Intergenerational Altruism, Ricardian Equivalence and the Relevance of Distributional Policy

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Abstract: In recent years there has been a growing controversy concerning the effective role of distributional policy in economies in which agents hold altruistic preferences. We show that in order to establish cross sectional Distributional Neutrality households require information on consanguinity links prior to making their decisions on consumption and bequests. If such information is not available due to the veil of time, then the Distributional Neutrality property disappears. However Ricardian Equivalence under stated conditions, remains and is therefore a property independent of DN.

Summary: In recent years there has been a growing controversy concerning the effective role of distributional policy in economies in which agents hold altruistic preferences. In an overlapping generations model with altruistic behaviour, Bernheim and Bagwell(1987) have purported to show the irrelevance not only of fiscal and monetary policies which switch resources between generations (Ricardian Equivalence (RE)); but also of policies which switch resources between members of the same generation (cross sectional Distributional Neutrality (DN)); and argue that DN is a necessary consequence of RE. In the present paper we show that in order to establish DN households require information on consanguinity links prior to making their decisions on consumption and bequests. If such information is not available due to the veil of time, then the DN property disappears. However Ricardian Equivalence, under stated conditions, remains and is therefore a property independent of DN. We show that for RE, households must know only that they will be related to a given number of households in the future, whereas for DN they also require precise details of the characteristics of these households. Thus the Bernheim-Bagwell arguments concerning the irrelevance of all redistributional policies arising out of intra-family linkages appear somewhat misplaced, and the theoretical case for Ricardian Equivalence is rather more robust than previously supposed. However, we show that for growth models in which intra-family linkages are explicitly introduced rather different results are generated than are found in the more usual formulation of the overlapping generations model, e.g. A Nash equilibrium with undercapitalization in steady growth. In this respect it becomes more difficult to accept results which flow from models which postulate families with asexual reproduction; i.e. the assumption found most often in the overlapping generations literature.

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I Introduction

In recent years there has been a growing controversy concerning the effective role of distributional policies in economies in which agents hold altruistic sentiments. In particular stemming from the seminal contribution of Barro(1974), work has centred on the circumstances under which government actions via fiscal or monetary policy, which affect household circumstances, may be neutralized through counterveiling action of the households The original neutrality result noted by Barro concerned an overlapping generations model, in which if each generation acted altruistically, at least with respect to its immediate heirs, then debt funding, under certain circumstances, would have no effect on the consumption stream of the "extended" family; i.e. each family would act in a similar (although not identical) fashion to an infinite-lived individual.

The validity of this neutrality proposition (Ricardian Equivalence) and its relevance to economic policy can be questioned from a number of directions; e.g. the Barro solution relies on "interior" solutions to the optimization decision. Intuitively, if family members are pushed to the limit of their adjustment capabilities, and the government continues to push, intertemporal adjustment of real resources may be accomplished. Furthermore, the inclusion of an explicit role for expectations and uncertainty within the Barro model may also lead to a modification of the neutrality result.

However the stance could be taken that any of these modifications would substantially alter the nature of Barro's original model. An alternative critique of the neutrality proposition has been followed by Bernheim and Bagwell (1988), Abel and Bernheim(1991). The approach taken in these papers is to argue that within Barro's own framework, the model implies neutrality with respect to a great number of alternative government actions, in addition to debt manipulation. Discussions of the neutrality results may be found in Barro(1989a),(1989b), and Bernheim(1989).

The intuition behind the Bernheim and Bagwell critique can be seen as follows. In the Barro intertemporal family chain, as governments attempt to (marginally) switch resources from one period to another, they are offset by (marginal) adjustments made by families to maintain the original intertemporal balance of consumption. As Bernheim and Bagwell note individuals need have no immediate concern for the welfare of others, other than their immediate successors; this is enough to ensure that any independent change in the circumstances of a descendant would be sufficient to imply a modification in the consumption behaviour of the current generation, as the required adjustments fan back through time. Bernheim and Bagwell then argue that in addition to such an intertemporal consanguinity chain, a *cross sectional* consanguinity chain exists, since procreation (usually) involves relationships between unrelated individuals. If such individuals have altruistic feelings in respect at least of their immediate successors, then again fanning forwards and backwards along and across generations, chains are built up to encompass the entire population. Such cross-sectional chains imply a wide variety of neutrality results, extending not only to the distributional effects of taxes and subsidies, but also, for example, to the relevance of the price system for allocation of resources between households.

