

# Optimization of Joint Progressive Source and Channel Coding for MIMO Systems

Meesue Shin<sup>1</sup>, Laura Toni<sup>2</sup>, Sang-Hyo Kim<sup>3</sup> and Seok-Ho Chang<sup>1</sup>

<sup>1</sup>Computer Science and Engineering, Dankook University, Yongin 448-701, South Korea

<sup>2</sup>Electronic and Electrical Engineering, University College London (UCL), London WC1E 7JE, UK

<sup>3</sup>Information and Communication Engineering, Sungkyunkwan University, Suwon 16419, South Korea

**Abstract**—The optimization of joint source and channel coding for a sequence of numerous progressive packets is a challenging problem. Further, the problem becomes more complicated if the space-time coding is also involved with the optimization in a multiple-input multiple-output (MIMO) system. This is because the number of ways of jointly assigning channels codes and space-time codes to progressive packets is much larger than that of solely assigning channel codes to the packets. This paper applies a parametric approach to address that complex joint optimization problem in a MIMO system. We use the parametric methodology to derive some useful theoretical results, and then exploit those results to propose an optimization method where the joint assignment of channel codes and space-time codes to the packets can be optimized in a packet-by-packet manner. As a result, the computational complexity of the optimization is exponentially reduced, compared to the conventional exhaustive search. The numerical results show that the proposed method significantly improves the peak-signal-to-noise ratio performance of the rate-based optimal solution in a MIMO system.

## I. INTRODUCTION

Recently, there has been significant demand for the transmission of multimedia services over wireless channels, which has motivated intense research into cross-layer optimization design [1]. Progressive sources, such as embedded images or scalable video [2], [3], employ a manner of transmission such that the quality of the decoded source improves when the number of successfully received bits increases. However, such advances in source coders have made the source bitstream very susceptible to impairments in mobile fading channels.

Multiple-input multiple-output (MIMO) technology is an important advance in wireless communications in terms of the link reliability and data rate. Spatial diversity schemes, such as orthogonal space-time block codes (OSTBC), improve reliability by extracting the diversity gain to combat signal fading from the channels. Spatial multiplexing schemes use a layered approach to increase the data rate. One popular example is the vertical Bell Laboratories layered space-time (V-BLAST) architecture, where independent data signals are transmitted over antennas to increase the data rate.

In this paper, we study the optimization of joint source, channel, and space-time coding of progressive sources in such a MIMO system. Progressive source encoders produce data with gradual differences in the importance of their bitstreams. We consider the system where the bitstream is taken from the progressive source encoder, and is transformed into a sequence

of  $L$  packets. Such a system is depicted in Fig. 1. Each of those  $L$  progressive packets can be encoded with different channel codes and modulations in a similar way to the works in [4]–[9]. Further, each packet can be encoded with different space-time codes [10]–[15], to achieve the best end-to-end performance as measured by the expected distortion of the source. We assume that all the encoded packets have the same time duration,  $T_{\text{pkt}}$ , and the same signal bandwidth,  $W_{\text{pkt}}$ . We let  $u_i$  denote the spectral efficiency (bits/s/Hz) of the  $i$ th packet that has been encoded by a given channel code and modulation ( $1 \leq i \leq L$ );  $u_i$  is determined by the code rate of the channel code, and by the alphabet size of the modulation. We let  $v_i$  denote the spatial multiplexing rate of the  $i$ th packet that has been encoded by a space-time code. Then, the number of information (or source) bits in the  $i$ th packet is expressed as  $u_i v_i T_{\text{pkt}} W_{\text{pkt}}$ . As we increase either the spectral efficiency,  $u_i$ , or the spatial multiplexing rate,  $v_i$ , the variance of the quantization error from the source coder decreases, but the probability of the packet error, caused by signal fading and noise from the channels, increases.

Let  $N_{\text{se}}$  denote the number of candidate spectral efficiencies considered in a system. The number of possible assignments of  $N_{\text{se}}$  spectral efficiencies to a sequence of  $L$  progressive packets is  $N_{\text{se}}^L$ , which exponentially grows as  $L$  increases. As an example, for the transmission of a  $512 \times 512$  progressive image with a rate of 1 bit-per-pixel (bpp), a sequence of  $L = 128$  packets is considered in [4]. Further, in the MIMO system depicted in Fig. 1, if each packet can be encoded with different space-time codes (e.g., V-BLAST, OSTBC, and two-layer diagonal BLAST (D-BLAST)), which offer different spatial multiplexing rates, the assignment of spatial multiplexing rates as well as spectral efficiencies to  $L$  packets yields a more complicated optimization problem. To address this matter, for a single-input single-output (SISO) system, there have been many studies about the optimal assignment of spectral efficiencies to a sequence of progressive packets [4]–[9]. For a MIMO system, however, those studies do not immediately indicate how to jointly assign spectral efficiencies and spatial multiplexing rates to progressive packets.

There have been some researches [13]–[15] to address that complex optimization problem in a MIMO system. Those works have focused on significantly reducing the number of ways to assign space-time codes to progressive packets,

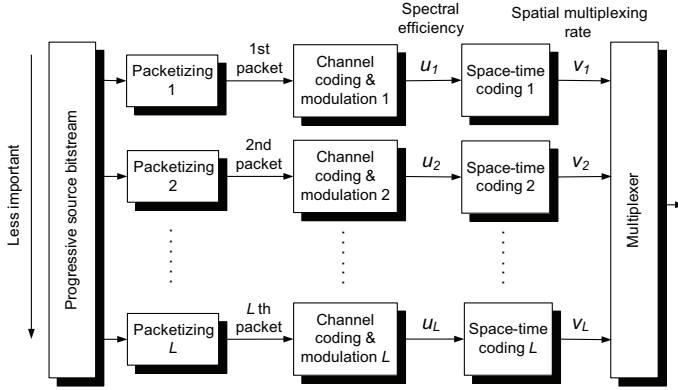


Fig. 1. A progressive source transmission in a MIMO system.  $u_i$  denotes the spectral efficiency (bits/s/Hz), and  $v_i$  denotes the spatial multiplexing rate of the  $i$ th packet ( $1 \leq i \leq L$ ).

but do not provide a complete solution for how to jointly assign the spectral efficiencies and spatial multiplexing rates to the packets. In [13]–[15], it was shown that performance of the progressive transmission in MIMO systems is sensitive to the way space-time codes are assigned to a sequence of packets. However, to our knowledge, solutions for such joint optimization problem in a MIMO system have not been presented yet in the literature.

