

# Seismic performance of structural systems equipped with buckling-restrained braces

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## ABSTRACT

Buckling-restrained braces (BRBs) are often employed for the seismic retrofit of existing systems and the design of new systems given their significant contribution in terms of stiffness and added damping. However, since BRBs are characterized by a low lateral post-elastic stiffness, their use may lead to excessive residual deformations that may jeopardize the reparability of the building. Moreover, accumulation of plastic deformations in the BRBs may endanger the capability of withstanding multiple earthquakes and aftershocks. The objective of this paper is to provide insight into the performance and residual capacity of dual systems made of BRB frames coupled with moment-resisting frames. This study considers a simplified single degree of freedom model which permits to investigate a wide range of configurations. A non-dimensional formulation of the equation of motion is introduced, the statistic of the normalized peak and residual displacements and cumulative ductility of the system is evaluated under a set of ground motion records. Different values of the BRB target maximum ductility and different coupled frame properties are considered. The results of this study provide useful information for the preliminary design of dual systems made of BRB frames coupled with moment-resisting frames.

## 1 INTRODUCTION

Buckling-restrained braced frames (BRBFs) are a type of minimal damage structure where the horizontal seismic-induced forces and the seismic energy are respectively resisted and dissipated by elastoplastic passive energy dissipation devices named buckling-restrained braces (BRBs), e.g. Soong and Spencer 2002, Christopoulos and Filiatrault 2006. The use of such devices is gaining popularity as lateral resisting system in seismic areas to be employed both for new constructions and rehabilitation of existing buildings. In BRBs, a sleeve provides buckling resistance to an unbonded core that resists axial stress. As buckling is prevented, the core of the BRB can develop axial yielding in compression in addition to that in tension, ensuring an almost symmetric hysteretic behaviour.

While the large and stable dissipation capacity of BRBs has been proven by many experimental studies (e.g. Black et al. 2002, Merritt et al.

2003), their low post-yield stiffness may result in inter-story drift concentration, e.g. Zona et al. 2012, and large residual interstory drifts. The latter problem is associated with high repair costs and disruption of the building use or occupation (Erochko et al. 2010). Sabelli et al. 2003 studied the seismic performance of BRBs reporting that residual drifts values are on average in the range of 40 to 60% of the maximum drift. Usually, values of residual drifts less than 0.5% are deemed acceptable for building frames since they would allow building reparability with little difficulties, e.g. doors, windows and elevators would still be functional (Iwata et al. 2006, McCormick et al. 2008). However, BRBF designed according to the codes may exhibit residual drift values higher than this limit even under the design basis earthquake. In addition, the performance under aftershocks may also be jeopardized by excessive residual drifts due to the main shock.

This issue, which may impair the cost-effectiveness of BRBFs, could be avoided by using special steel moment-resisting frame

(SMRF) in parallel with the BRBF to create a dual system configuration (Kiggins and Uang 2006, Ariyaratana and Fahnestock 2011, Baiguera et al. 2016). The ASCE/SEI 7-10 considers the situation of a dual system that combines a stiff primary seismic force-resisting system (e.g. BRBFs) with a SMRFs, as schematically represented in Figure 1. According to ASCE/SEI 7-10, the SMRF in dual systems should be capable of resisting at least 25% of the prescribed seismic force. Kiggins and Uang 2006 investigated the seismic response of a 3-storey and a 6-storey BRBFs with and without a parallel SMRFs designed to resist the 25% of the design base shear, showing that the SMRF in parallel allows to reduce the residual drifts by about 50%, while providing similar performances in terms of peak inter-story drift demand. The efficiency of dual BRBF-SMRF systems is also demonstrated in Ariyaratana and Fahnestock 2011 while using as case study a 7-story frame. BRBs are also employed to enhance the lateral strength, stiffness as well as the dissipation capacity of existing reinforced concrete (RC) buildings (Freddi et al. 2013, Di Sarno and Manfredi 2010). RC frames and BRBs braces also form a dual system, with the former often contributing to more the 25% of the total base shear.

