

Self-Interference Distribution over Full-Duplex Multi-User MIMO Channels

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Abstract—We consider the case where a reference full-duplex (FD) node (e.g., base station), equipped with arbitrary number of transmit/receive antennas, utilizes generalized linear beamformers to simultaneously communicate with multiple FD radios (e.g., user equipments). The fading coefficients for the residual self-interference (SI) channels are drawn from the complex Gaussian distribution with arbitrary mean and variance. Here, it is not possible to directly derive the distribution of the bi-directional channel power gain. As a result, we adopt the method of moments in order to provide a new Gamma approximation for the residual SI distribution over FD multi-user MIMO Rician fading channels. The proposed theorem holds under arbitrary linear precoder/decoder design, number of antennas and streams, and SI cancellation capability. The validity of the theoretical findings is confirmed via extensive simulations of the entire transmit/receive processing chain.

I. INTRODUCTION

The emergence of multiple-input multiple-output (MIMO) technology has revolutionized wireless communications [1], [2]. MIMO provides entirely new degrees of freedom, allowing for the implementation of key technologies and techniques such as diversity and multiplexing in the spatial domain. As a result, significant performance improvements have been achieved through exploiting MIMO in modern wireless systems. This trend is expected to continue going forward, with the emergence of new MIMO architectures such as the large-scale antenna system (LSAS) [3], [4].

Existing wireless systems have been designed and dimensioned under a strict separation of the transmit and the receive functions, referred to as half-duplex (HD) mode [5]. This is typically achieved through orthogonal radio frequency (RF) partitioning, such as time-division duplex (TDD) and frequency-division duplex (FDD), where the different communications directions occur over distinct time slots and frequency bands, respectively. The motivation behind HD design has largely been to avoid the overwhelming self-interference (SI), which arises from the bi-directional operation [6].

With the rapid increase in traffic, under limited RF resources, however, there has been an enormous incentive for an overhaul in the wireless system architecture [7]. A prominent candidate solution involves the transmission and reception of

information over the same RF resources, i.e., full-duplex (FD) mode [8], [9]. There is a growing surge of interest in this topic, as in theory, FD has the potential to double the spectral efficiency compared to HD. This trend is in line with recent major advances in SI suppression techniques [10]–[12]. Broadly speaking, SI can be tackled using isolation in space [13], [14], digital/analog subtraction [15], [16], and as a combination of both [17]. Several real-time point-to-point prototypes of FD radio nodes have been demonstrated in practice recently [18].

Despite pioneering efforts, the study of FD mode in multi-user MIMO setups remains an open problem [19]. Most existing works in the literature rely on simplifying assumptions, particularly concerning the SI. A long-standing limitation involves using the Rayleigh distribution for the bi-directional fading channel model [20]–[22]. This approach, whilst improving on the common perfect SI cancellation assumption, differs from experimental measurements which have shown that the residual SI channel undergoes Rician fading [10], [18]. As a result, a rigorous characterization of the residual SI statistics over FD multi-user MIMO Rician fading channels is critical for facilitating performance analysis and optimization.

In this work, we focus on a reference FD multi-antenna node functioning as a transmitter and a receiver with respect to an arbitrary number of FD terminals over the same time and frequency. The complex Gaussian distribution is accordingly employed in order to capture the characteristics of the residual SI under arbitrary SI cancellation capability. By applying linear precoding and decoding, it is not feasible to derive the statistics of the bi-directional channel power gain directly. As a result, we exploit the method of moments in order to develop a unified mathematical expression for the distribution of the residual SI over FD multi-user MIMO Rician fading channels. The theoretical findings are validated via extensive simulations of the entire RF transmit/receive processing chain.

Notation: \mathbf{X} is a matrix; \mathbf{x} is a vector; T , \dagger , and $+$ are the transpose, conjugate-transpose, and pseudo-inverse operations; $\mathbb{E}\{\cdot\}$ is the expected value; $\text{Var}\{\cdot\}$ is the variance; $\mathcal{P}(\cdot)$ is the probability density function (pdf); $|\cdot|$ is the modulus; $\|\cdot\|$ is the norm; $I_0(\cdot)$ is the zeroth-order Bessel function of the first kind, respectively.