In this paper, we use a parametric approach to address this complex joint optimization problem. Specifically, we employ a parametric model of the operational distortion-rate function of the source, which is based on the distortion-rate characteristic of the optimal source coder for the input source of independent and identically distributed (i.i.d.) Gaussian random variables. We use the parametric methodology to derive some useful theoretical results, and then exploit those results to propose an optimization method where the joint assignment of spectral efficiencies (i.e., channel codes and modulations) and spatial multiplexing rates (i.e., space-time codes) to progressive packets can be optimized in a packet-by-packet manner. As a result, the computational complexity involved in the optimization decreases exponentially relative to a conventional exhaustive search. Our work can be applied to the state-of-the-art wireless communication systems such as 3GPP Long Term Evolution and 5G systems based on orthogonal frequency division multiplexing (OFDM) for the delivery of progressive images.

## II. PRELIMINARIES

First, we describe the evaluation of the expected distortion of the progressive source. The system takes a compressed progressive bitstream from the source encoder, and transforms it into a sequence of packets with error detection capability. Then, as shown in Fig. 1, the packets are encoded using channel codes, modulations and space-time codes. At the receiver, if a packet has been correctly received, decoding of the next packet is considered by the source decoder. Otherwise, the decoding stops and the source is reconstructed from only

the correctly decoded packets.

Let  $p(u_i, v_i)$  denote the probability of the packet error with a spectral efficiency  $u_i \in \mathcal{R} = \{R_1, R_2, \dots, R_{N_{se}}\}$ , and spatial multiplexing rate  $v_i \in \mathcal{C} = \{C_1, C_2, \dots, C_{N_{smr}}\}$ , where  $N_{se}$  is the number of candidate spectral efficiencies, specified by channel codes and modulations, and  $N_{smr}$  is the number of candidate spatial multiplexing rates specified by the space-time codes employed in a system. We let  $b(u_i, v_i)$  denote the number of information (or source) bits in the packet that employs  $u_i \in \mathcal{R}$  and  $v_i \in \mathcal{C}$ . Recall that we have  $b(u_i, v_i) = u_i v_i T_{\text{pkt}} W_{\text{pkt}}$ , where  $T_{\text{pkt}}$  and  $W_{\text{pkt}}$  are the time duration and the signal bandwidth of the coded packet, respectively. Regarding  $\mathcal{R}$  and  $\mathcal{C}$ , it is assumed that for  $u_i < u_j$  ( $u_i, u_j \in \mathcal{R}$ ) and  $v_i \in \mathcal{C}$ , we have  $p(u_i, v_i) < p(u_j, v_i)$  [5], and that for  $v_i < v_j$  ( $v_i, v_j \in \mathcal{C}$ ) and  $u_i \in \mathcal{R}$ , we have  $p(u_i, v_i) < p(u_i, v_j)$  [11].

Let  $D_{1,2,\dots,k}(u_1, u_2, \dots, u_k; v_1, v_2, \dots, v_k)$  be the expected distortion of the progressive source for the event where  $u_i \in \mathcal{R}$  and  $v_i \in \mathcal{C}$  are assigned to the  $i$ th packet ( $i = 1, 2, \dots, k$ ) in a sequence of  $k$  packets. We denote the operational distortion-rate function of the source by  $f(x)$ . From the aforementioned decoding rule of the progressive codes,  $D_{1,2,\dots,k}(u_1, u_2, \dots, u_k; v_1, v_2, \dots, v_k)$  can be expressed as

$$D_{1,2,\dots,k}(u_1, u_2, \dots, u_k; v_1, v_2, \dots, v_k) = \sum_{n=0}^k f\left(\sum_{i=1}^n b(u_i, v_i)\right) P_{c,n}, \quad (1)$$

where  $f(\sum_{i=1}^n b(u_i, v_i))$  is the distortion of the source for the case where the first  $n$  packets in a sequence of  $k$  packets are used for the source decoding, and  $P_{c,n}$  is the probability that no decoding errors occur in the first  $n$  packets with an error in the next one. For  $1 \leq n \leq k-1$ ,  $P_{c,n}$  is given by  $P_{c,n} = p(u_{n+1}, v_{n+1}) \prod_{i=1}^n (1 - p(u_i, v_i))$ ; the probability of an error in the first packet is  $P_{c,0} = p(u_1, v_1)$ , and the probability that all  $k$  packets are correctly decoded is  $P_{c,k} = \prod_{i=1}^k (1 - p(u_i, v_i))$ . Then, (1) can be rewritten as

$$D_{1,2,\dots,k}(u_1, u_2, \dots, u_k; v_1, v_2, \dots, v_k) = \sum_{n=0}^{k-1} f\left(\sum_{i=1}^n b(u_i, v_i)\right) p(u_{n+1}, v_{n+1}) \prod_{i=1}^n (1 - p(u_i, v_i)) + f\left(\sum_{i=1}^k b(u_i, v_i)\right) \prod_{i=1}^k (1 - p(u_i, v_i)), \quad (2)$$

where we have used the definitions of  $\sum_{i=I_1}^{I_2} a(i) \triangleq 0$  and  $\prod_{i=I_1}^{I_2} a(i) \triangleq 1$  for an arbitrary function  $a(i)$ .

We consider a MIMO system with  $N_t$  transmit and  $N_r$  receive antennas. A space-time codeword,  $\mathbf{S} = [\mathbf{s}_1 \mathbf{s}_2 \dots \mathbf{s}_{T_s}]$  of size  $N_t \times T_s$  is transmitted over  $N_t$  transmit antennas and  $T_s$  symbol durations. The  $N_r \times 1$  received signal vector,  $\mathbf{y}_l$  ( $1 \leq l \leq T_s$ ), can be expressed as

$$\mathbf{y}_l = \mathbf{H}\mathbf{s}_l + \mathbf{n}_l, \quad (3)$$

where  $\mathbf{s}_l$  is an  $N_t \times 1$  transmitted signal vector,  $\mathbf{H}$  is the  $N_r \times N_t$  channel matrix, and  $\mathbf{n}_l$  is an  $N_r \times 1$  zero-mean com-

plex Gaussian vector with  $\mathcal{E}[\mathbf{n}_k \mathbf{n}_l^H] = \sigma_n^2 \mathbf{I}_{N_r} \delta(k-l)$ , where  $(\cdot)^H$  denotes Hermitian operation. We assume that the entries of  $\mathbf{H}$  are i.i.d  $\sim \mathcal{CN}(0, 1)$ , and that  $\mathbf{H}$  is random but constant over the duration  $T_s$  of a codeword. Let  $\gamma_s := \mathcal{E}[|(\mathbf{s}_k)_i|^2]/\sigma_n^2$  denote the signal-to-noise ratio (SNR) per symbol, where  $(\mathbf{s}_k)_i$  is the  $i$ th component of the transmit signal vector  $\mathbf{s}_k$  ( $i = 1, \dots, N_t$ ). Let  $N_s$  denote the number of symbols packed within a space-time codeword  $\mathbf{S}$ . The spatial multiplexing rate is defined as  $N_s/T_s$ .

