Generalized Regular Form Based SMC for Nonlinear Systems With Application to a WMR

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***Abstract*—In this paper, a generalized regular form is proposed to facilitate sliding mode control (SMC) design for a class of nonlinear systems. A novel nonlinear sliding surface is designed using implicit function theory such that the resulting sliding motion is globally asymptotically stable. Sliding mode controllers are proposed to drive the system to the sliding surface and maintain a sliding motion thereafter. Tracking control of a two-wheeled mobile robot is considered to underpin the developed theoretical results. Model-based tracking control of a wheeled mobile robot is first transferred to a stabilization problem for the corresponding tracking error system, and then the developed theoretical results are applied to show that the tracking error system is globally asymptotically stable even in the presence of matched and mismatched uncertainties. Both experimental and simulation results demonstrate that the developed results are practicable and effective.**

***Index Terms*—Generalized regular form, mobile robots, nonlinear systems, nonlinear sliding surfaces, sliding mode control (SMC), tracking control.**

I. Introduction

SLIDING mode control (SMC) is a powerful technique because of its fast convergence and strong robustness [1], [2]. The invariance property of systems in the sliding mode to matched uncertainties and parameter variations [3] has motivated numerous applications of sliding mode techniques to nonlinear systems including multimachine power systems [4], direct-drive robot system [5], induction motor [6], power converters [7], and wheeled mobile robot (WMR) systems [8]. The concept of the SMC is also used to observer design and fault detection [9]. Moreover, it has been demonstrated that the sliding mode approach can be applied to control systems with mismatched uncertainties, see for example [10]–[13]. In [14], the bounds on the uncertainties are estimated using adaptive techniques. However, the uncertainties are inevitably assumed to satisfy a linear growth condition in order to adaptively compensate the parameter uncertainty. In [11], by using an extended disturbance observer with a modified time-varying sliding surface, a novel SMC is applied to stabilize an SISO system with continuous external disturbance that does not vanish at the origin. Ultimate boundedness of the system is guaranteed and the obtained ultimate bound can be further reduced by choosing appropriate design parameters. However, the structure of the system is restricted, which makes the method difficult to extend to the MIMO case. The method proposed by Niu *et al.* [15] also shows the strong robustness of SMC for systems with an uncertain input distribution where the considered systems are linear with nonlinear disturbances. In [16], SMC for general nonlinear stochastic systems has been investigated. It is shown that for some special nonlinear stochastic systems, LMIs can be used for controller design. Furthermore, this method can also be applied for nonlinear uncertain stochastic systems with state-delay based on a T–S fuzzy modeling and control approach [17]. With the SMC above, the system is usually required to be in regular form or to be transferred into such a form for analysis. It should be noted that the transformation matrix for linear systems can be easily obtained by basic matrix theory. However, for nonlinear systems, it is very difficult to find a diffeomorphism to transfer a nonlinear system into the traditional regular form. Moreover, the associated conditions may be too strong to be applied for most general nonlinear systems, (see, for example [18] and reference therein). In this paper, a generalized regular form is proposed for a class of nonlinear systems, which includes the traditional regular form as a special case. Therefore, the developed results can be applied to a wide class of systems.

The WMR is increasingly used for both industrial and service purposes due to its flexible mobility [19]. Although it is not necessary to satisfy Brockett's well-known necessary condition [20] if the reference trajectory does not involve stabilization to a rest configuration [21], it is challenging to use PID control or linear control methods to obtain desired tracking performance for WMR systems because of the inherent nonlinearity caused by the nonholonomic constraints. This has motivated the development of nonlinear control approaches for trajectory tracking of WMR systems. In existing work considering mobile robot systems [22], [23], the controller for the kinematic model is based on the back-stepping method proposed in [24]. In [8], the kinematic controller based on the back-stepping technique was

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simplified and mismatched uncertainty is not considered. Due to the dynamic behaviour of the linear and steering velocities in implementation, the proposed control scheme requires the actuator to reduce the tracking error in practice [24]. Therefore, actuator dynamic control design is inevitably required in many control approaches to improve the system performance [8], [22], [23].

In a driftless nonholonomic system, since the uncertainties mainly come from the input channel, SMC can be a very powerful tool owing to the invariance of the sliding mode dynamics to matched uncertainty. An SMC scheme for trajectory tracking with polar coordinates has been previously proposed by Yang and Kim [25]. However, due to hardware limitations, the designed controller did not exhibit the expected tracking performance in practice. In both [23] and [8], SMC strategies were used in the dynamic layer. Although the simulation results in [8], [23] show robustness against matched uncertainties, the SMC was only applied to the dynamic model, which only ensures that the reference velocities can be tracked. In [26], sliding mode techniques were applied to a WMR system using a feedback linearization approach and results have been obtained not only for the tracking control problem but also for regulation tasks. However, this requires that the propulsive force of the WMR can be measured as one of the states in the system so that the strict relative degree condition required for feedback linearization can be satisfied. This is very difficult to implement from the practical point of view. In [27], SMC was applied to the kinematic model of a WMR. However, the system can only be controlled in a local domain to avoid the singularity. In most of the existing work for the WMR, mismatched uncertainties are not considered. However, in the presence of drift of the wheels, the uncertain drift term will result in mismatched uncertainties. Therefore, it is necessary to consider WMR systems with mismatched uncertainties to ensure high tracking performance.

In this paper, a generalized regular form is proposed for a class of nonlinear control systems, which is an extension of the traditional/classical regular form for SMC design. This is an extension of the traditional/classical regular form for sliding mode design. Then, a novel nonlinear sliding surface is designed associated with the generalized regular form such that the corresponding sliding mode dynamics are globally asymptotically stable using implicit function theory. Robust sliding mode controllers are designed to guarantee that the considered system is driven to the sliding surface in finite time and remains on it thereafter even in the presence of matched and mismatched uncertainties. All the uncertainties are assumed to be bounded by known functions and the bounds on the uncertainties are fully used to reduce the effects of the uncertainties. The developed results are tested by model-based tracking control of a WMR with a differential driving mechanism through simulation and experiment. The tracking error dynamics are derived initially, and then the developed results are applied to the error system to demonstrate the developed strategies. Experimental and simulation results on the WMR show that the proposed controller is insensitive to matched uncertainties, and can tolerate a certain level of mismatched uncertainties in both theory and application.

II. System Description

Consider a class of nonlinear systems with matched and mismatched uncertainties described by

$$\dot{x} = \mathcal{F}(t,x) + \mathcal{G}(t,x)(u + \Phi(t,x)) + \Psi(t,x) \tag{1}$$

where $x \in \mathcal{R}^n$ and $u \in \mathcal{R}^m$ are the state variables and control inputs, respectively. The nonlinear vector $\mathcal{F}(\cdot) \in \mathcal{R}^n$ and the input matrix function $\mathcal{G}(\cdot) \in \mathcal{R}^{n\times m}$ are known with full rank for $x \in \mathcal{R}^n$ and $t \in \mathcal{R}^+$. The terms $\Phi(\cdot)$ and $\Psi(\cdot)$ denote the matched and mismatched uncertainties, respectively. It is assumed that all the nonlinear functions are smooth enough so that the existence of the solution of system (1) is guaranteed.

