Wear, 80 (1982) 377 - 380

Short Communication

Purely elastic contributions to friction

A. M. STONEHAM and A. H. HARKER
Theoretical Physics Division, Atomic Energy Research Establishment, Harwell, Oxon. OX11 ORA (Gt. Britain)

(Received July 10, 1981; in revised form June 18, 1982)

We discuss contributions to the friction between rough perfectly elastic media. One mechanism could well describe the friction of diamond on diamond in air.

It is well known that friction is dominated by processes which involve bonding, plastic deformation, ploughing and wear. The purpose of this note is to show that even when these contributions are small, as may be the case for diamond in air, purely elastic processes still give finite friction.

All that is required for friction is that, when one body moves over another, energy should be dissipated. If a point load moves on the surface of a solid it will radiate sound waves, and the eventual degradation of those sound waves into heat constitutes a dissipation mechanism. The contact between asperities on two bodies results in time-varying point loads, and we have considered two extreme models. In model 1 we consider a particular asperity on body A moving over a smooth flat region of body B; the load on A is constant but the moving load on B will radiate sound waves into B. Model 2 involves intermittent contact between asperities on A and asperities on B; each contact will radiate pulses of sound into both A and B.

We consider the sliding of similar bodies of density ρ, shear modulus μ and compression and shear wave velocities C_p and C_s respectively. We define $\beta = C_p/C_s$; β is related to Poisson's ratio ν by

$$\beta = \left\{ \frac{2(1-\nu)}{1-2\nu} \right\}^{1/2}$$

We suppose that the bodies slide relative to each other with a velocity V and that the contacts may be represented by discs of radius a with pressures P on the discs. We ignore lateral forces caused by bonding and local slopes of the surfaces and assume a single contact radius rather than a distribution of sizes; both these simplifications could be removed quite easily. The pressure P is unknown but it is obviously limited by the yield stress of the material.

0043-1648/82/0000-0000/$02.75

For model 1 two cases occur, depending on the nature of the substrate. Examples which are soluble analytically and which illustrate the two classes of behaviour are (a) an elastic isotropic half-space as substrate and (b) a substrate of Einstein oscillators.

For an asperity modelled by a disc with a normal pressure P moving over the isotropic elastic substrate (case 1(a)) we may use the results of Eason [1] to deduce that the energy dissipation (and hence the friction coefficient $\mu_{1(a)}$) is identically zero for velocities below the velocity of sound. This striking result means that, in the steady state that we consider, the work done in depressing the substrate at the front of the moving disc is exactly balanced by the energy recovery at the rear. A necessary (but possibly not sufficient) condition for this is that all Fourier components of the displacement travel at least as fast as the asperity. All models of which we are aware show finite energy dissipation (hence finite friction) for asperity velocities exceeding sound velocities. Parallels with the phenomenon of Cerenkov radiation [2] are evident.

We may expect finite friction in model 1 whenever the sound velocity tends to zero for some wavelengths. This follows from comments on case 1(a) and from parallels with situations such as the resistance to ship motion on water. Case 1(b) illustrates this explicitly. For a substrate of Einstein oscillators with an elastic constant σ_0 and an asperity stress with a maximum value σ_{max}, we find that

$$\mu_{1(b)} = A\sigma_{max}/\sigma_0 \qquad (1)$$

Here A is dimensionless and of order unity (mathematical details will be given elsewhere: the precise value depends on geometric factors involving the bulk and surface densities of the oscillators, on the precise time dependence of the applied stress and on how the stress is presumed to fall off with depth into the bulk). We suspect that the form of eqn. (1) has much wider validity. If so, purely elastic friction requires that σ_{max} is less than the stresses for plastic deformation or for fracture so that $\mu_{1(b)}$ could be as large as 0.1 but is unlikely to become much higher. We note that the observed friction of diamond on diamond in air is characterized by values of μ in the range 0.05 - 0.1 [3].