### III. THE PROPOSED JOINT OPTIMIZATION METHOD

The joint optimization problem for a sequence of  $k$  progressive packets in a MIMO system is to find the set of spectral efficiencies  $u_1, u_2, \dots, u_k \in \mathcal{R}$  and spatial multiplexing rates  $v_1, v_2, \dots, v_k \in \mathcal{C}$  that minimizes the expected distortion given by (2). Due to the high nonlinearity of  $f(x)$  and  $p(x, y)$  in the expected distortion, convex optimization techniques are not exploited to find the optimal solution. To address this matter, we consider a parametric model of the operational distortion-rate characteristic of the source. When a sequence of i.i.d. Gaussian random variables with zero mean and variance of  $\sigma^2$  are encoded at bit rate  $x$  using an optimal source coder, the distortion of the source is given by [16]

$$d(x) = \sigma^2 2^{-2x}. \quad (4)$$

Although the distortion-rate bound, given by (4), can only be achieved with no constraint on the coding length, the operational distortion-rate function of a practical source coder usually shows the same exponential rate decay of  $2x$  at a high bit rate [11].

Instead of the actual operational distortion-rate function of the source, denoted by  $f(x)$ , we take into account the parametric distortion-rate function:

$$f^p(x) = \sigma^2 2^{-\alpha x}, \quad \alpha \geq 2 \quad (5)$$

where  $\alpha$  is a parameter that is free to be adjusted for the optimization, and  $\sigma^2 (\neq 0)$  is a constant that does not affect the optimization (this will be described in detail in this section). The function in (5) parameterizes the distortion-rate characteristic of the source  $d(x)$ , given by (4), to include a wide range of low-to-high bit rates  $x$  for a practical image source coder. Suppose that instead of  $f(x)$ ,  $f^p(x)$  is employed for the computation of the expected distortion. Then, from (2), the resulting expected distortion can be expressed as

$$\begin{aligned} D_{1,2,\dots,k}^p(u_1, u_2, \dots, u_k; v_1, v_2, \dots, v_k; \alpha) = & \\ \sum_{n=0}^{k-1} \sigma^2 \left( \prod_{i=1}^n g(b(u_i, v_i)) \right) p(u_{n+1}, v_{n+1}) \prod_{i=1}^n (1 - p(u_i, v_i)) & \\ + \sigma^2 \prod_{i=1}^k g(b(u_i, v_i)) \prod_{i=1}^k (1 - p(u_i, v_i)), & \end{aligned} \quad (6)$$

where  $g(x) \triangleq 2^{-\alpha x}$ . Let  $\mathbf{s}_{1,2,\dots,k} = [u_1, u_2, \dots, u_k; v_1, v_2, \dots, v_k]$  indicate a solution (or assignment) where a spectral efficiency  $u_i \in \mathcal{R}$  and a spatial multiplexing rate  $v_i \in \mathcal{C}$  are assigned to the  $i$ th packet ( $i = 1, 2, \dots, k$ ).

From here onwards, we refer to the parametric distortion-based optimal solution as the one that minimizes the expected distortion of the source with the parametric distortion-rate function  $f^p(x)$  employed as given by (6). In the following, we derive some theoretical results for the parametric distortion-based optimal solution. Based on them, we will propose an efficient optimization method for the joint source, channel, and space-time coding of progressive packets.

**Theorem 1:** For some integer  $L$  in the range of  $L \geq 2$ , suppose that  $\mathbf{s}_{1,2,\dots,L-1} = [r_2^*, r_3^*, \dots, r_L^*; c_2^*, c_3^*, \dots, c_L^*]$  is a parametric distortion-based optimal assignment of spectral efficiencies and spatial multiplexing rates to  $L-1$  progressive packets. That is,

$$\begin{aligned} D_{1,2,\dots,L-1}^p(r_2^*, r_3^*, \dots, r_L^*; c_2^*, c_3^*, \dots, c_L^*; \alpha) & \\ \leq D_{1,2,\dots,L-1}^p(r_2, r_3, \dots, r_L; c_2, c_3, \dots, c_L; \alpha) & \\ \text{for any } r_2, \dots, r_L \in \mathcal{R} \text{ and } c_2, \dots, c_L \in \mathcal{C}. & \end{aligned} \quad (7)$$

Then, if

$$\begin{aligned} D_{1,2,\dots,L}^p(r_1^*, r_2^*, \dots, r_L^*; c_1^*, c_2^*, \dots, c_L^*; \alpha) & \\ \leq D_{1,2,\dots,L}^p(r_1, r_2, \dots, r_L; c_1, c_2, \dots, c_L; \alpha) & \\ \text{for any } r_1 \in \mathcal{R} \text{ and } c_1 \in \mathcal{C}, & \end{aligned} \quad (8)$$

we obtain

$$\begin{aligned} D_{1,2,\dots,L}^p(r_1^*, r_2^*, \dots, r_L^*; c_1^*, c_2^*, \dots, c_L^*; \alpha) & \\ \leq D_{1,2,\dots,L}^p(r_1, r_2, \dots, r_L; c_1, c_2, \dots, c_L; \alpha) & \\ \text{for any } r_1, \dots, r_L \in \mathcal{R} \text{ and } c_1, \dots, c_L \in \mathcal{C}. & \end{aligned} \quad (9)$$

In other words,  $\mathbf{s}_{1,2,\dots,L} = [r_1^*, r_2^*, \dots, r_L^*; c_1^*, c_2^*, \dots, c_L^*]$  is a parametric distortion-based optimal solution for  $L$  progressive packets.

*Proof:* The proof of this theorem as well as the proofs of all other results are not included here due to space limitations, but they can be found in [17].  $\square$

Theorem 1 tells us that if the parametric distortion-rate function  $f^p(x)$ , given by (5), is used to compute the expected distortion, the joint assignment of the spectral efficiencies and spatial multiplexing rates to progressive packets can be optimized in a packet-by-packet manner. This optimization method will be described in detail in this section.

