non-topographic feature targets and would vary with the nature of the terrain.

As explained above, our *Reduced Targets Strategy* reduces the visibility index of observers by an amount approximately equal to the non-topographic feature targets potentially visible to them. This kind of uncertainty, arising due to a sparse targets set, is closely similar to the uncertainties related to *Object Generalization* (Weibel and Dutton, 1999). To our knowledge, this kind of uncertainty has not been widely addressed in the visibility studies literature. In general, however, finding the location of visibly dominant observers has more practical use than their exact visibility indices (Franklin, 2000). Therefore, the aim is to evaluate whether the overall visibility index pattern is realistic, albeit abstracted. The following section proposes two simple methods based on an iterative comparison between the AVI and EVI of observers for assessing this uncertainty.

Methodology

In visibility analysis, a target is considered visible if an LOS from an observer can be drawn to it without its being obstructed by an intermediate point (an exception is provided by Wang (2000b), who used reference planes to establish the visible areas). The most common approach in previous *Reduced Targets Strategy* based optimization methods (e.g., see Franklin *et al.* (1994)) has been to draw the LOS from an observer to an arbitrary small number of randomly located targets on the terrain (Figure 1). In the current work, we propose that the visibility index of an observer be computed by drawing the LOS only to a topographic feature (Figure 1). Of course, the underlying assumption of this proposal is that the terrain is not devoid of topographic features. This is true for mountainous terrain except in upland plateaus, although in which case the visibility indices will be mostly similar. Our methodology for the computation of visibility indices using topographic feature targets consists of three steps: (1) extract the topographic features, (2) compute the visibility index of each point using the topographic features as targets, and (3) assess the uncertainty in the visibility index.

An Experiment

Step 1—Extraction of Topographic Features

Many approaches have been proposed for the automated extraction of topographic features from DEMs and TINs (Greysukh, 1967; Peucker and Douglas, 1975; Evans, 1979; Takahashi *et al.*, 1995; Wood, 1998). A detailed treatment of this topic is beyond the scope of this work. We decided to use the extraction method of Wood (1998), based on the advantages he outlined against the other methods and partly due to its easy availability in the user-friendly freeware software LandSerf.

It is clear that the success of our *Reduced Targets Strategy* depends upon the accuracy of the non-trivial topographic feature classification. It is well known that most automated topographic feature extraction methods are vulnerable to the noise in the DEM (Jenson and Domingue, 1988) and, most importantly, have scale dependency limitations (Wood, 1999). While smoothing the DEM before extracting the features can eliminate the first limitation, the latter seems to remain a difficult conceptual problem yet to be completely solved. Due to scale dependency, the automated feature extraction identifies features only at a certain scale (e.g., features of a fixed geographic extent), while features at other scales remain undetected. Therefore, the assessment of an appropriate scale for the particular DEM requires iterating through a number of *feature extraction scales* (e.g., in LandSerf, one could achieve this by iterating with a different window or kernel sizes for the feature extraction and visual verification).

Finally, although the fundamental topographic features are a significantly reduced number of targets, there may still be too many for certain terrains, e.g., large desiccated DEMs, and thus lead to a long visibility index computation time. Two simple ways of reducing the number of topographic features are to (1) resample the topographic features set by a certain skip interval and (2) limit the topographic features to certain scales. A detailed treatment of the sampling methods is beyond the scope of this paper but we will demonstrate the use of the first method later.

Step 2—Visibility Analysis

The study areas for the current work are the 100-m resolution DEMs of the Cairngorms (5548 points) in Scotland (Figure 2a) and the central part of the Isle of Man (16335 points) (Figure 3a). Note that this methodology could also be easily applied to an irregular terrain model such as a TIN. The visibility analysis was carried out in ArcView GIS developed by ESRI, and all the parameters were the defaults of the *Visibility Request* in ArcView. In the experiment, the observer eye level is at 1 m above the local ground level and the targets are at local ground level. The observer is capable of seeing from ground zero to infinity (i.e., no horizon culling), across the full range of azimuths, and from the zenith to nadir. The experiments were done on a 1-GHz Intel-Pentium processor-based personal computer, with 256 MB RAM. We recorded the CPU time taken by ArcView for each visibility computation.

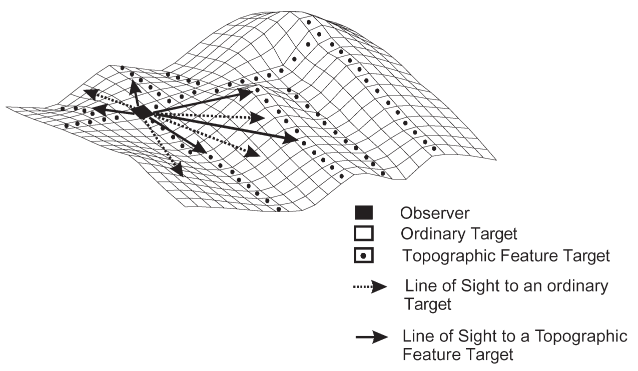

Figure 1. Two types of observer-target relationships in the Reduced Targets Strategy, i.e., line of sights drawn to random non-topographic feature targets and line of sights drawn to topographic features, i.e., peaks, passes, pits, ridges, and channels.