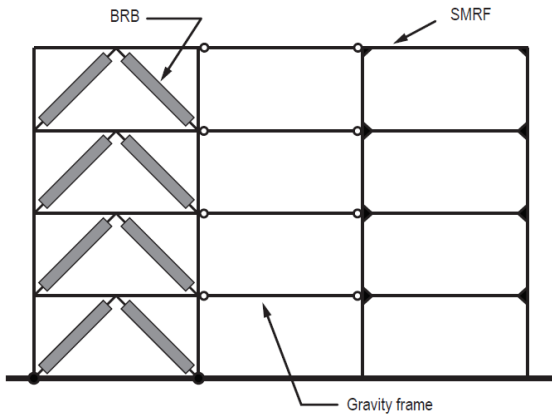


Figure 1. Schematic dual system combining buckling-restrained braced frames (BRBF) and special moment resisting frames (SMRF)

These studies evaluated the efficiency of dual BRBF-SMRF systems by considering only few case studies, without providing general indications on the influence on the seismic performance of values of important parameters such as the shear ratio, the stiffness ratio and target design ductility of the two systems. In this work, a comprehensive parametric investigation is carried out to shed light on this behavioural aspect, and provide useful recommendations for preliminary design. The problem is analysed by assuming that both the BRBF and the SMRF can be described as single degree of freedom (SDOF)

systems. While this approach is not suitable for describing the behaviour of complex multi-level frames, it allows to derive a non-dimensional formulation of the problem and highlight the few characteristic parameters that control the seismic performance. The variation of these parameters permits to explore the performance of a wide ranges of configurations under a set of ground motion records representative of the uncertainty of the seismic input.

Engineering demand parameters (EDPs) of interest include the peak normalized response, the normalized residual displacements, and the cumulative ductility demand in the BRBs. These EDPs are evaluated in correspondence of the design condition, where the BRBF and SMRF attain simultaneously their target ductility capacity.

## 2 PROBLEM FORMULATION

### 2.1 Single Degree of Freedom System

The equation of motion governing the seismic response of a SDOF system representative of a dual system, as represented in Figure 2 can be expressed as:

$$m\ddot{u}(t) + c_f\dot{u}(t) + f_f + f_b = \ddot{u}_g(t) \quad (1)$$

where  $m$  and  $c_f$  denote respectively the mass and the viscous damping constant of the system,  $f_f$  the resisting force of the frame,  $f_b$  the resisting force of the BRB,  $\ddot{u}_g(t)$  the ground acceleration input.

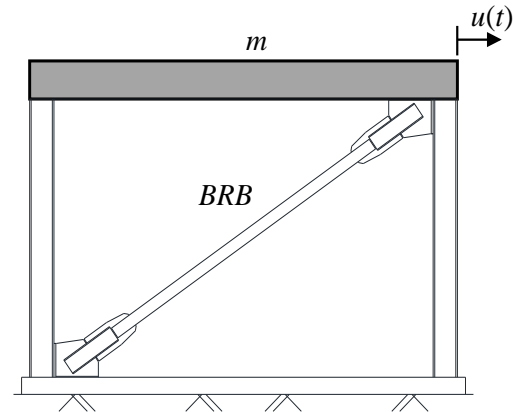


Figure 2. SDOF dual system with BRB

The frame is assumed to have an elastoplastic behaviour, with initial stiffness  $k_f$ , yield displacement  $u_{fy}$  and ductility capacity  $\mu_{fc}$  as reported in Figure 3. The BRB system has a constitutive law described by the model of Zona and Dall'Asta 2012. This model is characterized by many parameters which describe for example the hardening and the hysteretic behaviour.

However, to keep the problem as simple as possible, most of them are assumed as fixed and the BRB hysteretic behaviour is controlled only by the initial stiffness  $k_b$ , the yield displacement  $u_{by}$  and ductility capacity  $\mu_{bc}$ . These parameters are the one which exhibit significant variation from device to device and they are the design parameters explicitly reported in catalogues. The two models working in parallel, as reported in Figure 3 are representative of the dual SDOF system.