Assumption 1: There exist known continuous non-negative functions $\delta(t,x)$ and $\mu(t,x)$ such that the mismatched uncertainty $\Psi(t,x)$ and the matched uncertainty $\Phi(\cdot)$ in system (1) satisfy

$$\|\Psi(t,x)\| \le \delta(t,x) \tag{2}$$

$$\|\Phi(t,x)\| \le \mu(t,x). \tag{3}$$

Remark 1: Assumption 1 requires that the bounds on the uncertainties are known. These will be employed in the control design to reject/reduce the effects of the uncertainties.

For further analysis, partition $\mathcal{F}(\cdot)$, $\mathcal{G}(\cdot)$, and $\Psi(\cdot)$

$$\mathcal{F}(t,x) := \begin{bmatrix} \mathcal{F}_1(t,x) \\ \mathcal{F}_2(t,x) \end{bmatrix} \tag{4}$$

$$\mathcal{G}(t,x) := \begin{bmatrix} \mathcal{G}_1(t,x) \\ \mathcal{G}_2(t,x) \end{bmatrix} \tag{5}$$

$$\Psi(t,x) := \begin{bmatrix} \Psi_1(t,x) \\ \Psi_2(t,x) \end{bmatrix} \tag{6}$$

where $\mathcal{F}_1(\cdot) \in \mathcal{R}^{n-m}$, $\mathcal{F}_2(\cdot) \in \mathcal{R}^m$, $\mathcal{G}_1(\cdot) \in \mathcal{R}^{(n-m)\times m}$, $\mathcal{G}_2(\cdot) \in \mathcal{R}^{m\times m}$, $\Psi_1(\cdot) \in \mathcal{R}^{n-m}$, and $\Psi_2(\cdot) \in \mathcal{R}^m$. Then from the partitions (4)–(6), the system (1) can be rewritten as

$$\dot{x}_1 = \mathcal{F}_1(t,x) + \mathcal{G}_1(t,x)\big(u + \Phi(t,x)\big) + \Psi_1(t,x) \tag{7}$$

$$\dot{x}_2 = \mathcal{F}_2(t,x) + \mathcal{G}_2(t,x)\big(u + \Phi(t,x)\big) + \Psi_2(t,x) \tag{8}$$

where $x_1 \in \mathcal{R}^{n-m}$, $x_2 \in \mathcal{R}^m$, and $x = \mathrm{col}(x_1, x_2)$. Since $\mathcal{G}(\cdot) \in \mathcal{R}^{n\times m}$ is full rank for $x \in \mathcal{R}^n$ and $t \in \mathcal{R}^+$, without loss of generality, it is assumed that $\mathcal{G}_2(t,x)$ is nonsingular in $(t,x) \in \mathcal{R}^+ \times \mathcal{R}^n$.

Choose the sliding function $\sigma(x)$ as follows:

$$\sigma(x) = Kx_2 + \varphi(x_1, x_2) \tag{9}$$

where $K = \mathrm{diag}\{k_1, k_2, \ldots, k_m\}$ with $k_i > 0$ for $i = 1, 2, \ldots, m$, $\varphi(\cdot)$ is a known class $\mathcal{C}^1$ function and each entry of the Jacobian matrix $[\frac{\partial \varphi}{\partial x_2}]_{ij}$ for $i, j = 1, 2, \ldots, m$ is bounded.

Remark 2: There is no general way to design the function $\varphi(x_1, x_2)$ for a general nonlinear system since the function is dependent on the system dynamics. However, for a specific system, system knowledge can be used in conjunction with the assumptions to establish a design. It should be noted that the sliding function (9) proposed in this paper includes both the linear sliding function $\sigma(x) = Cx$ where $C \in \mathcal{R}^{m\times n}$ is a constant matrix, and the nonlinear sliding function in the form of $\sigma(x) = x_2 + \vartheta(x_1)$ where $\vartheta(\cdot) \in \mathcal{R}^m$ as special cases.

For the sliding function in (9), the sliding surface is described by

$$\mathcal{S} = \{x \in \mathcal{R}^n | \quad \sigma(x) = 0\}. \tag{10}$$

Definition 1: System (7) and (8) with the sliding function defined in (9) is called the generalized regular form of system (1) if the function $\mathcal{G}_1(\cdot)$ defined in (5) satisfies

$$\mathcal{G}_1(t,x)|_{x\in\mathcal{S}} = 0. \tag{11}$$

Remark 3: It should be emphasized that the classical regular form requires that $\mathcal{G}_1(t,x) = 0$ for all $t \geq 0$ and $x \in \mathcal{R}^n$ (see, e.g., [1], [18]), whereas the generalized regular form defined above requires that $\mathcal{G}_1(t,x) = 0$ only for all $t \geq 0$ and $x \in \mathcal{S}$. It is clear to see that the classical regular form is a special case of the generalized regular form defined above as $\mathcal{S}$ is just a surface in $\mathcal{R}^n$. From the Frobenius Theorem, the distribution spanned by the column vectors of the input matrix $\mathcal{G}(\cdot)$ is completely integrable if and only if the distribution is involutive (e.g., see [28]). This implies that the classical regular form may not exist for a nonlinear system. In contrast, the generalized regular form may exist and, thus, to develop a sliding mode theory associated with the proposed generalized regular form is valuable since the proposed method can be applied in cases where the classical regular form is not available.

Define function matrices $\Gamma_\mathcal{G}(t,x)$ and $\Gamma_\mathcal{F}(t,x)$ as

$$\Gamma_\mathcal{G}(t,x) := \frac{\partial\sigma}{\partial x}\mathcal{G}(t,x) = K\mathcal{G}_2(t,x) + \frac{\partial\varphi}{\partial x}\mathcal{G}(t,x) \tag{12}$$

$$\Gamma_\mathcal{F}(t,x) := \frac{\partial\sigma}{\partial x}\mathcal{F}(t,x) = K\mathcal{F}_2(t,x) + \frac{\partial\varphi}{\partial x}\mathcal{F}(t,x) \tag{13}$$

where $\mathcal{F}(\cdot)$, $\mathcal{F}_2(\cdot)$, $\mathcal{G}(\cdot)$, and $\mathcal{G}_2(\cdot)$ are defined in (4) and (5) and $\sigma(\cdot)$ is defined in (9). The following assumption is imposed on system (7) and (8).

Assumption 2: The function matrix $\Gamma_\mathcal{G}(t,x)$ defined in (12) is nonsingular for $x \in \mathcal{R}^n$ and $t \in \mathcal{R}^+$

Remark 4: Assumption 2 is a limitation on the input distribution matrix $\mathcal{G}(t,x)$ and the designed sliding surface $\sigma(x)$ in (9). It is required to guarantee that the system can be driven to the sliding surface (10). Since $\mathcal{G}_2(\cdot)$ is nonsingular, it is straight forward to see from (13) that Assumption 2 usually can be satisfied by choosing an appropriate parameter K, and thus, this condition is not strict.

It should be noted that under condition (11), when the system (1) is limited to the sliding surfaces (10), the system (7) has the following form:

$$\dot{x}_1 = \mathcal{F}_1(t,x)|_{x\in\mathcal{S}} + \Psi_1(t,x)|_{x\in\mathcal{S}}. \tag{14}$$

The objective now is to study under what conditions system (14) is the sliding mode dynamics of system (1) with respect to the sliding surface (10). Therefore, it is necessary to guarantee that there exists a unique solution of the functional equation $\sigma(x) = 0$ for x_2 in terms of x_1. The following lemma is introduced to facilitate further analysis.

