

# Reference Dependence and Market Competition\*

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## Abstract

This paper studies the implications of consumer reference dependence for market competition. If consumers take some product (e.g., the first product they consider) as the reference point when evaluating others and exhibit loss aversion, then the more “prominent” firm whose product is taken as the reference point by more consumers will randomize over a high and a low price. We also find that loss aversion in the price dimension intensifies the price competition while that in the product dimension softens the price competition. Consumer reference dependence can also shape firms’ advertising strategies. If advertising increases product prominence, *ex ante* identical firms may differentiate their advertising intensities and asymmetric prominence between firms can arise as an equilibrium outcome.

**Keywords:** advertising, loss aversion, price competition, prominence, reference dependence

**JEL classification:** D11, D43, L13, M37

## 1 Introduction

Economists have recently shown great interest in studying the market implications of non-standard consumer behavior (Ellison (2006), for instance). A branch of this literature investigates how firms may respond to consumers’ reference-dependent

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preferences. Although the research of psychology and behavioral economics has accumulated abundant evidence of reference dependence with loss aversion (see, for example, Kahneman and Tversky (1979,2000), and DellaVigna (2008)), our understanding of its market implications is still insufficient. A recent work by Heidhues and Köszegi (2008) shows that, if consumers take their rational expectation of transaction results as the reference point, then consumer loss aversion can give rise to price stickiness in the market. However, in many circumstances, people encounter and consider products sequentially, and the first product they consider may become the reference point when they value later ones. For example, the product which a consumer is currently using may become the reference point when she comes to value new alternatives; the product which a consumer saw in an advertisement may become the reference point when she shops in a store where other options are also present. Hence, it is desirable to investigate how such consumer reference dependence displayed in sequential consideration could affect market competition. We will show that this kind of consumer reference dependence can cause price variation in the market (e.g., some firm uses the high-low promotional pricing strategy), and it can also induce *ex ante* identical firms to differentiate their advertising intensities.

Our assumption of the reference point is supported by the empirical findings concerning the order effect such as the status quo bias, the ballot order effect, and the default effect. They basically indicate that the option which is considered first could be favored disproportionately even if there is *little* cost involved in moving to options considered later.<sup>1</sup> One explanation is that people tend to regard the first option as the reference point when they come to value later ones, and they display loss aversion in the sense later options' relative disadvantages are weighed more than their relative advantages (Tversky and Kahneman (1991), for instance). Thus, all else equal, the early option may outperform later ones.

We consider a duopoly model where firms supply differentiated products and consumers consider products sequentially. Consumers take the first product as the reference point when they value the second one and exhibit loss aversion: when they pay a price higher than the reference price or buy a product less well matched than the reference product, they will suffer an extra psychological loss on top of

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<sup>1</sup>For example, Hartman et al. (1991) reported evidence that consumers who had experienced a highly reliable electrical service were unwilling to accept a low reliability option (with a lower rate) which is currently being experienced by another group of consumers, and consumers who had experienced low reliability exhibit a similar status quo bias. (See more evidence about status quo bias in Samuelson and Zeckhauser (1988), and Kahneman et al. (1991).) Ho and Imai (2006) and Meredith and Salant (2007) observed that being listed first on the ballot paper can significantly increase a candidate's vote share. Johnson et al. (1993) and Madrian and Shea (2001) found that the automobile insurance or pension scheme which is assigned to consumers as a default is eventually chosen with a significantly higher probability than other options.

the standard intrinsic utility loss.<sup>2</sup> We also allow that one firm might be more “prominent” than the other in the sense that the prominent product is more likely to be considered first (and so taken as the reference product) by consumers.<sup>3</sup>

Section 2 investigates the pricing implication of consumer reference dependence. With consumer reference dependence, firms’ price choices have a direct bearing on consumers’ price sensitivity. If the prominent firm charges a lower price than its rival, loss aversion in the price dimension makes consumers *more* price sensitive; on the contrary, if the prominent firm charges a higher price, loss aversion in the product dimension makes consumers *less* price sensitive. Thus, the prominent firm’s demand curve has an inward kink at its rival’s price, while the other firm’s demand curve has an outward kink at the prominent firm’s price. With this new function for price, the prominent firm has an incentive to randomize its price over a high and a low one, while the other firm will charge a constant medium price. All else equal, the prominent firm enjoys an advantage over its rival: it occupies a (weakly) larger market share and earns a higher profit. We also find that the reference-dependence effects in different dimensions have opposite impacts on the price competition: loss aversion in the price dimension intensifies the price competition, while that in the product dimension weakens the price competition.

Section 3 examines the advertising implication of consumer reference dependence. It is often believed that advertising can influence the order in which consumers consider products, and a more heavily advertised product is more likely to be considered first by consumers.<sup>4</sup> If advertising increases product prominence in this way, we show that consumer reference dependence can cause differentiated advertising strategies among *ex ante* identical firms. In particular, it can happen that one firm advertises and the other does not. Therefore, consumer reference dependence can lead to endogenous asymmetric prominence in the market. The reason is that a greater prominence difference between the two firms can boost each firm’s profits. Making a firm relatively more prominent can induce it to rely more on those consumers who favor its product and so charge the high price more frequently, which softens the price competition.

Our finding that the more prominent firm in the market has an incentive to vary

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<sup>2</sup>The reference-dependent effect does not always require that people possess the reference product for a long time, though it might be more pronounced in that case. For example, in most of experimental studies on the status quo bias and the endowment effect, the time of possessing the object is rather short and sometimes subjects only possess the object mentally.

<sup>3</sup>The prominent product could be the default option, the product which is more heavily advertised, the product which is recommended intensively or displayed more visibly in the store, or the product which has already been in the market when new products enter.

<sup>4</sup>Advertising can function in this way by, for example, attracting consumer attention. This view is different from the traditional views of informative advertising and persuasive advertising. See, for instance, Hann and Moraga-Gonzalez (2009) for more discussion.

its price (for example, it holds sales more frequently) may help explain the empirical observation that the national brand usually has a more variable price than the private brand in supermarkets (if consumers tend to take the national brand as the reference point). For example, Slade (1998) documents that the major-manufacturer prices of saltine crackers are more volatile than the private-label prices. Muller et al. (2006) provide similar evidence that the average number of price changes is significantly smaller for private label products than for national brands.

There has been some research in the marketing literature which shows the relevance of reference-dependent preferences for consumer choices. (See, for example, Putler (1992) and Hardie et al. (1993).) However, they do not study how firms might respond to such non-standard consumer behavior. In a dynamic monopoly setting, Fibich et al. (2007) (and other papers cited therein) show that, if consumers regard the historical price as the reference price, the firm should charge a constant price if the effect of a price rise is greater than a same-size price reduction (i.e., if loss aversion prevails); while if the effect of a gain is greater, the firm should price cyclically.

Little research has studied consumer reference dependence in more competitive markets. Heidhues and Köszegi (2008) is the first paper in this direction. Following the framework proposed in Köszegi and Rabin (2006), they assume that consumers' reference points are their rational expectations of transaction results. They then show that loss aversion can give rise to price stickiness.<sup>5</sup> Suppose that consumers expect to pay some fixed price before they enter the market. If a firm now sets a price higher than that, due to loss aversion, its demand will become *more* price responsive. Thus, each firm's demand curve has an outward kink at the expected price, which can induce all firms to actually charge that price for a range of cost conditions. Our paper takes a different assumption of the formation of consumer reference point and offers a qualitatively different prediction. The main reason is that in Heidhues and Köszegi (2008) consumers' reference point is their equilibrium expectation, and so no firm's actual decision can influence it; while our reference point is a real product in the market, so firms can manipulate it directly. The formation of consumers' reference points is usually context dependent. Their model may be more suitable for the market where consumers have sufficient purchase experience, while ours may be more suitable for the market where consumers purchase products infrequently.<sup>6</sup>

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<sup>5</sup>Their companion paper Heidhues and Köszegi (2005), among other results, shows a similar result in a monopoly setting.

<sup>6</sup>Other recent papers which study the implications of the reference-dependence effect (in a broader sense) in other circumstances include, for example, Compte and Jehiel (2007) (prior offers as reference points in sequential bargaining), Eliaz and Spiegler (2007) (the default alternative as the reference point in forming consideration sets), Hart and Moore (2008) (contracts as reference points in *ex post* trading relationship), and Rosenkranz and Schmitz (2007) (reserve prices in auctions as reference points in deciding on bidding strategies).

The reference-dependence effect in our model can be regarded as a particular kind of switching costs: moving from the reference product to the other involves psychological costs if the latter is relatively inferior in at least one aspect. However, it is rather different from traditional switching cost models (for example, Farrell and Klemperer (2007)) in both specifications and consequences. We will further discuss this difference in Section 4.

Finally, our paper is related to the recent work on prominence in search markets where consumers are assumed to sample the prominent product first in a sequential search process. If products are homogenous, Arbatskaya (2007) shows that the more prominent firm will charge a higher price. If products are differentiated and consumers search both for price and product fitness, Armstrong, Vickers, and Zhou (2009) show that the more prominent firm will actually charge a lower price. We take a similar assumption about prominence, but in our model the advantage of being prominent is from consumer reference dependence rather than exogenous search costs, and the more prominent firm is predicted to charge sometimes a higher and sometimes a lower price. Our discussion on advertising and prominence is related to Hann and Moraga-Gonzalez (2009). They take a similar assumption that the more heavily advertised products are more likely to be sampled first by consumers. In a search model with differentiated products, they show that *ex ante* symmetric firms will advertise at the same level and so there will be no prominence in equilibrium. While we find that, if the advantage of prominence is from consumer reference dependence, *ex ante* symmetric firms tend to differentiate their advertising efforts.

## 2 Reference Dependence and Pricing

This section examines how consumer reference dependence affects firms' pricing strategies. We start from the relatively simple case where all consumers take the same product as the reference product.

### 2.1 Model

There are two firms (1 and 2) in an industry, each supplying a single distinct brand at a constant unit cost which we normalize to zero. They set prices  $p_1$  and  $p_2$  simultaneously. Consumers have diverse tastes for the two brands. We model this scenario via the Hotelling linear city. A consumer's taste is represented by the parameter  $x$  which is distributed on the interval  $[0, 1]$  according to a cumulative distribution function  $F(x)$  which is differentiable and has a positive density  $f(x)$ . Firm 1 is located at the endpoint  $x = 0$  and firm 2 is at the other endpoint  $x = 1$ . For a consumer at  $x$ , the match utility of product 1 is  $v - x$ , and that of product 2 is  $v - (1 - x)$ , where  $v$  is the gross utility of the product and is assumed to be

sufficiently large such that the whole market is covered in equilibrium. There is a unit mass of consumers, each of them having unit demand for one product.