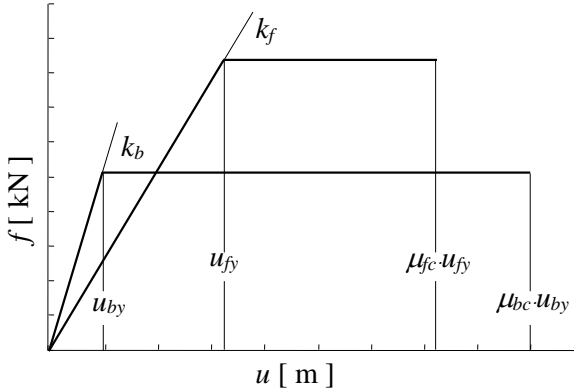


Figure 3. Constitutive laws of the dual systems

Such a model can describe a wide range of structural configurations, e.g. the case of BRBFs combined with SMRFs to form a dual system (Kiggins and Uang 2006, Ariyaratana and Fahnestock 2011, Baiguera et al. 2016) or retrofit applications involving BRBs inserted into existing RC frames (Freddi et al. 2013, Di Sarno and Manfredi 2010).

The seismic input is characterized by significant uncertainty affecting not only its intensity, but also the duration and frequency content. As usual in Performance Based Earthquake Engineering, the uncertainty of the seismic input is treated by introducing a seismic intensity measure (IM) (Shome et al. 1998, Freddi et al. 2017) whose statistical description is the object of the hazard analysis. The ground motion randomness for a fixed intensity level,  $im$ , usually denoted as record-to-record variability, can be described by selecting a set of ground motion realizations characterized by a different duration and frequency content and scaling these records to the common  $im$  value. The system response for a ground motion with an intensity  $im$  can be expressed as:

$$m\ddot{u}(t) + c_f\dot{u}(t) + f_f + f_b = im \cdot \ddot{u}_g(t) \quad (2)$$

where  $\ddot{u}_g(t)$  denotes the ground motion records scaled such that  $im = 1$  for that record.

The choice of an appropriate IM for the problem should be driven by criteria of

efficiency, sufficiency, and hazard computability (Shome et al. 1998, Freddi et al. 2017, Tubaldi et al. 2015, Galasso et al. 2015), and, in this paper, the spectral acceleration,  $Sa(\omega_0, \xi)$ , at the fundamental circular frequency of the system,  $\omega_0$ , and for the damping factor  $\xi$  is employed as IM.

## 2.2 Non-dimensional formulation of the equation of motion

Based on Eqn.(2), the maximum relative displacement of the system,  $u_{max}$ , can be expressed as:

$$u_{max} = f(m, c_f, k_f, u_{fy}, k_b, u_{by}, im) \quad (3)$$

The 8 variables appearing in Eqn.(3) have dimensions:  $[u_{max}] = L$ ,  $[m] = M$ ,  $[c_f] = MT^{-1}$ ,  $[k_f] = ML^{-2}$ ,  $[u_{fy}] = L$ ,  $[k_b] = ML^{-2}$ ,  $[u_{by}] = L$ ,  $[im] = LT^{-2}$  where the 3 physical dimensions are the time  $T$ , the mass  $M$ , and the length  $L$ . By applying the Buckingham  $\Pi$ -theorem (Barenblatt 1987), Eqn.(3) can be conveniently reformulated in terms of dimensionless parameters, denoted as  $\Pi$ -terms identifying the parameters that control the seismic response of the system and also reducing the number of variables. The problem involves 3 physical dimensions and 8 dimensional variables, thus, only  $8 - 3 = 5$   $\Pi$  dimensionless parameters are needed. By selecting the systems mass  $m$ , the seismic intensity measure  $im$ , and the initial frame stiffness  $k_f$  as repeating variables, the  $\Pi$ -terms can be derived and after manipulation, the following alternative set of  $\Pi$ -terms can be obtained:

$$\begin{aligned} \Pi_u &= \frac{u_{max} \omega_0^2}{im} \\ \mu_f &= \frac{u_{max}}{u_{fy}} \\ \mu_b &= \frac{u_{max}}{u_{by}} \\ \xi &= \frac{c_f}{2m\omega_0} \\ \alpha &= \frac{f_b}{f_f} \end{aligned} \quad (4)$$

where  $\omega_0^2 = (k_b + k_f)/m$  denotes the square of the circular frequency of the SDOF dual system.