Lemma 1 (see [29]): Assume that $f : \mathcal{R}^p \times \mathcal{R}^m \longmapsto \mathcal{R}^m$ is a continuous mapping and it is continuously differentiable with respect to the variable $\xi \in \mathcal{R}^m$. If there exists a constant $d > 0$ such that

$$\left|\left[\frac{\partial f}{\partial \xi}\right]_{ii}\right| - \sum_{j\neq i}\left|\left[\frac{\partial f}{\partial \xi}\right]_{ij}\right| \geq d, \quad i = 1,\dots,m \tag{15}$$

for any $(z,\xi) \in \mathcal{R}^p \times \mathcal{R}^n$ where $\left[\frac{\partial f}{\partial \xi}\right]_{ij}$ denotes the ij th entry of the Jacobian matrix $\frac{\partial f}{\partial \xi}$ and $p = n - m$, then there exists an unique mapping $g : \mathcal{R}^p \longmapsto \mathcal{R}^m$ such that $f(z, g(z)) = 0$. Moreover, this mapping $g(\cdot)$ is continuous. Furthermore, if $f(\cdot)$ is a class $\mathcal{C}^1$ function, then $g(\cdot)$ is a class $\mathcal{C}^1$ function.

Lemma 2: Under condition (11), there exists a function $g : \mathcal{R}^{n-m} \to \mathcal{R}^m$ such that when system (7) is constrained to the sliding surface (10), the dynamical system (7) can be described by

$$\dot{x}_1 = \mathcal{F}_1^s(t,x_1) + \Psi_1^s(t,x_1) \tag{16}$$

where

$$\mathcal{F}_1^s(t,x_1) = \mathcal{F}_1(t,x)|_{x_2=g(x_1)} \tag{17}$$

$$\Psi_1^s(t,x_1) = \Psi_1(t,x)|_{x_2=g(x_1)} \tag{18}$$

if $K = \text{diag}\{k_1, k_2, \dots, k_m\}$ in (9) satisfies

$$k_i \geq \varepsilon + \sum_{j=1}^{m}\sup\left|\left[\frac{\partial\varphi}{\partial x}\right]_{ij}\right|, \quad i = 1,2,\dots,m \tag{19}$$

where ε is a positive constant.

Proof: When system (7) is limited to the sliding surfaces (10), it follows from condition (11) that the system (7) can be described by (14).

From (9) and (19)

$$\begin{aligned}\left|\left[\frac{\partial\sigma}{\partial x_2}\right]_{ii}\right| &= \left|k_i + \left[\frac{\partial\varphi}{\partial x_2}\right]_{ii}\right| \geq k_i - \left|\left[\frac{\partial\varphi}{\partial x_2}\right]_{ii}\right| \\ &\geq \varepsilon + \sum_{j=1}^{m}\sup\left|\left[\frac{\partial\varphi}{\partial x}\right]_{ij}\right| - \left|\left[\frac{\partial\varphi}{\partial x_2}\right]_{ii}\right| \\ &= \varepsilon + \sum_{\substack{j=1\\ j\neq i}}^{m}\sup\left|\left[\frac{\partial\varphi}{\partial x}\right]_{ij}\right|\end{aligned} \tag{20}$$

for $i = 1,2,\dots,m$. This implies that

$$\left|\left[\frac{\partial\sigma}{\partial x_2}\right]_{ii}\right| - \sum_{\substack{j=1\\ j\neq i}}^{m}\left|\left[\frac{\partial\sigma}{\partial x_2}\right]_{ij}\right| \geq \varepsilon, \quad i = 1,2,\dots,m. \tag{21}$$

Then from Lemma 1, there exists a unique class $\mathcal{C}^1$ function $x_2 = g(x_1)$ satisfying $\sigma(x_1, g(x_1)) = 0$.

The analysis above shows that $x_2 = g(x_1)$ when $x \in \mathcal{S}$. Hence, the result follows by substituting $x_2 = g(x_1)$ into the right-hand side of the (14). ■

III. Sliding Motion Analysis and Control Design

A. Stability Analysis of the Sliding Mode

Assumption 3: There exists a continuously differentiable Lyapunov function $V(t,x_1) : \mathcal{R}^+ \times \mathcal{R}^{n-m} \longmapsto \mathcal{R}$ satisfying the inequalities

$$\varsigma_1(\|x_1\|) \le V(t,x_1) \le \varsigma_2(\|x_1\|) \tag{22}$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1}\mathcal{F}_1^s(t,x_1) \le -\varsigma_3(\|x_1\|) \tag{23}$$

$$\left\|\frac{\partial V}{\partial x_1}\right\| \le \varsigma_4(\|x_1\|) \tag{24}$$

where the functions $\varsigma_i(\cdot)$ for $i=1,2,3,4$ are continuous class $\mathcal{K}$ functions, and $\mathcal{F}_1^s(\cdot)$ is given in (17).

Remark 5: Assumption 3 implies that the nominal system of the sliding mode dynamics (16) is asymptotically stable. The conditions (22)–(24) are developed from the well-known converse Lyapunov Theorem (see [30]).

From Assumption 1, it is straightforward to see that the mismatched uncertainty $\Psi_1^s(t,x_1)$ in (16) satisfies

$$\|\Psi_1^s(t,x_1)\| \le \gamma(t,x_1) \tag{25}$$

where $\gamma(\cdot)$ is a known positive continuous function, which is assumed to satisfy $\gamma(t,0)=0$ such that the origin is the invariant equilibrium point of the sliding mode dynamics (14).

Theorem 1: Under condition (11) in Definition 1 and Assumptions 1 and 3, the sliding mode (16) is globally uniformly asymptotically stable if there exists a continuous nondecreasing function $w : \mathcal{R}^+ \longmapsto \mathcal{R}^+$ satisfying $w(r) > 0$ for $r > 0$ and $w(r) \to \infty$ when $r \to \infty$ such that for any $x_1 \in \mathcal{R}^{n-m}$

$$w(\|x_1\|) \le \varsigma_3(\|x_1\|) - \varsigma_4(\|x_1\|)\gamma(t,x_1). \tag{26}$$

Proof: Consider the Lyapunov candidate function $V(\cdot)$ satisfying Assumption 3 for system (16). The time derivative of $V(\cdot)$ along the trajectory of system (16) is given by

$$\begin{aligned}\dot V &= \frac{\partial V}{\partial t} + \left(\frac{\partial V}{\partial x_1}\right)^\tau (\mathcal{F}_1^s(t,x_1) + \Psi_1^s(t,x_1))\\ &\le \frac{\partial V}{\partial t} + \left(\frac{\partial V}{\partial x_1}\right)^\tau \mathcal{F}_1^s(t,x_1) + \left\|\left(\frac{\partial V}{\partial x_1}\right)^\tau\right\| \|\Psi_1^s(t,x_1)\|\\ &\le -\varsigma_3(\|x_1\|) + \varsigma_4(\|x_1\|)\gamma(t,x_1)\\ &\le -w(\|x_1\|)\end{aligned} \tag{27}$$

where the conditions (22)–(24) are used above. Hence, the conclusion follows. ■

Remark 6: It should be pointed out that condition (26) shows the limitation on the mismatched uncertainty $\Psi(t,x)$ in system (1) through the bounds $\gamma(t,x_1)$ in (25). It should be noted that:

1) $\gamma(t,x_1)$ is the bound on $\Psi_1^s(t,x_1)$ [see (25)];
2) $\Psi_1^s(t,x_1)$ is the contribution from the function $\Psi_1(t,x)$ when the system is on the sliding surface [see (18)];
3) $\Psi_1(t,x)$ is a subcomponent of $\Psi(t,x)$ [see (6)].