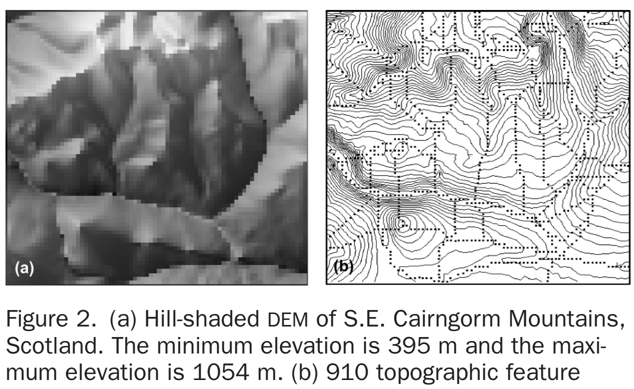

Figure 2. (a) Hill-shaded DEM of S.E. Cairngorm Mountains, Scotland. The minimum elevation is 395 m and the maximum elevation is 1054 m. (b) 910 topographic feature targets.

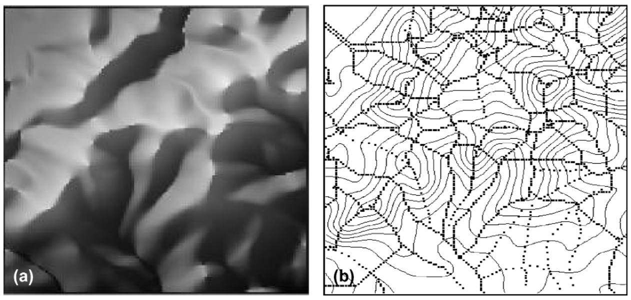

Figure 3. (a) Hill-shaded DEM of Central Isle of Man. The minimum elevation is 0 m and the maximum elevation is 553 m. (b) 2007 topographic feature targets.

Step 3—Uncertainty Assessment
The only previous example known to us, which dealt with the estimation of uncertainty in a similar *Reduced Targets Strategy*, is that of Franklin *et al.* (1994). They compared the visibility indices of an arbitrary number of spatially distributed points in the terrain, computed from their exhaustive R2-visibility algorithm (similar to our Golden Case), with their optimized methods. Though the results are encouraging, their sampling methods (i.e., the selection of the test points) could not be regarded as formal and objective for two reasons. First, because there is no prior knowledge about the statistical distribution of the visibility pattern, it is not possible to estimate the number of random points required to fully capture the sensitivity of the visibility index distribution of a terrain. However, the choice of the number of random points is critical, because it will dictate our computation time. Second, because visibility is a directional property, then the spatial location of random points could bias the uncertainty estimation. Later we provide examples that suggest that the visibility pattern is highly dependent upon the spatial distribution and number of the random points.

We propose the following two methods for the uncertainty assessment based on a slight modification of the Franklin *et al.* (1994) methods:

Method 1: Spatial Correlation between AVI and EVI
In this method, an assessment of the overall visibility pattern indicated by the EVI of the terrain points is done in the following two ways:

Type 1: AVI vs. EVI of the Topographic Feature Observers—

- Compute the AVI of the topographic features by drawing the LOS from each topographic feature to all the terrain points. Normalize the AVI and EVI of the topographic features, by scaling them between their respective minimum and maximum indices, in order to suppress the effect of target sample size on the indices. The normalization also reveals the visibility dominance of the observers.
- Calculate the correlation coefficient between the AVI and EVI of the topographic features. The correlation coefficient should suggest the similarity between the two visibility patterns. This method is similar to Franklin *et al.* (1994) except that the definition of our test points is objective and more *natural.* However, statistically it remains only an approximate test, especially when using exceptional terrains, where the topographic features are not distributed uniformly across the terrain.

Type 2: AVI vs. EVI of Random Observers—
Unlike the Type 1 method, this method is relatively more exhaustive but also more time-consuming. It is an abridged form of the Monte-Carlo method of uncertainty modeling and involves an iterative comparison between the AVI and EVI of a set of random observers but with the important exception that no subsequent model parameter estimation is done in this method. The steps are as follows:

(1) Estimate the number of observers to be distributed randomly on the terrain: As mentioned before, because there is no prior knowledge about the AVI distribution, it is non-trivial to determine the optimal number of random observers sufficient to capture the visibility pattern. We propose, without formal proof, that randomly placed observers, equal in number to the number of unique EVI, would be sufficient if we assume that
- No part of the study area is hidden from the topographic features. Thus, a histogram of the EVI (computed using topographic features) represents unique viewsheds, and
- Random observers do not form clusters.

