Nonlinear Sliding Mode Control of a Two-Wheeled Mobile Robot System

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Abstract: This paper presents a trajectory tracking control scheme for a two-wheeled mobile robot using sliding mode techniques. The stability of the designed sliding mode dynamics is analysed and reachability of the sliding mode is guaranteed in a given region with the proposed controller. A robot system including a micro-controller, the Arduino Due based on the ARM Cortex-M3, is used to implement the proposed control algorithm. Two DC motors controlled by PWM signals are used as actuators to implement the proposed feedback control. Simulation results are presented and compared with the results of practical experiments.

Keywords: sliding mode control; trajectory tracking; nonholonomic systems; mobile robot.

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1 Introduction

With the development of modern technology, the required performance for control systems in reality becomes increasingly complex. As a consequence, linear control theory including linearisation techniques cannot get desired performance since most real systems exhibit nonlinearity. This has motivated study on nonlinear control systems based on nonlinear control methodology Yan & Zhang (1997); Yan et al. (2014).

As a typical nonholonomic system, a wheeled mobile robot (WMR) system is nonlinear, and the motion control of WMR systems is full of challenges Umar et al. (2014). Although it has been proved by Thuilot et al. (1996) that it is not necessary for the trajectory tracking control to satisfy the well known necessary conditions proposed by Brockett (1983) if the reference trajectory does not involve rest configurations, it is still very difficult to use linear control methods to get desired tracking performance for WMR systems because of the inherent nonlinearity caused by the nonholonomic constraints. The development of nonlinear control approaches for the trajectory tracking control of WMR systems becomes necessary and important.

Most early work on trajectory tracking control was based on kinematic models since kinematic models strictly relate to the nonholonomic constraints. Samson & Ait-Abderrahim (1991) firstly designed a trajectory tracking control scheme based on Lyapunov stability theory. However, as the nonlinear feedback control law obtained by Samson & Ait-Abderrahim (1991) is directly based on the Lyapunov function, the design procedure is complicated and the resulting controller is difficult to implement in practice. In Kim & Oh (1999), input-ouput linearization techniques were used to linearize the system so that linear feedback can be applied to the tracking system. However, due to the approximation of the dynamics, the tracking performance is not as expected because the approximated dynamics cannot model the real WMR system well. One of choices to improve the control performance on the kinematic model is to use back-stepping control approach (see, e.g. Fierro & Lewis (1995)), which has been widely used by many authors for tracking control design for WMR (see, e.g. Fukao et al. (2000) and Chen et al. (2009)). In Asif et al. (2014), the kinetic controller based on the back-stepping technique was simplified when compared with previous work. However, because of the dynamic behaviour of the linear and steering velocities, it is mentioned by Fierro & Lewis (1995) that the proposed control scheme also requires feedback control of the dynamic model to reduce the tracking error in practice. Therefore, dynamic model based control design is inevitably required in many control approaches to improve system performance (see, e.g. Fukao et al. (2000), Chen et al. (2009) and Asif et al. (2014)).

Sliding mode control is a leading robust control method as it is completely insensitive to those uncertainties acting in the input channel, the so-called matched uncertainty, when the system is in the sliding motion (see, e.g. Edwards & Spurgeon (1998)). This approach has been widely employed (see, e.g. Mu et al. (2015); Fukushima et al. (2015); Zhao et al. (2015); Zhen et al. (2014)). Moreover, the sliding mode approach can also be used to deal with systems in the presence of unmatched uncertainty under suitable conditions even for time delay systems (see, e.g. Yan et al. (2013) and Yan et al. (2014)). Therefore, sliding mode control techniques can be a very powerful solution to the problem of trajectory tracking control in practical systems.

