  R Griffith  P Howitt
        Competition and Innovation: An inverted U relationship?