In other words, with these assumptions, we suppose that each viewshed will be assigned at least one test-observer site.

(2) Distribute a number of random observers equal to the number of unique EVI, found in step (1), spatially across the terrain. We used the Random Point Generator ArcView Extension developed by Jennes (2001).

(3) Compute the AVI of the random observers by drawing the LOS to all the points on the terrain. Normalize the AVI and EVI of the random observers as previously done.

(4) Calculate the correlation coefficient between the AVI and EVI of the random observers.

(5) Repeat steps 2 through 4 a number of times. Again, due to the lack of any prior information about the statistical distribution of the AVI, statistically it is difficult to decide upon a specific number of iterations. In a practical exercise, it would ultimately depend upon the amount of time available to the researcher for the experiment.

(6) Choose the lowest and highest correlation coefficient as indicators for the worst- and the best-case approximation of AVI.

Method 2: Error in the Estimated Visibility Indices
In the previous methods, the AVI to EVI correlation coefficients give an indication of the reliability of EVI in representing the spatial pattern of visibility dominance. However, these do not reveal the amount of approximation in the EVI. A simple method for measuring the uncertainty in the EVI is an average ratio of EVI over AVI, as follows:

$$\text{Average Error (\%)} = \pm \frac{\sum_{i=1}^{n} \frac{|x_i' - x_i|}{x_i}}{n} \times 100 \ldots \quad (1)$$

where x_i' is the normalized EVI of an observer i, x_i is the normalized AVI, and n is the number of observers. Note that normalized AVI and EVI are used to ensure that the approximation in visibility dominance is revealed.

Significance of the Topographic Features
As mentioned in the introduction, a *Reduced Targets Strategy* based on a small number of random non-topographic feature targets could also reduce the visibility index computation time. Therefore, in order to validate the uniqueness and benefit of our choice of targets, we wanted to ensure that they would be better than the same number of random targets spatially distributed across the terrain. One of the ways of verifying the significance of the topographic features as targets is to compare the quality of the visibility pattern produced by an equal and decreasing number of topographic feature targets and the random targets. In this work, we used the skip interval method of generalizing our topographic feature set and gradually kept increasing the skip interval. Some other suitable guides for the minimum number of test points could be the number of point topographic features (peaks, pits, and passes), a satisfactory level of accuracy, and the maximum permissible computation time.

For each set of topographic feature targets, we generated four sets of equal numbers of random targets. The quality of

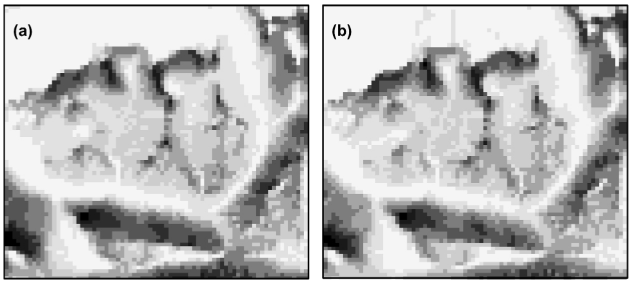

Figure 4. Comparison between the (a) Golden case based visibility dominance and (b) topographic features based estimated visibility dominance in the Cairngorm study area. Darker colored areas have more visual dominance than lighter colored areas.

the visibility dominance pattern produced from the topographic feature targets was then compared with the one produced from the random targets. The comparison was based on the two types of uncertainty estimation methods described in the previous section.

Results

Automated Extraction of the Topographic Features

After iterating with various window (kernel) sizes followed by visual inspection, we found that the 5 by 5 (500 m by 500 m) window and 3 by 3 (300 m by 300 m) window are suitable for extracting most linear (ridge, channel) and point (peak, pit, pass) topographic features, present in the Cairngorm (Figure 2b) and Isle of Man (Figure 3b) DEM, respectively, where 910 and 2007 topographic features have been extracted as the optimal targets. However, as mentioned previously, the number of topographic features extracted from the DEM depends upon the size of the filter window. Therefore, different window sizes will produce different numbers of topographic features. In future work, it would be interesting to investigate the change in the EVI pattern and its correlation with the AVI with varying extraction scales.