It should be noted that sliding mode control has been well applied in mobile robot control. A coordinated control scheme based on a leader follower approach is developed for the control of cooperative autonomous mobile robots by Defoort et al. (2008) which enables formation stabilisation and ensures the collision avoidance. An integral sliding mode control strategy is applied to Heisenberg system, and the developed results have been successfully applied in mobile robot control by Defoort et al. (2009). A sliding mode control scheme for the trajectory tracking control with polar coordinates has been previously proposed by Yang & Kim (1999). Due to hardware limitation, the controller experienced chattering problems and did not exhibit the expected tracking performance in practice. In both Chen et al. (2009) and Asif et al. (2014), a sliding mode control strategy was used in the dynamic layer. Although the simulation results in both cases show robustness against matched uncertainties, the sliding mode control was only applied on the dynamic model, which only ensures that the reference velocities can be tracked. In Belhocine et al. (2003), sliding mode techniques were applied to a WMR system using a feedback linearization approach and some excellent results have been obtained for not only the tracking control problem but also for regulation tasks. However, it requires that the propulsive force of the WMR can be measured as one of the states in the system such that the strict condition of relative degree for the feedback linearization can be satisfied. This is very difficult to implement from the practical point of view. Even if it is possible, it inevitably increases the cost and and complexity of the system. In Lee et al. (2009), sliding mode control was applied to the kinematic model of a WMR following a local coordinate transformation. However, the controller is described in implicit form which is very inconvenient for practical implementation.

In this paper, a sliding mode controller is proposed for a 2WMR system. Asymptotic tracking of trajectories based on the kinematic model of the system is considered. A new sliding surface is designed to guarantee the stability of the proposed sliding motion. Then a sliding mode control law is proposed to guarantee the reachability condition is satisfied so that the system attains and maintains the required sliding motion. The implementation of the control on a 2WMR using DC motors as actuators is carried out and thus it is demonstrated that implementation of the proposed scheme is straightforward. The experimental results achieved are consistent with the simulation results and show that the proposed approach is effective.

The remain of this paper is organised as follows. In Section 2 the WMR hardware is described. A model of the WMR is developed in Section 3. The sliding mode control design is presented and analysed in Section 4, and Section 5 contains simulation and experimental results. Section 6 concludes the paper.

2 Hardware description

This section describes the hardware design of the robot system.

The overview of the 2WMR built at the University of Kent is shown in Fig.1. Two DC motors are used as the actuator in the right and left side of the robot body for differential driving with encoder assembled on the shaft. In order to obtain accurate motion to estimate the coordinates, a rate gyroscope is also used. Thus the two encoders and the rate gyro together offer a relative accurate estimation of the position of the robot and feedback of the rotational velocities of each motor. Then, the motors are independently drove by two H-bridge MOSFET-based motor drivers controlled by the micro-controller with two separate pulse-width-modulation signals. The micro-controller applied on the robot is a 32-b micro-controller board. The programming of the micro-controller is user-friendly with C and C++ languages, and the sampling frequency for the main control unit is 100 Hz in the implementation which is sufficient for this application. The phototype of the two-wheeled mobile robot can be described in Fig. 2

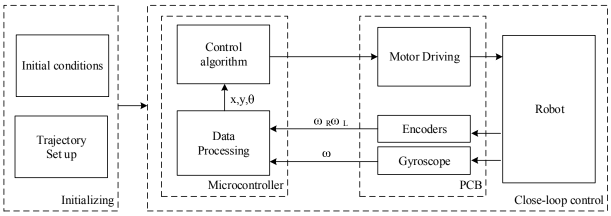

Figure 1 System overview for the 2WMR

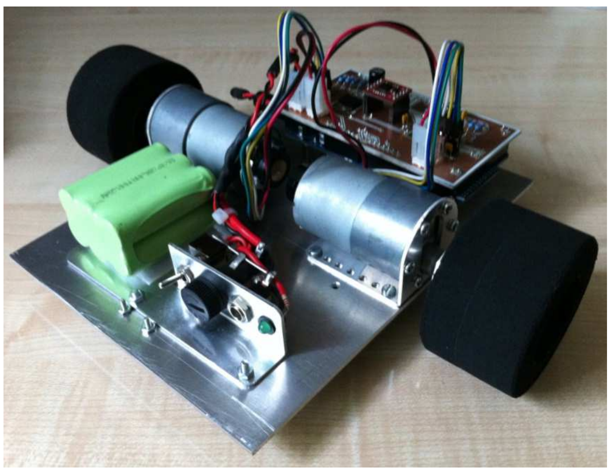

Figure 2 Phototype of the WMR

3 Modelling of the WMR

The following result is for the modelling analysis in the following subsection.