We introduce reference dependence by considering a sequential-consideration scenario where consumers consider products one by one, and a product’s price and match utility are discovered *only* when it is considered. We assume that a consumer’s valuation of the second product will be influenced by the first product she considered. Specifically, a consumer will take the first product as the reference point,<sup>7</sup> and will over weigh the second product’s relative disadvantage (higher price or lower match utility) in the spirit of loss aversion. We also assume that consumers do not intentionally choose the order in which they consider products, and they may just follow some natural presentation order of products or be guided by firms’ marketing activities. (We will discuss more sophisticated consumers in Section 4.) We temporarily suppose that *all* consumers will consider product 1 first (which is the default option, for instance), and we call it the reference product. We will treat a more flexible setting later.

Consumer preferences are specified as follows. Given the prices  $p_1$  and  $p_2$ , a consumer at  $x$  values product 1, the reference product, in the standard way:

$$v - x - p_1.$$

However, her valuation of product 2 is

$$v - (1 - x) - p_2 - (\lambda_p - 1) \max\{0, p_2 - p_1\} - (\lambda_t - 1) \max\{0, 1 - 2x\},$$

where the first three terms represent the standard intrinsic surplus of product 2 and the other two terms capture the potential reference-dependent “loss utility”.<sup>8</sup> The loss-aversion parameters  $\lambda_p > 1$  and  $\lambda_t > 1$  measure the strength of the reference-dependence effects in the price dimension and the product (or taste) dimension, respectively.<sup>9</sup> If  $\lambda_p = \lambda_t = 1$ , the reference-dependence effects disappear and we

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<sup>7</sup>In our model, the reference point is an individual product. An alternative specification of the reference point could be a weighted average of all products a consumer has considered before making a purchase decision. A product is more prominent if consumers put more weight on it. Our main results carry over to that case qualitatively. However, the assumption that the reference point is from the market rather from outside (for example, some “ideal” product already in a consumer’s mind before she enters the market) is important.

<sup>8</sup>We have made two simplifications in this specification of reference-dependent preferences: (i) we have normalized the psychological “gain utility” in the prospect theory to zero; (ii) we have used a linear loss utility function. One can consider a general setting in which the valuation of product 2 takes the form of  $v - (1 - x) - p_2 - u_p(p_2 - p_1) - u_t(1 - 2x)$ . Our main insights apply as long as the functions  $u_i$ ,  $i = p, t$ , have the following properties: (i)  $u_i$  is an increasing and differentiable function (except at zero), and  $u_i(0) = 0$ ; (ii)  $\lim_{\Delta \rightarrow 0^+} \frac{u_i(\Delta)}{u_i(-\Delta)} > 1$ .

<sup>9</sup>Although our main results are neutral to in which dimension consumers have more severe loss aversion, some empirical research (Hardie et al. (1993), for instance) suggests that loss aversion is more severe in the product dimension than in the price dimension.

return to the standard Hotelling model. An implicit assumption we have made is that the loss utility occurs separately in the price dimension and the taste dimension. This separability assumption is important in our model as in many other applications of reference dependence.<sup>10</sup> It is psychologically reasonable and has been extensively adopted in the literature of prospect theory (see, for example, Kahneman and Tversky (2000)).

To highlight how reference dependence could benefit the reference product, we focus on the case with a symmetric distribution of consumers (i.e.,  $F(1-x) = 1 - F(x)$ ). That is, there is no systematic quality difference between the two products. We also assume away any possible explicit costs involved in moving from one product to the other. Introducing such costs will bring firm 1 with an extra advantage.

Now we are ready to derive each firm's demand function. If firm 1 charges a lower price than firm 2, all consumers at  $x \leq \frac{1}{2}$  will buy product 1, and those at  $x > \frac{1}{2}$  will buy product 1 only if the gain from product 2's higher match utility is less than the loss (including the psychological part) from its higher price, i.e., only if  $2x - 1 < \lambda_p(p_2 - p_1)$ . Hence, firm 1's demand is

$$q_1(p_1, p_2) = F\left[\frac{1}{2} + \frac{\lambda_p}{2}(p_2 - p_1)\right] \text{ if } p_1 < p_2. \quad (1)$$

Consumers are now more price sensitive than in the orthodox model where  $\lambda_p = 1$ , because the attractiveness of firm 1's lower price has been amplified by consumer loss aversion in the price dimension.

If firm 1 charges a higher price than firm 2, all consumers at  $x > \frac{1}{2}$  will eventually buy product 2, and those at  $x < \frac{1}{2}$  will choose product 1 only if the loss (including the psychological part) from product 2's lower match utility exceeds the gain from its lower price, i.e., only if  $\lambda_t(1 - 2x) > p_1 - p_2$ . They now become less price sensitive, because the unattractiveness of firm 1's higher price has appeared less important relative to the unattractiveness of product 2's lower match utility. Firm 1's demand function is thus

$$q_1(p_1, p_2) = F\left[\frac{1}{2} + \frac{1}{2\lambda_t}(p_2 - p_1)\right] \text{ if } p_1 > p_2. \quad (2)$$

Expressions (1) and (2) imply that firm 1's demand is more price responsive at  $p_1 < p_2$  than at  $p_1 > p_2$ , and hence the demand curve has an *inward* kink at  $p_1 = p_2$  (see the first graph in Figure 1 below which illustrates the case with uniform  $x$ ).

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<sup>10</sup>If the loss utility occurs only for the net (standard) surplus of the product, then reference dependence will not affect demand functions at all. If the second product has a higher surplus, there is no loss utility and the consumer will buy it eventually; if the second product has a lower surplus, the consumer will not switch to it anyway no matter whether the loss utility is present or not.

Firm 2's demand is  $q_2 = 1 - q_1$ . Explicitly, using the symmetry of  $F$ , we have

$$q_2(p_1, p_2) = \begin{cases} F[\frac{1}{2} + \frac{\lambda_p}{2}(p_1 - p_2)] & \text{if } p_2 > p_1 \\ F[\frac{1}{2} + \frac{1}{2\lambda_t}(p_1 - p_2)] & \text{if } p_2 < p_1 \end{cases}. \quad (3)$$

When  $p_2 > p_1$ , the unattractiveness of firm 2's higher price will be amplified by loss aversion since consumers regard  $p_1$  as the reference price. When  $p_2 < p_1$ , the attractiveness of its lower price to the marginal consumer at  $x < \frac{1}{2}$  will be reduced by her extra aversion to product 2's lower match utility. Hence, firm 2's demand is more price responsive at  $p_2 > p_1$  than at  $p_2 < p_1$ , which implies that  $q_2$  has an *outward* kink at  $p_2 = p_1$  (see the second graph in Figure 1 below).

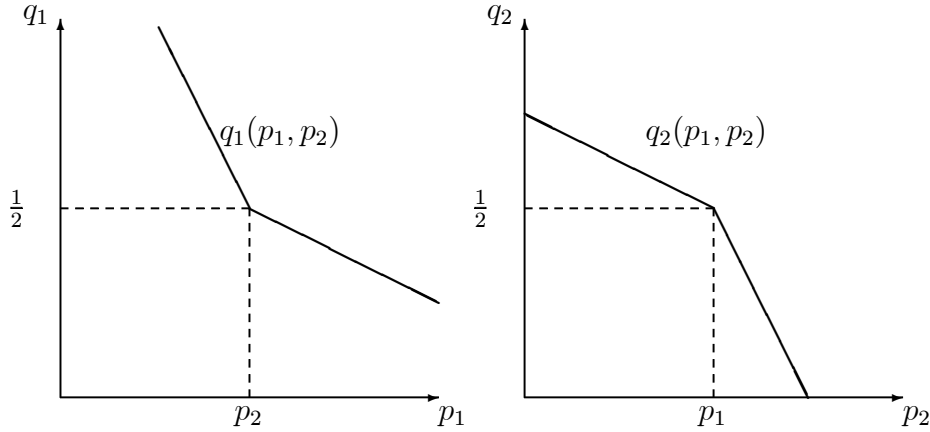


Figure 1: An Illustration of Demand Functions

In sum, compared to the orthodox case, demand becomes more price responsive if the reference product is relatively cheaper, and it becomes less price responsive if the reference product is relatively more expensive. Two additional properties of the demand function deserve mention. First,  $q_1 > q_2$  if and only if  $p_1 < p_2$ . In effect, reference dependence does not affect each firm's demand if they charge the same price.<sup>11</sup> However, it still affects the price sensitivity at that point. Second, at any fixed price pair  $p_1 \neq p_2$ , both firms' demand curves have the same slope since  $q_2 = 1 - q_1$  and  $\frac{\partial q_i}{\partial p_i} = -\frac{\partial q_i}{\partial p_j}$ .<sup>12</sup> In the following, we denote by  $\pi_i(p_1, p_2) \equiv p_i \cdot q_i(p_1, p_2)$  the profit function of firm  $i$ .

<sup>11</sup>It may be more realistic to consider a setting where the reference firm is given some advantage even if the two firms charge the same price. One possible modelling way (which does not cause demand discontinuity) is to assume that the consumer must incur an exogenous cost if she moves to the second product. Our main results remain qualitatively so long as this cost is relatively small relative to the reference dependence effect.

<sup>12</sup>Notice that this property does not depend on the assumption of a symmetric distribution of consumers.



## 2.2 Equilibrium

We now derive the Nash equilibrium of the price competition. First of all, both firms charging the same price is not an equilibrium. Given a positive price of firm 2, firm 1's demand has an inward kink at this price, which means that its profit function has a local minimum at this point. Hence, charging the same price will never be firm 1's best response. Second, we can also exclude the possibility of asymmetric pure-strategy equilibria. Thus, we have the following proposition. (All omitted proofs are presented in the Appendix.)