Visibility Analysis and Uncertainty Assessment

Because our study areas are small, we have been able to obtain the *Golden Case* visibility patterns of our study areas (Figures 4a and 5a). These visibility index patterns are now the ideal standards, i.e., the AVI. The visibility indices have been normalized (as previously) to assess the relative visibility dominance of the points in the visibility pattern. Figures 4b and 5b show the pattern of the EVI over the two terrains, and it is clear from the figures that the overall pattern of the visibility indices is similar to the Golden Case. In fact, as indicated by the correlation coefficients, there is 97 percent and 82 percent overall correlation between the AVI and EVI of the Cairngorm and Isle of Man DEMs, respectively (Figures 6a and 7a). The ridges and peaks have high visibility indices

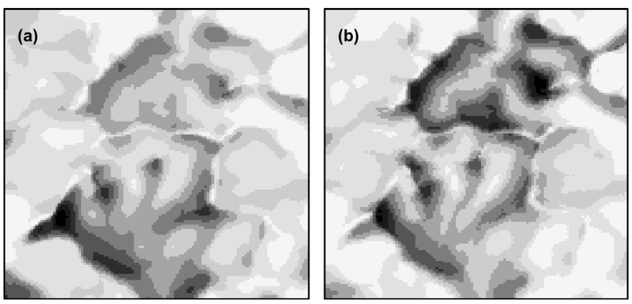

Figure 5. Comparison between the (a) Golden case based visibility dominance and (b) topographic features based estimated visibility dominance in the Isle of Man study area. Darker colored areas have more visual dominance than lighter colored areas.

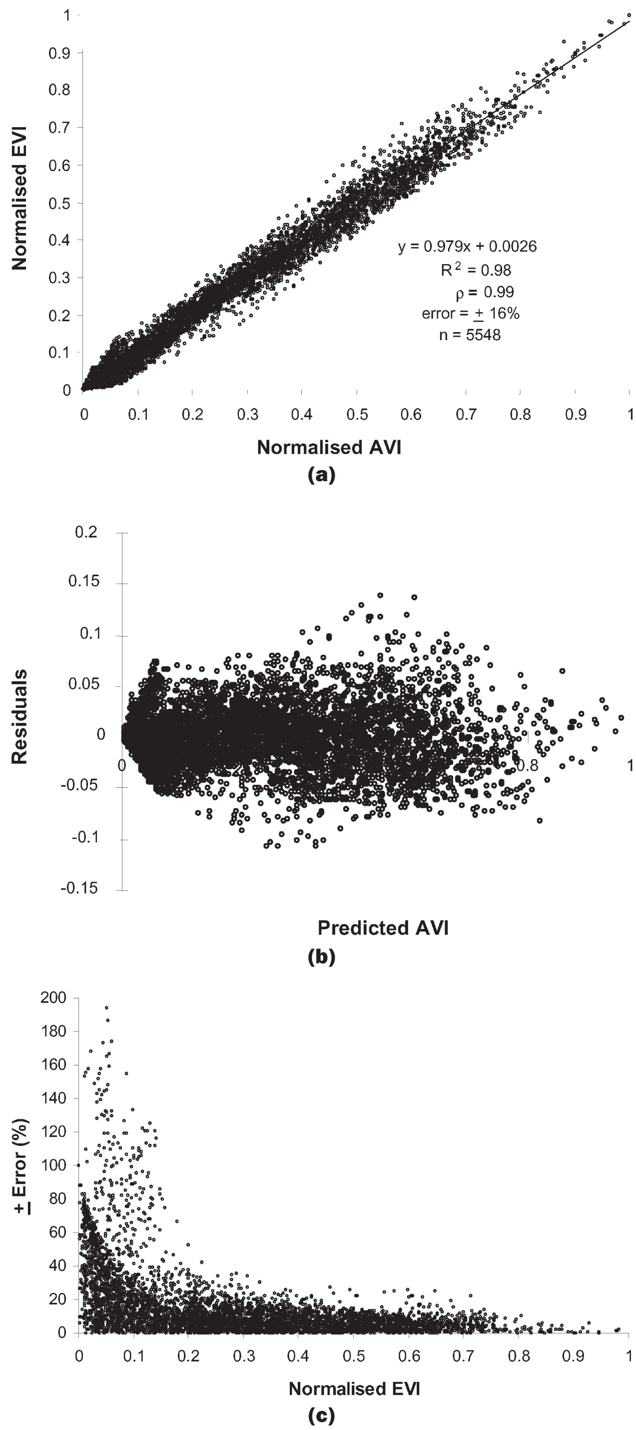

Figure 6. Uncertainty assessment in the EVI in the Cairngorm study area based on (a) AVI to EVI plot of all Cairngorm points, (b) residuals based on the linear regression between AVI and EVI, and (c) errors in the EVI.