Lemma 3.1: *Assume that $X = X(t) \in \mathcal{R}^n$ and $Z = Z(t) \in \mathcal{R}^n$ are continuous in $t \in \mathcal{R}^+$, and $T(X) \in \mathcal{R}^{n\times n}$ is a functional matrix with*

$$Z = T(X)X \tag{1}$$

in $X \in \mathcal{R}^n$. Then $\lim_{t\to\infty} X(t) \to 0$ if $\lim_{t\to\infty} Z(t) \to 0$ when $T(X)$ is nonsingular and bounded in $X \in \mathcal{R}^n$.

Proof. since $T(X)$ is nonsingular, it is straightforward to see that

$$\|X\| = \|T^{-1}(X)Z\| \le \|T^{-1}(X)\|\|Z\| \tag{2}$$

Since $T(X)$ is bounded, there exists a positive constant M such that

$$\|T^{-1}(X)\| \le M \tag{3}$$

Then

$$\|X\| \le \|T^{-1}(X)\|\|Z\| \le M\|Z\| \tag{4}$$

Hence the conclusion follows. ■

Consider a two-WMR with the generalized n-vector coordinates $q = \mathrm{col}(q_x, q_y, \theta) \in \mathcal{R}^n$ as shown in Fig.3. The Pfaffian nonholonomic constraint that the WMR cannot shift laterally is (see e.g. Oriolo et al. (2002))

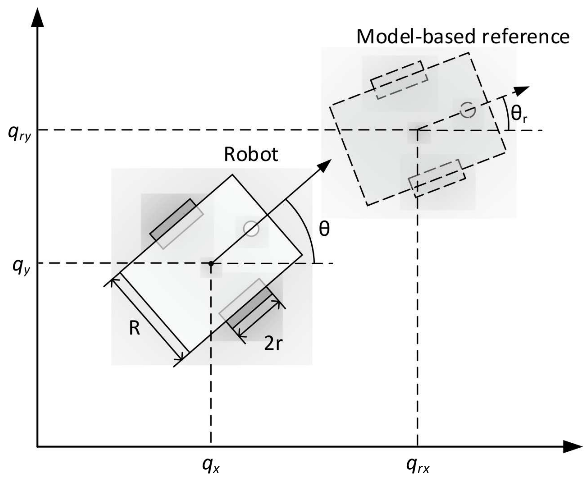

Figure 3 Configuration of the robot

$$A(q)\dot{q} = 0 \tag{5}$$

where

$$A(q) = \left[\sin\theta - \cos\theta \; 0\right] \tag{6}$$

From Oriolo et al. (2002), by expressing all the feasible motion of WMR as a linear combination of vector fields $S(q)$

$$S(q) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix}$$

which spans the null space of matrix $A(q)$, the so-called kinematic model can be obtained as (see e.g. Oriolo et al. (2002))

$$\dot{q} = S(q)u \tag{7}$$

where $u = \mathrm{col}(v, \omega)$, and v and ω are the linear velocity and the steering velocity of the WMR respectively

For the differential-driving mechanism, v and ω can be derived from the rotational velocity of two wheels as follows (see e.g. Oriolo et al. (2002))

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{R} & -\frac{r}{R} \end{bmatrix} \begin{bmatrix} \omega_R \\ \omega_L \end{bmatrix} \tag{8}$$

where ω_R and ω_L denote the rotational velocity of the wheels on the right side and left side respectively. r and R represent the radius of the wheel and the width of the robot respectively as shown in Fig.3.

Remark 3.1: In this paper, the actual commands for the WMR in Fig.3 are the angular velocities (ω_R, ω_L) defined in (8) (e.g. see Oriolo et al. (2002)), which is implemented by motors with voltage as input signals. This is also known as the dynamical model, e.g. see Fukao et al. (2000), Chen et al. (2009) and Asif et al. (2014). Since the mapping between these velocities is one-to-one, the pair of velocities for the robot (v, ω) with $u = \mathrm{col}(v, \omega)$ are implemented by two DC motors generating angular velocities (ω_R, ω_L) defined in (8).