**Lemma 2:** Consider an integer  $i$  in the range of  $1 \leq i \leq L-1$ . If  $\mathbf{s}_{1,2,\dots,L-i+1} = [r_i^*, r_{i+1}^*, \dots, r_L^*; c_i^*, c_{i+1}^*, \dots, c_L^*]$  is a parametric distortion-based optimal assignment of spectral efficiencies and spatial multiplexing rates to  $L-i+1$  progressive packets, i.e.,

$$\begin{aligned} D_{1,2,\dots,L-i+1}^p(r_i^*, r_{i+1}^*, \dots, r_L^*; c_i^*, c_{i+1}^*, \dots, c_L^*; \alpha) & \\ \leq D_{1,2,\dots,L-i+1}^p(r_i, r_{i+1}, \dots, r_L; c_i, c_{i+1}, \dots, c_L; \alpha) & \\ \text{for any } r_i, \dots, r_L \in \mathcal{R} \text{ and } c_i, \dots, c_L \in \mathcal{C}, & \end{aligned} \quad (10)$$

then, for some integer  $j$  in the range of  $i+1 \leq j \leq L$ , we have

$$\begin{aligned} D_{1,2,\dots,L-j+1}^p(r_j^*, r_{j+1}^*, \dots, r_L^*; c_j^*, c_{j+1}^*, \dots, c_L^*; \alpha) & \\ \leq D_{1,2,\dots,L-j+1}^p(r_j, r_{j+1}, \dots, r_L; c_j, c_{j+1}, \dots, c_L; \alpha) & \\ \text{for any } r_j, \dots, r_L \in \mathcal{R} \text{ and } c_j, \dots, c_L \in \mathcal{C}. & \end{aligned} \quad (11)$$

In other words,  $\mathbf{s}_{1,2,\dots,L-j+1} = [r_j^*, r_{j+1}^*, \dots, r_L^*; c_j^*, c_{j+1}^*, \dots, c_L^*]$  is a parametric distortion-based optimum for  $L-j+1$  progressive packets.  $\square$

Lemma 2 tells us that if a parametric distortion-based optimal solution for a given number of packets (or, equivalently, a given transmission rate in bpp) has been obtained, an optimal solution for a smaller number of packets (or a lower transmission rate) can be immediately found without additional computation. As an example, if  $\mathbf{s}_{1,2,3} = [r_1^*, r_2^*, r_3^*; c_1^*, c_2^*, c_3^*]$  is a parametric distortion-based optimum for a sequence of three packets ( $L = 3$ ), then  $\mathbf{s}_{1,2} = [r_2^*, r_3^*; c_2^*, c_3^*]$  is an optimum for a sequence of two packets ( $L = 2$ ), and  $\mathbf{s}_1 = [r_3^*; c_3^*]$  is an optimum for a single packet ( $L = 1$ ). Lemma 2 is used to prove the subsequent Lemma 4, Theorem 5, and Corollary 6.

**Lemma 3:** For an integer  $L$  in the range of  $L \geq 2$ , we have

$$\begin{aligned} & D_{1,2,\dots,L}^{\text{P}}(r_1, r_2, \dots, r_L; c_1, c_2, \dots, c_L; \alpha) \\ & < D_{1,2,\dots,L-1}^{\text{P}}(r_1, r_2, \dots, r_{L-1}; c_1, c_2, \dots, c_{L-1}; \alpha) \\ & \text{for any } r_1, \dots, r_L \in \mathcal{R} \text{ and } c_1, \dots, c_L \in \mathcal{C}. \end{aligned} \quad (12)$$

$\square$

Lemma 3 is used in the proof of Lemma 4.

**Lemma 4:** Consider some integers  $i, j$  in the range of  $1 \leq i \leq L-1$  and  $i+1 \leq j \leq L$ . If  $\mathbf{s}_{1,2,\dots,L-i+1} = [r_i^*, r_{i+1}^*, \dots, r_L^*; c_i^*, c_{i+1}^*, \dots, c_L^*]$  is a parametric distortion-based optimum for  $L-i+1$  progressive packets (that is, if (10) holds), we obtain

$$\begin{aligned} & D_{1,2,\dots,L-i+1}^{\text{P}}(r_i^*, r_{i+1}^*, \dots, r_L^*; c_i^*, c_{i+1}^*, \dots, c_L^*; \alpha) \\ & < D_{1,2,\dots,L-j+1}^{\text{P}}(r_j^*, r_{j+1}^*, \dots, r_L^*; c_j^*, c_{j+1}^*, \dots, c_L^*; \alpha). \end{aligned} \quad (13)$$

$\square$

Lemma 4 is employed in the proof of Theorem 5, which derives some constraints on the search space of  $\mathcal{R}$  and  $\mathcal{C}$  when we find a parametric distortion-based optimal solution.

**Theorem 5:** Consider an integer  $i$  in the range of  $1 \leq i \leq L-1$ . If  $\mathbf{s}_{1,2,\dots,L-i+1} = [r_i^*, r_{i+1}^*, \dots, r_L^*; c_i^*, c_{i+1}^*, \dots, c_L^*]$  is a parametric distortion-based optimum for  $L-i+1$  progressive packets (that is, if (10) holds), then for every integer  $j$  in the range of  $i+1 \leq j \leq L$ , at least one of the following three conditions holds:

$$\text{i) } r_i^* < r_j^*, \quad \text{ii) } c_i^* < c_j^*, \quad \text{iii) } r_i^* = r_j^*, \quad c_i^* = c_j^*. \quad (14)$$

That is, we obtain at least  $L-i$  constraints on  $r_i^*$  or  $c_i^*$  of the  $i$ th packet.  $\square$

Corollary 6 follows immediately from Theorem 5.

**Corollary 6:** If  $\mathbf{s}_{1,2,\dots,L} = [r_1^*, r_2^*, \dots, r_L^*; c_1^*, c_2^*, \dots, c_L^*]$  is a parametric distortion-based optimum for  $L$  progressive packets, i.e.,

$$\begin{aligned} & D_{1,2,\dots,L}^{\text{P}}(r_1^*, r_2^*, \dots, r_L^*; c_1^*, c_2^*, \dots, c_L^*; \alpha) \\ & \leq D_{1,2,\dots,L}^{\text{P}}(r_1, r_2, \dots, r_L; c_1, c_2, \dots, c_L; \alpha) \\ & \text{for any } r_1, \dots, r_L \in \mathcal{R} \text{ and } c_1, \dots, c_L \in \mathcal{C}, \end{aligned} \quad (15)$$

then, for every integer  $i, j$  in the range of  $1 \leq i \leq L-1$  and  $i+1 \leq j \leq L$ , at least one of the following three conditions

holds:

$$\text{i) } r_i^* < r_j^*, \quad \text{ii) } c_i^* < c_j^*, \quad \text{iii) } r_i^* = r_j^*, \quad c_i^* = c_j^*. \quad (16)$$

That is, we obtain at least  $(L^2 - L)/2$  constraints on  $r_1^*, r_2^*, \dots, r_{L-1}^*$  or  $c_1^*, c_2^*, \dots, c_{L-1}^*$ .  $\square$

By Corollary 6, we are able to reduce the search space of  $\mathcal{R}$  and  $\mathcal{C}$  when finding a parametric distortion-based optimal solution.