The parameters,  $\mu_f$  and  $\mu_b$  denote the ductility demand of the frame and the BRB respectively, while  $\Pi_u$  denotes the displacement demand normalized with respect to  $im/\omega_0^2$ . It is

noteworthy that by considering  $S_a(\omega_0, \xi)$  as  $IM$ , the non-dimensional response  $\Pi_u$ , can be interpreted to as the displacement amplification factor being the ratio between  $u_{\max}$  and the pseudo-spectral displacement  $S_d(\omega_0, \xi) = S_a(\omega_0, \xi)/\omega_0^2$ . The parameter  $\alpha$  (Freddi et al. 2013) is the ratio between the strength capacity of the bracing system and that of the frame. While the parameters  $\mu_f$ ,  $\mu_b$  and  $\Pi_u$  depend on the response of the system through  $u_{\max}$ ,  $\alpha$  and  $\xi$  are independent from the response. Other response parameters of interest such as the normalized cumulative plasticity demand of the BRB  $\mu_{b,cum}$ , and the normalized residual displacement of the system  $\mu_{res}$ , can be expressed as:

$$\mu_{b,cum} = \frac{u_{bp,cum}}{u_{by}} = f(\mu_f, \mu_b, \xi, \Pi_u, \alpha) \quad (5)$$

$$\mu_{res} = \frac{u_{res}\omega_0^2}{im} = f(\mu_f, \mu_b, \xi, \Pi_u, \alpha)$$

It is noteworthy that the system response in terms of these EDPs depends on the characteristics of the input via the circular frequency  $\omega_0$ . In fact, seismic inputs with the same intensity  $im$  but with different characteristics propagate differently and have different effects on systems with different natural frequencies  $\omega_0$ . This was demonstrated in Tubaldi et al. 2015 by considering SDOF systems with nonlinear viscous dampers but the same reasoning holds for the problem considered in this study. Alternatively, the ratio  $\omega_0/\omega_g$  between the bare system frequency and a frequency synthetically representing the ground motion frequency content could be considered. This approach was followed by Karavasilis et al. 2011 and Malaga 2015 in an attempt to extend the dimensional analysis of bilinear systems under pulse-type excitations to the case of excitations without distinct pulses.

### 3 PERFORMANCE ASSESSMENT METHODOLOGY

The objective of the proposed methodology is to evaluate how the coupled system behaves in correspondence of the design condition, i.e., when the design earthquake strikes the coupled system whose properties are defined by prefixed performance criteria. Account is made of the fact that the BRBs are designed to control the

imposed seismic demand, whereby an optimal condition corresponds to the BRBs and the frame reaching simultaneously their target ductility capacity under severe earthquake intensities (Freddi et al. 2013, Zona et al. 2012). This way the maximum exploitation of the system dissipation capacity is ensured and the design criterion imposes a constraint on the values that can be assumed by the non-dimensional problem parameters.

By assuming a target ductility capacity  $\mu_{bc}$  for the BRB, and a target ductility capacity  $\mu_{fc}$  for the frame, the design condition is attained when  $\mu_b = \mu_{bc}$  and at the same time  $\mu_f = \mu_{fc}$  under the design earthquake input. In design practice, this condition is ensured by considering a deterministic performance measure (Dall'Asta et al. 2016), i.e. by considering the mean demand obtained for the different earthquake inputs describing the record-to-record variability effects.

Given the system properties independent from the response  $\omega_0$ ,  $\alpha$ ,  $\mu_{fc}$ ,  $\mu_{bc}$ ,  $\xi$ , the design condition can be found by the following optimization problem: find the value  $\Pi_u^*$  of the normalized displacement demand such as  $\bar{\mu}_f = \mu_{fc}$  and  $\bar{\mu}_b = \mu_{bc}$ , where the over score denotes the mean across the samples, and thus  $\bar{\mu}$  denotes the mean ductility demand. The following procedure can be applied to ensure the attainment of the design condition under the set of records employed to describe the seismic input:

1. Select arbitrary values of  $\bar{u}_{\max}^*$  and  $m$ , e.g.  $\bar{u}_{\max}^* = 1m$  and  $m = 1ton$ . The corresponding non-dimensional parameter values are:

$$c_f = 2m\omega_0\xi$$

$$u_{fy} = \bar{u}_{\max}^* / \bar{\mu}_f$$

$$u_{by} = \bar{u}_{\max}^* / \bar{\mu}_b$$

$$k_f = \frac{\omega_0^2 m}{1 + \alpha u_{fy} / u_{by}}$$

$$k_b = (\alpha u_{fy} / u_{by}) k_f$$

2. Scale the records to a common value of the intensity measure e.g.  $im = 1$ ;
3. Perform nonlinear dynamic analyses for the different records;
4. Evaluate the mean system displacement response  $\bar{u}_{\max}$ . If  $\bar{u}_{\max}$  is equal to the

target value  $\bar{u}_{\max}^*$ , then  $\Pi_u = \Pi_u^*$  where  $\Pi_{\max}^* = \bar{u}_{\max}^* \omega_0^2 / im$ , and go to step 5. Otherwise multiply  $im$  by the ratio  $\bar{u}_{\max}^* / \bar{u}_{\max}$  and restart by step 2. This procedure corresponds to a linear interpolation between the relation  $\bar{u}_{\max}$  and  $im$ ;

