Supporting information

MILP-based approaches for Medium-Term Planning and Scheduling in Multiproduct Multistage Continuous Plants

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Table S1. Production rate (tons/week)

r_{mi}	**i1**	**i2**	**i3**	**i4**	**i5**	**i6**	**i7**
m1	110	70	110	90	120	80	130
m2	80	120	100	90	100	120	120
m3	90	100	100	90	110	110	120

Table S2. Changeover times (min)

t_{mij}	**j1**	**j2**	**j3**	**j4**	**j5**	**j6**	**j7**
m1.i1		45	45	45	60	80	30
m1.i2	55		55	40	60	80	80
m1.i3	60	100		100	75	60	80
m1.i4	60	100	30		45	45	45
m1.i5	60	60	55	30		35	30
m1.i6	75	75	60	100	75		100
m1.i7	80	100	30	60	100	85	
m2.i1		60	45	80	45	30	50
m2.i2	60		30	50	55	70	100
m2.i3	30	50		80	60	90	50
m2.i4	70	100	50		60	80	30
m2.i5	50	60	45	40		75	60
m2.i6	90	80	100	70	45		70
m2.i7	50	80	40	55	70	100	
m3.i1		70	100	55	30	60	55
m3.i2	40		70	35	60	75	90
m3.i3	30	25		80	70	55	40
m3.i4	70	100	90		80	100	55
m3.i5	80	90	45	60		75	60
m3.i6	35	45	80	90	55		85
m3.i7	100	65	85	55	30	45	
m1.i1		45	45	45	60	80	30
m1.i2	55		55	40	60	80	80
m1.i3	60	100		100	75	60	80
m1.i4	60	100	30		45	45	45
m1.i5	60	60	55	30		35	30
m3.i6	35	45	80	90	55		85
m3.i7	100	65	85	55	30	45	

Table S3. Demand forecast (tons)

D_{ciw}	w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12	w13	w14	w15	w16	w17	w18
c1.i1	5				5				5				5				5	
c1.i3	2	2	2	2	3	3	3	3	3	2	2	2	2	3	3	3	3	3
c5.i1	5				5				5				5				5	
c5.i3	2	2	2	2	3	3	3	3	3	2	2	2	2	3	3	3	3	3
c2.i4	3		3		3		3		3		3		3		3		3	
c2.i5	5		5		5		5		5		5		5		5		5	
c6.i4	3		3		3		3		3		3		3		3		3	
c6.i5	5		5		5		5		5		5		5		5		5	
c3.i2	4				4				4				4				4	
c3.i7			5										5					
c7.i2	4				4				4				4				4	
c7.i7			5										5					
c9.i2	4				4				4				4				4	
c9.i7			5										5					
c4.i1	7				7				7				7				7	
c4.i2		5		5			5		5			5		5			5	
c4.i3	5			5			5			5			5			5		
c4.i4	10					10			10					10			10	
c4.i5	11		11		11		11		11		11		11		11		11	
c4.i6	8			8			8			8			8			8		
c4.i7	4		4		4		4		4		4		4		4		4	
c8.i1	7				7				7				7				7	
c8.i2		5		5			5		5			5		5			5	
c8.i3	5			5			5			5			5			5		
c8.i4	10					10			10					10			10	
c8.i5	11		11		11		11		11		11		11		11		11	
c8.i6	8			8			8			8			8			8		
c8.i7	4		4		4		4		4		4		4		4		4	
c10.i1	7				7				7				7				7	
c10.i2		5		5			5		5			5		5			5	
c10.i3	5			5			5			5			5			5		
c10.i4	10					10			10					10			10	
c10.i5	11		11		11		11		11		11		11		11		11	
c10.i6	8			8			8			8			8			8		
c10.i7	4		4		4		4		4		4		4		4		4	

Table S4. Production rate (tons/week)

r_{mi}	**i1**	**i2**	**i3**	**i4**	**i5**	**i6**	**i7**	**i8**	**i9**	**i10**	**i11**	**i12**	**i13**	**i14**	**i15**	**i16**	**i17**	**i18**	**i19**	**i20**
m1	170	220	170	185	150	165	240	240	245	155	230	200	170	210	255	205	180	185	200	165
m2	220	180	175	255	160	250	195	235	180	145	155	230	150	190	235	195	175	180	160	260
m3	150	155	220	155	235	210	175	200	195	230	210	210	235	215	195	210	255	160	255	255