**Lemma 1** *Given a symmetric distribution of consumers, there is no pure-strategy Nash equilibrium.*

We will then show that, under regularity conditions, the game has a mixed-strategy equilibrium in which firm 1 charges a low price  $p_1^L$  with probability  $\mu \in (0, 1)$  and a high price  $p_1^H$  with probability  $1 - \mu$ , and firm 2 charges a medium price  $p_2 \in (p_1^L, p_1^H)$  for sure. The intuition is as follows. Given firm 2's price, firm 1 can either charge a lower price to make consumers more price sensitive and then earn a large market share, or charge a higher price to exploit those consumers who strongly favor its product and become less price sensitive due to loss aversion in the taste dimension.<sup>13</sup> Although these two strategies are equally profitable in equilibrium, firm 1 will not adopt either of them predictably. Otherwise, firm 2 would either be attempted to charge a low price to steal business when  $p_1^H$  applies, or be forced to match  $p_1^L$  to protect its own market share. Either situation will lower firm 1's profit, so firm 1 has an incentive to randomize its price and keep firm 2 guessing.

Given firm 1's mixed pricing strategy, let

$$q_2^e(p) \equiv \mu \cdot q_2(p_1^L, p) + (1 - \mu) \cdot q_2(p_1^H, p) \quad (4)$$

be firm 2's expected demand function. It has two outward kinks at  $p_1^L$  and  $p_1^H$  which divide it into three segments (see a uniform example in Figure 2 below). The regularity conditions we need are:

**Assumption 1** (i)  $f(x)$  is logconcave; (ii) for  $\mu \in (0, 1)$  and  $0 < p_1^L < p_1^H$ , each segment of firm 2's expected profit function  $p \cdot q_2^e(p)$  is quasi-concave.

The logconcavity condition is satisfied by many well-known (truncated if necessary) distributions. (See, for example, Bagnoli and Bergstrom (2005).) A sufficient condition for (ii) is that each segment of  $q_2^e(p)$  is logconcave. Although both  $q_2(p_1^L, p)$  and

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<sup>13</sup>This argument does not apply to firm 2. Given fixed  $p_1$ , if firm 2 charges a higher price, consumers will become more price sensitive, which will induce firm 2 to lower its price; if firm 2 charges a lower price than  $p_1$ , the marginal consumer who must have a strong taste for product 1 will become less price sensitive, which will induce firm 2 to raise its price.

$q_2(p_1^H, p)$  in (4) are logconcave under condition (i), a weighted average of them may not be. So we need condition (ii) separately.<sup>14</sup> Notice that the uniform distribution ( $F(x) = x$ ) satisfies both conditions in Assumption 1 since then each segment of  $q_2^e$  is linear.

**Proposition 1** *Given a symmetric distribution of consumers and Assumption 1, there exists a mixed-strategy equilibrium in which firm 1 charges a low price  $p_1^L$  with probability  $\mu$  and a high price  $p_1^H$  with probability  $1 - \mu$ , and firm 2 charges a medium price  $p_2 \in (p_1^L, p_1^H)$  for sure. The quadruplet  $(\mu, p_1^L, p_1^H, p_2)$  satisfies the following conditions:*

- (i)  $p_2 = \arg \max_p p \cdot q_2^e(p)$ ;
- (ii)  $p_1^L = \arg \max_{p \leq p_2} \pi_1(p, p_2)$ , and  $p_1^H = \arg \max_{p \geq p_2} \pi_1(p, p_2)$ ;
- (iii)  $\pi_1(p_1^L, p_2) = \pi_1(p_1^H, p_2)$ .

Conditions (i) and (ii) define each firm's best response given its rival's strategy, and condition (iii) means that firm 1 is indifferent between charging the low and the high price. A potential complication is, when the reference dependence effect is sufficiently strong, firm 1 may serve the whole market when it charges the low price  $p_1^L$ . As we show in the proof, such an equilibrium with a corner solution can actually occur. However, in the main text of this paper, we focus on the interior-solution equilibrium in which no firm captures all consumers (which requires a relatively small reference-dependence effect).<sup>15,16</sup>

In the uniform-distribution case, we can explicitly solve the equilibrium:

$$p_1^L = \frac{1 + 1/\sqrt{\lambda_p \lambda_t}}{2} \sqrt{\frac{\lambda_t}{\lambda_p}} < p_2 = \sqrt{\frac{\lambda_t}{\lambda_p}} < p_1^H = \frac{1 + \sqrt{\lambda_p \lambda_t}}{2} \sqrt{\frac{\lambda_t}{\lambda_p}} \quad (5)$$

and  $\mu = \frac{1}{1 + \sqrt{\lambda_p \lambda_t}}$ . Figure 2 below illustrates this equilibrium, where the kinked curves are firms' demand curves, and  $\pi_i$  is firm  $i$ 's iso-profit curve.

<sup>14</sup>However, one can show that condition (i) implies condition (ii) (or more precisely, the global concavity of firm 2's profit function) if  $\frac{f'(0)}{f(0)} < \frac{4}{1 + p_1^H \lambda_p}$ .

<sup>15</sup>A sufficient condition for the uniqueness of equilibrium is that, on top of Assumption 1, for any possible mixed pricing strategy of firm 1, firm 2's expected profit function will be globally quasi-concave. It has no simple primitive conditions, but it is satisfied at least in the uniform-distribution case as we will show below.

<sup>16</sup>As we show in the proofs of Lemma 1 and Proposition 1, the assumption of symmetric distribution of consumers can be replaced by a weaker one:  $F(\frac{1}{2}) = \frac{1}{2}$ . In effect, we can further show that for fixed  $\lambda_p, \lambda_t > 1$ , there exists  $\varepsilon > 0$  such that, when  $|F(\frac{1}{2}) - \frac{1}{2}| < \varepsilon$ , there is no pure-strategy equilibrium and a mixed-strategy equilibrium as in Proposition 1 exists.

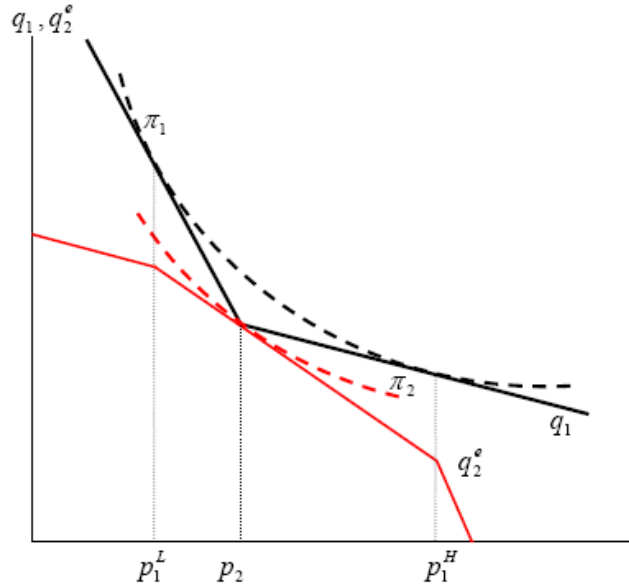


Figure 2: An Illustration of the Equilibrium

**The benefit of being the reference firm.** We now discuss the reference firm’s advantage over its rival due to the reference dependence effect.

**Proposition 2** *Given a symmetric distribution of consumers and Assumption 1, in the (interior-solution) mixed-strategy equilibrium we have identified, (i) firm 1 on average occupies a (weakly) larger market share than firm 2 (i.e.,  $q_2^e \leq \frac{1}{2}$ ), and they share the market equally if and only if the distribution is uniform; (ii) firm 1 earns a strictly higher profit than firm 2.*

These results are intuitive. As we can see from the demand function, if the reference firm charges a higher price than its rival, the shrink of its market share is mitigated by consumer loss aversion in the taste dimension; and if it charges a lower price, the expansion of its market share is amplified by consumer loss aversion in the price dimension. In either case, consumer reference dependence favors the reference firm.<sup>17</sup>

### 2.3 Heterogeneous reference products

We now allow for heterogenous reference products among consumers. Specifically, we suppose that  $\frac{1}{2} + \theta$  fraction of consumers will consider product 1 first and take it as the reference product, while  $\frac{1}{2} - \theta$  fraction of consumers will consider product 2 first and take it as the reference product. Without loss of generality, let  $\theta \in [0, \frac{1}{2}]$ . When  $\theta > 0$ , we say product 1 is more “prominent” than product 2, and  $\theta$  indicates the prominence difference between the two products.

<sup>17</sup>We can also show that firm 1 charges the high price more frequently (i.e.,  $\mu < \frac{1}{2}$ ) and product 1 is on average more expensive than product 2 (i.e.,  $\mu p_1^L + (1 - \mu)p_1^H > p_2$ ).

Each firm now has two demand sources: consumers who regard its product as the reference point and those who regard its rival's product as the reference point. If  $p_1 < p_2$ , firm 1's demand function becomes

$$q_1(p_1, p_2) = \left(\frac{1}{2} + \theta\right)F\left[\frac{1}{2} + \frac{\lambda_p}{2}(p_2 - p_1)\right] + \left(\frac{1}{2} - \theta\right)F\left[\frac{1}{2} + \frac{1}{2\lambda_t}(p_2 - p_1)\right].$$

The first part is similar as before, and the second part is because  $\frac{1}{2} - \theta$  of consumers take the relatively more expensive product as the reference point and so become less price sensitive. Similarly, if  $p_1 > p_2$ , we have

$$q_1(p_1, p_2) = \left(\frac{1}{2} + \theta\right)F\left[\frac{1}{2} + \frac{1}{2\lambda_t}(p_2 - p_1)\right] + \left(\frac{1}{2} - \theta\right)F\left[\frac{1}{2} + \frac{\lambda_p}{2}(p_2 - p_1)\right].$$

Define

$$h \equiv \left(\frac{1}{2} + \theta\right)\lambda_p + \left(\frac{1}{2} - \theta\right)\frac{1}{\lambda_t}; \quad l \equiv \left(\frac{1}{2} - \theta\right)\lambda_p + \left(\frac{1}{2} + \theta\right)\frac{1}{\lambda_t}. \quad (6)$$

Then, when  $p_1$  approaches to  $p_2$  from below, the absolute value of firm 1's demand slope is

$$\lim_{p_1 \rightarrow p_2^-} \left| \frac{\partial q_1}{\partial p_1} \right| = \frac{h}{2} f\left(\frac{1}{2}\right);$$

while when  $p_1$  approaches to  $p_2$  from above, it becomes

$$\lim_{p_1 \rightarrow p_2^+} \left| \frac{\partial q_1}{\partial p_1} \right| = \frac{l}{2} f\left(\frac{1}{2}\right).$$

For  $\theta \in (0, \frac{1}{2}]$  (i.e., when firm 1 is more prominent than firm 2), we have  $h > l$  and so  $q_1$  has an inward kink at firm 2's price. Firm 2's demand function can be treated similarly and it has an outward kink at firm 1's price.