5. Evaluate the statistics of  $\mu_{res}$  and of  $\mu_b$ .

Steps 1-4 ensure that the design condition of the frame and the BRBs attaining simultaneously their performance target under the design earthquake input is achieved.

## 4 PARAMETRIC STUDY

### 4.1 System properties

The performance of the systems corresponding to different values of  $\omega_0$ ,  $\alpha$ ,  $\mu_{fc}$ ,  $\mu_{bc}$ ,  $\xi$  is studied in this section considering the constraint posed by the attainment of the design condition, which corresponds to  $\Pi_u = \Pi_u^*$ .

The parameter  $\omega_0$  is varied in a range corresponding to a vibration period  $T_0 = 2\pi/\omega_0$  in the range between 0s and 4s. The strength ratio  $\alpha$  assumes the values in the range between 0 and 100. The lower bound  $\alpha = 0$  represents the case of the bare frame, whereas the upper bound represents the case of frame with pinned connections where the horizontal stiffness and resistance is provided only by the BRB. The parameter  $\mu_{fc}$  assumes values in the range between 1 and 4. The case  $\mu_{fc} = 1$  corresponds to a design condition where the frame behaves in its elastic range under the design earthquake. The case  $\mu_{fc} = 4$  corresponds to the a highly ductile behaviour of the frame under the design earthquake. The parameter  $\mu_{bc}$  assumes values in the range between 5 and 20. Values of 15-20 are typical ones for the ductility capacity of a BRB device. In some situations, such as the seismic retrofit of RC frames (Freddi et al. 2013), the BRB device is arranged in series with an elastic brace exhibiting adequate over-strength. This leads to reduced values of the ductility capacity which may attain the lower bound of 5 for a very flexible elastic brace (Ragni et al. 2011). The value of 2% is assumed for the damping factor  $\xi$  in this study.

### 4.2 Seismic input description

A set of 28 ground motions is considered in the parametric study to describe the record-to-record variability. The records have been selected from the PEER strong motion database (FEMA P695) on the basis of three fundamental parameters: site class, source distance, and magnitude. Ground motions associated with site class B, as defined in Eurocode 8, source-to-site distance,  $R$ , greater than 10km, and a moment magnitude,  $M_w$ , in the range between 6.0 and 7.5 are considered.

The record number is deemed sufficient to obtain accurate response estimates, given the efficiency of the intensity measure employed (Shome et al. 1998). This set of records has been employed also for similar parametric studies in Tubaldi et al. 2015.

### 4.3 Parametric study results

Figure 4 shows the median value of the normalized peak displacement demand  $\Pi_u^*$  vs the base shear ratio  $\alpha$ , for different values of the target BRB ductility  $\mu_{bc}$ . The different figures refer to different values of  $T_0$  and of the target frame ductility  $\mu_{fc}$ . All the curves attain the same value for  $\alpha = 0$  (SMRF only), and in particular for  $\mu_{fc} = 1$  they attain a value of about 1. This result is expected, since for  $\alpha = 0$  the response is not dependent on the BRBs ductility capacity, and for  $\mu_{fc} = 1$  the system behaves (on average) elastically, so that the inelastic displacement coincides with the elastic one. On the other hand, for  $\alpha = 0$  and  $\mu_{fc} = 4$ , a simple bilinear oscillator is obtained and  $\Pi_u$  can be significantly different than 1. In particular, higher values of the normalized peak displacement  $\Pi_u^*$  are observed for low values of the period  $T_0$ . In the case of dual system ( $\alpha > 0$ ), for low periods and increasing values of  $\alpha$ , the normalized peak displacement increases, whereas for high periods  $\Pi_u^*$  remains almost constant and slightly less than 1.