Table S5. Changeover times (min)

t_{mij}	j1	j2	j3	j4	j5	j6	j7	j8	j9	j10	j11	j12	j13	j14	j15	j16	j17	j18	j19	j20
m1.i1		30	85	60	40	40	35	45	85	25	60	100	65	100	80	30	70	30	40	70
m1.i2	55		45	45	30	30	65	85	35	70	80	40	25	60	30	90	40	40	65	75
m1.i3	70	55		50	25	45	20	45	30	70	65	80	40	70	80	70	40	25	25	70
m1.i4	60	20	80		25	30	60	80	30	20	65	70	50	45	35	35	30	95	50	80
m1.i5	40	30	80	25		35	20	40	60	30	30	45	45	45	95	100	45	50	80	50
m1.i6	90	25	75	20	65		20	20	50	60	70	35	50	40	30	95	50	30	50	50
m1.i7	40	95	35	40	25	50		25	50	45	35	25	65	60	20	80	95	65	65	45
m1.i8	65	75	60	30	70	60	30		95	35	70	80	95	35	40	35	35	70	75	25
m1.i9	30	55	35	75	80	30	50	75		85	65	95	20	35	25	60	30	75	25	55
m1.i10	80	55	60	20	60	55	95	30	30		20	30	20	20	85	65	20	35	95	45
m1.i11	65	40	70	30	55	50	85	60	50	65		95	45	20	95	65	45	95	80	25
m1.i12	100	65	30	70	45	90	90	20	45	70	65		20	85	95	60	40	60	20	80
m1.i13	60	80	75	70	25	95	75	95	85	70	75	75		80	65	20	55	20	75	35
m1.i14	35	85	100	80	75	20	40	85	85	85	35	55	20		45	55	45	30	85	50
m1.i15	30	55	40	90	25	50	45	55	70	70	45	25	90	35		90	55	25	50	50
m1.i16	50	25	80	85	20	55	40	90	20	75	95	90	90	90	50		60	85	65	25
m1.i17	65	65	75	30	45	45	60	45	30	75	60	65	75	75	20	85		75	30	65
m1.i18	70	35	45	70	75	50	30	20	20	95	95	95	85	30	20	60	30		100	85
m1.i19	40	25	50	45	25	65	55	50	90	35	35	60	70	45	30	95	40	25		45
m1.i20	20	85	35	50	40	55	75	65	30	30	45	65	40	20	75	70	45	80	30	
m2.i1		75	70	85	60	40	60	50	25	85	45	25	65	20	80	90	65	55	75	60
m2.i2	90		80	30	40	75	55	95	95	90	45	55	65	90	30	70	80	65	20	85
m2.i3	40	55		45	65	65	60	65	95	45	80	95	95	40	45	35	30	75	40	80
m2.i4	70	40	75		20	90	20	50	45	75	50	60	45	75	95	55	35	60	45	95
m2.i5	25	60	50	35		60	65	45	40	60	60	20	50	95	50	30	55	50	35	70
m2.i6	20	60	20	60	85		25	80	40	35	55	45	60	25	90	20	85	80	70	50
m2.i7	50	90	95	65	45	55		50	50	25	85	60	75	85	50	90	50	55	75	70
m2.i8	65	30	65	65	75	35	30		90	55	25	90	45	100	55	60	35	35	70	80
m2.i9	65	95	55	50	85	85	20	90		20	70	20	50	70	65	40	25	20	30	55