Assume the reference trajectory is model based. Then the differential equations of the reference trajectory with reference coordinates $q_r = \mathrm{col}(q_{xr}, q_{yr}, \theta_r)$ and reference velocities $u_r = \mathrm{col}(v_r(t), \omega_r(t))$ are given by the following dynamics

$$\begin{bmatrix} \dot{q}_{xr} \\ \dot{q}_{yr} \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & 0 \\ \sin\theta_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r(t) \\ \omega_r(t) \end{bmatrix} \tag{9}$$

where $v_r(t) \neq 0$, which implies that the reference trajectory does not have rest configuration (see, e.g. Thuilot et al. (1996)).

Then the objective of the model-based tracking control is to design a controller u for system (7) such that

$$\lim_{t \to \infty} \|q_r - q\| = 0$$

where $q_r = \mathrm{col}(q_{xr}, q_{yr}, \theta_r)$ is the reference trajectory created by (9).

Introduce a diffeomorphism $T : \mathcal{R}^3 \longrightarrow \mathcal{R}^3$ with $q_e = T(q)$ as (see e.g. Lee et al. (2009))

$$q_e := \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \tilde{T}(q)(q_r - q) \tag{10}$$

where $q = \mathrm{col}(x_c, y_c, \theta)$, $q_r = \mathrm{col}(x_r, y_r, \theta_r)$ and

$$\tilde{T}(q) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

It is straightforward to verify that the inverse $\tilde{T}^{-1}(q)$ with

$$\tilde{T}^{-1}(q) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

is bounded with $\|\tilde{T}^{-1}(q)\| \leq 1$. Then, from Lemma 3.1, when $\lim_{t \to \infty} \|q_e\| = 0$,

$$\lim_{t \to \infty} \|q_r - q\| = 0 \tag{13}$$

Since x_{22} represents the angular error between the robot and the reference, without loss of generality, let $|x_{22}| \leq \pi$. By direct computation, it follows from (7) and (9) that the differential equation of the new error system can be described by

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} v_r \cos\theta_e \\ v_r \sin\theta_e \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & y_e \\ 0 & -x_e \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{14}$$

Therefore, the model-based reference tracking control problem based on the kinematic model (7) is equivalent to determining a feedback control law to stabilise the new error system (14) to the origin.

4 Control Design for the WMR

Consider the error system (14). A sliding surface is designed for the system (14). Then the stability of the corresponding sliding motion is analysed. In the following, the tracking problem is considered based on the limitation to the reference trajectory: $v_r > 0$ and $v_r < 0$. The case $v_r > 0$ is mainly considered, and the other case $v_r < 0$ can be obtained directly by slightly modifying the case $v_r > 0$.

If $v_r > 0$, consider the system (14) in the domain

$$\Omega = \{(x_e, y_e, \theta_e)^\tau | x_e > -c_1(1 + y_e^2), y_e \in \mathcal{R}, |\theta_e| \leq \pi\} \tag{15}$$

where c_1 is a designed positive parameter and $c_1 > 0.5$.

Choose the sliding function $\sigma = \mathrm{col}(\sigma_1^l, \sigma_2^l)$ as follows

$$\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} c_1\theta_e + \tan^{-1}(y_e) \\ x_e \end{bmatrix} \tag{16}$$

4.1 Stability of the sliding mode

When the sliding motion takes place, it is straightforward to verify from $\sigma = 0$ that

$$\begin{cases} x_e = 0 \\ \theta_e = -\frac{\tan^{-1}(y_e)}{c_1} \end{cases} \tag{17}$$

It is clear that $|\frac{\tan^{-1}(y_e)}{c_1}| < \pi$

Substituting (17) into (14), the regular form can be obtained as

$$\dot{y}_e = v_r \sin(-\frac{\tan^{-1}(y_e)}{c_1}) \tag{18}$$

Choose the Lyapunov function

$$V = \frac{1}{2} y_e^2 \tag{19}$$

Then the derivative of (19) is given by

$$\dot{V} = y_e \dot{y}_e = -v_r \sin(-\frac{\tan^{-1}(y_e)}{c_1}) y_e \tag{20}$$

It is straightforward from (20) to verify that the derivative of the selected Lyapunov function is negative definite. Therefore, the sliding motion of system (14) with the sliding surface $\sigma = 0$ is asymptotically stable.