Therefore, inequality (26) represents the limitation on the bounds of the subcomponent $\Psi_1(\cdot)$ of $\Psi(\cdot)$ when $\Psi_1(\cdot)$ is on the sliding surface instead of the uncertainty $\Psi(\cdot)$ in the whole space $x \in \mathcal{R}^n$.

Remark 7: For systems with mismatched disturbances, which do not vanish at the origin or in the presence of mismatched external disturbances $d(t)$ and which do not vanish when time t goes to infinity, the problem is particularly challenging. In this case, usually only ultimate bounded results can be obtained under appropriate conditions unless other techniques such as adaptive control are used to identify the disturbance [13]. In this paper, global asymptotic stabilization is considered where it is required that the mismatched disturbances vanish at the origin, which is reflected in (25) where $\gamma(t,0)=0$.

B. Reachability

From Assumption 2, $\Gamma_{\mathcal{G}}(t,x)$ is nonsingular. Consider the control law

$$\begin{aligned}u(t,x) = &-\Gamma_{\mathcal{G}}^{-1}(t,x)\Gamma_{\mathcal{F}}(t,x) - \Gamma_{\mathcal{G}}^{-1}(t,x)\text{sgn}\big(\sigma(x)\big)\\ &\cdot\left\{\left\|\frac{\partial\sigma}{\partial x}\right\|\delta(t,x) + \|\Gamma_{\mathcal{G}}(t,x)\|\mu(t,x) + \eta\right\}\end{aligned} \tag{28}$$

where $\Gamma_{\mathcal{G}}(\cdot)$ and $\Gamma_{\mathcal{F}}(\cdot)$ are defined in (12) and (13), respectively, $\delta(\cdot)$ and $\mu(\cdot)$ satisfy (2) and (3), respectively, and $\eta > 0$ is a constant parameter selected to define the reaching behavior.

Theorem 2: Consider the nonlinear system (7) and (8). Under Assumptions 1 and 2, the control (28) is able to drive system (1) to the sliding surface (10) in finite time and maintain a sliding motion on it thereafter.

Proof: From (9),

$$\begin{aligned}\dot\sigma(x) &= \frac{\partial\sigma}{\partial x}\Big(\mathcal{F}(t,x) + \Psi(t,x)\Big) + \frac{\partial\sigma}{\partial x}\mathcal{G}(t,x)(u + \Phi(t,x))\\ &= \Gamma_{\mathcal{F}}(t,x) + \Gamma_{\mathcal{G}}(t,x)\Big(u + \Phi(t,x)\Big) + \frac{\partial\sigma}{\partial x}\Psi(t,x).\end{aligned} \tag{29}$$

Substituting the control in (28) into (29),

$$\begin{aligned}&\sigma^\tau(x)\dot\sigma(x)\\ &= \sigma^\tau(x)\left\{\frac{\partial\sigma}{\partial x}\Psi(t,x) + \Gamma_{\mathcal{G}}(t,x)\Phi(t,x)\right\} -\\ &\quad \sigma^\tau(x)\text{sgn}(\sigma(x))\left\{\left\|\frac{\partial\sigma}{\partial x}\right\|\delta(t,x) + \|\Gamma(t,x)\|\mu(t,x) + \eta\right\}\\ &\le \|\sigma(x)\|\left\{\left\|\frac{\partial\sigma}{\partial x}\Psi(t,x)\right\| + \|\Gamma(t,x)\Phi(t,x)\|\right.\\ &\quad \left. - \left\|\frac{\partial\sigma}{\partial x}\right\|\delta(t,x) - \|\Gamma(t,x)\|\mu(t,x) - \eta\right\}.\end{aligned} \tag{30}$$

From Assumption 1,

$$\begin{aligned}\left\|\frac{\partial\sigma}{\partial x}\Psi(t,x)\right\| &\le \left\|\frac{\partial\sigma}{\partial x}\right\|\|\Psi(t,x)\|\\ &\le \left\|\frac{\partial\sigma}{\partial x}\right\|\delta(t,x)\end{aligned} \tag{31}$$

$$\begin{aligned}\|\Gamma(t,x)\Phi(t,x)\| &\le \|\Gamma(t,x)\|\|\Phi(t,x)\|\\ &\le \|\Gamma(t,x)\|\mu(t,x).\end{aligned} \tag{32}$$

Substituting inequalities (31) and (32) into (30) yields

$$\sigma^{\tau}(x)\dot{\sigma}(x) \le \|\sigma(x)\|\Bigg\{\left\|\frac{\partial\sigma}{\partial x}\Psi(t,x)\right\| - \left\|\frac{\partial\sigma}{\partial x}\right\|\delta(t,x) + \|\Gamma(t,x)\Phi(t,x)\| - \|\Gamma(t,x)\|\mu(t,x) - \eta\Bigg\} \le -\eta\|\sigma(x)\|. \tag{33}$$

Hence, the conclusion follows. ■

IV. Application to a WMR System

A. Problem Formulation

Consider a WMR with differential driving mechanism. As the wheels of the robot may drift, which may result in mismatched uncertainty, it is necessary to consider mismatched disturbances. From [31], the kinematic model of the WMR can be described by

$$\dot{q} = \begin{bmatrix}\cos\theta_c & 0\\ \sin\theta_c & 0\\ 0 & 1\end{bmatrix}\Big(u + \phi(t,q)\Big) + \psi(t,q) \tag{34}$$

where $q = \text{col}(q_x, q_y, \theta_c) \in \mathcal{R}^3$ is the state with coordinates (q_x, q_y) on the $x - y$ plane and the heading angle θ_c, $u = \text{col}(v, \omega)$ is the control input where v is the linear velocity and ω is the steering velocity, $\phi(\cdot) \in \mathcal{R}^2$ includes all uncertainties in the input channel (i.e., the matched uncertainty) and the term $\psi(\cdot) \in \mathcal{R}^3$ denotes the mismatched uncertainty.

Without loss of generality, it is assumed that $\psi(\cdot)$ has the form $\psi(t,q) := \text{col}(\psi_1(t,q), \psi_2(t,q), 0)$ where $\psi_1(\cdot) \in \mathcal{R}$ and $\psi_2(\cdot) \in \mathcal{R}$. Note that the third component of $\psi(\cdot)$ is assumed to be zero. If it is not zero, then it can be included in the matched uncertainty $\phi(\cdot)$ in (34).

Assume that the reference trajectory is model based, and it is given by the following dynamic system:

$$\begin{bmatrix}\dot{q}_{xr}\\ \dot{q}_{yr}\\ \dot{\theta}_r\end{bmatrix} = \begin{bmatrix}\cos\theta_r & 0\\ \sin\theta_r & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_r(t)\\ \omega_r(t)\end{bmatrix} \tag{35}$$

where $q_r = \text{col}(q_{xr}, q_{yr}, \theta_r)$ is the reference trajectory and $u_r = \text{col}(v_r(t), \omega_r(t))$ is the reference control with $v_r \neq 0$. Then the objective of the model-based tracking control is to design a controller u for the system (34) such that $\lim_{t\to\infty}\|q_r - q\| = 0$ where $q = \text{col}(q_x, q_y, \theta_c) \in \mathcal{R}^3$ is the state of the system (34) and $q_r = \text{col}(q_{xr}, q_{yr}, \theta_r)$ is the reference trajectory created by (35).