m2.i10	30	65	85	20	45	45	55	40	90		85	70	95	80	60	80	85	65	65	65
m2.i11	65	95	60	95	50	85	20	55	50	20		25	65	60	70	90	35	50	30	90
m2.i12	70	85	25	100	65	20	65	30	45	75	25		55	70	55	45	90	30	90	55
m2.i13	55	30	25	35	45	90	85	85	55	90	95	25		45	85	45	35	95	45	90
m2.i14	30	55	40	40	25	50	90	40	25	30	50	20	20		30	25	85	50	50	95
m2.i15	35	45	70	35	25	50	55	30	60	90	20	60	80	95		75	90	40	80	85
m2.i16	55	70	50	45	40	40	55	95	45	20	85	75	90	45	35		20	90	25	85
m2.i17	55	20	30	75	40	70	65	95	60	65	65	20	95	65	20	60		30	50	90
m2.i18	75	45	100	20	70	85	40	95	75	70	65	60	30	85	90	75	45		45	80
m2.i19	30	75	95	55	40	55	50	45	70	85	80	90	55	25	95	85	95	65		20
m2.i20	50	20	60	25	90	35	70	40	85	35	60	95	75	35	90	30	50	80	95	
m3.i1		95	40	90	90	95	80	70	80	60	75	75	90	80	35	65	90	65	20	90
m3.i2	65		45	50	40	35	85	30	95	20	75	25	55	45	35	90	55	95	25	55
m3.i3	75	55		30	25	85	50	25	90	30	30	60	55	90	75	30	45	65	65	70
m3.i4	20	30	25		45	65	85	75	75	45	35	90	85	45	95	40	70	95	95	50
m3.i5	25	25	70	45		65	65	70	95	25	30	65	80	85	45	70	90	50	40	95
m3.i6	45	50	95	65	90		40	35	65	60	65	45	50	80	20	45	80	55	25	25
m3.i7	45	35	25	55	55	85		55	95	25	50	90	75	90	60	40	40	45	30	85
m3.i8	45	50	20	35	70	40	30		50	30	85	50	90	40	80	50	50	40	50	25
m3.i9	75	75	25	95	40	45	90	65		50	95	70	95	75	55	90	55	55	50	25
m3.i10	95	85	75	95	45	80	45	60	30		45	25	95	60	70	75	20	35	90	25
m3.i11	70	25	20	80	95	20	30	100	30	25		35	80	90	80	50	80	70	80	65
m3.i12	70	25	65	55	45	45	25	70	60	30	50		20	90	35	85	25	95	65	75
m3.i13	85	65	20	90	60	35	90	80	75	70	60	50		65	25	95	60	75	85	70
m3.i14	25	40	25	60	35	80	100	45	85	80	45	35	60		45	40	100	40	25	95
m3.i15	40	45	50	85	55	95	55	90	60	90	35	30	80	45		20	60	60	90	55
m3.i16	45	25	70	95	20	45	90	55	50	75	70	25	60	90	100		60	70	80	90
m3.i17	30	60	20	55	60	75	80	90	40	90	55	90	35	25	75	90		20	55	70
m3.i18	75	45	20	45	45	90	100	35	45	55	70	20	90	20	40	25	55		65	55
m3.i19	60	30	50	60	95	65	55	75	75	25	50	30	75	65	45	25	70	80		50
m3.i20	85	65	70	80	70	25	80	70	85	90	85	95	20	95	20	25	80	40	90	