Based on Theorem 1 and Corollary 6, a parametric distortion-based optimal solution for  $L$  progressive packets can be obtained in the following way.

**Step 1:** Choose the best parameter  $\alpha^*$  of the parametric model of the distortion-rate function,  $f^{\text{P}}(x) = \sigma^2 2^{-\alpha x}$ , as follows:

$$\begin{aligned} \alpha^* = & \arg \min_{\alpha \in \{\alpha_1, \alpha_2, \dots, \alpha_Q\}} D_{1,2,\dots,L}(r_1^*(\alpha), r_2^*(\alpha), \dots, r_L^*(\alpha); \\ & c_1^*(\alpha), c_2^*(\alpha), \dots, c_L^*(\alpha)), \end{aligned} \quad (17)$$

where  $D_{1,2,\dots,L}(\cdot)$  is the expected distortion of the source, given by (2), employing the actual distortion-rate function  $f(x)$ . For a given parameter  $\alpha \in \{\alpha_1, \alpha_2, \dots, \alpha_Q\}$ ,  $r_i^*(\alpha)$  and  $c_i^*(\alpha)$  can be obtained through Steps 2–4 below.

From  $\alpha^*$  chosen in (17), we obtain a parametric distortion-based optimal solution of  $r_1^*(\alpha^*), r_2^*(\alpha^*), \dots, r_L^*(\alpha^*)$  and  $c_1^*(\alpha^*), c_2^*(\alpha^*), \dots, c_L^*(\alpha^*)$ .

**Step 2:** In order to find  $r_i^*(\alpha)$  and  $c_i^*(\alpha)$  for a given parameter  $\alpha$ , the packet index  $i$  is initialized as  $i = L$ .

**Step 3:** If  $i = L$ ,

$$r_L^*(\alpha), c_L^*(\alpha) = \arg \min_{r_L \in \mathcal{R}, c_L \in \mathcal{C}} D_1^{\text{P}}(r_L; c_L; \alpha), \quad (18)$$

otherwise (i.e.,  $1 \leq i \leq L-1$ ),

$$\begin{aligned} & r_i^*(\alpha), c_i^*(\alpha) = \\ & \arg \min_{r_i \in \mathcal{R}, c_i \in \mathcal{C}} D_{1,2,\dots,L-i+1}^{\text{P}}(r_i, r_{i+1}^*(\alpha), \dots, r_L^*(\alpha); \\ & c_i, c_{i+1}^*(\alpha), \dots, c_L^*(\alpha); \alpha), \end{aligned} \quad (19)$$

subject to at least one of the three constraints :

$$\begin{aligned} & \text{i) } r_i^*(\alpha) < r_j^*(\alpha); \quad \text{ii) } c_i^*(\alpha) < c_j^*(\alpha); \quad \text{iii) } r_i^*(\alpha) = r_j^*(\alpha), \\ & c_i^*(\alpha) = c_j^*(\alpha), \text{ for every integer } j \text{ in the range of} \\ & i+1 \leq j \leq L, \end{aligned} \quad (20)$$

where  $D_{1,2,\dots,L-i+1}^{\text{P}}(\cdot)$  ( $1 \leq i \leq L$ ) is the expected distortion of the source with parametric distortion-rate function  $f^{\text{P}}(x)$ , and is given by (21) at the top of the next page.

**Step 4:** Set  $i = i-1$ . If  $i = 0$ , we have obtained  $r_1^*(\alpha), r_2^*(\alpha), \dots, r_L^*(\alpha)$ , and  $c_1^*(\alpha), c_2^*(\alpha), \dots, c_L^*(\alpha)$  for a given parameter  $\alpha$ ; thus go to Step 1. Otherwise, go to Step 3.  $\square$

We first describe Steps 2–4. Eq. (19) follows from Theorem 1. In (18) and (19), it is shown that the joint assignment of spectral efficiencies and spatial multiplexing rates to  $L$  packets is optimized in a packet-by-packet manner; that is, for the  $i$ th packet, only two optima  $r_i^*(\alpha)$  and  $c_i^*(\alpha)$  are exhaustively searched ( $1 \leq i \leq L$ ), from which the global minimum of  $D_{1,2,\dots,L}^{\text{P}}(\cdot)$  can be attained. The expected distortion in (18) and (19) of Step 3, given by (21) at the top of the next page, can be derived from (6). From (18), (19), and (21), it is seen

$$\begin{aligned}
D_{1,2,\dots,L-i+1}^p & (r_i, r_{i+1}^*(\alpha), \dots, r_L^*(\alpha); c_i, c_{i+1}^*(\alpha), \dots, c_L^*(\alpha); \alpha) = \sigma^2 p(r_i, c_i) + \sigma^2 g(b(r_i, c_i)) (1 - p(r_i, c_i)) \\
& \times \left[ \sum_{n=1}^{L-i} \left( \prod_{k=2}^n g(b(r_{i+k-1}^*(\alpha), c_{i+k-1}^*(\alpha))) \right) p(r_{i+n}^*(\alpha), c_{i+n}^*(\alpha)) \prod_{k=2}^n (1 - p(r_{i+k-1}^*(\alpha), c_{i+k-1}^*(\alpha))) \right. \\
& \left. + \prod_{k=2}^{L+1-i} g(b(r_{i+k-1}^*(\alpha), c_{i+k-1}^*(\alpha))) \prod_{k=2}^{L+1-i} (1 - p(r_{i+k-1}^*(\alpha), c_{i+k-1}^*(\alpha))) \right]. \tag{21}
\end{aligned}$$

that the selection of  $r_i^*(\alpha)$  and  $c_i^*(\alpha)$  does not depend on how large  $\sigma^2 (\neq 0)$  is, but depends on how large  $\alpha (\geq 2)$  is. To emphasize this, in Steps 1–4, we have used the notation  $r_i^*(\alpha)$  and  $c_i^*(\alpha)$  instead of  $r_i^*$  and  $c_i^*$ , respectively. The constraint given by (20), which reduces the search space of  $\mathcal{R}$  and  $\mathcal{C}$  for the evaluation in (19), is based on Corollary 6. Consequently, following Steps 2–4, the number of ways to assign  $N_{\text{se}}$  spectral efficiencies and  $N_{\text{smr}}$  spatial multiplexing rates to  $L$  packets, of which the expected distortions need to be evaluated for the optimization, is given by  $N_{\text{steps } 2-4} \leq N_{\text{se}} N_{\text{smr}} L$ , where the inequality follows from the constraint given by (20). Note that the number of possible assignments for an exhaustive search is  $(N_{\text{se}} N_{\text{smr}})^L$ .