Bernheim and Bagwell therefore argue that since such consequences are not observed in the real world, the basic formulation of altruistic preferences as embodied in the overlapping generations model of Barro(1974) must be fatally flawed.
The implications of these type of criticisms are therefore that models of economic growth which do not explicitly model family chains may give misleading results concerning household behaviour; in particular with regard to the effectiveness of government policy; and with respect to the specification of the long run growth properties of the economy. In the present paper we argue that in order to establish Distributional Neutrality households require information on consanguinity links prior to making their decisions on consumption and bequests. If such information is not available due to the veil of time, then the Distributional Neutrality property disappears. However Ricardian Equivalence, under stated conditions, remains and is therefore a property independent of Distributional Neutrality. We show that for Ricardian Equivalence, households must know only that they will be related to a given number of households in the future, whereas for Distributional Neutrality they also require precise details of the characteristics of these households. In the case of growth models in which intra-family linkages are explicitly introduced rather different results are generated than are found in the more usual formulation of the overlapping generations model, e.g. a Nash equilibrium with undercapitalization in steady growth. In this respect results which flow from models which postulate families with asexual reproduction, the assumption found most often in the overlapping generations literature, are not robust.
The paper is organized as follows. In Section II we briefly reprise the Distributional Neutrality and Ricardian Equivalence results for the two generational model established by Bernheim and Bagwell(1988) and Abel and Bernheim(1991). We then develop an alternative two generational model in which household behaviour is based on marriage probabilities of progeny. In Section III we present the implications for the steady state behaviour of an infinite period overlapping generations model. Finally in Section IV we briefly recapitulate the results and interpretations.

II The Two Generational Model

We assume that each person lives for two periods, childhood and adulthood. During each period each individual is a member of a single household; but of a different household as between the two periods. In the childhood period, the individual is a member of their parents' household; consisting of their parents and siblings; in adulthood of their own household, consisting of spouse and children. Each household is taken as the basic decision making unit in household matters. Given its resources, to be defined below, each household determines its consumption, and the amount it wishes to transfer to the children of that household in the form of inheritances. The disposition of the consumption of the household between its members is not considered here, only its totality. Likewise we abstract from lifecycle considerations regarding savings, consumption, and fertility choice; i.e. the number of children in each household is exogenous; and in the simplest case is two.
The only difference of the model from the standard overlapping generations model is therefore that of household formation; i.e. the marriage partners chosen by the children of a given household, to form the next generations set of households. In the constant population case we shall assume that each household has a male and female child, and that new households are formed by intermarriage between the children of different households. An apparently innocuous assumption , and one which broadly accords with reality; however

which as we shall see, has rather profound implications for the evolution and equilibrium of the economy.
We shall initially consider the neutrality results established by Bernheim and Bagwell(1988), and further developed by Abel and Bernheim(1991) for the two period case. The distinguishing feature of the Abel,Bernheim,Bagwell approach is the construction of a cross sectional "consanguinity" chain, which builds up across the population as a result of intermarriage between the siblings of different households. The linkage between households is generated as follows: Consider a model with only two generations, with n households in each generation. Each household of the first generation has two children; who subsequently marry children of other households to form the households of the second generation. A number of consanguinity chains may of course be postulated, however the chain considered by Abel and Bernheim has the following structure: Household i of the first generation, h_i, is assumed to be linked to Household i, H_i, and Household $i+1$,H_{i+1}, of the second generation $(i < n)$.The nth household of the first generation is linked to household n and household 1, of the second generation.
Note, of course, this is not the only feasible chain that could be constructed; e.g. the smallest set of complete chains occur when both children of household j marry the children of household $j+1$, where j are the odd integers (assuming even n, and no brother-sister households).
Now consider the optimal choices for the ith household of the first generation. The only resources available to this household are wage payments which it may spend on its own consumption and transfers to each of its children, i.e.,

$$w_i^1 = c_i^1 + k_{i,i}^2 + k_{i,i+1}^2 \tag{1}$$

whilst for households of the second generation, their resources are the inheritances from their parents plus their wage income; all these resources are consumed, and so,

$$w_i^2 + k_{i,i}^2 + k_{i-1,i}^2 = c_i^2 \tag{2}$$

The utility of a household of the first generation is,

$$U_i = U(c_i^1) + \mu[U(c_i^2) + U(c_{i+1}^2)] \tag{3}$$

$\mu > 0$. Whereas the utility of a household of the second generation is,

$$U_i = U(c_i^2) \tag{4}$$

In order to determine its optimal choice of consumption and bequests, household i has to make assumptions regarding the bequests of other households, i.e. of households $i-1$, and $i+1$, since the welfare of its childrens' households depend on the value of these bequests. We shall focus attention on the competitive Nash equilibrium, in which each household maximizes its welfare under the assumption that the bequests of other households are given. Thus the Nash equilibrium has the property that, $k_{i,i}^*, k_{i,i+1}^*$ solves,

$$\max_{k_{i,i},k_{i,i+1}} \{U(c_i^1) + \mu[U(c_i^2) + U(c_{i+1}^2)]\} \tag{5}$$

for all i, subject to the constraints 1,2.
The first order conditions are,

$$\frac{\partial U(c_i^1)}{\partial c_i^1} = \mu \frac{\partial U(c_i^2)}{\partial c_i^2} \tag{6}$$

$$\frac{\partial U(c_i^1)}{\partial c_i^1} = \mu \frac{\partial U(c_{i+1}^2)}{\partial c_{i+1}^2} \tag{7}$$