4.2 Reachability of the sliding mode

Define the input u as

$$u = -E^{-1}\left\{ J_n \begin{bmatrix} v_r \cos\theta_e \\ v_r \sin\theta_e \\ \omega_r \end{bmatrix} + \begin{bmatrix} \eta_1 \mathrm{sgn}(\sigma_1) \\ \eta_2 \mathrm{sgn}(\sigma_2) \end{bmatrix} \right\} \tag{21}$$

where $\sigma_1(y_e, \theta_e)$ and $\sigma_2(x_e)$ defined in (16), η_1,η_2 are positive reaching gains, J_n is the Jacobian Matrix of the sliding functions defined by

$$J_n = \begin{bmatrix} 0 & \frac{1}{1+y_e^2} & c_1 \\ 1 & 0 & 0 \end{bmatrix} \tag{22}$$

and the matrix E is defined by

$$E = \begin{bmatrix} 0 & -(c_1 + \frac{x_e}{1+y_e^2}) \\ -1 & y_e \end{bmatrix} \tag{23}$$

It should be noticed that E is invertible for $q_e \in \Omega$ where Ω is defined in (15).

Remark 4.1: The limitation of x_e in the domain Ω is to ensure that the invertible matrix E always exists. Since θ_e is periodic in the coordinates, an appropriate equivalent θ_e can always be found in the defined domain Ω.

Theorem 1: *Consider the WMR system (14) in the domain Ω. The controller (21) drives the system (14) to the sliding surface $\sigma = 0$ where $\sigma(\cdot)$ is defined in (16) and maintains a sliding motion on it.*

Proof. Rewrite the derivatives of the sliding surface in the following form:

$$\begin{bmatrix} \dot\sigma_1 \\ \dot\sigma_2 \end{bmatrix} = J_n \begin{bmatrix} v_r \cos\theta_e \\ v_r \sin\theta_e \\ \omega_r \end{bmatrix} + Eu \tag{24}$$

Then substituting (21) into (24), it follows that

$$\begin{bmatrix} \dot\sigma_1 \\ \dot\sigma_2 \end{bmatrix} = \begin{bmatrix} -\eta_1 \mathrm{sgn}(\sigma_1) \\ -\eta_2 \mathrm{sgn}(\sigma_2) \end{bmatrix} \tag{25}$$

It is clear that

$$\sigma^\tau \dot\sigma = -\eta_1 \sigma_1 \mathrm{sgn}(\sigma_1) - \eta_2 \sigma_2 \mathrm{sgn}(\sigma_2) \leq \varepsilon \|\sigma\| \tag{26}$$

where $\varepsilon > 0$.

Thus the results follow. ■

If $v_r < 0$, then choose a sliding surface $\sigma = \mathrm{col}(\sigma_1^l, \sigma_2^l)$ as follows

$$\sigma = \begin{bmatrix} c_1\theta_e - \tan^{-1}(y_e) \\ x_e \end{bmatrix} \tag{27}$$

Then following the analysis above, it is straightforward to obtain the required result for tracking in reverse by slightly modifying the case $v_r > 0$.

Remark 4.2: From sliding mode theory, the controller developed from the proposed nonlinear sliding surface can stabilise the system only locally because the matrix E is singular when $x_e = -c_1(1 + y_e^2)$.

4.3 Implementation of the control with DC motors

In order to implement the control algorithm within the robot, two DC motors are used as actuators. The wheels on each side of the robot are driven independently, since the linear velocity and steering velocity correspond to differential driving. The relationship between the velocities of the robot and the rotational velocities of the wheels is given in (8). Pulse-width-modulation techniques are used to adjust the supply voltage so that the micro-controller can control the rotational velocities of each wheel independently. The rotational velocity of a motor with no load according to the input voltage adjusted by a PWM signal with 100 Hz sampling rate is shown in Table1.

Table 1 Rotational velocities with no load according to the duty cycle of the PWM signals.