Therefore, compared to the single-reference-product case, we should not expect any qualitative changes to take place. The counterparts of Lemma 1 and Proposition 1 can be proved similarly but with heavier notation. Although the counterpart of Proposition 2 has not been established completely, we conjecture it would hold. We will verify it in the uniform-distribution case below, and we can also verify it in the limit case with  $\lambda_p = \lambda_t$  approaching to one.<sup>18</sup>

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<sup>18</sup>If  $\lambda_p = \lambda_t = 1 + \varepsilon$  with  $\varepsilon \approx 0$ , equilibrium prices can be approximated as

$$\begin{aligned} p_1^L &\approx p[1 - \theta\varepsilon + (3\theta^2 - (1 - \theta)/2 - A)\varepsilon^2], \\ p_1^H &\approx p[1 + \theta\varepsilon + (3\theta^2 - (1 + \theta)/2 - A)\varepsilon^2], \\ p_2 &\approx p[1 + (2\theta^2 - 1/2)\varepsilon^2], \end{aligned}$$

and  $\mu \approx \frac{1}{2}[1 - (\theta + A/\theta)\varepsilon]$ , where  $p = 1/f(\frac{1}{2})$  is the equilibrium price in the standard Hotelling model and  $A = \frac{\theta^2 p^3}{16} f''(\frac{1}{2})$ . The complication in the limit analysis is that we need the second-order price approximation to show our results. All details are available from the author.

A special case is  $\theta = 0$ , i.e., when the two firms are equally prominent. In this symmetric case,  $h = l$  and so the demand functions are smooth everywhere. We then have a symmetric pure-strategy equilibrium with

$$p^* = \frac{2}{(\lambda_p + 1/\lambda_t)f(1/2)}. \quad (7)$$

However, any extent of prominence difference between the two firms will overturn this pure-strategy equilibrium.

## 2.4 The uniform-distribution case

We continue our analysis in the uniform-distribution case. By using the notation introduced in (6), the two firms' demand functions can now be written as

$$q_1 = \begin{cases} [1 + h(p_2 - p_1)]/2 & \text{if } p_1 < p_2 \\ [1 + l(p_2 - p_1)]/2 & \text{if } p_1 > p_2 \end{cases}; \quad q_2 = \begin{cases} [1 + h(p_1 - p_2)]/2 & \text{if } p_2 > p_1 \\ [1 + l(p_1 - p_2)]/2 & \text{if } p_2 < p_1 \end{cases}.$$

Since  $h > l$  for  $\theta > 0$ , consumers are more price sensitive when the prominent product is cheaper than the other. When firm 1 uses the mixed strategy, firm 2's expected demand function is

$$q_2^e(p_2) = \frac{1}{2}[1 + \mu h(p_1^L - p_2) + (1 - \mu)l(p_1^H - p_2)]$$

for  $p_2 \in [p_1^L, p_1^H]$ .

In this uniform setting, the following condition ensures an interior-solution equilibrium:

$$h < 9l, \text{ or } \frac{5\theta - 2}{5\theta + 2}\lambda_p\lambda_t < 1. \quad (8)$$

This condition is easier to hold for a smaller  $\theta$ . In particular, when  $\theta = \frac{1}{2}$ , it holds if  $\lambda_p\lambda_t < 9$ ; when  $\theta \leq \frac{2}{5}$ , it holds for any  $\lambda_p$  and  $\lambda_t$ .

**Proposition 3** *In the uniform-distribution case with  $\theta > 0$  and condition (8),*

(i) *there exists a unique equilibrium in which firm 1 charges a low price  $p_1^L$  with probability  $\mu = 1/(1 + \sqrt{h/l})$  and a high price  $p_1^H$  with probability  $1 - \mu$ , and firm 2 charges a medium price  $p_2$ , where*

$$p_1^L = \frac{1}{2}\left(\frac{1}{h} + \frac{1}{\sqrt{hl}}\right) < p_2 = \frac{1}{\sqrt{hl}} < p_1^H = \frac{1}{2}\left(\frac{1}{l} + \frac{1}{\sqrt{hl}}\right); \quad (9)$$

(ii) *in this equilibrium, the two firms share the market equally, but firm 1 earns a higher profit than firm 2:*

$$\pi_1 = \frac{1}{8}\left(\frac{1}{\sqrt{h}} + \frac{1}{\sqrt{l}}\right)^2 > \pi_2 = \frac{1}{2\sqrt{hl}}. \quad (10)$$

Therefore, similar results to Proposition 2 have been established in this heterogeneous-reference-product setting (with the uniform distribution).<sup>19</sup>

**The impact of loss aversion.** We now examine how the degree of loss aversion affects market competition in the uniform case. The following result indicates that the reference-dependence effects in different dimensions affect market competition in opposite directions.

**Proposition 4** *In the uniform-distribution case, all prices and profits decrease with  $\lambda_p$  but increase with  $\lambda_t$ .*

Since both  $h$  and  $l$  increase with  $\lambda_p$  but decrease with  $\lambda_t$ , this result can be readily seen from (9) and (10). More severe loss aversion in the price dimension will provide the prominent firm with more incentive to lower its prices, which intensifies the price competition and reduces profits. Hence, loss aversion in the price dimension is pro-competitive. However, more severe loss aversion in the taste dimension will induce the prominent firm to rely more on the consumers who have a strong taste for its product and so charge higher prices, which softens the price competition and improves profits. Hence, loss aversion in the taste dimension is anti-competitive. If  $\lambda_p$  and  $\lambda_t$  are correlated, the impact of loss aversion will be kind of combination of these two effects.

### 3 Reference Dependence and Advertising

In many circumstances, the order in which consumers consider products can be influenced by sellers' marketing efforts. For example, consumers may first notice the product which is heavily advertised or displayed prominently in the store; people often first click through the links displayed at the top of a webpage (e.g., the sponsor links). We have known that with consumer reference dependence, the prominent firm enjoys an advantage over its rival. Does that imply that firms will have an intense advertising competition (or more generally, marketing competition) in order to acquire prominence? This section considers an extended model in which firms compete first in advertising and then in price. If the relatively more heavily advertised product is more likely to be considered first (and so become the reference point), we will show that firms may actually differentiate their advertising intensities in order to soften the price competition, and *asymmetric* prominence between firms can arise as an equilibrium outcome.

We first extend our model. At the first stage, firms choose their advertising intensities simultaneously. If firm  $i$  advertises at intensity  $a_i$ ,  $i = 1, 2$ , then  $\frac{1}{2} +$

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<sup>19</sup>Beyond the uniform setting, we conjecture that for any  $\theta \in (0, 1/2]$  firm 1 would have a strictly greater market share than firm 2, as Proposition 2 has suggested.

$\theta(a_1, a_2)$  fraction of consumers will consider product 1 first and the other  $\frac{1}{2} - \theta(a_1, a_2)$  fraction of consumers will consider product 2 first. The function  $\theta(a_1, a_2) \in [-\frac{1}{2}, \frac{1}{2}]$  describes firm 1's relative prominence in the market. We assume that  $\theta$  increases in  $a_1$  and decreases in  $a_2$ , and  $\theta(a_1, a_2) = -\theta(a_2, a_1)$ , i.e., the advertising technology is symmetric between firms. (In particular, symmetry implies  $\theta(a, a) = 0$ .) Let  $c(a_i)$  be the advertising cost function and  $c' > 0$ . At the second stage, observing the outcome of advertising competition (which is summarized by the prominence index  $\theta$ ), firms choose prices simultaneously. For simplicity, we consider the uniform-distribution setting.

To analyze this extended game, we need to know how  $\theta$  affects each firm's profit in the pricing subgame. Let  $\pi_i(\theta)$  be firm  $i$ 's profit for a given  $\theta$ .

**Lemma 2** *In the uniform-distribution case, both  $\pi_1(\theta)$  and  $\pi_2(\theta)$  increase with  $\theta$  if  $\theta > 0$  and decrease with  $\theta$  if  $\theta < 0$ .*

That is, a greater prominence difference between the two firms will benefit both of them in the pricing subgame. The reason is, when more consumers consider the prominent firm first, it will rely more on those consumers who favor its product and so charge the high price more frequently,<sup>20</sup> which will further soften the price competition. The figure below is an example with  $\lambda_p = \lambda_t = 2$  and  $\theta$  varying from 0 to  $1/2$ , where the higher curve is  $\pi_1(\theta)$  and the lower one is  $\pi_2(\theta)$ .<sup>21</sup>

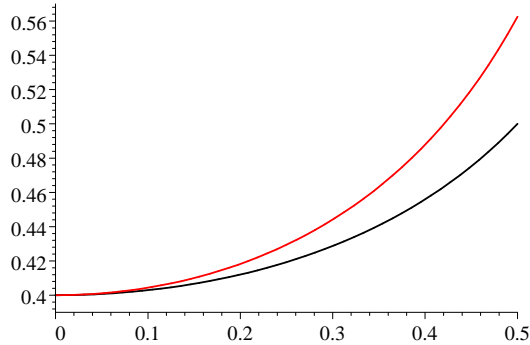


Figure 3: Profits and  $\theta$

Given the symmetry of our setting, we denote by  $\pi(a_i, a_j) \equiv \pi_i(\theta(a_i, a_j))$  firm  $i$ 's gross profit function when it advertises at intensity  $a_i$  and firm  $j$  advertises at intensity  $a_j$ . Then Lemma 2 implies that, for fixed  $a_j$ ,  $\pi(a_i, a_j)$  increases in  $a_i$  if

<sup>20</sup>It is easy to see that  $1 - \mu = \sqrt{h/l} / (1 + \sqrt{h/l})$  increases with  $\theta$  since  $h$  increases and  $l$  decreases with  $\theta$ .

<sup>21</sup>We conjecture that our result would hold even for general distributions. For example, in the limit case with  $\lambda_p = \lambda_t = 1 + \varepsilon$  with  $\varepsilon \approx 0$ , using the results in footnote 18, we can approximate equilibrium profits as  $\pi_1(\theta) \approx \frac{p}{2} + (\frac{3}{2}\theta^2 - \frac{1}{4})p\varepsilon^2$  and  $\pi_2(\theta) \approx \frac{p}{2} + (\theta^2 - \frac{1}{4})p\varepsilon^2$  for  $\theta > 0$ . Both of them are increasing in  $\theta$ .