Figure 5 shows the median value of the normalized residual displacement demand  $\mu_{res}$  vs the base shear ratio  $\alpha$ , for different values of the target BRB ductility  $\mu_{bc}$ . The different figures refer to different values of  $T_0$  and of the target frame ductility  $\mu_{fc}$ . It can be observed that when the system behaves linearly ( $\alpha = 0$ ,  $\mu_{fc} = 1$ ), the residual displacements are zero. Obviously, adding in parallel to a linear system a nonlinear

one ( $\alpha > 0$  in Figure 5 (a, c, e)) results in an increase of residual displacements. This increase is higher for higher values of the target BRB ductility  $\mu_{bc}$  and for lower vibration periods. On the other hand, if the frame exhibits a nonlinear behaviour with a target ductility  $\mu_{fc} = 4$ , then it is characterized by high residual drifts of the order

of 50-60% of the peak ones, and adding in parallel the BRBs ( $\alpha > 0$  in Figure 5 (b, d, f)) does not increase them. It is noteworthy that the values of  $\mu_{res}$  for  $\alpha = 0$  are consistent with the ones observed in Ruiz-García and Miranda 2006 on bilinear oscillators.

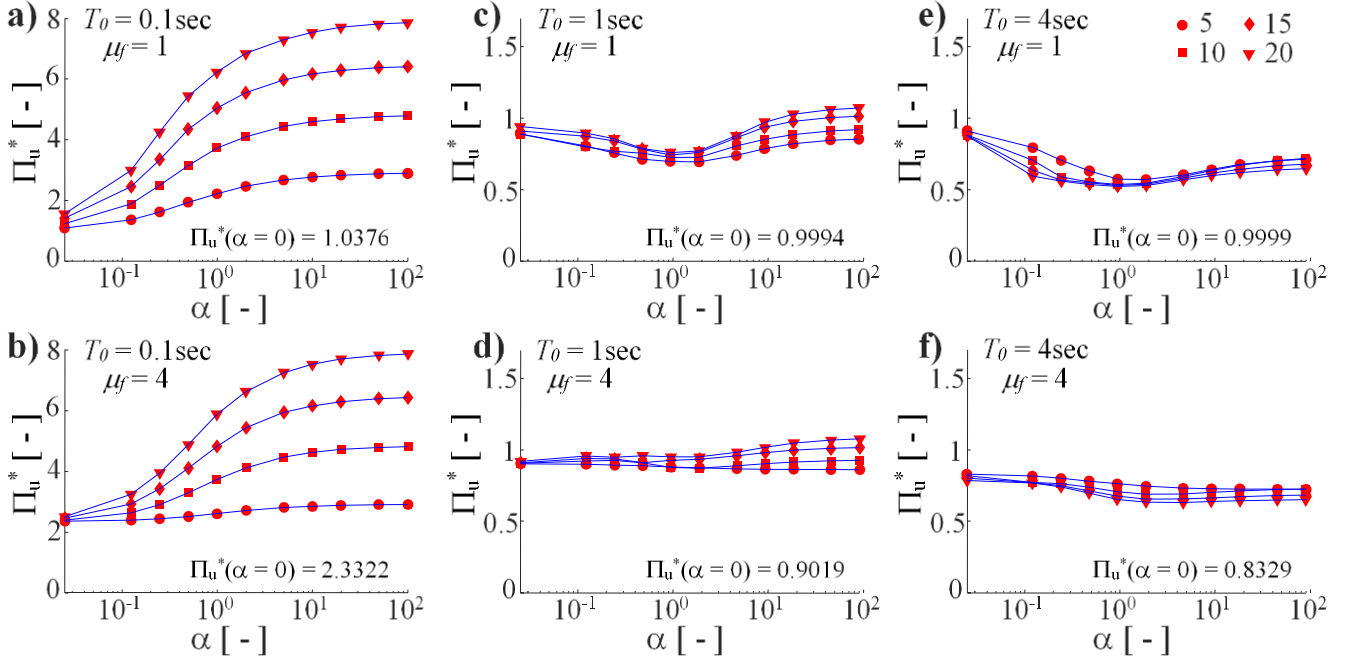


Figure 4. Median value of the normalized peak displacement demand  $\Pi_u^*$  vs the base shear ratio  $\alpha$ , for different values of  $T_0$  (0.1, 1 and 4s), of  $\mu_{fc}$  (1 and 4) and of  $\mu_{bc}$  (5, 10, 15 and 20)