Table S6. Demand forecast (tons)

D_{ciw}	w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12	w13	w14	w15	w16	w17	w18
c1.i1	5				5				5				5				5	
c1.i3	2	2	2	2	3	3	3	3	3	2	2	2	2	3	3	3	3	3
c1.i11		7	7	7		7	7	7		7	7	7		7	7	7		7
c1.i14		15				5				15				5				15
c1.i17	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
c1.i18					20					20					20			
c1.i19	15			15		15			15		15			15		15		
c1.i20		10	10				10	10				10	10				10	10
c5.i1	5				5				5				5				5	
c5.i3	2	2	2	2	3	3	3	3	3	2	2	2	2	3	3	3	3	3
c5.i11		7	7	7		7	7	7		7	7	7		7	7	7		7
c5.i14		15				5				15				5				15
c5.i17	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
c5.i18					20					20					20			
c5.i19	15			15		15			15		15			15		15		
c5.i20		10	10				10	10				10	10				10	10
c2.i4	3		3		3		3		3		3		3		3		3	
c2.i5	5		5		5		5		5		5		5		5		5	
c2.i8		12				12		12				12		12				12
c2.i12			15		15				15		15				15		15	
c2.i15				9						9						9		
c6.i4	3		3		3		3		3		3		3		3		3	
c6.i5	5		5		5		5		5		5		5		5		5	
c6.i8		12				12		12				12		12				12
c6.i12			15		15				15		15				15		15	
c6.i15				9						9						9		
c3.i2	4				4				4				4				4	
c3.i7			5										5					
c3.i10		6		6		6		6		6		6		6		6		6
c3.i13		9		9		9		9		9		9		9		9		9
c7.i2	4				4				4				4				4	
c7.i7			5										5					
c7.i10		6		6		6		6		6		6		6		6		6
c7.i13		9		9		9		9		9		9		9		9		9
c9.i2	4				4				4				4				4	
c9.i7			5										5					
c9.i10		6		6		6		6		6		6		6		6		6
c9.i13		9		9		9		9		9		9		9		9		9
c4.i1	7				7				7				7				7	
c4.i2		5		5			5		5			5		5			5	
c4.i3	5			5			5			5			5			5		
c4.i4	10					10			10					10			10	
c4.i5	11		11		11		11		11		11		11		11		11	
c4.i6	8			8			8			8			8			8		
c4.i7	4		4		4		4		4		4		4		4		4	
c4.i8	1	1	1	3	3	3	1	1	1	3	3	3	1	1	1	3	3	3
c4.i9	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
c4.i10			3		3	3		3		3	3		3		3	3		3
c4.i11		15				15				15				15				15
c4.i12			10				10				10				10			
c4.i13				12				12				12				12		
c4.i14			10				10				10				10			
c4.i15		15				15				15				15				15
c4.i16				11							11							11
c8.i1	7				7				7				7				7	
c8.i2		5		5			5		5			5		5			5	
c8.i3	5			5			5			5			5			5		
c8.i4	10					10			10					10			10	
c8.i5	11		11		11		11		11		11		11		11		11	
c8.i6	8			8			8			8			8			8		
c8.i7	4		4		4		4		4		4		4		4		4	
c8.i8	1	1	1	3	3	3	1	1	1	3	3	3	1	1	1	3	3	3
c8.i9	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
c8.i10			3		3	3		3		3	3		3		3	3		3
c8.i11		15				15				15				15				15
c8.i12			10				10				10				10			
c8.i13				12				12				12				12		
c8.i14			10				10				10				10			
c8.i15		15				15				15				15				15
c8.i16				11							11							11
c10.i1	7				7				7				7				7	
c10.i2		5		5			5		5			5		5			5	
c10.i3	5			5			5			5			5			5		

D_{ciw}	**w1**	**w2**	**w3**	**w4**	**w5**	**w6**	**w7**	**w8**	**w9**	**w10**	**w11**	**w12**	**w13**	**w14**	**w15**	**w16**	**w17**	**w18**
c10.i4	10					10			10					10			10	
c10.i5	11		11		11		11		11		11		11		11		11	
c10.i6	8			8			8			8			8			8		
c10.i7	4		4		4		4		4		4		4		4		4	
c10.i8	1	1	1	3	3	3	1	1	1	3	3	3	1	1	1	3	3	3
c10.i9	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
c10.i10			3		3	3		3		3	3		3		3	3		3
c10.i11		15				15				15				15				15
c10.i12			10				10				10				10			
c10.i13				12				12				12				12		
c10.i14			10				10				10				10			
c10.i15		15				15				15				15				15
c10.i16				11							11							11

Table S7. Product prices ($/ton)

	i1	**i2**	**i3**	**i4**	**i5**	**i6**	**i7**	**i8**	**i9**	**i10**	**i11**	**i12**	**i13**	**i14**	**i15**	**i16**	**i17**	**i18**	**i19**	**i20**
PS_{ciw}	10	12	13	12	15	10	8	14	7	15	11	9	13	7	12	14	10	16	5	11

Table S8. Comparison of results for different problem instances with 20% variation in demands