$a_i > a_j$  and decreases in  $a_i$  if  $a_i < a_j$ , and so it reaches its minimum at  $a_i = a_j$ . Firm  $i$ 's net profit function, when its rival advertises at  $a_j$ , is

$$\pi(a_i, a_j) - c(a_i).$$

We report possible advertising equilibria in the following proposition:

**Proposition 5** *Suppose  $a^* = \arg \max_a [\pi(a, 0) - c(a)]$ , a firm's best response when the other firm does not advertise, exists uniquely.*

(i) *If  $a^* = 0$ , the pure-strategy advertising equilibrium is symmetric with  $a_1 = a_2 = 0$ .*

(ii) *If  $a^* > 0$  and  $\arg \max_a [\pi(a, a^*) - c(a)] = 0$ , the pure-strategy advertising equilibria are asymmetric with  $a_i = a^*$  and  $a_j = 0$ ,  $i \neq j$ .*

(iii) *If  $a^* > 0$  and  $\arg \max_a [\pi(a, a^*) - c(a)] > a^*$ , there is no pure-strategy advertising equilibrium.*

The proof is simple. In light of Lemma 2, both firms advertising at positive intensities cannot be an equilibrium outcome. Otherwise at least the firm which is advertising (weakly) less intensively would improve its profit by reducing its advertising investment unilaterally and further enlarging the prominence difference. Hence, in a pure-strategy advertising equilibrium, the two firms will either both not advertise or differentiate their advertising efforts such that one advertises and the other does not. If  $a^* = 0$ , then both firms not advertising is the only pure-strategy equilibrium. If  $a^* > 0$  and  $\arg \max_a [\pi(a, a^*) - c(a)] = 0$ , then in equilibrium one firm advertises at  $a^*$  and the other does not advertise at all.<sup>22</sup> If  $a^* > 0$  and  $\arg \max_a [\pi(a, a^*) - c(a)] > a^*$ , no pure-strategy advertising equilibrium can be sustained. (Notice that it is impossible that  $\arg \max_a [\pi(a, a^*) - c(a)] \in (0, a^*]$  since  $\pi(a, a^*)$  decreases in  $a \in [0, a^*]$  and  $c'(a) > 0$ .)

Two comments are in order. First, whether  $a^* > 0$  or  $a^* = 0$  depends on the properties of  $\pi(a, 0)$  (or the  $\theta$  function) and  $c(a)$ . Given  $\pi(a, 0)$ ,  $a^* > 0$  is more likely for less costly advertising. In our uniform setting, we can show that a sufficient condition for  $a^* > 0$  is  $c'(0) = 0$  and  $c''(0) = 0$  (e.g.,  $c(a) = ka^3$ ). Second, when the pure-strategy equilibrium fails to exist, the advertising competition game has mixed-strategy equilibria if all payoff functions are continuous and the feasible advertising intensities are bounded. However, exploring mixed-strategy advertising equilibria requires more structures of  $\theta(a_1, a_2)$  and  $c(a_i)$ .<sup>23</sup>

**An example.** To illustrate our main points, we present an example with a discrete advertising choice. Suppose each firm can only choose to advertise or not

<sup>22</sup>In this case, there also exists a symmetric mixed-strategy advertising equilibrium, where two *ex ante* identical firms still differentiate their advertising intensities.

<sup>23</sup>One general result we can derive is that, if there is any symmetric mixed-strategy advertising equilibrium and  $\left| \frac{\partial \pi(0,0)}{\partial a_i} \right| < \infty$ , then each firm does not advertise with a strictly positive probability.



( $a_i \in \{0, 1\}$ ), and the advertising cost is  $c > 0$ . If one firm advertises and the other does not, then all consumers will take the advertised product as the reference product (i.e.,  $\theta(1, 0) = 1/2$ ); if both firms advertise or neither of them does, then each product will be equally likely to be the reference product (i.e.,  $\theta(0, 0) = \theta(1, 1) = 0$ ). By using the profit expressions in (10), it is not difficult to show: (i) when the advertising cost is relatively small such that

$$c < \frac{1}{8} \left[ 1/\sqrt{\lambda_p} + \sqrt{\lambda_t} \right]^2 - \frac{1}{\lambda_p + 1/\lambda_t}, \quad (11)$$

$a^* = 1$  and in equilibrium one firm advertises and the other does not;<sup>24</sup> (ii) when condition (11) fails to hold,  $a^* = 0$  and in equilibrium both firms do not advertise. (Case (iii) in Proposition 5 has been automatically excluded in this binary-choice example.)

Another two observations follow immediately in this example. First, if  $\lambda_p = \lambda_t = \lambda$ , then the right-hand side of (11) increases with  $\lambda$ .<sup>25</sup> That is, for a given advertising cost, the asymmetric advertising equilibrium is more likely to emerge for a higher degree of loss aversion. Second, when (11) holds, there is also a symmetric mixed-strategy advertising equilibrium. However, both firms earn less in that equilibrium compared to in the pure-strategy one.

## 4 Discussion

**Sophisticated consumers.** In our model, the order in which consumers consider products is specified exogenously or manipulated by firms' marketing efforts. One issue is whether our mixed-strategy pricing equilibrium can still be sustained if consumers realize their own reference-dependent preferences and if they can fully control their own consideration orders.

First of all, as we have pointed out, if  $\theta = 0$  (i.e., if the two firms are equally prominent), we have a symmetric pure-strategy pricing equilibrium given in (7). In turn, such a pricing equilibrium supports a random consumer consideration order. (Remember that consumers do not know a product's match utility until they consider it, so consumers are *ex ante* identical.) Therefore, for sophisticated consumers, a symmetric equilibrium always exists. However, as the following proposition shows, the mixed-strategy pricing equilibrium (with  $\theta = 1/2$ ) can also be sustained for a range of loss-aversion parameters. That is, if all consumers consider some firm first, this firm will randomize its price; given this firm is randomizing its price, consumers actually prefer to consider it first. We focus on the uniform-distribution case.

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<sup>24</sup>Notice that the first term in the right-hand side of (11) is greater than  $\frac{1}{2}\sqrt{\lambda_t/\lambda_p}$  while the second term is less than  $\frac{1}{2}\sqrt{\lambda_t/\lambda_p}$ . Thus, the right-hand side of (11) is always positive.

<sup>25</sup>The right-hand side of (11) increases with  $\lambda_t$  but varies with  $\lambda_p$  non-monotonically.

**Proposition 6** *Suppose consumers are sophisticated and can choose their own consideration orders.*

(i) *There always exists a symmetric equilibrium in which consumers consider products in a random order and each firm charges  $p^* = 2/(\lambda_p + 1/\lambda_t)$ .*

(ii) *If consumers do not include the psychological loss utility in their welfare calculation and*

$$\lambda_p + 1/\lambda_t > 2, \quad (12)$$

*or if they do and*

$$(\lambda_p^2 \lambda_t - 1)[(\lambda_p \lambda_t)^{3/2} - 1] < 4\lambda_p \lambda_t (\lambda_p - 1)(\sqrt{\lambda_p \lambda_t} + 1), \quad (13)$$

*there also exist two asymmetric equilibria in which all consumers consider one firm first and firms use the pricing strategies specified in (5).*

The range of parameters for asymmetric equilibria is non-trivial. For example, when  $\lambda_p = \lambda_t = \lambda$ , condition (12) always holds, and condition (13) holds for  $\lambda$  less than about 1.98.

In fact, the prominent firm charges on average a higher price than the other (see footnote 17). So our result implies that a consumer may actually prefer to consider the more expensive product first. The reason is that considering the more expensive product first can prevent consumers from being over “addicted” to the low price (due to loss aversion) at the expense of taste satisfaction.<sup>26</sup>

**More firms.** The situation with more than two firms may become complicated, depending on the evolution process of consumers’ reference points as they consider more products. However, if all consumers take some product (for example, the default option) as the reference point in evaluating *all* other products, a similar equilibrium as in the duopoly case will exist. That is, the reference firm will randomize its price while all other firms will charge a constant price. For example, in the Salop circular-city model with  $n$  firms and uniform distribution, there is an equilibrium in which the reference firm randomizes its price between  $p_1^L = \frac{1}{2n}(\frac{1}{\lambda_p} + \sqrt{\frac{\lambda_t}{\lambda_p}})$  and  $p_1^H = \frac{1}{2n}(\lambda_t + \sqrt{\frac{\lambda_t}{\lambda_p}})$ , while all other firms charge a constant price  $p_2 = \frac{1}{n}\sqrt{\frac{\lambda_t}{\lambda_p}}$ .

**Is loss aversion similar to switching costs?** The reference-dependence effect in our model can be regarded as a particular kind of switching costs. But it occurs only if the second product is relatively inferior to the first one in at least one aspect. Readers may wonder whether the results of this paper could be replicated in a setting

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<sup>26</sup>More precisely, two conflicting forces operate here: on the one hand, if a consumer considers the cheaper product first, she will become extra averse to paying a higher price due to loss aversion and so will be more likely to buy at the first firm, which of course can save her expected payment; on the other hand, this will also result in more severe product-choice distortion and so cause a greater expected taste loss. Essentially, conditions (12) and (13) guarantee that the latter negative effect dominates.

where there is an exogenous cost involved in switching from the first product to the second. In that setting, it is conceivable that the prominent firm will still earn more than the other, but we do not have the counterparts of other main results. Let us consider the uniform setting. Instead of the reference-dependence effect, we introduce an exogenous switching cost  $s$ . Then, with the same notation, the demand functions are:

$$q_1 = \frac{1}{2} + \theta s + \frac{p_2 - p_1}{2}; \quad q_2 = \frac{1}{2} - \theta s + \frac{p_1 - p_2}{2}.$$

It is ready to see that no firm will randomize its price. If  $s$  is appropriate such that we have an interior-solution equilibrium, equilibrium prices and profits are:

$$p_1 = 1 + \frac{2}{3}\theta s, \quad \pi_1 = \frac{1}{2}\left(1 + \frac{2}{3}\theta s\right)^2; \quad p_2 = 1 - \frac{2}{3}\theta s, \quad \pi_2 = \frac{1}{2}\left(1 - \frac{2}{3}\theta s\right)^2.$$

Clearly, making one firm more prominent benefits this firm but harms the other, and so our results concerning advertising and endogenous prominence cannot hold either.