II. SYSTEM MODEL

We consider the general scenario where a FD multi-antenna node is transmitting and receiving information with respect to multiple FD radios over the same time and frequency. We use M and N in order to denote the arbitrary number of RF transmit and receive chains at the reference FD multi-antenna node, respectively. The arbitrary number of FD radios, i.e., transmit/receive streams, is in turn represented with U . An immediate application example that can be highlighted here is a reference base station which communicates simultaneously in the downlink and the uplink with multiple user equipments, in FD mode. For simplicity, we assume $U < \min(N, M)$ holds, hence avoiding the need for channel assignment.

Let $\mathbf{g}_k \in \mathcal{C}^{1 \times M}$ and $\mathbf{h}_k \in \mathcal{C}^{N \times 1}$ denote the transmit and receive fading channels of the respective FD multi-antenna node with respect to the k -th FD radio, $k \in \mathcal{U} = \{1, \dots, U\}$. Further, the corresponding combined channels are respectively $\mathbf{G} = [\mathbf{g}_k^T]_{1 \leq k \leq U} \in \mathcal{C}^{U \times M}$ and $\mathbf{H} = [\mathbf{h}_k]_{1 \leq k \leq U} \in \mathcal{C}^{N \times U}$. Moreover, the cross-mode fading channel at the k -th FD radio from the j -th FD radio is denoted with q_{kj} , $j \in \mathcal{U} \setminus \{k\}$. Further, $\mathbf{F}_0 \in \mathcal{C}^{N \times M}$ and f_k denote the residual SI channels at the reference FD multi-antenna node and the k -th FD radio, respectively. The residual SI channels, \mathbf{F}_0 and f_k , are subject to Rician fading with independent and identically-distributed elements drawn from a complex Gaussian distribution $\mathcal{CN}(\mu, \nu^2)$ [23]. All other communications channels are assumed to undergo Rayleigh fading with independent and identically-distributed elements drawn from $\mathcal{CN}(0, 1)$ [24]. We assume channel state information (CSI) is available throughout.

In this work, we consider arbitrary linear beamforming design at the transmitter and the receiver of the reference FD multi-antenna node. Let $\mathbf{s} = [s_k]_{1 \leq k \leq U} \in \mathcal{C}^{U \times 1}$ denote the corresponding complex information vector of the reference node with respect to the neighboring sources (in either direction). Hence, the transmit signal vector under linear precoding at the reference FD multi-antenna node is constructed as $\mathbf{t} = \mathbf{V}\mathbf{s}$ where $\mathbf{V} = [\mathbf{v}_k]_{1 \leq k \leq U} \in \mathcal{C}^{M \times U}$ is the precoding matrix. Further, the linear receive filter at the reference FD multi-antenna node is denoted using $\mathbf{W} = [\mathbf{w}_k^T]_{1 \leq k \leq U} \in \mathcal{C}^{U \times N}$.

The received signal from the reference FD multi-antenna node at an arbitrary FD radio k^* , $k^* \in \mathcal{U}$, is given by

$$\begin{aligned} x_{k^*} = & \underbrace{\sqrt{p_0} \mathbf{g}_{k^*} \mathbf{v}_{k^*} s_{k^*}}_{\text{useful signal, } d-k^*} + \underbrace{\sqrt{p_0} \sum_{k=1, k \neq k^*}^U \mathbf{g}_k \mathbf{v}_k s_k}_{\text{multi-user interference, } dd-k^*} \\ & + \underbrace{\sum_{k=1, k \neq k^*}^U \sqrt{p_k} q_{k^*k} s_k}_{\text{cross-mode interference, } ud-k^*} + \underbrace{\sqrt{p_{k^*}} f_{k^*} s_{k^*}}_{\text{self-interference, } dsi-k^*} + \underbrace{\eta_{k^*}}_{\text{noise}} \end{aligned} \quad (1)$$

where p_0 is the reference FD multi-antenna node per-stream transmit power, p_k is the k -th FD radio transmit power, and η_{k^*} is the complex additive white Gaussian noise (AWGN), with mean zero and variance σ_d^2 , at the k^* -th FD radio, respectively. Next, the post-processing received signal from the k^* FD radio at the reference FD multi-antenna node is given by