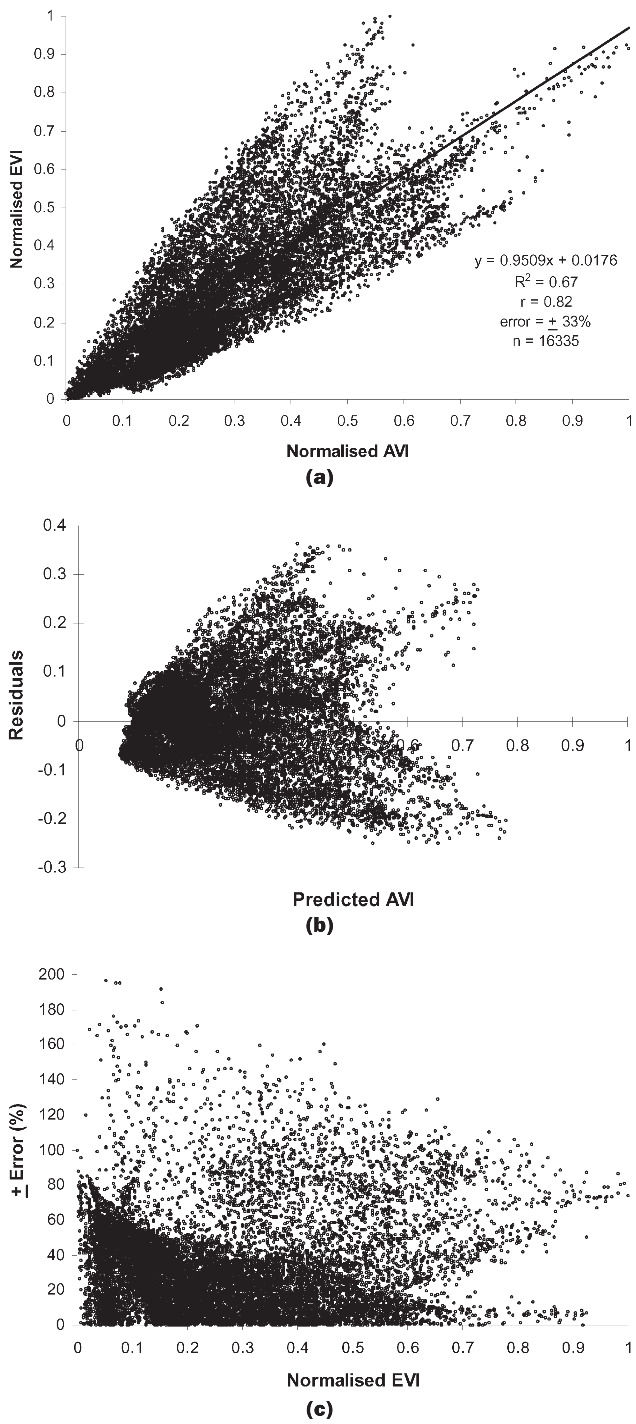

Figure 7. Uncertainty assessment in the EVI in the Isle of Man study area based on (a) AVI to EVI plot of all Isle of Man points, (b) residuals based on the linear regression between AVI and EVI, and (c) errors in the EVI. Note the distinct fork in both (a) and (b), indicating that the EVI has over estimated the visibility dominance of some points.

compared to the pits, passes, and channels. The plot of the AVI versus EVI for the Isle of Man DEM (Figure 7a) indicates that, in the case of some viewpoints, the optimization approach significantly overestimated their visibility dominance. A careful analysis of the spatial distribution of Figure 5b data revealed that these anomalous points are located around the main ridge structure in the central part of the DEM. The reason for this lies in the fact that the ridges make up a large proportion of the topographic features and therefore have a significantly better view of the topographic features (as also shown by Lee (1992)) compared to the other parts of the terrain. The plot of the residuals based on a linear regression supports this observation by the distinct fork-shape distribution of the residuals (Figure 7b). At the same time, it is evident from the range of the EVI that our optimized approach has significantly underestimated the visibility indices. The average error in the Cairngorm and Isle of Man DEMs are ±16 percent and ±33 percent, respectively. Further, Figures 6c and 7c show that the error varies according to the visibility dominance of the observer space, with the less dominant points having the bigger errors. The residuals versus the predicted AVI plots (Figures 6b and 7b) reveal an interesting dichotomy. In the case of the Cairngorm study area, the residuals are uniform but, in the case of Isle of Man study area, the residuals are strongly related to the visibility index magnitude. An implication of this observation is that the regression between AVI and EVI should only be used as a basis for testing similarity (e.g., using the correlation coefficient) but not for modeling visibility magnitudes.