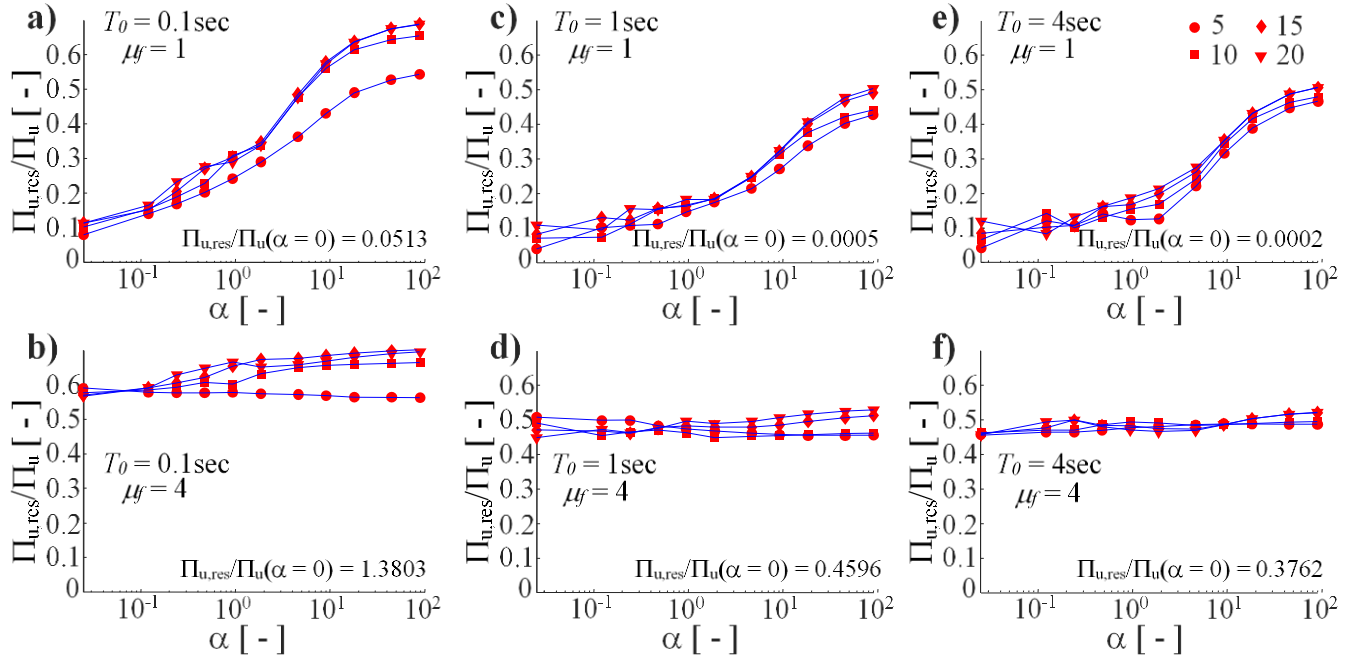


Figure 5. Median value of the residual displacement  $\mu_{res}$  vs the base shear ratio  $\alpha$ , for different values of  $T_0$  (0.1, 1 and 4s), of  $\mu_{fc}$  (1 and 4) and of  $\mu_{bc}$  (5, 10, 15 and 20)