Voltage	Counts	ω(rad/s)
1.088	2	0.062
2.175	10	0.31
3.263	21	0.66
4.35	31	0.97
5.438	41	1.28
6.525	51	1.59
7.613	61	1.91
8.697	71	2.22

To avoid unnecessary complexity of the control algorithm, two PI controllers are applied to the two motors to produce the desired inputs $u = \mathrm{col}(v, \omega)$ in the system (14) as designed in (21). The dynamical parameters of the motors are estimated by experiments based on their specification. With the same strategies proposed by Silva-Ortigoza et al. (2013), and assuming the two motors have the same parameters, the approximated dynamic model of the DC motor can be described by

$$\dot\omega_m = -15.385\omega_m + 3.846u_m \tag{28}$$

where ω_m is the rotational velocity of the motor and u_m is the adjusted voltage.

Define the PI controller for the motor as

$$u_m(t) = K_p e_m(t) + K_i \int_0^t e_m(t)dt \tag{29}$$

where $e_m(t)$ is the error between the expected and actual rotational velocities of motor. K_p is the proportional gain, and K_i is the integral gain for the motors on the right and left side. It should be noticed that the differences between the dynamics of the two motors are ignored and the PI controllers are simply implemented with identical controller gains.

With 100 Hz sampling rate, the parameter $K_p = 6.96$ and $K_i = 17.94$ are obtained by test. The sine wave tracking response shown in Figure 4 shows that the tracking performance is as expected.

Despite the dynamic variability and parameter variations in the motors, for example the slight differences in manufacturing between the two motors, the effect of the inductance, load changes and external forces, these may be ignored in the model as these uncertainties largely occur in the input channel of the kinematic model. The designed sliding mode control system can be made completely insensitive to such matched uncertainty as described in section 1.