MILP		Existing MILP models			Proposed MILP models			Sequential approaches	
		[M1]	**[M2]**	**[M3]**	**[H1]**	**[H2]**	**[H3]**	**RH**	**RH+II**
P1	**PROFIT ($)**	**5844**	**3561**	**2297**	**4509**	**6857**	**6106**	**6305**	**6970**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	126	380
	GAP (%)	31%	69%	80%	48%	19%	28%	-	-
	IP (%)	-	-39%	-61%	-23%	17%	4%	8%	19%
P2	**PROFIT ($)**	**5300**	**3590**	**1916**	**2757**	**6408**	**NFS**	**6111**	**6635**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	190	456
	GAP (%)	35%	68%	83%	67%	21%	-	-	-
	IP (%)	-	-32%	-64%	-48%	21%	-	15%	25%
P3	**PROFIT ($)**	**5058**	**3692**	**1770**	**5721**	**6288**	**6514**	**6485**	**6847**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	124	381
	GAP (%)	41%	68%	85%	33%	26%	24%	-	-
	IP (%)	-	-27%	-65%	13%	24%	29%	28%	35%
P4	**PROFIT ($)**	**5444**	**3561**	**1699**	**5724**	**6459**	**NFS**	**6237**	**6619**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	65	313
	GAP (%)	33%	68%	85%	30%	21%	-	-	-
	IP (%)	-	-35%	-69%	5%	19%	-	15%	22%
P5	**PROFIT ($)**	**7050**	**3822**	**3737**	**6746**	**7513**	**NFS**	**7214**	**7855**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	40	283
	GAP (%)	24%	69%	70%	26%	19%	-	-	-
	IP (%)	-	-46%	-47%	-4%	7%	-	2%	11%
P6	**PROFIT ($)**	**5432**	**4001**	**2411**	**6158**	**7230**	**5536**	**6508**	**6823**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	38	290
	GAP (%)	37%	66%	80%	30%	17%	36%	-	-
	IP (%)	-	-26%	-56%	13%	33%	2%	20%	26%
P7	**PROFIT ($)**	**5966**	**4024**	**3930**	**4557**	**7057**	**6258**	**6433**	**6870**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	43	295
	GAP (%)	31%	66%	66%	49%	18%	28%	-	-
	IP (%)	-	-33%	-34%	-24%	18%	5%	8%	15%
P8	**PROFIT ($)**	**5308**	**3216**	**1568**	**5099**	**5711**	**NFS**	**5735**	**6184**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	100	364
	GAP (%)	32%	70%	86%	37%	27%	-	-	-
	IP (%)	-	-39%	-70%	-4%	8%	-	8%	17%
P9	**PROFIT ($)**	**4797**	**3815**	**3386**	**5266**	**6670**	**NFS**	**6338**	**6767**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	54	312
	GAP (%)	44%	67%	71%	41%	22%	-	-	-
	IP (%)	-	-20%	-29%	10%	39%	-	32%	41%
P10	**PROFIT ($)**	**4513**	**2875**	**1375**	**4508**	**6076**	**NFS**	**5362**	**5828**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	119	379
	GAP (%)	40%	73%	87%	40%	19%	-	-	-
	IP (%)	-	-36%	-70%	0%	35%	-	19%	29%

TerminationCriteria = if any solution improve the objective function or time limit imposed of 1000 sec. %GAP = relative gap. %IP=improvement percent in comparison with model [M1]. NFS = no feasible solution.

Table S9. Comparison of results for different problem instances with 20% variation changeover times