Remark 8: Due to the complex nonlinearity in the nonholonomic WMR system, it is straightforward to see that not all trajectories can be tracked. Therefore, the trajectory in this paper is assumed to be model based. It should be noted that the initial misalignment of the WMR may result in different initial misalignment of the tracking error system. Such an effect can be included in the system uncertainty that can be overcome by redesign of the SMC if necessary.

Introduce a diffeomorphism $T : \mathcal{R}^3 \longrightarrow \mathcal{R}^3$ with $x = T(q)$ as (see, e.g., [27])

$$x := \begin{bmatrix}x_1\\ x_2\end{bmatrix} = \begin{bmatrix}x_1\\ x_{21}\\ x_{22}\end{bmatrix} = \tilde{T}(q)(q_r - q) := T(q) \tag{36}$$

where $x_1 \in \mathcal{R}$, $x_2 = \text{col}(x_{21}, x_{22}) \in \mathcal{R}^2$ and

$$\tilde{T}(q) = \begin{bmatrix}-\sin\theta_c & \cos\theta_c & 0\\ \cos\theta_c & \sin\theta_c & 0\\ 0 & 0 & 1\end{bmatrix}.$$

From (34)–(36), the dynamics of the new error system in x coordinates is given by

$$\begin{aligned}\dot{x} &= \begin{bmatrix}v_r(t)\sin\theta_r\cos\theta_c - v_r(t)\cos\theta_r\sin\theta_c\\ v_r(t)\cos\theta_r\cos\theta_c + v_r(t)\sin\theta_r\sin\theta_c\\ \omega_r(t)\end{bmatrix}\\ &\quad + \begin{bmatrix}0 & -\cos\theta_c(q_{rx} - q_x) - \sin\theta_c(q_{ry} - q_y)\\ -1 & -\sin\theta_c(q_{rx} - q_x) + \cos\theta_c(q_{ry} - q_y)\\ 0 & -1\end{bmatrix}\\ &\quad \cdot \Big(u + \hat{\phi}(t,x)\Big) + \Psi(t,x)\\ &= \underbrace{\begin{bmatrix}v_r(t)\sin x_{22}\\ v_r(t)\cos x_{22}\\ \omega_r(t)\end{bmatrix}}_{\mathcal{F}(t,x)} + \underbrace{\begin{bmatrix}0 & -x_{21}\\ -1 & x_1\\ 0 & -1\end{bmatrix}}_{\mathcal{G}(t,x)}\Big(u + \hat{\phi}(t,x)\Big)\\ &\quad + \Psi(t,x)\end{aligned} \tag{37}$$

where

$$\hat{\phi}(t,x) = \phi(t,q)|_{q=T^{-1}(x)}$$

$$\Psi(t,x) := \begin{bmatrix}\Psi_1(t,x)\\ \Psi_2(t,x)\end{bmatrix} = \frac{\partial T}{\partial q}\psi(t,q)|_{q=T^{-1}(x)}. \tag{38}$$

By direct calculation,

$$\frac{\partial T}{\partial q} = \Big(-\tilde{T}(q) + \hat{T}(x)\Big) \tag{39}$$

where

$$\hat{T}(x) = \begin{bmatrix}0 & 0 & -x_{21}\\ 0 & 0 & x_1\\ 0 & 0 & 0\end{bmatrix}.$$

Substituting (39) into (38) yields

$$\Psi(t,x) = -\tilde{T}(q)\psi(t,q)|_{q=T^{-1}(x)}. \tag{40}$$

Then it is straightforward to see that the mismatched uncertainty $\Psi(t,x)$ in the new error system (37) has the form

$$\Psi(t,x) = \begin{bmatrix}\Psi_1(t,x)\\ \Psi_2(t,x)\end{bmatrix} = \begin{bmatrix}\Psi_1(t,x)\\ \hline \Psi_{21}(t,x)\\ 0\end{bmatrix}.$$

Thus, system (37) can be described in the form (7) and (8) as follows

$$\dot{x}_1 = \underbrace{v_r(t)\sin x_{22}}_{\mathcal{F}_1(t,x)} + \underbrace{\begin{bmatrix} 0 & -x_{21} \end{bmatrix}}_{\mathcal{G}_1(t,x)} \Big(u + \Phi(t,x)\Big) + \Psi_1(t,x) \tag{41}$$

$$\dot{x}_2 = \underbrace{\begin{bmatrix} v_r(t)\cos x_{22} \\ \omega_r(t) \end{bmatrix}}_{\mathcal{F}_2(t,x)} + \underbrace{\begin{bmatrix} -1 & x_1 \\ 0 & -1 \end{bmatrix}}_{\mathcal{G}_2(t,x)} \Big(u + \Phi(t,x)\Big) \tag{42}$$

where $x_2 = \text{col}(x_{21}, x_{22}) \in \mathcal{R}^2$, $x_1 \in \mathcal{R}$ and

$$\Phi(t,x) := \hat{\phi}(t,x) - \Psi_2(t,x). \tag{43}$$

It is straightforward to verify that $\tilde{T}(q)$ is nonsingular and $\tilde{T}^{-1}(q)$ is bounded. From (36), $\|q_r - q\| \le \|\tilde{T}^{-1}(q)\|\,\|x\|$, which implies that $\lim_{t\to\infty}\|q_r - q\| = 0$ if $\lim_{t\to\infty}\|x\| = 0$. Therefore, the model-based reference tracking control problem for the kinematic model (34) has now been transformed to a stabilization problem for the error system (37). It remains to design a control u to stabilize the system (37) globally and asymptotically.

B. Control design

Assume that the reference trajectory only moves forward with $v_r(t) \ge \mathcal{V}_m$ where $\mathcal{V}_m$ is a positive constant such that a continuously differentiable feedback control law that asymptotically stabilizes the tracking error system exists [21], [32], and the reference velocities $(v_r(t), \omega_r(t))$ are bounded with $v_r(t) \le \mathcal{V}_x$ and $|\omega_r(t)| \le \mathcal{W}_x$ for any $t \in \mathcal{R}^+$. Furthermore, the mismatched and matched uncertainties $\Psi_1(t,x)$ and $\Phi(t,x)$ satisfy

$$\|\Psi_1(t,x)\| \le \underbrace{\sin^2(x_{22})\sqrt{x_{21}^2 + \alpha} + 0.1|x_1 x_{21}|\sqrt{x_{21}^2 + \alpha}}_{\delta(t,x)} \tag{44}$$

$$\|\Phi(t,x)\| \le \underbrace{0.5\|x\| + 0.6|v_r\omega_r|}_{\mu(t,x)} \tag{45}$$

where α is a positive constant satisfying $\alpha < \mathcal{V}_m^2$. Design the switching functions

$$\sigma(x) = \begin{bmatrix} k_1 x_{21} \\ k_2 x_{22} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{x_1}{\sqrt{c + x_1^2 + x_{21}^2}} \end{bmatrix}}_{\varphi(x_1, x_2)} \tag{46}$$

where $k_1 > 0$ and $k_2 > 1$ are design parameters and $c > 0$ is a constant. The sliding surface is described by