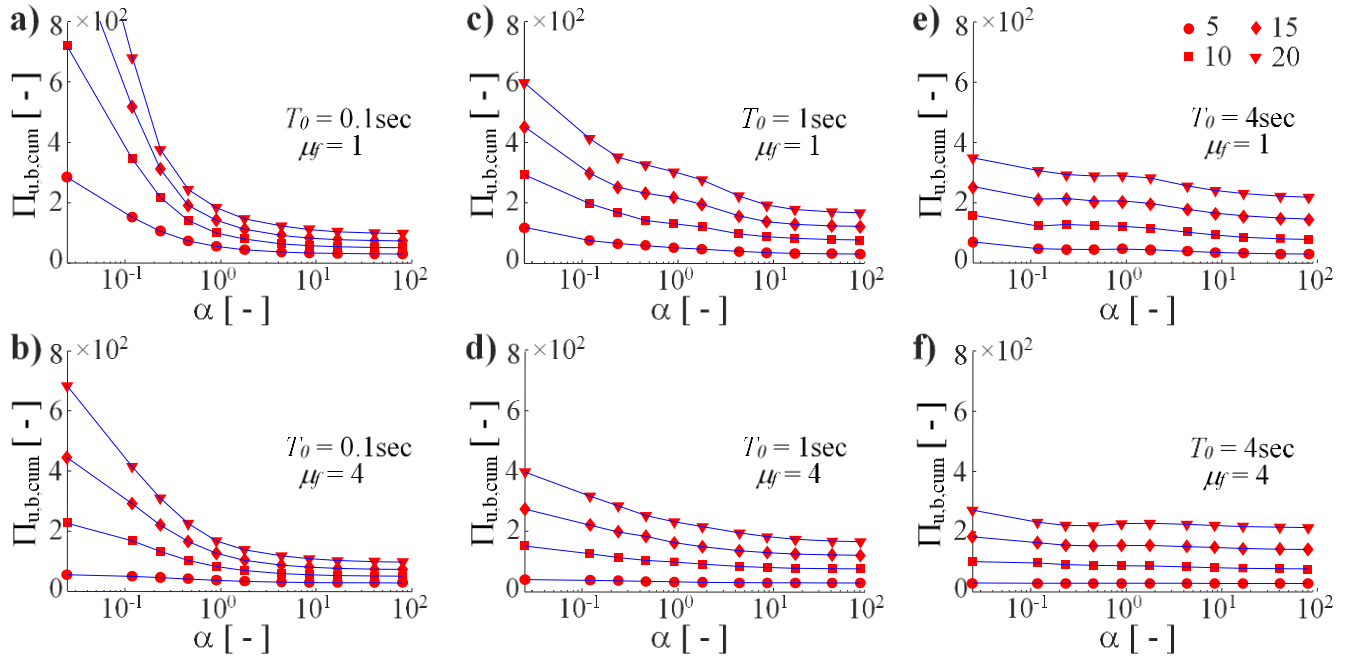


Figure 6. Median value of cumulative plastic ductility demand in the BRB  $\mu_{b,cum}$  vs the base shear ratio  $\alpha$ , for different values of  $T_0$  (0.1, 1 and 4s), of  $\mu_{fc}$  (1 and 4) and of  $\mu_{bc}$  (5, 10, 15 and 20)

Figure 6 shows the median value of the cumulative plastic ductility demand in the BRBs  $\mu_{b,cum}$  vs. the base shear ratio  $\alpha$ , for different values of the target BRB ductility  $\mu_{bc}$ . The different figures refer to different values of  $T_0$  and of the target frame ductility  $\mu_{fc}$ . In general, the cumulative ductility demand reduces by increasing  $\alpha$  because the system undergoes less cycles of vibrations. In other terms, by increasing  $\alpha$  the system becomes more non-linear and period elongation generally results in less cycles and less ductility accumulation under the same earthquake histories. In the case of pure BRBF (*i.e.*  $\alpha = 100$ ), the cumulative ductility increases with the target ductility level. This increase is different for the different period considered. The obtained trends are quite different from those observed in Choi and Kim 2006, showing that the accumulated ductility ratios are nearly constant in BRBFs with  $T_0 > 0.1$ s. Moreover, there is an almost linear relation between  $\mu_{b,cum}$  and  $\mu_{bc}$ . Thus, the curves  $\mu_{b,cum}/\mu_{bc}$  collapse into a single master-curve.

## 5 CONCLUSIONS

This paper presented the results of study on the seismic performance of dual systems consisting of BRB frames coupled with moment-resisting frames, designed according to a criterion which allows to control the maximum ductility demand on the BRB frame and the coupled frame. A single degree of freedom system assumption and

a non-dimensional problem formulation allow to estimate the response of wide range of configurations while limiting the number of simulations. This permits to evaluate how the system properties, and in particular the values of the ratio  $\alpha$  between the base shear of the BRB frame and the moment resisting frame, affect the median demand of normalized displacements, residual displacements, and cumulative BRB ductility. The study results provide information useful for the preliminary design of the coupled system, and for the performance assessment of existing frames coupled with BRBs.

## 6 ACKNOWLEDGEMENTS

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