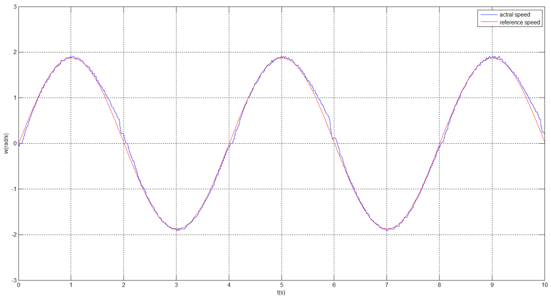

Figure 4 Sinewave response of the motor

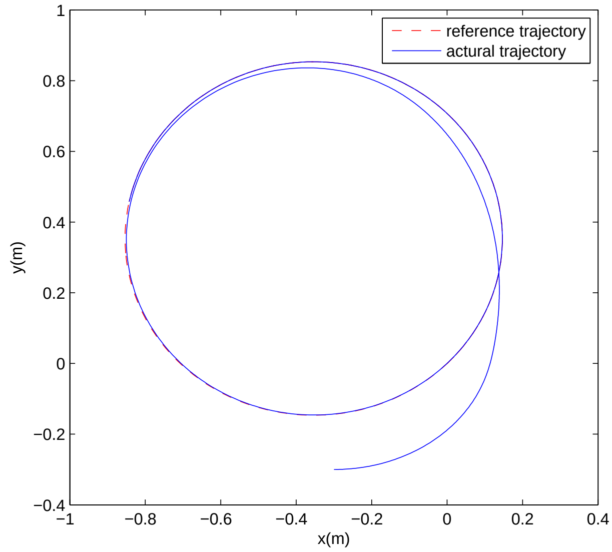

Figure 5 Simulated motion shown in the x-y phase plane

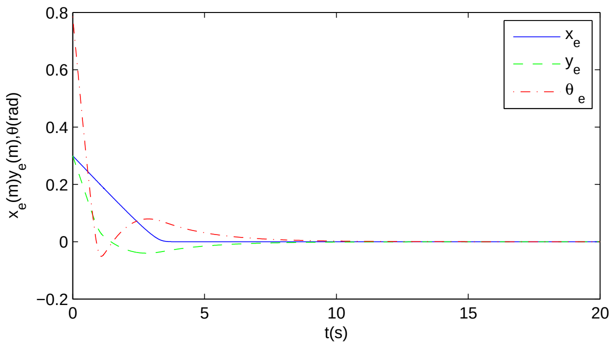

Figure 6 Time response of the error states

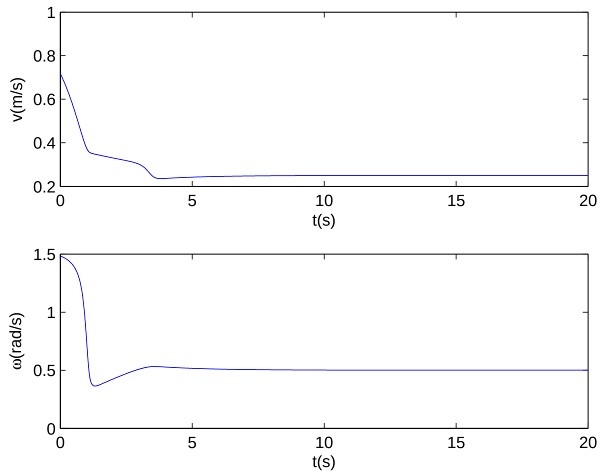

Figure 7 Time response of the velocities

5 Simulation and experimental results

In this section, both closed-loop simulation and experimental results are presented to test the behaviour of the sliding mode control proposed in Section 4. The simulations are implemented in MATLAB, and the real-time experiments are based on the Arduino Due board with the software configured according to the control design. The measured and estimated parameters are shown in Table 2

Table 2 The choice of options.

Parameters	value
radium of wheels r(m)	0.0315
Width between two wheels R(m)	0.09
Sliding surface parameter c_1	0.6
Reaching gain η_1	1.2
Reaching gain η_2	0.1
Boundary layer parameter δ_1	0.05
Boundary layer parameter δ_2	0.01

Remark 5.1: As in our design the two DC motors are identical, all the parameters of both motors, r, K_p and K_i, are the same.

5.1 Simulation results

The main simulation results with a circle reference trajectory with initial condition $q_r(0,0,\frac{\pi}{4})$, reference control pair $v_r = 0.25, \omega_r = 0.5$ and initial posture of the actual robot $q(-0.2,-0.3,0)$ are shown in Figures 5 to 7

Remark 5.2: The simulation environment is configured to represent the hardware. All the parameters are selected corresponding to the hardware constraints. The dynamics of the actuators, in this case the motors, are also incorporated in the simulation.

5.2 Experimental results

An image of the WMR during the circle tracking task is shown in Figure 8 and the actual motion is compared with simulation results in Figure 9.

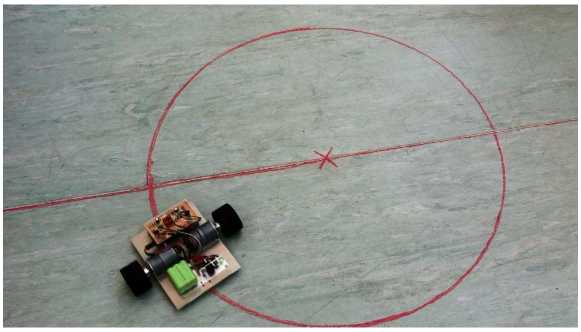

Figure 8 Tracking task experiments

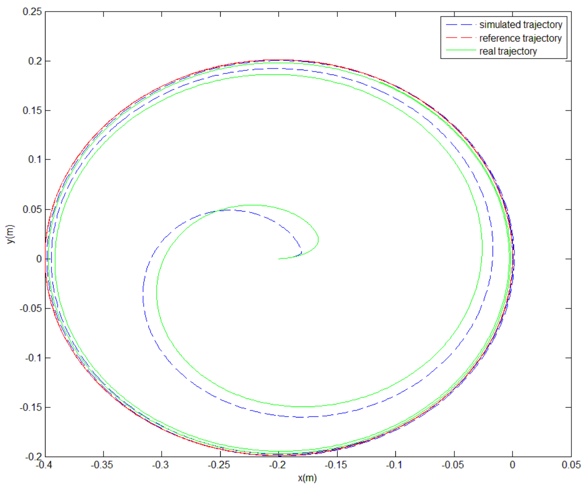

Figure 9 Comparison between experiments and simulations

From the experimental results, it is evident that although modelling error may exist, the robustness properties of the sliding mode control ensure the system exhibits the expected tracking performance. A small tracking error is achieved in practice.

6 Conclusion

In this paper, the trajectory tracking control design problem for a two-wheeled differential drive WMR has been considered. The proposed sliding mode controller has been successfully implemented on a real-time WMR platform. Both simulation results and experimental tests show that the proposed controller is straightforward to implement and exhibits good tracking performance.

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