$$\mathcal{S} = \{x \in \mathcal{R}^3 \,|\, \sigma(x) = 0\} \tag{47}$$

where $\sigma(x)$ is defined in (46). Then on the sliding surface (47), $x_{21} = 0$, and thus, from (41), $\mathcal{G}_1(t,x) = 0$. Therefore, system (41) and (42) has the generalized regular form. From $\mathcal{F}(\cdot)$ and $\mathcal{G}(\cdot)$ in (37) and by direct calculation

$$\Gamma_{\mathcal{F}}(t,x) := \frac{\partial \sigma}{\partial x}\mathcal{F}(t,x) = \begin{bmatrix} k_1 v_r \cos x_{22} \\ \frac{(c + x_{21}^2)v_r \sin x_{22} - x_1 x_{21} v_r \cos x_{22}}{\sqrt{c + x_1^2 + x_{21}^2}} + k_2\omega_r \end{bmatrix} \tag{48}$$

$$\Gamma_{\mathcal{G}}(t,x) := \frac{\partial \sigma}{\partial x}\mathcal{G}(t,x) = \begin{bmatrix} -k_1 & k_1 x_1 \\ \frac{x_1 x_{21}}{(c + x_1^2 + x_{21}^2)^{\frac{3}{2}}} - \frac{x_{21}}{(c + x_1^2 + x_{21}^2)^{\frac{1}{2}}} & -k_2 \end{bmatrix} \tag{49}$$

which is nonsingular when $k_2 \ge 1$. When system (41) is limited to the sliding surface (47), it can be described by

$$\dot{x}_1 = \underbrace{v_r(t)\sin\left(-\frac{x_1}{k_2\sqrt{c + x_1^2}}\right)}_{\mathcal{F}_1^s(t,x_1)} + \Psi_1^s(t,x_1) \tag{50}$$

where

$$\|\Psi_1^s(t,x_1)\| \le \underbrace{\sqrt{\alpha}\sin^2\left(\frac{x_1}{k_2\sqrt{c + x_1^2}}\right)}_{\gamma(t,x_1)}. \tag{51}$$

Therefore, system (50) with $\Psi_1^s(\cdot)$ satisfying (51) is the sliding mode dynamics associated with the sliding surface (47). For system (50), define the candidate Lyapunov function as $V(t,x_1) = \frac{1}{2}x_1^2$, then it is clear to see that

$$\underbrace{0.4x_1^2}_{\varsigma_1(t,x_1)} \le V(t,x_1) \le \underbrace{0.6x_1^2}_{\varsigma_2(t,x_1)}.$$

The time derivative of V along the trajectories of system (50) is given by

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x_1}\mathcal{F}_1^s(t,x_1) = v_r(t)\sin\left(-\frac{|x_1|}{k_2\sqrt{c + x_1^2}}\right)x_1 \le \underbrace{-\mathcal{V}_m\sin\left(\frac{|x_1|}{k_2\sqrt{c + x_1^2}}\right)|x_1|}_{\varsigma_3(|x_1|)} \tag{52}$$

$$\left\|\frac{\partial V}{\partial x_1}\right\| = \underbrace{|x_1|}_{\varsigma_4(|x_1|)}. \tag{53}$$

From $k_2 \ge 1 > \frac{2}{\pi}$, which implies

$$\frac{\tau}{k_2\sqrt{c + \tau^2}} < \frac{\pi}{2} \tag{54}$$

it is straightforward to see that $\varsigma_3(\tau)$ is a class $\mathcal{K}$ function. Thus

$$\begin{aligned}
&\varsigma_3(|x_1|) - \varsigma_4(|x_1|)\gamma(t, x_1) \\
&= \mathcal{V}_m \sin\left(\frac{x_1}{k_2\sqrt{1+x_1^2}}\right)|x_1| - \left(\sqrt{\alpha}\sin^2\left(\frac{x_1}{k_2\sqrt{c+x_1^2}}\right)\right)|x_1| \\
&\leq \left(\mathcal{V}_m \sin\left(\frac{x_1}{k_2\sqrt{1+x_1^2}}\right) - \sqrt{\alpha}\sin^2\left(\frac{x_1}{k_2\sqrt{c+x_1^2}}\right)\right)|x_1| \\
&= w(|x_1|) \qquad (55)
\end{aligned}$$

where

$$w(\tau) = \left(\mathcal{V}_m \sin\left(\frac{\tau}{k_2\sqrt{c+\tau^2}}\right) - \sqrt{\alpha}\sin^2\left(\frac{\tau}{k_2\sqrt{c+\tau^2}}\right)\right)\tau \qquad (56)$$

where $\tau \in \mathcal{R}^+$. Since $\mathcal{V}_m \geq \sqrt{\alpha} \geq \sqrt{\alpha}\sin(\frac{\tau}{k_2\sqrt{c+x_1^2}})$, it is clear that $w(\tau)$ is positive definite. Therefore, the conditions of Theorem 1 hold. By limiting the minimum reference velocity $\mathcal{V}_m = 0.01$, the kinematic controller $u = \text{col}(v, \omega)$ is described by

$$\begin{aligned}
u(t,x) = &-\Gamma_{\mathcal{G}}^{-1}(t,x)\Gamma_{\mathcal{F}}(t,x) - \Gamma_{\mathcal{G}}^{-1}(t,x)\text{sgn}\big(\sigma(x)\big)\cdot \\
&\left\{\left\|\frac{\partial\sigma}{\partial x}\right\|\delta(t,x) + \|\Gamma_{\mathcal{G}}(t,x)\|\mu(t,x) + 5\right\} \qquad (57)
\end{aligned}$$

where the uncertainties $\delta(\cdot)$ and $\mu(\cdot)$ for the WMR are defined in (44) and (45), respectively. $\sigma(x)$ for the WMR is defined in (46) with $k_1 = k_2 = 1$ and $c = 0.01$, and the corresponding $\Gamma_{\mathcal{G}}(\cdot)$ and $\Gamma_{\mathcal{F}}(\cdot)$ are defined in (48) and (49), respectively. Then, from Theorems 1 and 2, it is straightforward to see that systems (41) and (42) are globally asymptotically stable.

The performance of the proposed controller is tested with a smoothed sharp corner trajectory, which can be described by the following equations:

$$q_{rx}(t) = \begin{cases} 0 & t < 4 - \beta \\ \frac{\sqrt{(t+\beta-4)^2+\beta}-\sqrt{\beta}}{\sqrt{16+\beta}}, & t \geq 4 - \beta \end{cases} \qquad (58)$$

$$q_{ry}(t) = \begin{cases} 1 - \frac{\sqrt{(t-4)^2+\beta}}{\sqrt{16+\beta}}, & t < 4 \\ 1 & t \geq 4 \end{cases} \qquad (59)$$

where $\beta = 0.81$ is a positive parameter that smoothes the corner.

The initial point of the reference is $(0, 0, \frac{\pi}{2})$ and the initial point of the robot is chosen as $(0.5, 0.1, 2.17)$. The motion of the robot and the reference trajectory given by (58) and (59) are shown in Fig. 1. The time response of the tracking errors and the control signal (v, ω) shown in Figs. 2 and 3, respectively. From Fig. 3, it can be seen that the system is affected by the matched uncertainties at the corner. However, due to the complete robustness of SMC to matched uncertainties, the performance of the system is not affected. From Figs. 1–3, it is straightforward to see that the proposed approach is effective. It should be noted that due to the discontinuity of the sgn function, the control in reality may experience chattering [33]. To avoid such problems, the boundary-layer technique proposed in [34] has been introduced to reduce the chattering in the simulation and experiments presented in this paper.