We next describe Step 1. In (17), the best parameter  $\alpha^*$  is chosen to minimize the expected distortion of the source. Note that the actual distortion-rate function  $f(x)$  has been used to compute the expected distortion. In (17), due to the high nonlinearity of  $f(x)$ , we resort to an exhaustive search of  $\alpha^*$  that minimizes the expected distortion. If  $\alpha \geq 2$  is quantized into  $Q$  levels for the exhaustive search, Steps 2–4 should be repeated  $Q$  times. This is because for every  $\alpha \in \{\alpha_1, \alpha_2, \dots, \alpha_Q\}$ ,  $r_i^*(\alpha)$  and  $c_i^*(\alpha)$  ( $1 \leq i \leq L$ ) should be obtained to calculate  $D_{1,2,\dots,L}(\cdot)$ . Thus, the number of ways of assignment, of which the expected distortions need to be evaluated in Steps 1–4, can be expressed as

$$N_{\text{steps } 1-4} \leq Q N_{\text{se}} N_{\text{smr}} L, \tag{22}$$

where the inequality follows from the constraint of (20). Eq. (22) indicates that even in the worst case, the number of ways of assignment decreases exponentially from  $(N_{\text{se}} N_{\text{smr}})^L$  to  $Q N_{\text{se}} N_{\text{smr}} L$ , relative to an exhaustive search. We note that, however, a parametric distortion-based solution is suboptimal in terms of the expected distortion performance, while an exhaustive search obviously yields the optimal performance. In the next section, we will assess the performance of our solution.

#### IV. NUMERICAL EVALUATION

We numerically evaluate the performance of the proposed optimization method. We take three space-time codes into consideration: V-BLAST with an MMSE receiver, OSTBC with a decorrelator, and two-layer D-BLAST with a group zero-forcing receiver [18]. Group decoding is a recent decoding method which can be regarded as a compromise between zero-forcing and maximum likelihood decoding. The spatial multiplexing rates of OSTBC, two-layer D-BLAST,

and V-BLAST are denoted by  $C_1$ ,  $C_2$ , and  $C_3$ , respectively, and a set of candidate spatial multiplexing rates is given by  $\mathcal{C} = \{C_1, C_2, C_3\}$ . We have  $C_2 = 2N_t/(N_t + 1)$  and  $C_3 = N_t$  for  $N_t$  transmit antennas. In the complex OSTBC, the Alamouti code achieves  $C_1 = 1$  for  $N_t = 2$ , and  $C_1 = 3/4$  is the maximum achievable rate for  $N_t = 3$  or 4. For  $N_t \geq 5$ ,  $C_1 = 1/2$  is the maximum rate. We assume a MIMO system with  $N_r \geq N_t \geq 2$ . The strict inequality of  $C_1 < C_2 < C_3$  can be shown to hold for  $N_t \geq 2$ . In this evaluation, as an example, a set of candidate spectral efficiencies is chosen as  $\mathcal{R} = \{1, 2, 3, 4\}$  (bits/s/Hz). The channel is assumed to experience slow fading such that the fading coefficients remain nearly constant over a packet and are i.i.d. across different packets. We assume that the channel estimation at the receiver is perfect in such slow fading channels.

With suitably powerful channel codes, the error probability when not in outage is very small, and hence the outage probability is an accurate approximation of the actual probability of the packet error [19]. Let  $P_{\text{out},1}(\gamma_s)$ ,  $P_{\text{out},2}(\gamma_s)$ , and  $P_{\text{out},3}(\gamma_s)$  denote the outage probabilities of OSTBC, D-BLAST, and V-BLAST, respectively. For a given spectral efficiency  $R \in \mathcal{R}$  and a spatial multiplexing rate  $C_1 \in \mathcal{C}$ , the outage probability of OSTBC is given by  $P_{\text{out},1}(\gamma_s) = \Pr[\log_2(1 + \gamma_s \|\mathbf{H}\|_F^2 / C_1) < R]$  [13], where  $\|\cdot\|_F$  denotes the Frobenius norm of the matrix. For a given spectral efficiency  $R \in \mathcal{R}$  and a spatial multiplexing rate  $C_2 \in \mathcal{C}$ , the outage probability of D-BLAST provided by [18] is expressed as  $P_{\text{out},2}(\gamma_s) = \Pr[\log_2 \det(\mathbf{I}_{N_t} + N_t \gamma_s \mathbf{H}_1^H \mathbf{Q}_1^H \mathbf{Q}_1 \mathbf{H}_1) < T_s C_2 R / 2]$ , where  $\mathbf{H}_1$  and  $\mathbf{Q}_1$  are given by Eqs. (4) and (6) of [18], respectively. For the V-BLAST scheme, we consider pure spatial multiplexing [20] where data is split into several substreams, one for each transmit antenna, and each substream undergoes independent temporal coding to avoid complex joint decoding of substreams at the receiver. For this scheme, an outage event occurs when any of the substreams is in outage. Thus, the outage probability of V-BLAST, for a given spectral efficiency  $R \in \mathcal{R}$  and a spatial multiplexing rate  $C_3 \in \mathcal{C}$ , is given by  $P_{\text{out},3}(\gamma_s) = \Pr\left[\bigcup_{k=1}^{N_t} \{\log_2(1 + \gamma_k) < C_3 R / N_t\}\right]$  [20], where  $\gamma_k = 1/[(\mathbf{I}_{N_t} + \gamma_s \mathbf{H}^H \mathbf{H})^{-1}]_{kk} - 1$ , and  $[\cdot]_{kk}$  indicates the  $k$ th diagonal entry of a matrix. From the outage probability expressions given above, we calculate the probability of the packet error,  $p(R, C_i)$ , in (21).