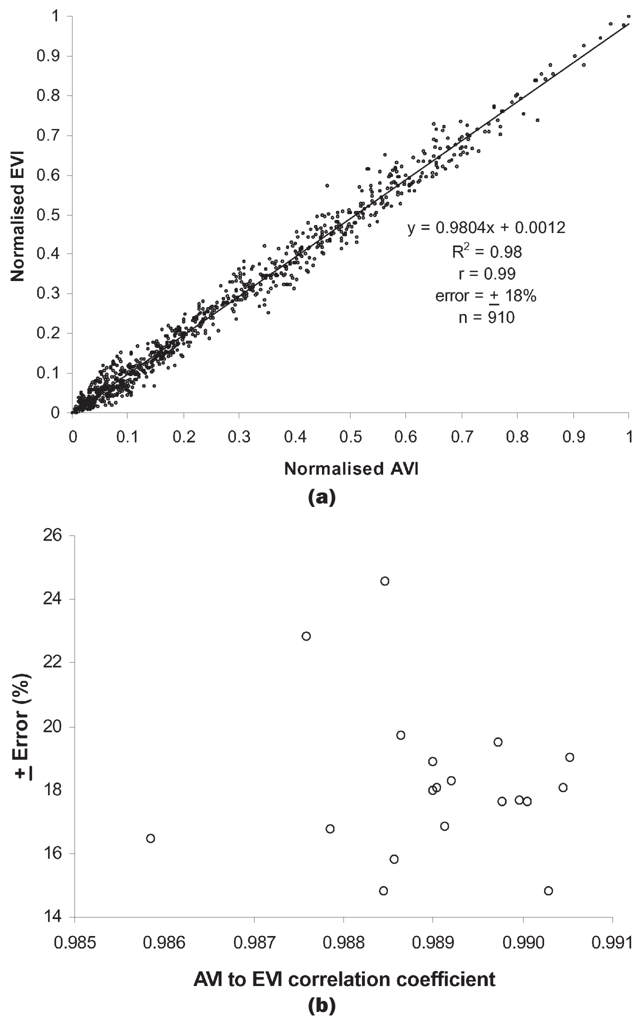

Figure 8. Uncertainty assessment in the EVI in the Cairngorm study area based on (a) AVI to EVI plot of the topographic features, and (b) AVI to EVI correlation coefficient versus errors at a set of random locations.

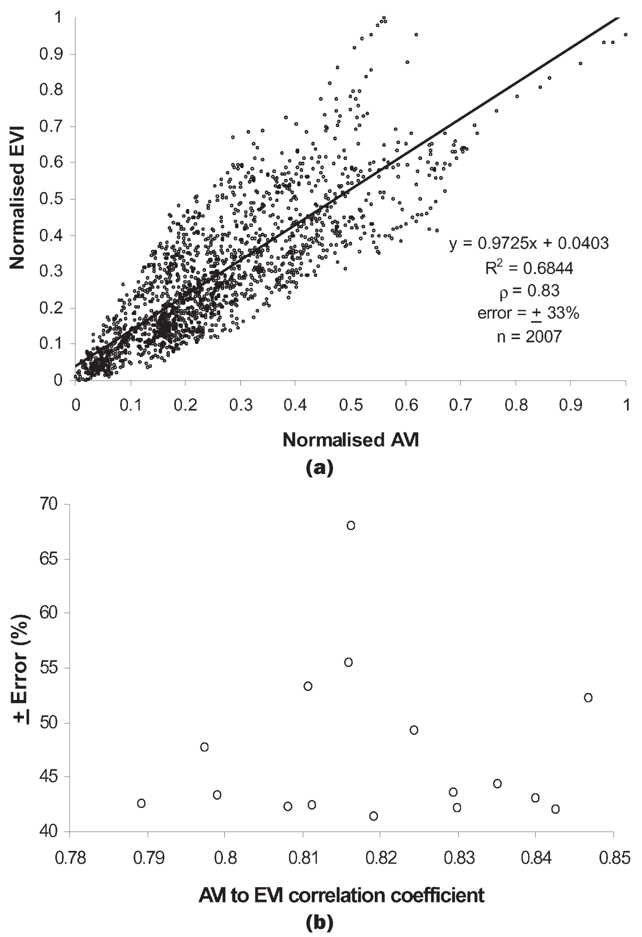

Figure 9. Uncertainty assessment in the EVI in the Isle of Man study area based on (a) AVI to EVI plot of the topographic features, and (b) AVI to EVI correlation coefficient versus errors at a set of random locations.