MILP		Existing MILP models			Proposed MILP models			Sequential approaches	
		[M1]	**[M2]**	**[M3]**	**[H1]**	**[H2]**	**[H3]**	**RH**	**RH+II**
P1	PROFIT ($)	**3679**	**3377**	**1474**	**6574**	**6366**	**NFS**	**6294**	**6845**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	122	376
	GAP (%)	57%	71%	87%	22%	25%	-	-	-
	IP (%)	-	-8%	-60%	79%	73%	-	71%	86%
P2	PROFIT ($)	**6158**	**3403**	**2443**	**5887**	**6836**	**NFS**	**6612**	**6882**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	87	349
	GAP (%)	28%	71%	79%	34%	20%	-	-	-
	IP (%)	-	-45%	-60%	-4%	11%	-	7%	12%
P3	PROFIT ($)	**5344**	**3599**	**3164**	**5361**	**6037**	**NFS**	**6244**	**6933**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	148	441
	GAP (%)	38%	69%	73%	38%	29%	-	-	-
	IP (%)		-33%	-41%	0%	13%	-	17%	30%
P4	PROFIT ($)	**5904**	**3681**	**2198**	**4842**	**NFS**	**NFS**	**6476**	**6813**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	59	343
	GAP (%)	31%	68%	81%	45%	-	-	-	-
	IP (%)	-	-38%	-63%	-18%	-	-	10%	15%
P5	PROFIT ($)	**NFS**	**3271**	**2663**	**5844**	**5150**	**NFS**	**6495**	**6933**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	40	327
	GAP (%)	-	72%	77%	33%	39%	-	-	-
	IP (%)	-	-	-	-	-	-	-	-
P6	PROFIT ($)	**5087**	**3957**	**2738**	**3582**	**6637**	**NFS**	**6481**	**6817**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	49	338
	GAP (%)	40%	66%	76%	58%	22%	-	-	-
	IP (%)		-22%	-46%	-30%	30%	-	27%	34%
P7	PROFIT ($)	**5504**	**3630**	**3568**	**5238**	**7048**	**NFS**	**6512**	**6813**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	78	368
	GAP (%)	35%	69%	69%	38%	17%	-	-	-
	IP (%)		-34%	-35%	-5%	28%		18%	24%
P8	PROFIT ($)	**5134**	**3483**	**2406**	**6065**	**6282**	**6165**	**6556**	**6923**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	95	375
	GAP (%)	40%	70%	79%	30%	26%	28%	-	-
	IP (%)		-32%	-53%	18%	22%	20%	28%	35%
P9	PROFIT ($)	**5705**	**3197**	**3284**	**6123**	**7016**	**6017**	**6467**	**7119**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	45	327
	GAP (%)	33%	72%	72%	28%	18%	30%	-	-
	IP (%)		-44%	-42%	7%	23%	5%	13%	25%
P10	PROFIT ($)	**5407**	**3309**	**2654**	**5923**	**6751**	**NFS**	**6460**	**6876**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	48	317
	GAP (%)	37%	71%	77%	33%	21%	-	-	-
	IP (%)		-39%	-51%	10%	25%		19%	27%

TerminationCriteria = if any solution improve the objective function or time limit imposed of 1000 sec. %GAP = relative gap. %IP=improvement percent in comparison with model [M1]. NFS = no feasible solution.

Table S10. Comparison of results for different problem instances with 20% variation production rates

MILP		Existing MILP models			Proposed MILP models			Sequential approaches	
		[M1]	**[M2]**	**[M3]**	**[H1]**	**[H2]**	**[H3]**	**RH**	**RH+II**
P1	PROFIT ($)	**2393**	**NFS**	**NFS**	**2650**	**1997**	**4704**	**3383**	**3701**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	179	481
	GAP (%)	27%	-	-	33%	32%	32%	-	-
	IP (%)	-	-	-	11%	-17%	97%	41%	55%
P2	PROFIT ($)	**NFS**	**NFS**	**NFS**	**2189**	**5688**	**5525**	**2233**	**2693**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	103	378
	GAP (%)	-	-	-	30%	26%	25%	-	-
	IP (%)	-	-	-	-	-	-	-	-
P3	PROFIT ($)	**5654**	**1449**	**3344**	**5760**	**5543**	**3007**	**6786**	**7153**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	103	383
	GAP (%)	24%	47%	42%	25%	24%	33%	-	-
	IP (%)	-	-74%	-41%	2%	-2%	-47%	20%	27%
P4	PROFIT ($)	**NFS**	**2609**	**3432**	**5327**	**3686**	**2103**	**6114**	**6452**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	148	437
	GAP (%)	-	44%	41%	26%	28%	42%	-	-
	IP (%)	-	-	-	-	-	-	-	-
P5	PROFIT ($)	**NFS**	**128**	**NFS**	**3167**	**5426**	**1192**	**4015**	**4424**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	111	380
	GAP (%)	-	50%	-	32%	22%	42%	-	-
	IP (%)	-	-	-	-	-	-	-	-
P6	PROFIT ($)	**NFS**	**1181**	**2836**	**4807**	**NFS**	**NFS**	**5468**	**6049**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	110	398
	GAP (%)	-	47%	43%	28%	-	-	-	-
	IP (%)	-	-	-	-	-	-	-	-
P7	PROFIT ($)	**NFS**	**NFS**	**NFS**	**1066**	**NFS**	**2998**	**2615**	**2832**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	180	456
	GAP (%)	-	-	-	43%	-	37%	-	-
	IP (%)	-	-	-	-	-	-	-	-
P8	PROFIT ($)	**NFS**	**NFS**	**NFS**	**NFS**	**4517**	**NFS**	**1694**	**2037**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	194	482
	GAP (%)	-	-	-	-	28%	-	-	-
	IP (%)	-	-	-	-	-	-	-	-
P9	PROFIT ($)	**NFS**	**351**	**NFS**	**4229**	**NFS**	**6017**	**4960**	**5516**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	267	552
	GAP (%)	-	49%	-	30%	-	23%	-	-
	IP (%)	-	-	-	-	-	-	-	-
P10	PROFIT ($)	**5407**	**NFS**	**NFS**	**NFS**	**6751**	**NFS**	**1936**	**2250**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	436	578
	GAP (%)	27%	-	-	-	17%	-	-	-
	IP (%)	-	-	-	-	25%	-	-64%	-58%