for $i = 1$ to n, where the $n+1$ household is household $n = 1$.
Given concavity assumptions on the utility functions these are sufficient to establish an equilibrium.
Equations 6,7, together with the constraints 1 and 2 form a system of $4n$ equations, in the $4n$ unknowns, i.e. the consumption levels of both generations, c_i^1, c_i^2; and the transfers made to each child $k_{i,i}, k_{i,i+1}$. However one of the equations is redundant, since the index $n+1 = 1$; thus we have an underdetermined system. However, summing over 1 and 2, we obtain,

$$\Sigma_{i=1}^n [c_i^1 + c_i^2 - w_i^1 - w_i^2] = 0 \tag{8}$$

Thus taking 1, 2,6,7, excepting the redundant equation, together with 8, we have a system of 2n equations, to determine the levels of consumption of both generations, $c_i^1, c_i^2, i = 1, \ldots, n$. With consumption levels thus determined, inspection of the constraints, 1,2 show us that bequest levels are undetermined up to an additive constant, provided bequests remain non-negative; i.e. in equilibrium, increasing one child's bequest at the expense of the other leaves consumption levels of both generations unchanged.
The important neutrality properties of the model noted by Abel,Bernheim and Bagwell are:
(i) Shifting resources from one generation to another, i.e. increasing w_i^1 at the expense of w_i^2 leaves the consumption equilibrium unchanged; provided we retain positive bequests. In this context this denotes the *Ricardian Equivalence* result.
(ii) Shifting resources between individuals of the same generation, e.g. increasing w_i^1 at the expense of w_{i+j}^1 again leaves the consumption equilibrium unchanged; provided we retain positive bequests. This result may be termed the cross sectional *Distributional Neutrality* result.
Both results follow from inspection of the equation system 1,2,6,78, excluding the redundant equation; i.e. the distribution of consumption depends only on the total availability of resources, $\Sigma[w_i^1 + w_i^2]$, not on its distribution.
Distributional neutrality in the Abel,Bernheim, Bagwell analyses thus results from the existence of a consanguinity chain spanning the entire population. One criticism of the approach is to deny the existence of such a chain; e.g. due to the existence of childless households, or independent circular chains within the population, e.g. along racial or religious lines. However whilst it may be doubted whether such an all encompassing chain would be generated in two generations, Abel and Bernheim would argue that the two generational model is a proxy for the multi-period case; in which chains spanning very large sections of the population, would undoubtedly be generated.
In the present paper we do not intend to question the existence of a such a chain, only its relevance in terms of the planning of the households which form part of the chain. A major distinction must be made between the *ex post* position of belonging to a chain and the *ex ante* position of planning consumption and bequests. We again consider the two generation model in which the first generation household has two children, but now in making it's disposition with regard to consumption and bequests is assumed to have no certain knowledge of the marriage partners of it's children; in other words whilst each household of generation i knows with certainty that it will be linked to two households of the next generation, it does not know with certainty which households.

We set out the model as follows. Consider the ith household of the first generation. The only resources available to this household are wage payments which it may spend on its own consumption, and bequests to each child, i.e.,

$$w_i^1 = c_i^1 + k_{i,1}^2 + k_{i,2}^2 \tag{9}$$

where $k_{i,1}^2, k_{i,2}^2$ denote respectively the transfers to the first (female)and second (male) child. The utility of household i of the first generation is,

$$U_i = U(c_i^1) + \mu[U_{i,1} + U_{i,2}] \tag{10}$$

whereas the utility of each child is simply

$$U_{i,1} = U(c_{i,1}^2); U_{i,2} = U(c_{i,2}^2) \tag{11}$$

The problem facing household i is therefore to choose c_i^1 together with $k_{i,1}^2, k_{i,2}^2$ to maximize 10; and it therefore has to make conjectures concerning the values of consumption of each of its childrens' households.

The consumption that each of the childrens' households attains is given by,

$$c_{i,m}^2 = w_{i,m}^2 + k_{i,m}^2 + \hat{k}_{i,m}^2 \quad m = 1, 2 \tag{12}$$

i.e. given the wage income of each child's household, the parent household can guarantee a consumption level of $w_{i,m}^2 + k_{i,m}^2$. The amount $\hat{k}_{i,m}^2$ is the estimated receipt of inheritance from the spouse of child m; which is unknown to households of the first generation when they make their optimising choice. We shall assume that,

$$\hat{k}_{i,1}^2 = \sum_{j=1} p_{i,j} k_{j,2}^2 \tag{13}$$

$$\hat{k}_{i,2}^2 = \sum_{j=1} p_{i,j} k_{j,1}^2 \tag{14}$$

where $p_{i,j}$ is the probability of a child of the jth household of generation 1 marrying a child of the ith household. The $p_{i,j}$ are the probability beliefs held by household i. There is no requirement that $p_{i,j} = p_{j,i}$, i.e. sets of inconsistent beliefs may be held. One possible set of beliefs is that $p_{i,j} = 1/(n-1), i \neq j, p_{i,i} = 0$, i.e. all possible marriages, except those of siblings, are equally probable, however this assumption is not essential. Given the assumptions on utilities and probabilities we see that both sexes are treated symmetrically in terms of bequests. The symmetric assumptions are not essential but simplifies the notation somewhat.