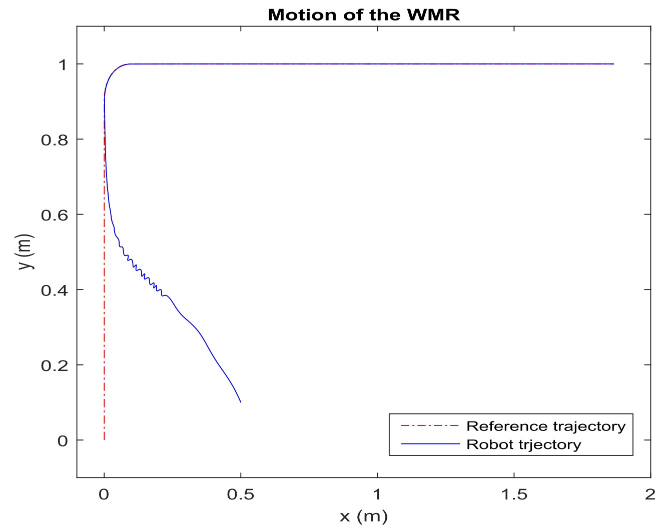

Fig. 1. Reference trajectory of the Lemniscate curve and the trajectory of the robot in the $x - y$ plane.

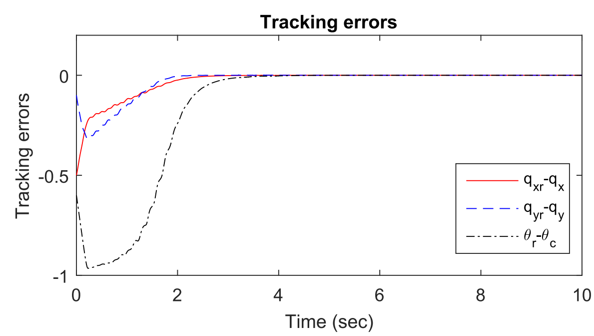

Fig. 2. Time response of the tracking errors.

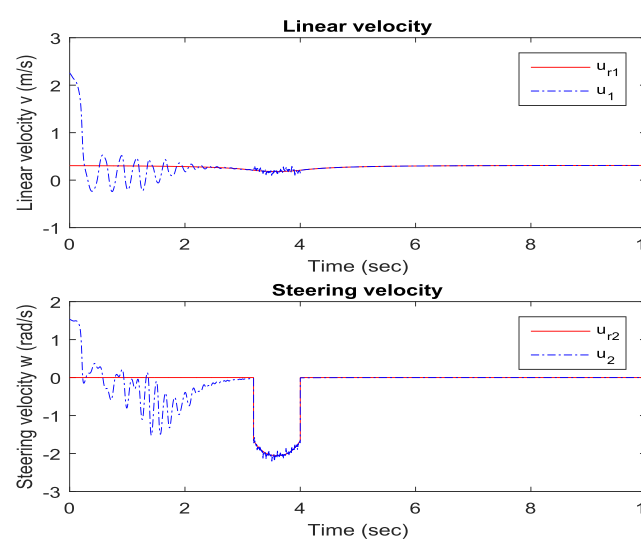

Fig. 3. Time response of the control pair (v, ω).

Remark 9: Uncertainties are added in the WMR simulation and bounds on the uncertainties are given to show the robustness of the proposed methodology. In the real system, the uncertainties will vary on a case-by-case basis and can be obtained by statistical data analysis or engineering experience.

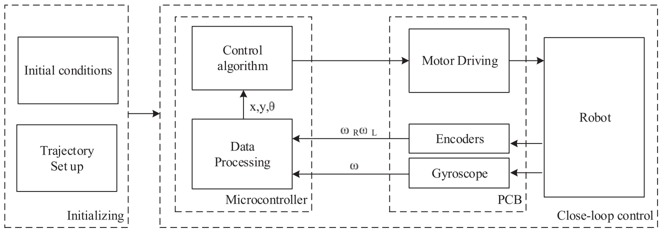

Fig. 4. System overview for the WMR.

V. Experimental Test

A low-cost WMR was built at the University of Kent for experimental testing, the overview of the system is shown in Fig. 4. Two wheels with a radius of 0.063 m are assembled on the right and left side equipped with 12V dc motors as actuators for differential driving. The size of the chassis is 20 cm(l/w) with a 12-V battery and electronics. A rate gyroscope and two encoders with 1600 pulses/turn assembled on the shaft of the motors are used to estimate the coordinates. It should be noted that the motors are independently driven by two H-bridge MOSFET-based motor drivers. The actual control signals are pulse-width-modulation (PWM) signals controlled by a microcontroller embedded in the robot. In order to obtain data from the controller, a bluetooth module is used to transfer data to the PC via a serial communication with cycle time of 10 ms.

A. Implementation of the Control With DC Motors

It should be noted that the control inputs of system (41) and (42) are the linear velocity v and the steering velocity ω. As assumed by other authors (e.g., see [31]), such a controller can be implemented directly using the differential driving mechanism to produce the desired inputs (v, ω) required by the controller (28). Two dc motors are used as actuators driving the wheels on each side of the robot independently. The relationship between the velocities of the robot (v, ω) and the rotational velocities of the wheels (ω_R, ω_L) can be described as follows (e.g., see [31]):

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \frac{1}{2} \begin{bmatrix} r & r \\ \frac{r}{b} & -\frac{r}{b} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} \tag{60}$$

where (ω_R, ω_L) denote the rotational velocities of the wheels on the right and left sides, respectively. r and b denote the radius of the wheel and width of the robot, respectively. The dynamics of the motor are also investigated to achieve the input (v, ω) required by controller (28). The model of the motor system can be described by (e.g., see [3])

$$\begin{bmatrix} \dot{\omega}_m \\ \dot{i}_m \end{bmatrix} = \begin{bmatrix} 0 & \frac{K_t}{J_m} \\ -\frac{K_e}{L_m} & -\frac{R_m}{L_m} \end{bmatrix} \begin{bmatrix} \omega_m \\ i_m \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_m} \end{bmatrix} u_v + \begin{bmatrix} -T_L \\ 0 \end{bmatrix} \tag{61}$$

$$y = \omega_m \tag{62}$$

where ω_m and i_m are the angular velocity and motor current, and y is the measured output. u_v denotes the input voltage

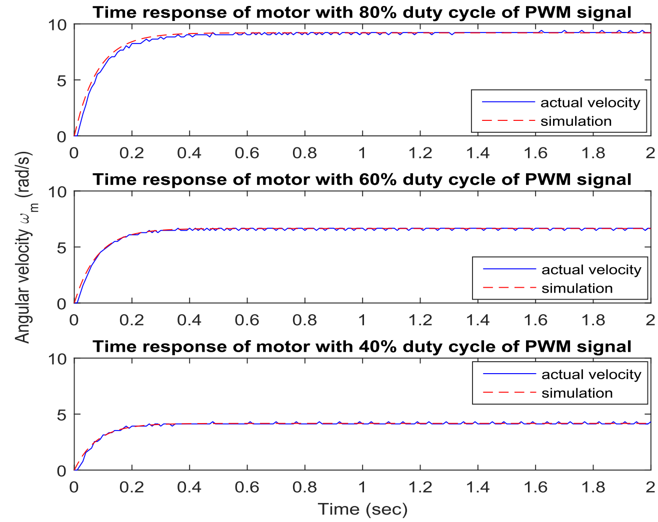

Fig. 5. Comparison between the actual time response of angular velocity of the motor and the simulation.