$$\begin{aligned} y_{k^*} = & \underbrace{\sqrt{p_{k^*}} \mathbf{w}_{k^*}^T \mathbf{h}_{k^*} s_{k^*}}_{\text{useful signal, } u-k^*} + \underbrace{\sum_{k=1, k \neq k^*}^U \sqrt{p_k} \mathbf{w}_k^T \mathbf{h}_k s_k}_{\text{multi-user interference, } uu-k^*} \\ & + \underbrace{\sqrt{p_0} \mathbf{w}_{k^*}^T \mathbf{F}_0 \mathbf{V} \mathbf{s}}_{\text{self-interference, } u,usi-k^*} + \underbrace{\mathbf{w}_{k^*}^T \boldsymbol{\eta}}_{\text{scaled noise}} \end{aligned} \quad (2)$$

where $\boldsymbol{\eta} \in \mathcal{C}^{N \times 1}$ is the circularly-symmetric complex AWGN vector with mean zero and covariance matrix $\sigma_u^2 \mathbf{I}_N$.

III. SIGNALS STATISTICS

The received signal-to-interference-plus-noise ratio (SINR) at the typical FD radio k^* from the reference FD multi-antenna node under consideration can be expressed as

$$\text{SINR}_{x_{k^*}} = \frac{\mathcal{X}_{k^*}}{\mathcal{I}_{dd-k^*} + \mathcal{I}_{ud-k^*} + \mathcal{I}_{dsi-k^*} + \sigma_d^2} \quad (3)$$

where $\mathcal{X}_{k^*} = pG_{k^*}$, $\mathcal{I}_{dd-k^*} = p_0 \sum_{k=1, k \neq k^*}^U G_k$, $\mathcal{I}_{ud-k^*} = \sum_{k=1, k \neq k^*}^U p_k Q_{k^*-k}$, and $\mathcal{I}_{dsi-k^*} = pF_{k^*}$ are the different signals involved, with channel power gains $G_{k^*} \triangleq |\mathbf{g}_{k^*} \mathbf{v}_{k^*}|^2$, $G_k \triangleq |\mathbf{g}_k \mathbf{v}_k|^2$, $Q_{k^*-k} \triangleq |q_{k^*k}|^2$, and $F_{k^*} \triangleq |f_{k^*}|^2$, respectively. Moreover, the received SINR from the k^* FD radio at the reference FD multi-antenna node is formulated as

$$\text{SINR}_{y_{k^*}} = \frac{\mathcal{Y}_{k^*}}{\mathcal{I}_{uu-k^*} + \mathcal{I}_{usi-k^*} + \sigma_u^2} \quad (4)$$

where we have $\mathcal{Y}_{k^*} = p_{k^*} H_{k^*}$, $\mathcal{I}_{uu-k^*} = \sum_{k=1, k \neq k^*}^U p_k H_k$, $\mathcal{I}_{usi-k^*} = pF_{0-k^*}$, $H_{k^*} \triangleq |\mathbf{w}_{k^*}^T \mathbf{h}_{k^*}|^2$, $H_k \triangleq |\mathbf{w}_k^T \mathbf{h}_k|^2$, and $F_{0-k^*} \triangleq \|\mathbf{w}_{k^*}^T \mathbf{F}_0 \mathbf{V}\|^2$.

With linear processing over MIMO Rayleigh fading channels, the signals statistics can be captured using the Gamma distribution [25]. For example, consider the reference FD multi-antenna node uses linear zero-forcing (ZF) beamforming for removing the multi-user interference in both communications directions by setting (i) the column vectors of the linear precoder \mathbf{V} equal to the normalized columns of $\mathbf{G}^+ = \mathbf{G}^\dagger (\mathbf{G}\mathbf{G}^\dagger)^{-1}$, and (ii) the row vectors of the linear decoder \mathbf{W} equal to the normalized rows of $\mathbf{H}^+ = (\mathbf{H}^\dagger \mathbf{H})^{-1} \mathbf{H}^\dagger$. With the projection of each intended channel vector onto the nullspace spanned by the multi-user interference, the useful channel power gains can be characterized as $G_{k^*} \sim \text{Gamma}(M-U+1, 1)$ and $H_{k^*} \sim \text{Gamma}(N-U+1, 1)$. Moreover, the cross-mode interfering channel power gain from the k -th FD radio is distributed as $Q_{k^*-k} \sim \text{Gamma}(1, 1) \equiv \text{Exp}(1)$, $k \in \mathcal{U} \setminus \{k^*\}$.