We focus attention on the competitive Nash equilibrium, in which each household maximizes its welfare under the assumption that the bequests of other households are given. Thus the Nash equilibrium has the property that, $k_{i,1}^{2*}, k_{i,2}^{2*}$ solves,

$$\max_{k_{i,1}^2, k_{i,2}^2} \{U(c_i^1) + \mu[U(c_{i,1}^2) + U(c_{i,2}^2)]\} \tag{15}$$

for all i, subject to the constraints 9,12-14

The first order conditions are,

$$\frac{\partial U(c_i^1)}{\partial c_i^1} = \mu \frac{\partial U(c_{i,1}^2)}{\partial c_{i,1}^2} \tag{16}$$

$$\frac{\partial U(c_i^1)}{\partial c_i^1} = \mu \frac{\partial U(c_{i,2}^2)}{\partial c_{i,2}^2} \tag{17}$$

for $i = 1$ to n. Given concavity assumptions on the utility functions these are sufficient to establish an equilibrium. Equations 16,17, together with the constraints (9), 9,12-14 form a system of $7n$ equations in $7n$ unknowns. Proof of the existence of the Nash equilibrium is provided in the Appendix.
What of the properties of the Nash equilibrium in terms of Ricardian Equivalence and Distributional Neutrality ?
In order to determine whether Ricardian equivalence exists we shall assume a fiscal measure which transfers an amount - from each household of the first generation to each household of the second generation. All of the households being fully aware of the tax/benefit measure.
Consider the set of equations determining consumption for household i of the first generation in the Nash equilibrium. Single asterisks denote choices of the households in relation to consumption and bequests prior to the fiscal perturbation, double asterisks relate to choices after the perturbation.

$$c_i^{1*} = w_i^1 - k_{i,1}^{2*} - k_{i,2}^{2*} \tag{18}$$

$$c_{i,1}^{2*} = w_{i,1}^2 + k_{i,1}^{2*} + \sum_j p_{i,j} k_{j,2}^{2*} \tag{19}$$

$$c_{i,2}^{2*} = w_{i,1}^2 + k_{i,2}^{2*} + \sum_j p_{i,j} k_{j,1}^{2*} \tag{20}$$

Under the fiscal perturbation, the new levels of consumption become,

$$c_i^{1**} = w_i^1 - k_{i,1}^{2**} - k_{i,2}^{2**} - - \tag{21}$$

$$c_{i,1}^{2**} = w_{i,1}^2 + k_{i,1}^{2**} + \sum_j p_{i,j} k_{j,2}^{2**} + - \tag{22}$$

$$c_{i,2}^{2**} = w_{i,1}^2 + k_{i,2}^{2**} + \sum_j p_{i,j} k_{j,1}^{2**} + - \tag{23}$$

Note, of course, that there are not $2n$ families in the second generation, each family in the first generation loses - and each family in the second generation gains - .
The equilibrium prior to the perturbation can be seen to be maintained if all bequests are reduced by - /2. Thus letting,

$$k_{i,m}^{2**} = k_{i,m}^{2*} - - /2 \tag{24}$$

and noting that $\sum p_{j,i}(- /2) = (- /2)$; we arrive at,

$$c_i^{1**} = w_i^1 - k_{i,1}^{2**} - k_{i,2}^{2**} - - = c_i^{1*} \tag{25}$$

$$c_{i,1}^{2**} = w_{i,1}^2 - k_{i,1}^{2**} + \sum_j p_{i,j} k_{j,2}^{2**} + - = c_{i,1}^{2*} \tag{26}$$

$$c_{i,2}^{2**} = w_{i,1}^2 - k_{i,2}^{2**} + \sum_j p_{i,j} k_{j,1}^{2**} + - = c_{i,2}^{2*} \tag{27}$$

with exactly the same consumption levels as prior to the fiscal change.
It is thus feasible for each households to retain the same consumption profile over time consequent on the fiscal perturbation; the question whether they would wish to maintain such a profile has to be resolved. However, we may note the "strategic equivalence" of the pre and post fiscal perturbation games, (see e.g. Bernheim and Bagwell(1988)). Thus provided the original equilibrium is insensitive to the perturbation of the individuals' constraints then Ricardian Equivalence is established.
Now turn to the problem of establishing distributional neutrality. Consider the case where we have a shift of resources of Ψ from household x to household y of the first generation. The consumption level of household x of the first generation would then become,