TerminationCriteria = if any solution improve the objective function or time limit imposed of 1000 sec. %GAP = relative gap. %IP=improvement percent in comparison with model [M1]. NFS = no feasible solution.

Table S11. Comparison of results for different problem instances with 20% variation production yields

MILP		Existing MILP models			Proposed MILP models			Sequential approaches	
		[M1]	**[M2]**	**[M3]**	**[H1]**	**[H2]**	**[H3]**	**RH**	**RH+II**
P1	PROFIT ($)	**1312**	**NFS**	**NFS**	**2330**	**NFS**	**2473**	**2665**	**3054**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	124	411
	GAP (%)	68%	-	-	44%	-	40%	-	-
	IP (%)	-			78%		88%	103%	133%
P2	PROFIT ($)	**NFS**	**NFS**	**NFS**	**NFS**	**1057**	**1072**	**1334**	**1403**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	78	347
	GAP (%)	-	-	-	-	44%	43%	-	-
	IP (%)	-	-	-	-	-	-	-	-
P3	PROFIT ($)	**8253**	**4476**	**5143**	**8037**	**8183**	**NFS**	**8329**	**8470**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	122	356
	GAP (%)	24%	68%	63%	26%	24%	-	-	-
	IP (%)	-	-46%	-38%	-3%	-1%	-	1%	3%
P4	PROFIT ($)	**8432**	**4346**	**5640**	**6919**	**NFS**	**NFS**	**8840**	**9098**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	77	368
	GAP (%)	25%	70%	62%	39%	-	-	-	-
	IP (%)	-	-48%	-33%	-18%	-	-	5%	8%
P5	PROFIT ($)	**2148**	**148**	**2007**	**4245**	**4995**	**4383**	**4636**	**5067**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	114	355
	GAP (%)	67%	98%	79%	37%	22%	32%	-	-
	IP (%)	-	-93%	-7%	98%	133%	104%	116%	136%
P6	PROFIT ($)	**3200**	**1096**	**2516**	**5743**	**NFS**	**NFS**	**6137**	**6273**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	74	341
	GAP (%)	58%	90%	77%	26%	-	-	-	-
	IP (%)	-	-66%	-21%	79%	-	-	92%	96%
P7	PROFIT ($)	**3455**	**NFS**	**NFS**	**3968**	**NFS**	**NFS**	**4289**	**4840**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	148	352
	GAP (%)	43%	-	-	34%	-	-	-	-
	IP (%)	-	-	-	15%	-	-	24%	40%
P8	PROFIT ($)	**NFS**	**NFS**	**NFS**	**NFS**	**NFS**	**NFS**	**NFS**	**NFS**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	1000	1000
	GAP (%)	-	-	-	-	-	-	-	-
	IP (%)	-	-	-	-	-	-	-	-
P9	PROFIT ($)	**1777**	**824**	**NFS**	**3733**	**NFS**	**3937**	**4536**	**4784**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	278	532
	GAP (%)	73%	92%	-	43%	-	40%	-	-
	IP (%)	-	-54%	-	110%	-	122%	155%	169%
P10	PROFIT ($)	**NFS**	**NFS**	**NFS**	**NFS**	**NFS**	**NFS**	**1735**	**4784**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	91	138
	GAP (%)	-	-	-	-	-	-	-	-
	IP (%)	-	-	-	-	-	-	-	-