## 5 Conclusion

This paper has examined the impact of consumer reference dependence displayed in sequential consideration on market competition. We have shown that the prominent firm whose product is more likely to become the reference point has an incentive to randomize between a high and a low price. Hence, consumer reference dependence can cause price variation in the market. We have also shown that consumer reference dependence can shape firms' advertising strategies. If advertising increases product prominence by influencing the order in which consumers consider products, *ex ante* identical firms may differentiate their advertising intensities. Taken together with the existing research such as Heidhues and Kőszegi (2008), our analysis indicates that the market implications of consumer reference dependence may be sensitive to the specification of reference points. In particular, it is crucial whether consumers' reference points are independent of, or influenced by, firms' actual decisions.

Several extensions deserve future study. First, it would be interesting to explore the impact of consumer reference dependence in a dynamic competition model where consumers purchase products repeatedly and the historical purchase might influence the reference point. Second, it is also desirable to consider the market with vertically differentiated products. Intuitively, making reference-dependent consumers consider a high-quality product first may decrease their price sensitivity and so soften the price competition. This may influence how a platform displays products with heterogenous qualities, how a multi-product firm launches its differentiated new products, and how firms differing in product qualities choose their advertising strategies.

# A Appendix

## A.1 Proof of Lemma 1

We prove a more general result:

**Claim 1** *If the distribution of consumers satisfies  $F(\frac{1}{2}) = \frac{1}{2}$  (of which the symmetric distribution is a special case), the price competition has no pure-strategy Nash equilibrium.*

Denote by  $q'_i(p_1, p_2)$  the partial derivative of firm  $i$ 's demand function with respect to  $p_i$ , and let  $\pi'_i(p_1, p_2) = q_i(p_1, p_2) + p_i \cdot q'_i(p_1, p_2)$ . If  $p_1 = p_2 = \hat{p} > 0$  were an equilibrium, we need

$$\lim_{p_1 \rightarrow \hat{p}^+} \pi'_1(p_1, \hat{p}) \leq 0 \leq \lim_{p_1 \rightarrow \hat{p}^-} \pi'_1(p_1, \hat{p}).$$

However, the inward kinked demand function of firm 1 implies

$$\lim_{p_1 \rightarrow \hat{p}^-} q'_1(p_1, \hat{p}) = -\frac{\lambda_p}{2} f\left(\frac{1}{2}\right) < \lim_{p_1 \rightarrow \hat{p}^+} q'_1(p_1, \hat{p}) = -\frac{1}{2\lambda_t} f\left(\frac{1}{2}\right),$$

which leads to

$$\lim_{p_1 \rightarrow \hat{p}^-} \pi'_1(p_1, \hat{p}) < \lim_{p_1 \rightarrow \hat{p}^+} \pi'_1(p_1, \hat{p}).$$

This is a contradiction, so  $p_1 = p_2 = \hat{p}$  cannot be an equilibrium.

Now suppose  $p_1 > p_2 > 0$  were an equilibrium. Since each firm's demand function will be smooth around its own equilibrium price in this hypothetical equilibrium,<sup>27</sup> we have

$$q_1 + p_1 q'_1(p_1, p_2) = q_2 + p_2 q'_2(p_1, p_2) = 0.$$

Since  $q'_1(p_1, p_2) = q'_2(p_1, p_2)$  for  $p_1 \neq p_2$ , we get

$$\frac{q_1(p_1, p_2)}{q_2(p_1, p_2)} = \frac{p_1}{p_2}.$$

On the other hand, when  $p_1 > p_2$ , we have  $q_1(p_1, p_2) < F(\frac{1}{2}) = \frac{1}{2} < q_2(p_1, p_2)$ . This is again a contradiction, so  $p_1 > p_2$  can neither be an equilibrium. Using the same logic, we can also exclude the possibility of  $p_1 < p_2$ .

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<sup>27</sup>It is impossible for firm 2 to serve the whole market at  $p_2 > 0$ , since firm 1 would then choose  $p_2 - \varepsilon$  to earn a positive profit. Hence, each firm's demand function must be smooth around its own price in this hypothetical equilibrium.

## A.2 Proof of Proposition 1

We show a more general result:

**Claim 2** *Given Assumption 1 and  $F(\frac{1}{2}) = \frac{1}{2}$ , there exists a mixed-strategy equilibrium in which firm 1 randomizes over  $p_1^L$  and  $p_1^H$  and firm 2 charges a constant price  $p_2 \in (p_1^L, p_1^H)$ .*

We prove this result by dealing with the interior-solution equilibrium and the corner-solution equilibrium separately.

**The interior-solution equilibrium.** We first rewrite the equilibrium conditions for an interior-solution equilibrium in Proposition 1. Define

$$z_L \equiv \frac{1}{2} + \frac{\lambda_p}{2}(p_2 - p_1^L); \quad z_H \equiv \frac{1}{2} + \frac{1}{2\lambda_t}(p_2 - p_1^H). \quad (14)$$

They are the locations of consumers who are indifferent between the two products when firm 1 charges  $p_1^L$  and  $p_1^H$ , respectively. Then the demand functions are

$$q_1(p_1^i, p_2) = F(z_i), \quad i = L, H; \quad q_2^e(p) = \mu [1 - F(z_L)] + (1 - \mu) [1 - F(z_H)].$$

Let  $F_i \equiv F(z_i)$  and  $f_i \equiv f(z_i)$ . Then firm 2's best response (i.e., condition (i) in Proposition 1) requires

$$\mu(1 - F_L) + (1 - \mu)(1 - F_H) = \frac{p_2}{2} \left( \mu \lambda_p f_L + \frac{1 - \mu}{\lambda_t} f_H \right), \quad (15)$$

and firm 1's best response (i.e., condition (ii) in Proposition 1) requires  $F_L = \lambda_p p_1^L f_L / 2$  and  $F_H = p_1^H f_H / (2\lambda_t)$ , which can be further written as

$$\frac{F_L}{f_L} + z_L = \frac{\lambda_p}{2} p_2 + \frac{1}{2}, \quad (16)$$

$$\frac{F_H}{f_H} + z_H = \frac{1}{2\lambda_t} p_2 + \frac{1}{2}, \quad (17)$$

by using  $p_1^L = p_2 + (1 - 2z_L) / \lambda_p$  and  $p_1^H = p_2 + \lambda_t(1 - 2z_H)$  from (14). The indifference condition (iii) in Proposition 1 is

$$p_1^L F_L = p_1^H F_H. \quad (18)$$

Equations (15)–(18) define an equilibrium if

- (a) they have a solution  $(\mu, p_2, z_L, z_H)$  with  $\mu \in (0, 1)$  and  $0 \leq z_H < \frac{1}{2} < z_L \leq 1$ ;
- (b) no firm has global profitable deviation given its rival's strategy.

We now show that conditions (a) and (b) are indeed satisfied under  $F(\frac{1}{2}) = \frac{1}{2}$  and Assumption 1. Let  $k_1 \equiv F(1)/f(1)$  and  $k_2 \equiv F(\frac{1}{2})/f(\frac{1}{2})$ . Since logconcave

$f$  implies logconcave  $F$ ,  $F(z)/f(z)$  is an increasing function and so  $k_1 > k_2$ . We consider two cases:

*Case 1: if the degree of loss aversion is relatively small such that  $\lambda_p \lambda_t < (k_1 + \frac{1}{2})/k_2$ , we have a mixed-strategy equilibrium with an interior solution.*

We first consider condition (a). First, since the left-hand side of (17) is an increasing function of  $z_H$  and the right-hand side is always positive, we have  $z_H > 0$ . Second, we prove  $z_H < \frac{1}{2} < z_L < 1$ . It is true if

$$\frac{2k_2}{\lambda_p} < p_2 < 2k_2 \lambda_t \quad (19)$$

by realizing that the left-hand sides of (16)–(17) are both increasing in  $z_i$ .<sup>28</sup> Now we show that (18) does have a solution satisfying (19) given (16)–(17). If  $p_2 = 2k_2/\lambda_p$ , then (16) and (17) imply  $z_L = \frac{1}{2}$  (i.e.,  $p_1^L = p_2$ ) and  $0 < z_H < \frac{1}{2}$  (i.e.,  $p_1^H > p_2$ ), so

$$\pi_1(p_1^H, p_2) = \max_{p \geq p_2} p \cdot q_1(p, p_2) > p_2 \cdot q_1(p_2, p_2) = \pi_1(p_1^L, p_2).$$

Similarly, if  $p_2 = 2\lambda_t k_2$ , then  $z_H = \frac{1}{2}$  (i.e.,  $p_1^H = p_2$ ) and  $\frac{1}{2} < z_L < 1$  (i.e.,  $p_1^L < p_2$ ), so

$$\pi_1(p_1^L, p_2) = \max_{p \leq p_2} p \cdot q_1(p, p_2) > p_2 \cdot q_1(p_2, p_2) = \pi_1(p_1^H, p_2).$$

Hence, the indifference condition  $\pi_1(p_1^H, p_2) = \pi_1(p_1^L, p_2)$  implies (19). (Notice that  $p_1^L$  and  $p_1^H$  are solved as functions of  $p_2$  from (16)–(17).)

Third, we show that (15) has a solution  $\mu \in (0, 1)$ . Using (16)–(17), we rewrite (15) as

$$\mu \left[ 1 - 2F_L - (z_L - \frac{1}{2})f_L \right] + (1 - \mu) \left[ 1 - 2F_H - (z_H - \frac{1}{2})f_H \right] = 0. \quad (20)$$

Then we solve

$$\mu = \frac{g_H}{g_H - g_L}, \quad (21)$$

where  $g_i \equiv 1 - 2F(z_i) - (z_i - \frac{1}{2})f(z_i)$ . Given  $z_H < \frac{1}{2} < z_L$  and  $F(\frac{1}{2}) = \frac{1}{2}$ , we have  $g_L < 0 < g_H$ , and so  $\mu \in (0, 1)$ .