$$c_x^{1**} = w_x^1 - k_{x,1}^{2**} - k_{x,2}^{2**} - \Psi \tag{28}$$

If c_x^{1**} is to remain equal to c_x^{1*} then $k_{x,1}^{2**} = k_{x,1}^{2*} - \Psi/2$ and $k_{x,2}^{2**} = k_{x,2}^{2*} - \Psi/2$. Thus for the related households of the second generation,

$$c_{x,1}^{2**} = w_{x,1}^2 + k_{x,1}^{2*} + \sum_j p_{j,x} k_{j,1}^{2**} - (\Psi/2) \tag{29}$$

$$c_y^{1**} = w_y^1 - k_{y,1}^{2**} - k_{y,2}^{2**} + \Psi \tag{30}$$

If c_y^{1**} is to remain equal to c_{yx}^{1*} then $k_{y,1}^{2**} = k_{y,1}^{2*} + \Psi/2$ and $k_{y,2}^{2**} = k_{y,2}^{2*} + \Psi/2$. And so,

$$c_{y,1}^{2**} = w_{y,1}^2 + k_{y,1}^{2*} + \sum_j p_{j,y} k_{j,1}^{2**} + (\Psi/2) \tag{31}$$

$$\sum_j p_{j,z} k_{j,1}^{2*} = \sum_j p_{j,z} k_{j,1}^{2**}; \;\; \sum_j p_{j,z} k_{j,2}^{2*} = \sum_j p_{j,z} k_{j,2}^{2**} \tag{32}$$

If these relations do not hold then quite clearly distributional neutrality fails. Let us assume however that these relations hold, it simply implies that households do not have any reason to favour a marriage of their progeny to either the progeny of household x or y.
Now consider the losing household, consumption of the second generation can only be maintained if,

$$\sum_j p_{j,x} k_{j,1}^{2**} - (\Psi/2) = \sum_j p_{j,x} k_{j,1}^{2*} \tag{33}$$

which may be written as,

$$p_{y,x} \Delta k_{y,1}^{2**} + \sum_{j \neq y} p_{j,x} \Delta k_{j,1}^{2**} = \Psi/2 \tag{34}$$

where,

$$\Delta k_{j,1}^{2**} = k_{j,1}^{2**} - k_{j,1}^{2*} \tag{35}$$

Since we assume $p_{x,x} = 0$, and for distributional neutrality we must have $\Delta k_{j,1}^{2**} = 0$ for all j other than y, then from 34 we must have for distributional neutrality,

$$p_{y,x} \Delta k_{y,1}^{2**} = \Psi/2 \tag{36}$$

However since $\Delta k_{y,1}^{2**} = \Psi/2$, this can only occur if $p_{y,x} = 1$. Thus only if the progeny of the gaining and losing families are expected to marry with probability 1 can distributional neutrality hold.
From an alternative perspective it may be argued that due to the nature of the expectations operator with respect to temporal transfers, the opportunity set of consumption facing households is invariant with respect to the temporal transfers which are used in the context of demonstrating Ricardian Equivalence. However opportunity sets do change when cross-sectional transfers are considered.
Finally, in this section we draw attention to the fact that whilst *ex ante,* the relationship between marginal utilities are determined according to 16, 17; ex post, of course, once the marriage partners are known, different values of these relationships hold. Thus a household in full knowledge of the marriage settlement would have wished to change its consumption and bequests, e.g., when the children marry relatively rich or poor spouses in relation to the average. Only in the symmetric case of an economy composed of identical families, would the ex post and ex ante values of 16 and 17 be identical. Even in the symmetric case, however the families could do better. It is well known that in the symmetric case bequests are smaller than would be the case if households recognised in their optimisation the matching bequest that other households make to their children. The sub-optimality of the Nash equilibrium is considered further in the next section.

III The Over-Lapping Generations Model

In the preceding sections we have considered the properties of Distributional Neutrality and Ricardian Equivalence in the two period model proposed by Abel, Bernheim and Bagwell. In the present section we wish to consider the applicability of Neutrality results relevant to the steady state properties of a multi-generational model. To this end we set up an almost standard OLG model; the principal difference between existing overlapping generations models and the model considered below concern the assumptions made with respect to family formation, and the generation of consanguinity chains. As we shall see such assumptions prove crucial to the steady state properties of the model.
The model of the economy is composed of families and firms. A family of generation t exists for two periods, $t, t+1$. A family only receives wage income in the first period of its existence; during which period it supplies inelastically one unit of labour for which it receives wage w_t. During the first period procreation occurs and $2(1+n)$ children are born; at the start of the second period the children of the family marry partners from the children of other families; and the new households created thus form generation $t+1$, which again exist for two periods, $t+1, t+2$; and so the process continues.
The decisions of the household of generation t concern the choice of level of consumption in periods $t, t+1$; and the bequests made to each of the children of the household.
Concerning the macroeconomy, the saving of the households of generation t adds to the capital stock of period $(t+1)$, which together with the labour supplied by generation $(t+1)$, produces the output of period $(t+1)$. The number of labour units at t is identical to the number of households of generation t. The number of households is assumed to grow at rate n. Hence the number of labour units at time t, given an initial number of households of N_o is $N_t = N_0 e^{nt}$.
Production in the economy takes place via a constant returns to scale production function;