IV. SELF-INTERFERENCE DISTRIBUTION

The exact residual SI distribution at the neighboring FD radios can be readily derived as in the following result.

Lemma 1. Consider the residual SI fading channel, f_k , of an arbitrary FD radio k , with single RF transmit/receive chain, following a complex Gaussian distribution with mean μ and standard deviation ν . The magnitude of this channel, $|f_k|$, is a Rician random variable with Rician factor K and fading attenuation Ω where $\mu \triangleq \sqrt{\frac{K\Omega}{K+1}}$ and $\nu \triangleq \sqrt{\frac{\Omega}{K+1}}$. The

corresponding residual SI channel power gain, $F_k = |f_k|^2$, is a non-central Chi-squared random variable with pdf

$$\mathcal{P}_{F_k}(x) = \frac{1+K}{\Omega} \exp\left(-\left(K + \frac{(1+K)x}{\Omega}\right)\right) I_0\left(2\sqrt{\frac{K(1+K)x}{\Omega}}\right). \quad (5)$$

The moment matching technique can be applied here to provide a Gamma approximation for the non-central Chi-squared distribution of the residual SI F_k in (5).

Lemma 2. *The residual SI channel power gain, of an arbitrary FD radio k , with single RF transmit/receive chain and fading coefficients drawn from the complex Gaussian distribution with mean μ and standard deviation ν , can be approximated using $F_k \sim \text{Gamma}(\kappa, \theta)$ where*

$$\kappa \triangleq \frac{(\mu^2 + \nu^2)^2}{(2\mu^2 + \nu^2)\nu^2} \quad (6)$$

and

$$\theta \triangleq \frac{(2\mu^2 + \nu^2)\nu^2}{\mu^2 + \nu^2}. \quad (7)$$

On the other hand, the residual SI channel power gain at the reference FD multi-antenna node equipped with an arbitrary number of RF transmit/receive chains, involves a sum of non-identically distributed positive random variables. For example, for the decoding of the k -th stream, $F_{0-k} = \left\| \mathbf{w}_k^T \mathbf{F}_0 \mathbf{V} \right\|^2$ where $\mathbf{w}_k^T \mathbf{F}_0 \mathbf{V} = \left[\sum_{i=1}^N \sum_{j=1}^M \{ \mathbf{w}_k^T \}_i \{ \mathbf{F}_0 \}_{i,j} \{ \mathbf{V} \}_{j,k} \right]_{1 \leq k \leq U}$.

It is therefore not possible to directly derive the exact distribution of $F_{0,k}$, $k \in \mathcal{U}$. In this work, we derive closed-form expressions for the exact first and second moments of F_{0-k} , $k \in \mathcal{U}$. We then apply the Gamma moment matching technique to develop a unified approximation for the residual SI distribution.

Theorem. *Consider the residual SI fading channel, \mathbf{F}_0 , of a reference FD multi-antenna node, with M transmit and N receive antennas, following a per-entry complex Gaussian distribution with mean μ and standard deviation ν . The reference FD multi-antenna node applies a linear precoder \mathbf{V} and a linear decoder \mathbf{W} to simultaneously transmit/receive information streams with respect to U neighboring FD radios. The residual SI channel power gain with arbitrary linear beamforming over FD multi-user MIMO Rician fading channels can be approximated using $F_{0,k} \sim \text{Gamma}(\kappa, \theta)$, $k \in \mathcal{U} = \{1, \dots, U\}$, where*

$$\kappa \triangleq \frac{U(N+1)(M-U+2)(\mu^2 + \nu^2)^2}{\left(2NM + \frac{U(M-U+2)}{(M+1)}(NM - N - M - 1)\right)\mu^4 + (N+1)(M+1)\nu^2(2\mu^2 + \nu^2)} \quad (8)$$

and

$$\theta \triangleq \frac{\left(2NM + \frac{U(M-U+2)}{(M+1)}(NM - N - M - 1)\right)\mu^4 + (N+1)(M+1)\nu^2(2\mu^2 + \nu^2)}{(N+1)(M-U+2)(\mu^2 + \nu^2)}. \quad (9)$$

Proof: The proof is omitted due to space limitations.

