Trends of growth and differentiation in street networks

Abstract.

Research in the area of Space syntax tends to be centred on static network representations of the built environment and its embedded social logic. Lacking the element of time, this synchronic representation cannot capture the dynamics of growth and change in urban systems. In this paper, we argue that the abstract values of space-time as a dual dimension play a key role as generators of city systems. Hence, we map urban growth and seek explanatory descriptions for the driving forces that characterise growth and differentiation in street networks. In two case studies; Manhattan and Barcelona, synchronic states of the growing systems are analysed. The states are separated by a short radius of time. The analysis leads to regularities that may help define the local and global processes that characterise urban growth marked by alternating periods of expansion and pruning in street networks.

Introduction

Instigated by the call to understand cities as matters of organised complexity (Jacobs, 1961), many theorisations were made on the description of such organisation and how that organisation came to be (Krafta, 1999; Portugali et al, 2011). The consequences of a proclaimed self-organised behaviour can -perhaps- be traced in the spatial signature of growth (Al_Sayed, 2013), and the invariant trends that characterise it. In line with Jacobs' call (1961), complexity theories of cities regarded urban systems as emergent products of complex adaptive processes that involve nonlinear interactions between different sets of variables. The definition of complex adaptive behaviour in urban systems was very much dependant on initial conditions and the actors that are put on display in simulation models (Allen et. al., 1977; Allen and Sanglier, 1979; Portugali et al, 2011). Until recently (Masucci et al, 2013), there has been little effort dedicated to outline the very trends that urban systems converge to in their growth patterns, where the majority of studies were more focused on comparing cities across different geographies (Carvalho and Penn, 2004; Bettencourt et al, 2007). Recently there has been a wide interest in the research community to empirically define the local mechanisms that generate the patterns we observe in cities (Al_Sayed et al, 2009; 2010; 2012; Strano et al, 2012; Barthelemy et al, 2013; Serra et al, 2016).

With the scope of identifying the local and global trends that cities display in their growth behaviour, an empirical investigation is held in this paper focusing on two case studies; Barcelona and Manhattan. Historical data on street networks is mapped and analysed using space syntax (Hillier and Hanson, 1984). The values of street network configurations are used to build statistical models of growth. The mapping reveals positive and reinforcing feedback mechanisms that operate through alternating periods of expansion and pruning in the urban grid.

Street network data

For the purpose of our investigation, street network maps were manually traced and extracted as a vector layer on top of a set of historical maps (see appendix 1) that record the history of urban development of Manhattan and Barcelona. The maps of Manhattan were observed in the years; 1642, 1661, 1695, 1728, 1755, 1767, 1789, 1797, 1808, 1817, 1836, 1842, 1850, 1880, 1920, and 2008. The sequential dates of the maps of Barcelona are; 1260, 1290, 1698, 1714, 1806, 1855, 1891, 1901, 1920, 1943, 1970 and 2008. Street network maps were drawn in such a way as to reduce the complexity of the street layout to the fewest and longest lines. *Axial*[1] lines were drawn manually. At each stage of growth, an *axial* line was drawn to match the exact coordinates of the *axial* line in the former stage. This rule was to take the preference when considering the fewest and longest lines of sight.

Tracing growth trends in historical data

The next two sections will be directed towards understanding and analysing dual historical transformations in the organic and uniform grids of Manhattan and Barcelona. While we take the synchronic representation of cities as networks interconnecting spatial elements, we map synchronic episodes of growth to trace their diachronic development (Griffiths, 2009). The assumption is that, the configurations of space are the main influential factors in the formation of cities. The synchronic view of space might be identified as a frame in time. Hence, a resultant diachronic model would be constituted of a sequence of synchronic models. We assume the presence of invariant features that mark the general trends in a diachronic model of urban growth, hence our analysis is aimed at identifying any regularity in the behaviour of urban systems as they expand and change.

In order to build an explanatory model of urban development we chose to analyse sequential synchronic episodes. In Space Syntax terms, these episodes might be represented by spatial networks that captures street configurations at each historical stage of growth. To calculate network distance we used the segment[2] representation of Space Syntax (Hillier and Iida, 2005). The representation is manually constructed as a vector layer on top of historical map data[3]. Considering

[1] The topological description of street networks might be simply defined as the fewest and longest lines of sight (axial lines) that cover all continuous spaces in an urban region. Each axial line will have a certain *connectivity* value (degree); that is the number of axial lines intersecting with it.