$$Y_t = F(K_t, N_t) \tag{37}$$

where Y_t denotes aggregate output, and K_t the aggregate capital stock. In terms of output per labour unit, y_t, we have,

$$y_t = f(k_t) \tag{38}$$

where k_t is the capital-labour ratio, and $f(.)$ satisfies the Inada conditions. We assume that wage rates and capital rental ratios are determined by their marginal products, i.e.,

$$w_t = f(k_t) - k_t f'(k_t) \tag{39}$$

$$r_t = f'(k_t) \tag{40}$$

Let us now turn to the decisions that households make with regard to their consumption and bequests. We shall assume that the utility of household i of generation t is given by,

$$\begin{aligned} V_t^i &= u^i(c_{1,t}^i) + (1+\alpha)^{-1} u^i(c_{2,t+1}^i) + (1+\beta^*)^{-1}(1+n)2V_{t+1}^i \\ &= u^i(c_{1,t}^i) + (1+\alpha^i)^{-1} u^i(c_{2,t+1}^i) + (1+\beta)^{-1} V_{t+1}^i \end{aligned} \tag{41}$$

where $\beta = (\beta^* - 1 - 2n)/2(n+1)$; $\alpha^i, \beta \geq 0$, and the utility functions are concave. α^i, β are respectively the rates of discount between the households own utility in adjacent periods, and the utility of the next generation.
Solving 41 recursively, we derive,

$$V_t^i = \sum_{j=0}^{\infty} (1+\beta)^{-j} [u^i(c_{1,t+j}^i) + (1+\alpha^i)^{-1} u^i(c_{2,t+j+1}^i)] \tag{42}$$

The constraints faced by household i in a generation born at time t are, during the first period,

$$c_{1,t}^i + s_t^i = w_t + b_t^i + g_{1,t}^i \tag{43}$$

i.e. consumption in the first period plus saving, must equal wage income plus the bequests that household has received, plus net government transfers received in the first period.
In the second period of life, each household faces the constraint,

$$c_{2,t+1}^i + (1+n)2b_{t+1}^i = (1+r_{t+1})s_t^i + g_{2,t+1} \tag{44}$$

i.e. consumption in the second period plus bequests made by the household must equal the value of accrued savings plus net government transfers received by the household in that period. The second term on the L.H.S. of 44 denotes that even in the absence of growth in the number of households, i.e. $n = 0$, the household still makes bequests to two households of the next generation, i.e. the households of its two children.
Expanding 42, a household from generation t, maximizes 45 w.r.t. s_t^i and b_{t+1}^i,

$$V_t^i = u^i(c_{1,t}^i) + (1+\alpha^i)^{-1} u^i(c_{2,t+1}^i) + (1+\beta)^{-1} u^i(c_{1,t+1}^i) + .. \tag{45}$$

The only additional variable that we have to define is the value of B_{t+1}^i, which enters into the determination of c_{t+1}^i; i.e. moving 43 up one period we have,

$$c_{1,t+1}^i + s_{t+1}^i = w_{t+1} + B_{t+1}^i + g_{t+1}^i \tag{46}$$

Now quite clearly from the perspective of a household of generation t, B_{t+1} cannot be taken as exogenous, as e.g. can w_{t+1}; neither can it be taken as equal to b_{t+1}; i.e. its own bequest to each of the households of its children. In fact, following the discussion in Section 2, B_{t+1} is assumed equal to the expected value of bequests received from parents of each of the households; therefore from household i's viewpoint;

$$B_{t+1} = b_{t+1} + E(b^j_{t+1}) \tag{47}$$

where $E(b^j_{t+1})$ is the expected value per child of bequests in the economy, excluding household i. We assume that household i takes the value of bequests by other households as fixed when choosing its own bequest; we therefore seek a Nash competitive equilibrium for the economy in relation to bequest levels.
The first order conditions for a maximum w.r.t. s^i_t and b^i_{t+1} are,

$$\frac{\partial u^i(c^i_{1,t})}{\partial c_{1,t}} = (1+\alpha^i)^{-1}\frac{\partial u^i(c^i_{2,t+1})}{\partial c_{2,t+1}}(1+r_{t+1}) \tag{48}$$

$$2(n+1)(1+\alpha^i)^{-1}\frac{\partial u^i(c^i_{2,t+1})}{\partial c_{2,t+1}} = (1+\beta)^{-1}\frac{\partial u^i(c^i_{1,t+1})}{\partial c_{1,t+1}} \tag{49}$$

under the assumption that bequests are positive.
Subst. for $\partial u^i(c^i_{2,t+1})/\partial c^i_{2,t+1}$ from 48 into 49, we thus have,