We evaluate the performance of the proposed optimization method using the source coder SPIHT [3] as an example, for 8 bpp 512 × 512 Lena and Pepper images with trans-

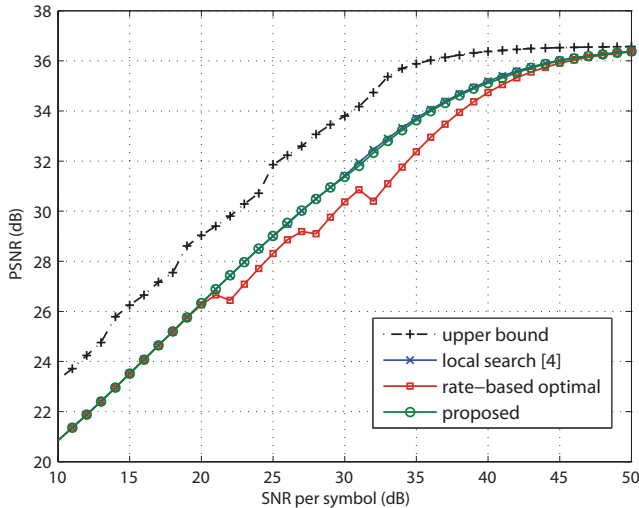


Fig. 2. The PSNR performance for the transmission of 8 bpp  $512 \times 512$  Lena image in SISO Rayleigh fading channels. The transmission rate is 0.5 bpp and the number of packets is  $L = 64$ .

mission rates of 0.5 and 1.0 bpp. The number of packets is chosen as  $L = 64$  and 128 for the 0.5 and 1.0 bpp rates, respectively.<sup>1</sup> The end-to-end performance is measured by the expected distortion,  $D_{1,2,\dots,L}(\cdot)$ . To compare the image quality, we use the peak-signal-to-noise ratio (PSNR) defined as  $10\log_{10}(255^2/D_{1,2,\dots,L}(\cdot))$  (dB). To find the best  $\alpha^*$  of the parametric distortion-rate function,  $f^p(x) = \sigma^2 2^{-\alpha x}$ , we quantize  $\alpha$  into  $Q$  bins for the range of  $2 \leq \alpha \leq M$ , such that the width of each bin is given by  $(M - 2)/Q$ . In our evaluation, we set  $M$  and  $Q$  to be 20 and 1, respectively.

To begin, we observe the PSNR performance of the proposed optimization method when employed in a SISO system. For this case, only spectral efficiencies are optimally assigned to progressive packets, and space-time codes are not employed. Recall that in a SISO system, there have been many studies about the optimal assignment of spectral efficiencies to progressive packets. The local search algorithm [4] and heuristic algorithm based on graph search [6] are among the best optimization methods in terms of the expected distortion performance (or, equivalently, mean squared-error performance). In [6], it is shown that the two algorithms provide nearly identical mean squared-error performances for progressive transmission, which are near optimal [4]. Fig. 2 shows the PSNR performances of the proposed method and the local search algorithm in a SISO system, in addition to showing the upper bound on the PSNR performance (or, equivalently, the lower bound on the expected distortion) that is presented in [4], [6] for reference. Fig. 2 also depicts the PSNR of a rate-based optimal solution [21], [22] that assigns the spectral efficiencies to packets in a way that the expected number of correctly decoded information bits is maximized, rather than minimizing the expected distortion. The search

<sup>1</sup>The transmission rates and the number of packets in this evaluation are the same as those used in [4], [6].

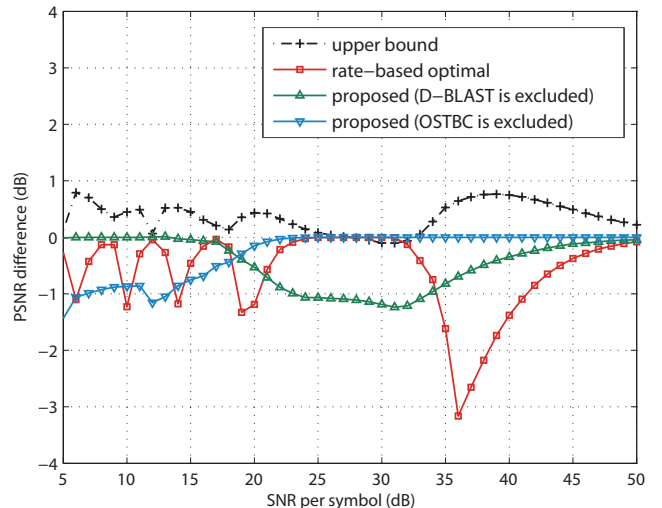


Fig. 3. The PSNRs of various optimization schemes, each subtracted by the PSNR of the proposed method, for the transmission of 8 bpp  $512 \times 512$  Lena image in  $2 \times 2$  MIMO i.i.d. Rayleigh fading channels. The transmission rate is 0.5 bpp and the number of packets is  $L = 64$ .

space of  $\mathcal{R}$  for a sequence of progressive packets is too large to find a global optimal solution through an exhaustive search. Thus, the PSNR achieved by an exhaustive search is not presented in Fig. 2. It is shown that the PSNR performance of the proposed method is seen to be close to that of the local search algorithm, and is significantly better than that of the rate-based optimal solution.<sup>2</sup>

Next, we evaluate the PSNR performance in a  $2 \times 2$  MIMO i.i.d. Rayleigh fading channels. The system parameters are set equal to those used in a SISO system. Note that in a SISO system, a rate-based optimal assignment of spectral efficiencies to progressive packets can be found in a packet-by-packet manner [22]. This can be immediately extended to a MIMO system such that a rate-based optimal assignment of spectral efficiencies and spatial multiplexing rates to the packets can be obtained using a packet-by-packet method. In other words, we are able to obtain a rate-based optimal solution even for the case where space-time codes are involved in the optimization.

On the other hand, the optimization algorithms that are proposed to minimize the expected distortion in a SISO system do not immediately tell us how to jointly assign the spectral efficiencies and spatial multiplexing rates to progressive packets. We are unaware of any algorithm in [4]–[9] that has been successfully extended to the case where space-time coding is also involved in the optimization. For this reason, we cannot compute and present the PSNR performances of those optimization algorithms. Fig. 3 shows the PSNR performances in  $2 \times 2$  MIMO systems. For better visual comparison, Fig. 3 depicts the PSNR difference between our solution and the others; that is, Fig. 3 shows the PSNRs of the upper bound and

<sup>2</sup>The numerical evaluations for a Pepper image and other transmission rates yield similar results, which are not depicted here for limited space.

a rate-based optimal solution, each subtracted by the PSNR of our solution. It is seen that the proposed method is able to significantly improve the performance of a rate-based optimal solution. Fig. 3 also shows the performance of a suboptimal case where a space-time code is excluded from a candidate set. It is seen that there is a significant PSNR gap between the two cases where a single space-time code has been excluded and not excluded. This indicates that when progressive packets are transmitted in a MIMO system, the PSNR performance would improve if a variety of space-time codes are employed to encode a sequence of packets. This motivated us to suggest an optimization method that is able to handle two or more space-time codes to transmit progressive packets in a MIMO system. For the PSNR performance of the proposed method shown in Fig. 3, (19) has been computed with the constraint given by (20). We note that the same set of spectral efficiencies and spatial multiplexing rates is obtained when (19) is computed with and without the constraint. That is, the constraint in (20) reduces the computational complexity involved with the optimization without losing any PSNR performance.