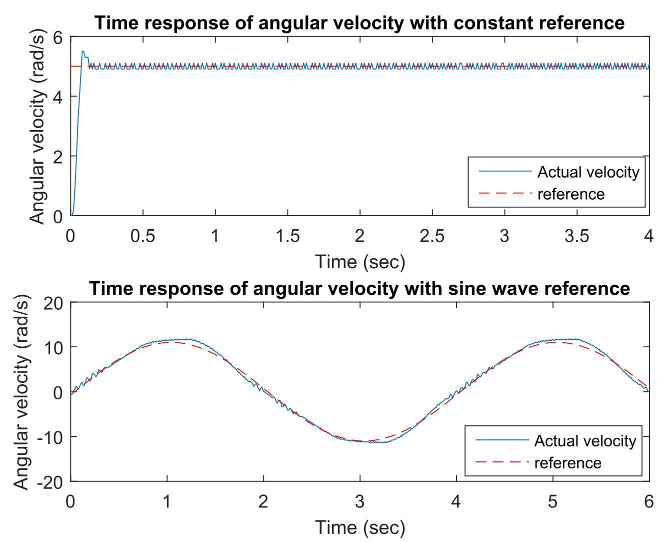

Fig. 6. Tracking performance of the motor control.

adjusted by the microcomputer with PWM techniques. Parameters J_m, L_m, K_t, K_e, and R_m denote the motor inertia, inductance, torque constant, back electromotive force constant and resistance, respectively. T_L is the external disturbance representing the effects of friction and the motor load.

Parameters identified through experiments with no-load are $J_m = 0.0012$ Kg $\cdot$ m^2, $L_m = 0.0054$ F, $K_t = 0.034$ N $\cdot$ m/A, $K_e = 1.04$ V $\cdot$ s/rad, and $R_m = 2.4\,\Omega$. The comparison between the model response (61) and the response of the actual motor is shown in Fig. 5. The experimental results when tracking a constant reference and sine wave reference signals are shown in Fig. 6. From the test results, it can be seen that although the system is affected by the limitation of the hardware, the tracking performance is as expected. Although the control performance of the motors may also be affected by parameter variations, the uncertainties caused by friction between the wheels and ground

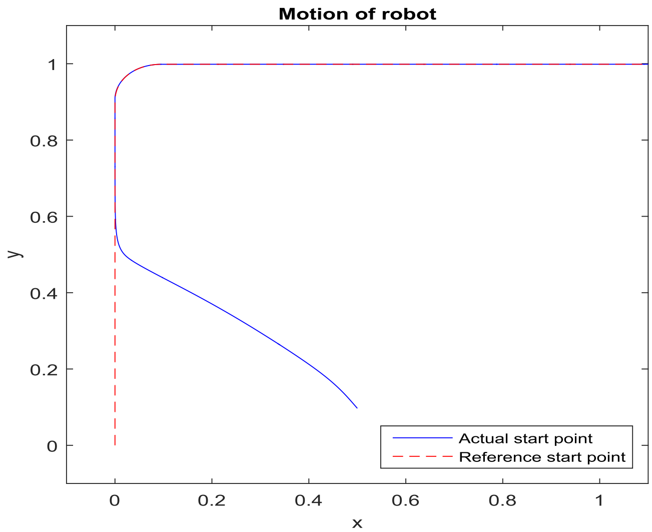

Fig. 7. Motion of robot in *x*–*y* plane in tracking task experiment.

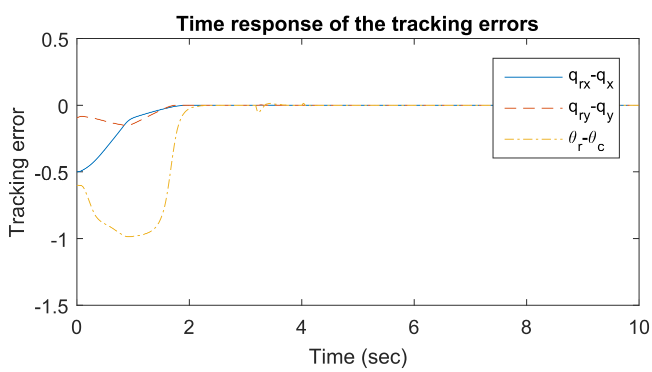

Fig. 8. Time response of tracking errors in tracking task experiment.

in the motor system will not affect the performance of the WMR system since the SMC is robust to matched uncertainties.

B. Experimental Results

The experimental results for the WMR are presented in this section. The control of the robot is designed with the same process described in Section IV-B and the control performance is tested with the reference curve described in (58) and (59), which denotes a smoothed right-angled curve.

The actual motion of the robot and the reference trajectory are shown in Fig. 7. The time response of the tracking errors is shown in Fig. 8, and the control signal is shown in Fig. 9. From Fig. 8, it is seen that the system experiences uncertainties caused by the hardware. However, the robot exhibits good tracking performance as shown in Fig. 7 due to the high robustness of the designed SMC.

From the experimental results, it is evident that although modeling error and noise may exist, the robustness properties of the SMC ensure that the system exhibits the expected tracking performance in the presence of uncertainties. It should be noted that the noise usually comes from the motors, and thus, it is matched.

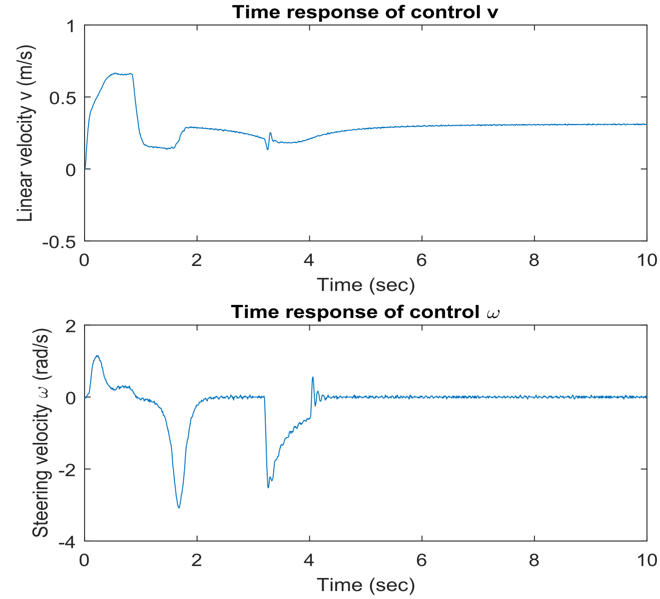

Fig. 9. Measured control input (v, ω) based on sensors data in tracking task experiment.

Since SMC is completely robust to matched uncertainty, good tracking accuracy is achieved in the experiments.

VI. Conclusion

This paper has proposed a novel generalized regular form for a class of nonlinear systems. Based on the generalized regular form, a novel sliding surface has been designed and global asymptotic stability of the corresponding sliding motion has been presented. A SMC scheme was designed to guarantee reachability of the sliding mode. The developed results have been applied to a WMR. Based on the WMR dynamics, a nonlinear sliding surface was formed and global asymptotic stability was exhibited. This application demonstrates that sliding mode techniques can be used to stabilize systems when the normal regular form is not available. Simulation and experimental results showed that the proposed results are effective.

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