[2] A finer-scale description of streets is the (segment lines); the uninterrupted street interjunctions that link two road intersections (Turner, 2000).

[3] see appendix 1

the two cases under studies, synchronic states of the growing systems were analysed. The states are separated by a short radius of time. For a more accurate representation, the streets were unlinked were there are multilevel interchanges in the road infrastructure[4]. Patterns of *Integration*[5] calculated within local (500m) and global (R2000m) will be examined. Of interest, is how values of integration distribute and change over time. Further to that, statistical distributions will be outlined for each stage to reveal whether the probability distributions of geometric network properties change as the system expands. This is of great value, since the distribution of road networks - particularly in what concerns the network geometry- was not studied before (Waters, 2006).

1. Syntactic analysis of Manhattan's growth

After acquiring historical map data for Manhattan[6], it was possible to map and analyse the growth of the street network. The axial and segment integration values of each historical stage of growth were computed in Depthmap (Turner, 2010), and the statistical distributions of segment integration maps were visualised using JMP (SAS/Stat software). In the maps and associated distribution plots, the values of angular segment *Integration* showed an increase in local and global *Integration* as the urban system grows (see Figure 1). Local *Integration* values were calculated within 500 metre radius which almost equals five minutes walking distance. The number of elements retaining high local *Integration* values dropped when the uniform grid was imposed. Spatially, higher values appeared to concentrate in downtown Manhattan and Washington Heights areas. During the later phases of growth, clusters with high local *Integration* values appeared to concentrate in areas that have grown organically over time. On a global scale and within a radius that captures a metric distance of 2000 metres from each segment element, there was a more recognisable increase in *Integration* values as the system grows (see Figure 1). The highest values concentrated in the Lower Manhattan area at the initial stages of growth, spreading in later stages to the midtown area.

Overall, the local and global *Integration* values of the segment elements proved to fit well into a *heavy tailed lognormal* distribution in most phases of growth (see Figure 1). As the urban system grew, the range of global *Integration* values was spreading more widely than the range of local *Integration* values. Local structures preserved their range of values with slight changes on the mean and median values. The peaks of the distribution plots for local *Integration* sharpened in later phases of growth. This was less evident when rendering global *Integration* values.

Further topological structural properties are listed below the distribution plots to reflect on how changes on the shape of the distribution associated a change on the topological structures. The

[4] An example for that is where bridges and tunnels connect to the street network.

[5] Integration is a special measure of angular closeness in the segment network. It is calculated by relating total angular depth in the neighbourhood to the node count (Turner, 2000). Segment integration was not normalised in this particular case

[6] see appendix 1

topological properties were defined by *axial Intelligibility* R^2 (R2, Rn) and *Synergy* R^2 (R2, R5). Along with the structural properties, the distribution shape defined by the mean, median and skewness showed interesting patterns where transitional states in the street network distinguished periods of rapid change. With these measures, two sharp changes were recognised; one in the years (1695, 1728) and the other in the years (1850, 1880). Both periods witnessed critical changes in the grid directionality and structure. These changes had an observable effect on the distribution shape as well as on *Intelligibility* and *Synergy* correlation coefficients. The goodness of fit measured by a Kolmogorov–Smirnov limits (KSL) test[7] acted as an indicator to sharp transitions where the D value suddenly dropped to the half. In this case, the drop marked an improvement in the degree to which the empirical distribution fits the reference distribution (the *lognormal* distribution). There was no particular rule that associated the rise and drop of *Intelligibility* and *Synergy* on one hand and the mean and median on the other hand. Generally, the structure of the grid was weakened when exposed to a massive addition of segment street elements within relatively shorter periods of time. After a sudden drop in *Intelligibility*, the system appeared to retrieve the structural properties it had before. In parallel, the distribution shape settled with no significant changes. These findings are significant as they indicate to an autonomous process that takes place in urban structures. This is mostly visible in the way the street network preserved certain patterns of persistence and change. When such patterns were disturbed by an artificial intervention, the system readapted its structure to retrieve its prior distribution patterns.

Overall, the analytical attempts to understand the spatial structure of Manhattan underlined several consistencies that were noticeable in the growth process of the urban region. On both local and global metric radii, segment angular *Integration* values increased as the system grew. The pace of change on the global scale was more rapid than that on the local scale. The latter was less affected after a certain stage, probably due to the scale of measurement. Where evident, the increase in *Integration* values seemed to be directional. It generally started from certain centres and developed in space and time. Another consistency was evident in how the street network presented sharp changes in Intelligibility and Synergy that appeared to synchronise with changes on the shape of statistical distributions. The sharp transitional stages were often associated with a massive addition of streets, and were characterized by a change on the fitness level of integration values to a *lognormal* distribution. These transitional states of the system were followed by a period where the urban system adapted back to its prior structural patterns to improve its fitness to a *lognormal* distribution.