We then consider condition (b). Under Assumption 1, firm 1's demand  $F(z_i)$  is logconcave in  $p_1^i$  and so its profit function must be quasi-concave. Thus, the necessary conditions (16)–(17) are also sufficient for optimization. For firm 2, it has no profitable deviation on  $[p_1^L, p_1^H]$  since Assumption 1 ensures that its profit function on this interval is quasi-concave. But does it have any incentive to deviate to  $p_2 < p_1^L$  or  $p_2 > p_1^H$ ? Under Assumption 1, its profit function is quasi-concave

<sup>28</sup>Notice that  $z_L < 1$  if  $p_2 < 2(k_1 + \frac{1}{2})/\lambda_p$ , which is a looser condition than  $p_2 < 2\lambda_t k_2$  given  $\lambda_p \lambda_t < (k_1 + \frac{1}{2})/k_2$ .

in each situation, and so a sufficient condition for neither case to be a profitable deviation is that

$$\lim_{p_2 \rightarrow (p_1^L)^-} \frac{\partial \pi_2^e}{\partial p_2} > 0, \quad \lim_{p_2 \rightarrow (p_1^H)^+} \frac{\partial \pi_2^e}{\partial p_2} < 0$$

where  $\pi_2^e = p \cdot q_2^e(p)$ . They are actually true because (i) Assumption 1 and  $p_2 \in (p_1^L, p_1^H)$  imply

$$\lim_{p_2 \rightarrow (p_1^L)^+} \frac{\partial \pi_2^e}{\partial p_2} \geq 0, \quad \lim_{p_2 \rightarrow (p_1^H)^-} \frac{\partial \pi_2^e}{\partial p_2} \leq 0,$$

and (ii) the two kinks of  $q_2^e$  at  $p_1^L$  and  $p_1^H$  are both outward.

*Case 2:* we also have a mixed-strategy equilibrium with an interior solution if  $f(1) \leq 2$  and the degree of loss aversion is medium such that  $(k_1 + \frac{1}{2})/k_2 < \lambda_p \lambda_t < k_1 f(\bar{z})/F(\bar{z})^2$ , where  $\bar{z} \leq \frac{1}{2}$  solves<sup>29</sup>

$$\frac{k_1}{k_1 + 1/2} = \frac{F(\bar{z})^2}{F(\bar{z}) + (\bar{z} - 1/2)f(\bar{z})}. \quad (22)$$

The same logic as in Case 1 applies except that now instead of (19) we need to show

$$\frac{2k_2}{\lambda_p} < p_2 < \frac{2k_1 + 1}{\lambda_p}.$$

(See footnote 28.) If  $p_2 = 2k_2/\lambda_p$ , the same proof as before implies  $\pi_1(p_1^L, p_2) < \pi_1(p_1^H, p_2)$ , where  $p_1^L$  and  $p_1^H$  are solved as functions of  $p_2$  from (16)–(17). It then suffices to show, if  $p_2 = 2(k_1 + \frac{1}{2})/\lambda_p$ , we have  $\pi_1(p_1^L, p_2) > \pi_1(p_1^H, p_2)$ . When  $p_2 = (2k_1 + 1)/\lambda_p$ , (16) implies  $z_L = 1$ . Then  $p_1^L = p_2 - 1/\lambda_p = 2k_1/\lambda_p$  and so  $\pi_1(p_1^L, p_2) = 2k_1/\lambda_p$ . Meanwhile, notice that the first-order condition for  $p_1^H$  to be the best response is  $F_H = p_1^H f_H/(2\lambda_t)$ , and so  $\pi_1(p_1^H, p_2) = p_1^H F_H = 2\lambda_t F_H^2/f_H$ . Then  $\pi_1(p_1^L, p_2) > \pi_1(p_1^H, p_2)$  if and only if  $k_1 > \lambda_p \lambda_t F_H^2/f_H$ . Since at  $p_2 = (2k_1 + 1)/\lambda_p$  condition (17) requires  $z_H$  to satisfy

$$\frac{1}{\lambda_p \lambda_t} (k_1 + \frac{1}{2}) = \frac{F_H}{f_H} + z_H - \frac{1}{2}, \quad (23)$$

the condition  $k_1 > \lambda_p \lambda_t F_H^2/f_H$  is further equivalent to

$$\frac{k_1}{k_1 + 1/2} > \frac{F_H^2}{F_H + (z_H - 1/2)f_H}. \quad (24)$$

From (22), we know that (24) holds if  $z_H > \bar{z}$  since the right-hand side is decreasing in  $z_H$ .  $z_H > \bar{z}$  is implied by  $\lambda_p \lambda_t < k_1 f(\bar{z})/F(\bar{z})^2$  and (23).

<sup>29</sup>One can verify that (22) has a solution  $\bar{z} \leq \frac{1}{2}$  if  $f(1) \leq 2$ . Let  $z_0$  be the solution to  $F(z_0) + (z_0 - \frac{1}{2})f(z_0) = 0$ . Using the fact  $f^2 > Ff'$  (which is implied by logconcave  $F$ ), one can check that the right-hand side of (22) is a decreasing and positive function on  $(z_0, \frac{1}{2})$  (which varies from  $\infty$  to  $\frac{1}{2}$ ) and a decreasing and negative function on  $(0, z_0)$ . Therefore,  $\bar{z} \in (z_0, \frac{1}{2}]$  when  $k_1 \geq \frac{1}{2}$  (i.e., when  $f(1) \leq 2$ ).

**The corner-solution equilibrium.** We claim that we have a mixed-strategy equilibrium with a corner solution if  $f(1) > 2$  and  $\lambda_p \lambda_t \geq (k_1 + \frac{1}{2})/k_2$  or if  $f(1) \leq 2$  and  $\lambda_p \lambda_t \geq k_1 f(\bar{z})/F(\bar{z})^2$ . Under Assumption 1, the following conditions guarantee a corner-solution equilibrium:

- (i)  $p_1^H = \arg \max_{p \geq p_2} p \cdot q_1(p, p_2)$ ;
- (ii)  $p_1^L = p_2 - 1/\lambda_p$  from  $z_L = 1$ ;
- (iii)  $p_1^L$  is the best response to  $p_2$  (i.e.,  $\frac{\partial}{\partial p} p q_1(p, p_2) \leq 0$  at  $p = p_1^L$ );
- (iv) The indifference condition  $p_1^L = \pi_1(p_1^H, p_2)$ ;
- (v)  $p_2$  is the best response to  $(p_1^L, p_1^H, \mu)$ . That is,

$$\frac{p_2}{2} \left[ \mu \lambda_p f(1) + \frac{1 - \mu}{\lambda_t} f_H \right] = (1 - \mu)(1 - F_H). \quad (25)$$

We need to show that the above conditions have a solution with  $0 < p_1^L < p_2 < p_1^H$  and  $\mu \in (0, 1)$ .

First, under Assumption 1, condition (i) is again equivalent to (17), so  $p_2 < p_1^H$  (or  $z_H < \frac{1}{2}$ ) is implied by  $p_2 < 2\lambda_t k_2$ . Second, condition (iii) requires  $1 - \lambda_p f(1) p_1^L / 2 \leq 0$ , or equivalently  $p_2 \geq (2k_1 + 1)/\lambda_p$  by using condition (ii). (Note that this condition also ensures  $p_1^L > 0$ ). Hence, we need to show

$$\frac{2k_1 + 1}{\lambda_p} \leq p_2 < 2k_2 \lambda_t. \quad (26)$$

If  $p_2$  tends to  $2\lambda_t k_2$ , then (17) implies  $z_H = \frac{1}{2}$  (i.e.,  $p_1^H = p_2$ ) and condition (iii) is satisfied, so

$$\pi_1(p_1^L, p_2) = p_1^L = \max_{p < p_2} p \cdot q_1(p, p_2) > p_2 \cdot q_1(p_2, p_2) = \pi_1(p_1^H, p_2).$$

If  $p_2$  tends to  $(2k_1 + 1)/\lambda_p$ , the same argument as in the above Case 2 yields that  $\pi_1(p_1^L, p_2) = p_1^L < \pi_1(p_1^H, p_2)$  if and only if  $k_1 < \lambda_p \lambda_t F_H^2 / f_H$ , where  $z_H$  is again determined by (23). Reversing the proof there proves this inequality. Thus, the indifference condition implies (26).

Finally, we prove that (25) has a solution  $\mu \in (0, 1)$ . When  $\mu = 1$ , the left-hand side of (25) is positive but the right-hand side is zero. When  $\mu = 0$ , the left-hand side is  $p_2 f_H / (2\lambda_t)$  and the right-hand side is  $1 - F_H$ . The left-hand side is smaller if  $k_2 < (1 - F_H) / f_H$  by using  $p_2 \leq 2k_2 \lambda_t$ . This inequality is actually true because  $(1 - F_H) / f_H$  decreases with  $z_H$  (which is from logconcave  $1 - F$ ),  $z_H < \frac{1}{2}$ , and  $F(\frac{1}{2}) = \frac{1}{2}$ .

### A.3 Proof of Proposition 2

We first show the following preliminary results:



**Lemma 3** Suppose  $f(x)$  is symmetric and logconcave on  $[0, 1]$ .

(i) The function

$$\alpha(x) \equiv \frac{F(x)^2}{F(x) + (x - 1/2)f(x)}$$

decreases on  $(z_0, \frac{1}{2})$  and increases on  $(\frac{1}{2}, 1]$ , where  $z_0$  solves  $F(z_0) + (z_0 - \frac{1}{2})f(z_0) = 0$ . For any  $\varepsilon \in (0, \frac{1}{2} - z_0)$ ,  $\alpha(\frac{1}{2} - \varepsilon) > \alpha(\frac{1}{2} + \varepsilon)$ .

(ii) The function

$$\beta(x) \equiv \frac{1/2 - F(x)}{(1/2 - x)f(x)}$$

is symmetric on  $[0, 1]$ . For the uniform distribution,  $\beta(x) = 1$ . Beyond this special case,  $\beta(x)$  strictly decreases on  $[0, \frac{1}{2})$  and strictly increases on  $(\frac{1}{2}, 1]$ .

**Proof.** (i) Logconcave  $f$  implies logconcave  $F$ , and so  $F/f$  is an increasing function. Hence,  $\alpha(x)$  is positive on  $(z_0, 1]$ . One can verify that  $\alpha'(x)$  has the sign of  $(x - \frac{1}{2})(2f^2 - Ff')$ . Since the second term must be positive for logconcave  $F$ ,  $\alpha(x)$  decreases on  $(z_0, \frac{1}{2})$  and increases on  $(\frac{1}{2}, 1]$ . Second, notice that

$$\alpha(1/2 - \varepsilon) = \frac{(1/2 - A)^2}{1/2 - A - B\varepsilon}, \quad \alpha(1/2 + \varepsilon) = \frac{(1/2 + A)^2}{1/2 + A + B\varepsilon},$$

where  $A = \frac{1}{2} - F(\frac{1}{2} - \varepsilon) = F(\frac{1}{2} + \varepsilon) - \frac{1}{2} > 0$  and  $B = f(\frac{1}{2} - \varepsilon) = f(\frac{1}{2} + \varepsilon)$ . Then  $\alpha(\frac{1}{2} - \varepsilon) > \alpha(\frac{1}{2} + \varepsilon)$  if and only if

$$(A + \frac{1}{4A})(1 + \frac{B}{A}\varepsilon) > \frac{1}{2}.$$

This inequality is always true since  $A + 1/(4A) \geq 1$ .