$$\frac{\partial u^i(c^i_{1,t})}{\partial c_{1,t}} = (1+r_{t+1})\{2(n+1)(1+\beta)\}^{-1}\frac{\partial u^i(c^i_{1,t+1})}{\partial c_{1,t+1}} \tag{50}$$

where r_{t+1} is determined by 40.
Concerning Ricardian equivalence : assume a social security scheme which takes q_t from each household of generation $t+1$, and transfers $(1+n)q_t$ to each household of the elderly generation alive at $t+1$. The relevant equations are 44, and 46; thus we have,

$$c_{2,t+1} + (1+n)2b_{t+1} + (1+n)q_t = (1+r_{t+1})s_t \tag{51}$$

$$c_{1,t+1} + s_{t+1} = w_{t+1} - q_t + B_{t+1} \tag{52}$$

As can be seen if each households increase bequests to each child by $q_t/2$, remembering the definition of B_{t+1} we see that consumption levels of each generation remain unchanged. Maintaining the same consumption path over time is therefore feasible for each family; desirability of maintaining that path relies again on the strategic equivalence of the pre and post perturbation games. Transfers between non-contiguous generations can of course be accommodated by appropriate chaining of transfers and consequent bequests between adjacent generations.
Now consider the case of Distributional Neutrality. The case we consider is where there is a lump sum transfer from household i to household j at time t. If the children of household i marry the children of household j, and the households know this prior to disposition of their bequests then no change in the consumption patterns results. However if uncertainty with respect to marriage partners exist, then if households keep to the same consumption profile prior to the transfer, then the budget constraints 43,44 are not satisfied. Strategic Equivalence of the pre and post budget games does not carry through for redistributional transfers of this type.

Finally, in this section we contrast the implications of the present model with those of the standard OLG model.
Let c_1^{i*}, c_2^{i*}, and k^*, denote the steady state values of $c_{1,t}^i, c_{2,t}^i$, and k_t. Since in steady state, $c_{1,t}^i = c_{1,t+1}^i = c_1^{i*}$,we have,

$$\frac{\partial u^i(c_{1,t}^i)}{\partial c_{1,t}} = \frac{\partial u^i(c_{1,t+1}^i)}{\partial c_{1,t+1}} = \frac{\partial u^i(c_1^{i*})}{\partial c_1} \tag{53}$$

and $r_{t+1} = f'(k_{t+1}) = f'(k^*) = r^*$, we have from 50,

$$(1 + r^*) = 2(n + 1)(1 + \beta) \tag{54}$$

How does this result compare with that for a command optimum ? The command optimum can be mimicked by assuming that households recognise that any bequest made to their childrens' households are matched by the bequests given to their children's spouses; eq. 47 thus becomes,

$$B_{t+1}^i = b_{t+1}^i + E(b_{t+1}^j) = 2b_{t+1}^i \tag{55}$$

and so the first order condition 49 now becomes,

$$(n + 1)(1 + \alpha^i)^{-1} \frac{\partial u^i(c_{2,t+1}^i)}{\partial c_{2,t+1}} = (1 + \beta)^{-1} \frac{\partial u^i(c_{1,t+1}^i)}{\partial c_{1,t+1}} \tag{56}$$

Consequently, in the steady state, $c_1^{i**}, c_2^{i**}, k^{**}$, we have,

$$(1 + r^{**}) = (n + 1)(1 + \beta) \tag{57}$$

i.e. the modified golden rule result.
Comparing 54 with 57, we note that $r^* > r^{**}$, i.e., the Nash competitive equilibrium suffers from under capitalization, i.e. the steady state capital stock is too low compared to that found under the modified golden rule. The under capitalized Nash equilibrium contrasts quite strongly with results in the standard overlapping generations model in which under capitalized equilibria are ruled out.
The inefficiency of equilibria in models with consanguinity chains between different families has of course been recognised in the literature. In particular we may note the literature stemming from the "isolation paradox", Sen(1967), Nerlove, Razin and Sadka (1984). However the implications for the standard growth models of such chains do not appear to have been widely recognised. Thus, for example, in the well known text of Blanchard and Fischer(1989) it is noted that, "The presence of a bequest motive implies that the steady state interest rate, $1 + r^*$, cannot be greater than the modified golden rule $(1 + n)(1 + R)$[where $R = \beta$ in our present notation]; the steady state capital stock cannot be too low" (*op. cit.*, p107) We have seen that this implication is no longer true for models in which all marriages are not between siblings. Likewise the statement "We can also see that it is not finite lives as such that generate possible inefficient equilibria, but the fact that future generations preferences do not affect current decisions. When parents incorporate their childrens' in their own utility function to an extent sufficient to cause the parents to make bequests, the equilibrium becomes efficient; the steady state of the economy is at the modified golden rule",(*op.cit.* p.107) is also not applicable in OLG models with non-sibling marriage. Inefficient equilibria exist with non-zero bequests.

Thus results which flow from models which postulate families with asexual reproduction; i.e. the assumption found most often in the overlapping generations literature, are not robust to alternative assumptions regarding family formation. Since asexual reproduction or marriage between siblings is not the norm in most societies doubt must be cast on the relevance of the equilibrium properties of models which assume such.