TerminationCriteria = if any solution improve the objective function or time limit imposed of 1000 sec. %GAP = relative gap. %IP=improvement percent in comparison with model [M1]. NFS = no feasible solution.

Table S12. Comparison of results for different problem instances

MILP		Existing MILP models			Proposed MILP models			Sequential approaches	
		[M1]	**[M2]**	**[M3]**	**[H1]**	**[H2]**	**[H3]**	**RH**	**RH+II**
P1	PROFIT ($)	**7363**	**5090**	**3793**	**6916**	**7907**	**NFS**	**7563**	**8136**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	34	280
	GAP (%)	25%	61%	71%	29%	19%	-	-	-
	IP (%)	-	-31%	-48%	-6%	7%	-	3%	10%
P2	PROFIT ($)	**8327**	**5583**	**5886**	**8607**	**9199**	**8057**	**8939**	**9392**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	75	343
	GAP (%)	26%	61%	59%	22%	17%	28%	-	-
	IP (%)	-	-33%	-29%	3%	10%	-3%	7%	13%
P3	PROFIT ($)	**8459**	**5831**	**3739**	**6687**	**8855**	**7802**	**8432**	**8731**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	52	289
	GAP (%)	16%	56%	72%	34%	12%	22%	-	-
	IP (%)	-	-31%	-56%	-21%	5%	-8%	0%	3%
P4	PROFIT ($)	**11193**	**8117**	**8167**	**10530**	**10759**	**NFS**	**10556**	**10997**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	51	294
	GAP (%)	12%	50%	50%	18%	16%	-	-	-
	IP (%)	-	-27%	-27%	-6%	-4%	-	-6%	-2%
P5	PROFIT ($)	**12538**	**9726**	**7836**	**12516**	**NFS**	**NFS**	**12209**	**12890**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	58	336
	GAP (%)	18%	48%	59%	19%	-	-	-	-
	IP (%)	-	-22%	-38%	0%	-	-	-3%	3%
P6	PROFIT ($)	**10280**	**7669**	**8661**	**9922**	**12408**	**NFS**	**11568**	**11940**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	182	463
	GAP (%)	31%	59%	53%	35%	17%	-	-	-
	IP (%)	-	-25%	-16%	-3%	21%	-	13%	16%
P7	PROFIT ($)	**6620**	**4404**	**3489**	**6398**	**NFS**	**NFS**	**7108**	**7496**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	233	500
	GAP (%)	32%	67%	74%	34%	-	-	-	-
	IP (%)	-	-33%	-47%	-3%	-	-	7%	13%
P8	PROFIT ($)	**9087**	**8671**	**7904**	**10024**	**10932**	**NFS**	**10710**	**11414**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	51	288
	GAP (%)	31%	47%	52%	24%	17%	-	-	-
	IP (%)	-	-5%	-13%	10%	20%	-	18%	26%
P9	PROFIT ($)	**8201**	**6406**	**5682**	**9492**	**8886**	**NFS**	**8924**	**9116**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	56	305
	GAP (%)	24%	54%	59%	12%	24%	-	-	-
	IP (%)	-	-22%	-31%	16%	8%	-	9%	11%
P10	PROFIT ($)	**9878**	**7262**	**6096**	**7641**	**10505**	**NFS**	**10538**	**10850**
	CPU (sec.)	1000	1000	1000	1000	1000	1000	62	322
	GAP (%)	23%	55%	62%	39%	16%	-	-	-
	IP (%)	-	-26%	-38%	-23%	6%	-	7%	10%

TerminationCriteria = if any solution improve the objective function or time limit imposed of 1000 sec. %GAP = relative gap. %IP=improvement percent in comparison with model [M1]. NFS = no feasible solution.