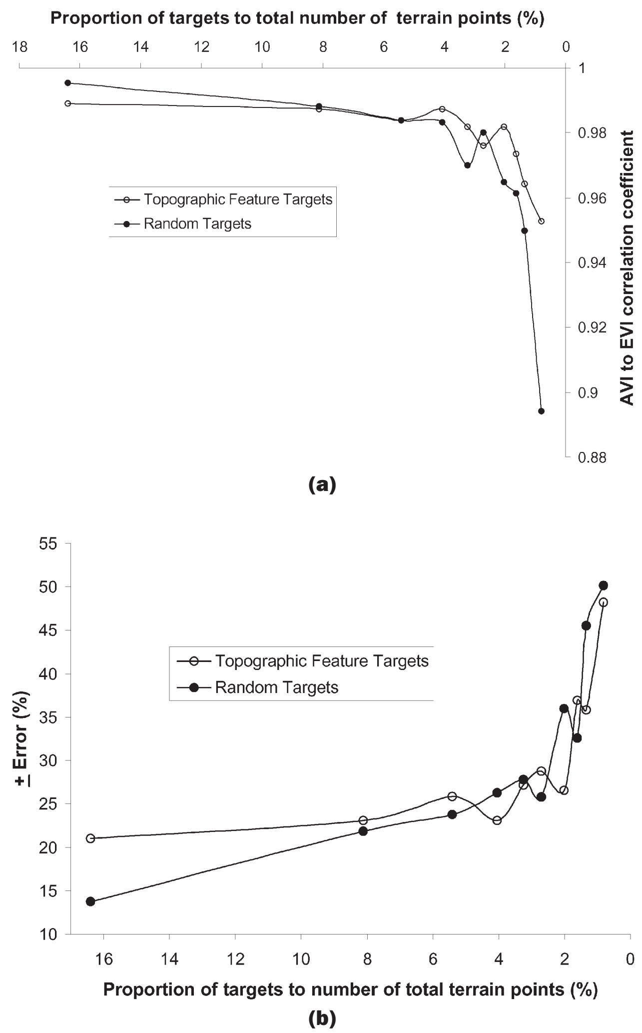

Figure 10. Validation of the significance of topographic features as optimal targets in the Cairngorm study area based on (a) comparison of the AVI to EVI correlation coefficients and (b) error in EVI for a decreasing number of topographic features and random targets.

Based on Method 1 for uncertainty estimation, Figures 8a and 9a show the relation between the AVI and EVI of the topographic features in the Cairngorm and Isle of Man DEMs, respectively. The strong correlation coefficients of 0.98 (Cairngorm) and 0.83 (Isle of Man) suggest that the optimization has successfully achieved representing the overall visibility pattern. To perform a more exhaustive assessment, we generated 16 sets of 414 (unique number of EVI in the Cairngorm study area) and 19 sets of 479 (unique number of EVI in the Isle of Man study area) spatially distributed random points in the Cairngorm and Isle of Man study areas, respectively. We then calculated the correlation coefficient between the AVI and EVI for each of these sets of random points. Figures 8b and 9b show the wide variation in the quality of the estimated visibility pattern at various points on the terrain, thus supporting the exercise to validate the quality of the estimated visibility iteratively.

Significance of the Fundamental Topographic Features

Figures 10a and 11a show the comparison between the correlation coefficients and average approximation of the AVI and EVI calculated with the topographic feature targets and the random targets, at various target numbers. To begin with, note the considerably small number of targets compared to the total number of terrain points and yet the high correlation between the AVI and EVI of the observers. The figures show that at high target numbers, by virtue of their wider spatial distribution, random targets could provide a better approximation of the visibility pattern than could the topographic features. However, as the number of targets is decreased, the quality of the approximation degrades rapidly in the case of random targets but, on the other hand, topographic features provide a more consistent and better approximation. This suggests that, at high target numbers, the better correlation between the AVI and EVI is a result of both the spatial distribution and the topographical significance of the reduced number of targets. However, at low numbers, the topographical significance will be a more useful basis for placing targets across the terrain. Therefore, it can be stated that one could reduce the number of the topographic features for the visibility computation for large terrains with a large number of topographic features, without the fear of losing any significant visibility pattern information. The plot of the errors in each case (Figures 10b and 11b) essentially support the results based on correlation coefficients.

An interesting aspect of Figures 10a and 11a is the intersection of the topographic feature target and random target