## V. CONCLUSIONS

The joint optimization of source, channel, and space-time coding for a series of numerous progressive packets is a challenging problem. To our knowledge, a feasible solution for this joint optimization problem has not yet been presented in the literature. This paper uses a parametric methodology to solve such a complex joint optimization problem. In the proposed method, employing a parametric distortion-rate function, we jointly optimize the assignment of spectral efficiencies and spatial multiplexing rates to progressive packets in a packet-by-packet manner. As a result, the computational complexity of the optimization is exponentially reduced, compared to an exhaustive search. Moreover, some constraints on the search space are derived to further reduce the complexity. The numerical results show that the proposed solution significantly improves the PSNR performance of a rate-based optimal solution in a MIMO system. In addition, the performance of our solution when applied to a SISO system is close to that of local search algorithm, one of the best optimization methods proposed for a SISO system.

Lastly, we note that our solution can be computed independently of a specific progressive source, once the best  $\alpha^*$  of the parametric function for that source has been chosen. Thus, if a single parameter  $\alpha^*$  is known to the receiver side, which requires only a small amount of overhead, our solution for spectral efficiencies and spatial multiplexing rates can then be recomputed at the receiver side. Therefore, the overhead information for the solution is unnecessary. In general, the number of parameters in a parametric model should remain small, because both the overhead and the optimization time grow with the number of parameters. In our approach, only a single parameter  $\alpha$  is taken to avoid such a problem.

The work in this paper has significance in terms of its impact on the area of multimedia communications, and deepens our understanding of joint source and channel coding problem in a

MIMO system. As a future work, we consider extending our joint source, channel, and space-time coding to progressive transmission in multiview settings where multiple cameras are coded and sent to clients in a progressive fashion.

## REFERENCES

- [1] S.-H. Chang, P. C. Cosman and L. B. Milstein, "Iterative channel decoding of FEC-based multiple description codes," *IEEE Trans. Image Process.*, vol. 21, pp. 1138–1152, Mar. 2012.
- [2] H. Schwarz, D. Marpe, and T. Wiegand, "Overview of the scalable video coding extension of H.264/AVC," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 17, pp. 1103–1120, Sept. 2007.
- [3] Y. Jin and H.-J. Lee, "A block-based pass-parallel SPIHT algorithm," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 22, pp. 1064–1075, Jul. 2012.
- [4] R. Hamzaoui, V. Stankovic, and Z. Xiong, "Fast algorithm for distortion-based error protection of embedded image codes," *IEEE Trans. Image Process.*, vol. 14, pp. 1417–1421, Oct. 2005.
- [5] L. Cao, "On the unequal error protection for progressive image transmission," *IEEE Trans. Image Process.*, vol. 16, pp. 2384–2388, Sep. 2007.
- [6] M. Fresia and F. Lavagetto, "Determination of optimal distortion-based protection in progressive image transmission: A heuristic approach," *IEEE Trans. Image Process.*, vol. 17, pp. 1654–1662, Sep. 2008.
- [7] A. Nosratinia, J. Liu, and B. Aazhang, "Source-channel rate allocation for progressive transmission of images," *IEEE Trans. Commun.*, vol. 51, pp. 186–196, Feb. 2003.
- [8] Z. Wu, A. Bilgin, and M. W. Marcellin, "Joint source/channel coding for image transmission with JPEG2000 over memoryless channels," *IEEE Trans. Image Process.*, vol. 14, pp. 1020–1032, Aug. 2005.
- [9] X. Pan, A. H. Banihashemi, and A. Cuhadar, "A fast trellis-based rateallocation algorithm for robust transmission of progressively coded images over noisy channels," *IEEE Trans. Commun.*, vol. 54, pp. 1–6, Jan. 2006.
- [10] S.-H. Chang, M. Rim, P. C. Cosman and L. B. Milstein, "Superposition MIMO coding for the broadcast of layered sources," *IEEE Trans. Commun.*, vol. 59, pp. 3240–3248, Dec. 2011.
- [11] S. Lin, A. Stefanov, and Y. Wang, "On the performance of space-time block-coded MIMO video communications," *IEEE Trans. Veh. Technol.*, vol. 56, pp. 1223–1229, May 2007.
- [12] T. Holliday, A. J. Goldsmith, and H. V. Poor, "Joint source and channel coding for MIMO systems: Is it better to be robust or quick?," *IEEE Trans. Inf. Theory*, vol. 54, pp. 1393–1405, Apr. 2008.
- [13] S.-H. Chang, P. C. Cosman and L. B. Milstein, "Optimal transmission of progressive sources based on the error probability analysis of SM and OSTBC," *IEEE Trans. Veh. Technol.*, vol. 63, pp. 94–106, Jan. 2014.
- [14] S.-H. Chang, J. P. Choi, P. C. Cosman and L. B. Milstein, "Optimization of multimedia progressive transmission over MIMO channels," *IEEE Trans. Veh. Technol.*, vol. 65, pp. 1244–1260, Mar. 2016.
- [15] S.-H. Chang, J. P. Choi, P. C. Cosman and L. B. Milstein, "Optimization of scalable broadcast for a large number of antennas," to appear in *IEEE Trans. Veh. Technol.*
- [16] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New York, NY, USA: Wiley, 2006.
- [17] M. Shin, L. Toni, S.-H. Kim, and S.-H. Chang, "Joint source, channel and space-time coding of progressive sources in MIMO systems," Available: <http://arxiv.org/abs/1610.04378>.
- [18] Y. Lu, W. Zhang, X.-G. Xia, "On diversity and multiplexing tradeoff of two-layer D-BLAST with group zero-forcing detection," *IEEE Trans. Commun.*, vol. 60, pp. 2255–2264, Aug. 2012.
- [19] A. Lozano and N. Jindal, "Transmit diversity vs. spatial multiplexing in modern MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 9, pp. 186–197, Jan. 2010.
- [20] K. Raj Kumar, G. Caire, and A. L. Moustakas, "Asymptotic performance of linear receivers in MIMO fading channels," *IEEE Trans. Inf. Theory*, vol. 55, pp. 4398–4418, Oct. 2009.
- [21] V. Chandé and N. Farvardin, "Progressive transmission of image over memoryless noisy channels," *IEEE J. Sel. Areas Commun.*, vol. 18, pp. 850–860, Jun. 2000.
- [22] V. Stankovic, R. Hamzaoui, and D. Saupé, "Fast algorithm for rate-based optimal error protection of embedded codes," *IEEE Trans. Commun.*, vol. 51, pp. 1788–1795, Nov. 2003.