IV Conclusions

In the above analysis we have constructed a model in which Ricardian Equivalence does not imply Distributional Neutrality. If such is accepted then it would appear that the dynastic framework would still remain an useful tool for studying public policy issues, and any conclusions derived within this framework need not necessarily be met with considerable scepticism.

Let us turn to the arguments in the Bernheim and Bagwell(1988) analysis concerning Ricardian Equivalence and Distributional Neutrality. That analysis dealt primarily with a world of certainty in which in the context of the above analysis there is a degenerate probability distribution with respect to the partners of progeny. In this case both Ricardian Equivalence and Distributional Neutrality occur under the stated assumptions. However Bernheim and Bagwell argue that allowing for uncertainty concerning future linkages both Ricardian Equivalence and Distributional Neutrality continue to hold as long as "for each pair of individuals, one can devise an algorithm that describes transfers as a function of realized linkages (e.g. marriage) and that connects this pair with probability one" (Bernheim and Bagwell,*op.cit.* p.332).

Undoubtedly Distributional Neutrality preserving algorithms do exist. One such proposed by Bernheim is to assume that children are distinguished by sex, and that all children marry spouses of opposite sex. Consequent on any redistribution across households, gaining households retain their consumption profiles and pass through their gains in larger bequests to female children; losing households also retain their consumption profiles and pass on their losses in smaller bequests to male children. When females in the gaining households marry males from losing households the gains and losses are cancelled out. Almost certainly every male descendant in a losing household will marry a female descendant from a gaining household at some time in the future hence Distributional Neutrality is preserved.

Alternatively, one can construct an algorithm which ensures Distributional Neutrality cannot exist, e.g. the assumption of primogeniture in which all gains and losses are pushed through in respect of male heirs. No offsetting marriages can then occur. Ricardian Equivalence however still results since any transfer between generations results in an offsetting change in the bequests to the male heir.

However the possible deployment of such an algorithm to ensure Distributional Neutrality is not a point of issue. We have shown that a model which implies Ricardian Equivalence need not imply Distributional Neutrality. Additional or alternative assumptions have to be deployed in order to maintain Distributional Neutrality in the presence of uncertainty regarding consanguinity chains. In particular the existence of a social convention relating to the allocation of bequests is an additional coordination requirement that would have to be placed on households. The distinction between the model proposed in this paper, and that proposed in the papers by Bernheim and Bagwell and Abel and Bernheim rests on the different assumptions employed with regard to the behaviour of consumers in the face of uncertainty.

Consider a tax of \$1 on every household in the U.S., the proceeds of the tax being allocated to one household, a compulsory lottery. If Ricardian Equivalence, and hence Distributional Neutrality, exists in the Bernheim model then Bernheim would have to argue that the recipient household of some \$60 million dollars would have unchanged consumption. A reductio ad absurdum indeed. The current proposed model would retain Ricardian Equivalence but imply that the consumption path of households in such circumstances as the above lottery would indeed change.

Analyses which question the relevance of distributional policies in overlapping generations models, in which households exhibit intergenerational altruism are themselves of doubtful relevance. We have argued that in order to establish cross sectional Distributional Neutrality households require information on consanguinity links prior to making their decisions on consumption and bequests. If such information is not available due to the veil of time, then the property of distributional neutrality disappears. However Ricardian Equivalence, under stated conditions, remains and is therefore a property independent of cross-sectional neutrality. To distinguish between the properties of Distributional Neutrality and Ricardian Equivalence, we note that for Ricardian Equivalence households must know that they will be related to a given number of households in the future, whereas for Distributional Neutrality they also require precise details of the characteristics of these households. Thus the Abel-Bernheim -Bagwell arguments concerning the irrelevance of all redistributional policies arising out of intra-family linkages may appear somewhat misplaced. Consequently, we argue that the theoretical case for Ricardian Equivalence is rather more robust than previously supposed.

Appendix

The following theorem is a modification to that proved by Abel and Bernheim(1989)
Theorem 1: For all wage profiles $(w_i^1, w_i^2), i = 1, \dots, n$,a Nash equilibrium exists.

Proof:(Abel and Bernheim(1991)). Let,

$$S_i = \{(k_{i,1}^2, k_{i,2}^2) \mid k_{i,1}^2 + k_{i,2}^2 \leq w_i^1 \text{ and } k_{i,1}^2, k_{i,2}^2 \geq 0\} \tag{A.1}$$

S_i is $h_i's$ strategy space; let s_i denote an element of S_i. Note that S_i is compact and convex. $h_i's$ utility is by assumption continuous in $s = (s_1, \dots, s_N)$, and quasi concave in s_i. Thus by Debreu's (1952) Social Equilibrium Existence Theorem, there exists a profile of strategies $(s_1^*, \dots, s_N^*)$ which satisfies our definition of equilibrium.

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