The unified expressions in the *Theorem* hold for arbitrary linear precoder/decoder design, number of transmit/receive antennas, number of streams, and Rician channel statistics (and in turn arbitrary residual SI cancellation capability).

V. PERFORMANCE ANALYSIS

In this section, we conduct Monte-Carlo simulations of the entire RF transmit/receive processing chain in order to assess the validity of the theoretical findings. Without loss of generality, consider a 16×8 multi-user MIMO Rician fading channel with mean $\frac{1}{2}$ and variance 1. The distributions of the residual SI power with linear ZF precoding and decoding are depicted with different number of transmit/receive streams in *Figure 1* and *Figure 2*. For the single-user MIMO case in *Figure 1*, the proposed *theorem* provides a near exact fit with respect to the empirical data. The proposed *Theorem* also yields a tight approximation of the experimental values for the multi-user MIMO case in *Figure 2*. In general, it should be noted that the goodness of the fit of the theoretical distribution with respect to the empirical results increases for smaller

mean μ , variance ν , and information streams U . On the other hand, increasing the number of transmit and receive antennas, for example in the context of massive MIMO, improves the tightness of the fit.

VI. SUMMARY

A rigorous study of the residual SI in the context of FD multi-user MIMO was provided. We considered the case where a reference FD multi-antenna node utilized linear processing in order to simultaneously function as a transmitter and a receiver with respect to multiple FD radios. The bi-directional fading channels were drawn from a complex Gaussian distribution with arbitrary mean and variance. We provided exact closed-form expressions for the first and second positive moments of the residual SI over Rician fading channels. The method of moments was then applied to derive a novel Gamma distribution for the residual SI. The main result from this paper can greatly assist with the performance analysis and optimization of FD multi-user MIMO wireless systems.

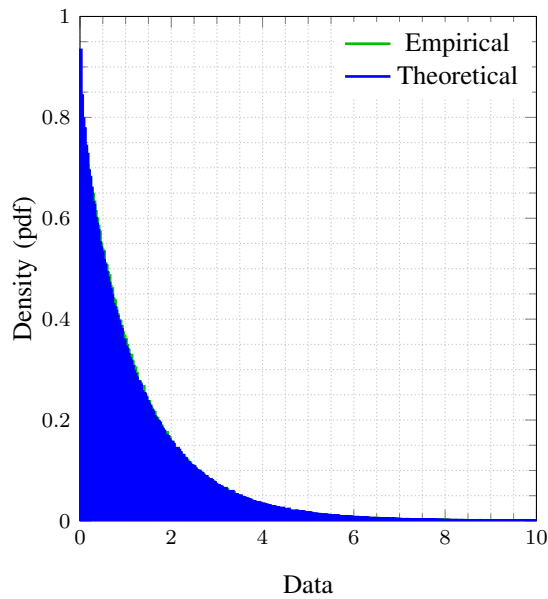


Figure 1. Simulation parameters are: $M = 16$, $N = 8$, $U = 1$, $\mu = \frac{1}{2}$, $\nu = 1$.

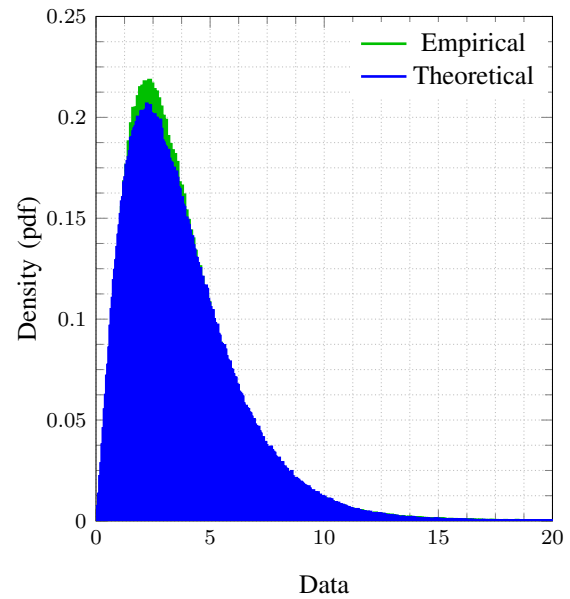


Figure 2. Simulation parameters are: $M = 16$, $N = 8$, $U = 3$, $\mu = \frac{1}{2}$, $\nu = 1$.

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