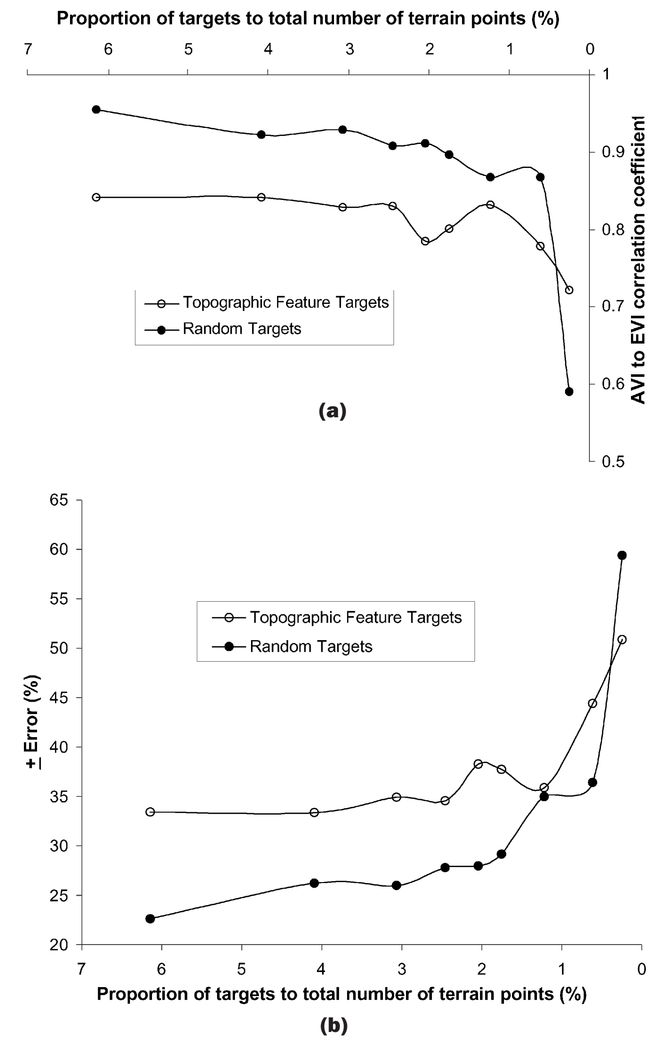

Figure 11. Validation of the significance of topographic features as optimal targets in the Isle of Man study area based on (a) comparison of the AVI to EVI correlation coefficients and (b) error in EVI for a decreasing number of topographic features and random targets.

correlation coefficient curves. It may indicate that in each terrain there are target numbers at which both the topographic feature targets and random targets could provide an equal level of spatial optimization. While there are situations in which this insight could be useful, for example, in deciding the optimal number of viewpoints for solving the time consuming minimum number of watchtowers problem (Lee, 1991), ironically it would not be possible to use this information without having done this iterative process.

Optimization of Computation Time

Figure 12 shows the linear relation between the CPU time usages versus the various magnitudes of visibility computations performed in the work. Computations here represent the product of the number of observers and the number of targets. The computation times for extracting the topographic features from the Cairngorm and Isle of Man DEMs in LandSerf were less than one second. As can be seen clearly, the time saved is substantial. However, the CPU time usage could be further optimized by combining the current approach with a *Reduced Targets Strategy* such as horizon culling.

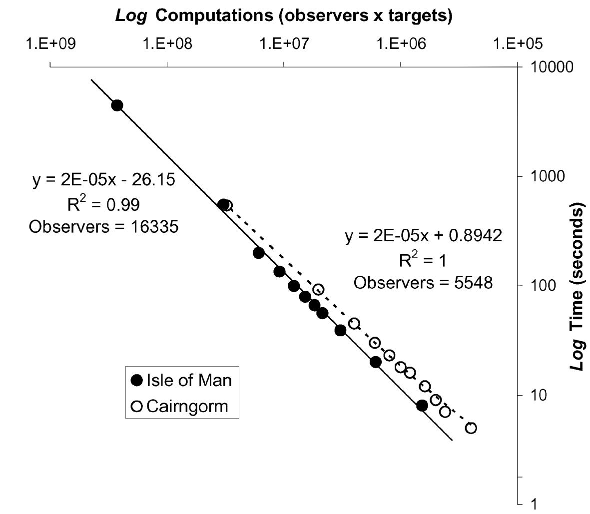

Figure 12. Linear relationship between the computation time and the computations.

Conclusion and Future Work

In this work, we have shown that the use of the fundamental topographic features as targets, as part of the *Reduced Targets Strategy*, can be used to decrease the visibility computation time substantially without any significant visibility information loss. This approach is especially useful for a fast approximation of visibility dominance in mountainous terrain. The reduced sampling of the targets on the terrain, however, introduces an uncertainty in the visibility indices of the observers on the terrain.

In the current work, the use of the correlation coefficient and the simple EVI to AVI ratio as measures of a visibility pattern quality and uncertainty provides only a global pattern matching, but visibility is a directional property. We anticipate developing ways in which we could estimate the visual integrity in our optimized approach. Although our observation that, at certain numbers, both topographic features targets and random targets would produce a similar quality of visibility estimation is based on thorough experimentation of the current study areas, experiments with other DEMs will be useful for fully validating this empirical observation.

The current work has also brought up a number of interesting questions, which could be investigated in future work. While the residuals between AVI and EVI in the case of the Cairngorm study area are uniform, in the case of Isle of Man study area, residuals appear to be dependent upon the visibility index magnitude. There can be many reasons for this anomaly, and one may even be able to model the non-uniform residuals in individual cases using a non-linear regression model. However, we believe it is more important to realize that visibility, as a property of terrain location, could not be modeled because it is derived only after an LOS test with other locations. Therefore, it is invariant of the local properties (e.g., elevation, slope, aspect) and global properties (e.g., geographic setting, i.e., faults, etc.) of a location. Thus, based on the current study, we believe that the regression between AVI and EVI only provides the information about the similarity or the amount of approximation.

Finally, two relatively straightforward extensions of the current work include (1) the combination of the *Reduced Observers Strategy* (e.g., horizon culling) and the proposed *Reduced Targets Strategy* for visibility index computation on very DEM and (2) the understanding of the effect of the topographic feature extraction scale on the computed visibility pattern.

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