Systems for which mechanism 1(b) applies need a sound velocity that is zero for some wavelength. An important case includes *layered* systems [4], Lamb waves giving an extreme case. The existence of surface layers, or of modified or contaminated surface regions, may therefore be an important contributing feature of this elastic friction term. We may also make a more general conjecture. For the elastic system (case 1(a)) at velocities less than the velocity of sound it is impossible to tell the direction of motion from a snapshot of the displacement field ($u(r)$ at a given t) alone. For case 1(b) systems (where the ship-on-water case is the easiest to visualize) the direction is easily identified. We conjecture that, in general, this feature of the displacement field distinguishes between the cases of zero (case 1(a)) and finite (case 1(b)) μ.

For model 2 we use the results of Miller and Pursey [5] in a manner similar to the treatment of impact by Hunter [6]. For a disc oscillating on a half-space with frequency f, energy is radiated at the rate

$$\frac{(E_\mathrm{p} + E_\mathrm{s} + E_\mathrm{R})(2\pi f)^2(\pi a^2 P)^2}{\rho C_\mathrm{p}^2}$$

The coefficients E_p, E_s and E_R correspond to compression, shear and Rayleigh waves respectively; they take the values 0.083, 0.311 and 0.814 for a Poisson ratio of 1/4. We extract the energy radiated during brief contacts between asperities by assuming a characteristic time variation in pressure during each contact and calculating the corresponding frequency spectrum. The general form of the result is not particularly sensitive to the form of the time variation: we assume a gaussian form

$$P(t) = P_0 \exp\left\{-\left(\frac{t}{T}\right)^2\right\}$$

and then the energy radiated by each brief contact is

$$E = 0.27\,\frac{(\pi a^2 P_0)^2}{\rho C_\mathrm{p}^3 T}$$

If the "collision time" T is taken to be a/V, and we evaluate the number of collisions made by each asperity to find a time-averaged contact force, the energy radiated into each medium leads to

$$\mu_{\mathrm{F}2} = 0.6\,\frac{P_0}{\mu_\mathrm{L}}\,\frac{V}{C_\mathrm{p}} \tag{2}$$

Here μ_L is the Lamé elastic constant. The velocity-dependent term is very small; μ_2 is reduced to less than $\mu_{1(\mathrm{b})}$ by roughly the ratio of the asperity velocity to the velocity of the compression waves.

The friction coefficients in both models have particularly simple forms. The velocity-dependent term is, as expected, very small. If typical experimental values for diamond are used in model 1, the contact pressure P is found [2] to be of the same order of magnitude as measured values of the compressive strength of diamond.

In the absence of plastic deformation, the topography of rubbing surfaces dictates the time-dependent forces and hence the friction. We have shown how friction can arise from purely elastic effects. This involves some subtle issues (notably the distinction between cases 1(a) and 1(b)) but may account for the observed friction of diamond on diamond in air. Our models can be extended to include bonding, elastic anisotropy and more general topographies.

We are indebted to Professor D. Tabor and Dr. J. A. Greenwood for a most helpful correspondence.

1 G. Eason, The stresses produced in a semi-infinite solid by a moving surface force, *Int. J. Eng. Sci.*, *2* (1965) 581 - 609.
2 A. M. Stoneham and A. H. Harker, Friction between non-bonding elastic solids without plastic deformation, *AERE Rep. TP 891*, 1981 (Atomic Energy Research Establishment).
3 M. Seal, The friction and wear of diamond, *Proc. R. Soc. London, Ser. A*, *248* (1958) 379.
M. Casey and J. Wilks, The friction of diamond sliding on polished cube faces of diamond, *J. Phys. D*, *6* (1973) 1772.
D. Tabor, Adhesion and friction. In J. E. Field (ed.), *The Properties of Diamond*, Academic Press, New York, 1979, p. 325.
4 K. F. Graff, *Wave Motion in Elastic Solids*, Oxford, 1975.
L. D. Landau and E. M. Lifshitz, *Theory of Elasticity*, Pergamon, Oxford, 1970, Sections 24 and 25.
5 G. F. Miller and H. Pursey, On the partition of energy between elastic waves in a semi-infinite solid, *Proc. R. Soc. London, Ser. A*, *233* (1955) 55 - 69.
6 S. C. Hunter, Energy absorbed by elastic waves during impact, *J. Mech. Phys. Solids*, *5* (1957) 162 - 171.