(ii) The symmetry of  $\beta(x)$  is from the symmetry of  $f$ . Clearly,  $\beta(x) = 1$  for the uniform distribution. We now show that  $\beta(x)$  strictly decreases on  $[0, \frac{1}{2})$  for non-uniform distributions. Since  $f$  is logconcave and symmetric, it must increase on  $[0, \frac{1}{2})$ , and so  $F$  is convex on  $[0, \frac{1}{2})$ . One can show that, for  $x < \frac{1}{2}$ ,  $\beta'(x) < 0$  if

$$\frac{1/2 - F}{1/2 - x} < f + (\frac{1}{2} - F)\frac{f'}{f}.$$

The left-hand side is increasing on  $[0, \frac{1}{2})$  since  $F$  is convex on this interval, while the right-hand side is decreasing because its derivative has the sign of  $ff'' - (f')^2 < 0$  (which is implied by logconcave  $f$ ). We further observe that, when  $x$  tends to  $\frac{1}{2}$ , both sides tend to  $f(\frac{1}{2})$ . Therefore, the above inequality must hold for  $x < \frac{1}{2}$ . ■

We continue to prove our main results. We first show

$$\frac{1}{2} - z_H < z_L - \frac{1}{2}, \tag{27}$$

where  $z_i$  is defined in (14). To prove this, we rewrite the indifference condition (18) as

$$\frac{F_L^2/f_L}{F_H^2/f_H} = \lambda_p \lambda_t \quad (28)$$

by using firm 1's first-order conditions  $F_L = \lambda_p p_1^L f_L/2$  and  $F_H = p_1^H f_H/(2\lambda_t)$ . On the other hand, (16)–(17) imply

$$\frac{F_L/f_L + (z_L - 1/2)}{F_H/f_H + (z_H - 1/2)} = \lambda_p \lambda_t. \quad (29)$$

Then, from (28)–(29), we have  $\alpha(z_L) = \alpha(z_H)$ . The result (27) then follows from result (i) in Lemma 3.<sup>30</sup>

(i) We now show that firm 1 on average occupies a larger market share. Using  $\mu = g_H/(g_H - g_L)$  and the definition of  $g_i$  in (21), one can check that  $q_2^e = 1 - \mu F_L - (1 - \mu)F_H \leq \frac{1}{2}$  if and only if

$$(F_L - \frac{1}{2})g_H \geq -(\frac{1}{2} - F_H)g_L,$$

or equivalently  $\beta(z_H) \leq \beta(z_L)$ . For the uniform distribution,  $\beta(z_H) = \beta(z_L)$  and so  $q_2^e = \frac{1}{2}$ . Beyond this special case,  $\beta(z_H) < \beta(z_L)$  because of result (ii) in Lemma 3 and result (27).

(ii) Given firm 2's price  $p_2$ , firm 1 can at least earn  $p_2/2$  by charging the same price. Thus,  $\pi_1 > p_2/2 \geq \pi_2$ , where the second inequality is because  $q_2^e \leq \frac{1}{2}$ .

## A.4 Proof of Proposition 3

(i) Given  $p_2$ , firm 1's best responses are<sup>31</sup>

$$p_1^L = \frac{1}{2h} + \frac{p_2}{2}; \quad p_1^H = \frac{1}{2l} + \frac{p_2}{2}.$$

Combining them with firm 1's indifference condition we can solve those equilibrium prices in (9). From firm 2's best response, we can further pin down  $\mu = 1/(1 + \sqrt{h/l})$ . Given  $h > l$ , it is ready to see that  $p_1^L < p_2 < p_1^H$ . For this configuration to be an equilibrium, we need further to ensure that no firm captures all consumers. Firm 1's demands are  $q_1^L = \frac{1}{4}(1 + \sqrt{h/l})$  and  $q_1^H = \frac{1}{4}(1 + \sqrt{l/h})$  when it charges  $p_1^L$  and  $p_1^H$ , respectively, so condition (8) implies that both demands are less than one.

We then prove the uniqueness. Since firm 2's profit function is strictly concave for any fixed  $p_1$  in this uniform setting, its expected profit function, corresponding to any mixed pricing strategy of firm 1, must be also strictly concave. Firm 2 will therefore never randomize its price in equilibrium. Now suppose firm 2 charges  $p_2$

<sup>30</sup>Notice that (17) and the definition of  $z_0$  in part (i) of Lemma 3 imply  $z_H > z_0$ .

<sup>31</sup>In the uniform setting, all necessary conditions are also sufficient.

in equilibrium. Since firm 1's demand function is linear when  $p_1 < p_2$  or  $p_1 > p_2$ , it has at most two best response prices. Since there is no pure-strategy equilibrium, the proposed mixed-strategy equilibrium must be the unique one.

(ii) The proof is straightforward and so omitted.

## A.5 Proof of Lemma 2

Due to the symmetry of our model, we only need to prove the case with  $\theta > 0$  (i.e., when firm 1 is more prominent than firm 2). In the pricing subgame with uniform distribution, firm 1 and firm 2 have profits  $\pi_1 = (1/\sqrt{h} + 1/\sqrt{l})^2/8$  and  $\pi_2 = 1/(2\sqrt{hl})$ , respectively (see (10)).  $\pi_1$  increases with  $\theta$  because the derivative of  $1/\sqrt{h} + 1/\sqrt{l}$  with respect to  $\theta$  has the sign of  $1/(l\sqrt{l}) - 1/(h\sqrt{h}) > 0$ , and  $\pi_2$  increases with  $\theta$  because  $hl = (\lambda_p + 1/\lambda_t)^2/4 - \theta^2(\lambda_p - 1/\lambda_t)^2$  decreases with  $\theta$ .

## A.6 Proof of Proposition 6

The proof of (i) is trivial. For (ii), we consider two cases separately, depending on whether consumers include the psychological loss utility in their welfare calculation.

(ii-a) We first consider the case where consumers do not take into account the loss utility. Without loss of generality, suppose all consumers consider product 1 first such that a mixed-strategy pricing equilibrium as in (5) exists. We now show that under condition (12) consumers do obtain higher surplus if they consider product 1 first given the two firms' pricing strategies. Let  $\Delta_L \equiv p_2 - p_1^L$  and  $\Delta_H \equiv p_1^H - p_2$ , and define

$$x_L \equiv \frac{1}{2} + \frac{\lambda_p}{2} \Delta_L; \quad x_H \equiv \frac{1}{2} - \frac{1}{2\lambda_t} \Delta_H.$$

Then, for a consumer considering product 1 first, if its price is  $p_1^i$ ,  $i = L, H$ , she will buy product 1 if and only if she finds out that her taste location is less than  $x_i$ . Hence, the sum of her expected taste loss and payment is

$$\alpha_i = \int_0^{x_i} (x + p_1^i) dx + \int_{x_i}^1 (1 - x + p_2) dx.$$

Since firm 1 is using the mixed pricing strategy, the expected surplus of a consumer who considers product 1 first is  $v - \mu\alpha_L - (1 - \mu)\alpha_H$ . Similarly, if we let

$$y_L \equiv \frac{1}{2} + \frac{1}{2\lambda_t} \Delta_L; \quad y_H \equiv \frac{1}{2} - \frac{\lambda_p}{2} \Delta_H,$$

the expected surplus of a consumer who considers product 2 first is  $v - \mu\beta_L - (1 - \mu)\beta_H$ , where

$$\beta_i = \int_0^{y_i} (x + p_1^i) dx + \int_{y_i}^1 (1 - x + p_2) dx$$

is the sum of expected taste loss and payment when firm 1 charges  $p_1^i$ . Therefore, considering product 1 first is better if and only if

$$\mu\alpha_L + (1 - \mu)\alpha_H < \mu\beta_L + (1 - \mu)\beta_H. \quad (30)$$

One can verify

$$\alpha_L - \beta_L = \underbrace{\int_{y_L}^{x_L} (2x - 1)dx}_{\text{extra taste loss}} - \underbrace{\int_{y_L}^{x_L} \Delta_L dx}_{\text{payment saving}} = \frac{M}{4}\Delta_L^2,$$

where  $M \equiv (\lambda_p - 1/\lambda_t)(\lambda_p + 1/\lambda_t - 2)$ . Similarly,

$$\alpha_H - \beta_H = \int_{y_H}^{x_H} (2x - 1 + \Delta_H)dx = -\frac{M}{4}\Delta_H^2.$$

If (12) holds, then  $M > 0$  and so condition (30) holds if and only if  $\mu\Delta_L^2 < (1 - \mu)\Delta_H^2$ , or equivalently,

$$\frac{\mu}{1 - \mu} < \frac{\Delta_H^2}{\Delta_L^2} = \lambda_p\lambda_t.$$

This inequality must be true since  $\mu = 1/(1 + \sqrt{\lambda_p\lambda_t}) < \frac{1}{2}$ .

(ii-b) Now consider the case where consumers take into account the loss utility.

The same logic applies except that now

$$\begin{aligned} \hat{\alpha}_L &= \alpha_L + (\lambda_p - 1) \int_{x_L}^1 \Delta_L dx, & \hat{\alpha}_H &= \alpha_H + (\lambda_t - 1) \int_{x_H}^{1/2} (1 - 2x) dx; \\ \hat{\beta}_L &= \beta_L + (\lambda_t - 1) \int_{1/2}^{y_L} (2x - 1) dx, & \hat{\beta}_H &= \beta_H + (\lambda_p - 1) \int_0^{y_H} \Delta_H dx. \end{aligned}$$

No matter which firm consumers consider first, they now suffer extra loss utility. A straightforward but lengthy calculation shows that considering product 1 is better if and only if

$$\mu\Delta_L[2(\lambda_p - 1) - (\lambda_p^2 - 1/\lambda_t)\Delta_L] < (1 - \mu)\Delta_H[2(\lambda_p - 1) - (\lambda_p^2 - 1/\lambda_t)\Delta_H].$$

By using the price expressions in (5), it is further equivalent to (13).

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