[7] KSL test measures here a distance between the empirical distribution function of the segment Integration values and the cumulative distribution function of the lognormal distribution

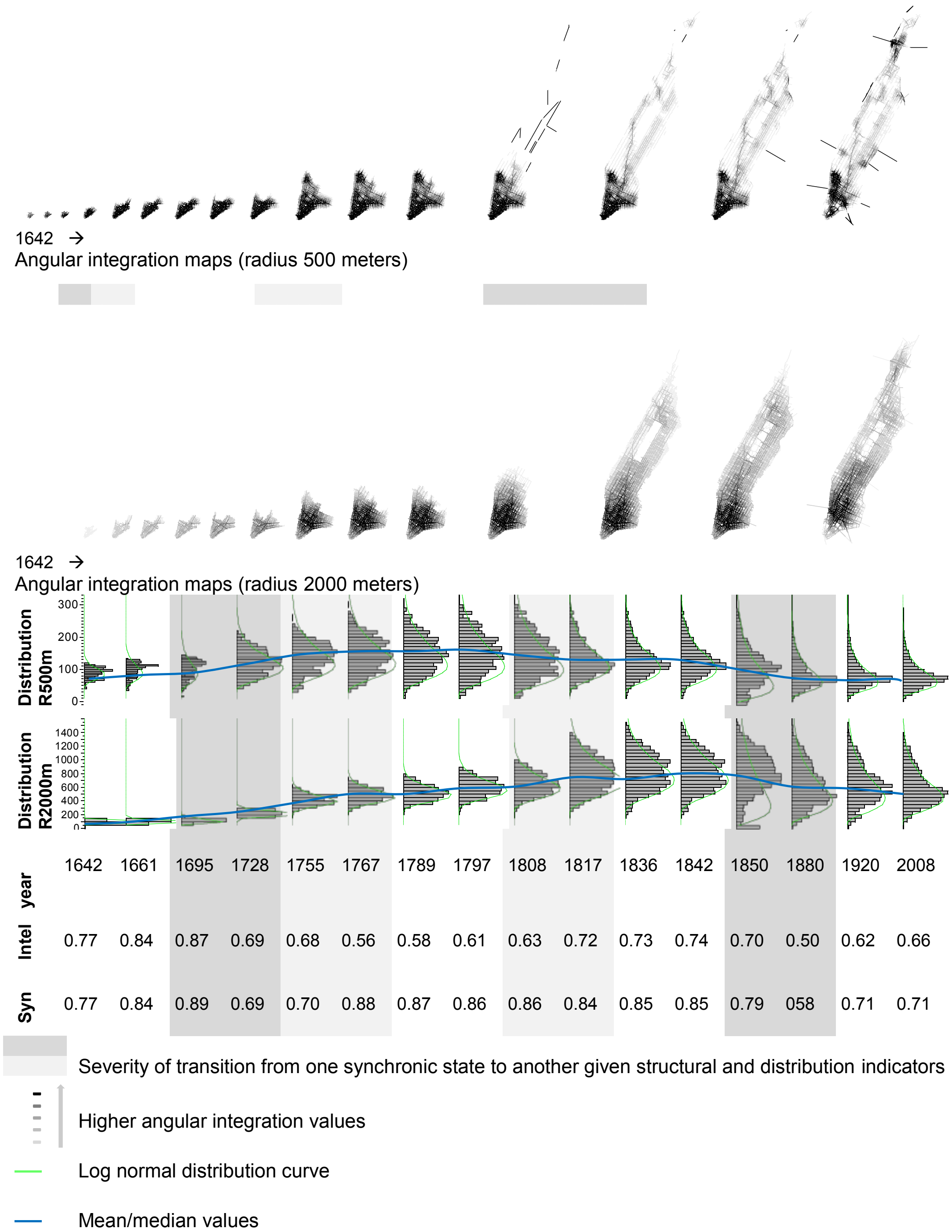


Figure 1 Comparing angular Integration maps (radius 500 and radius 2000 meters), their distribution plots, and the axial Intelligibility R2 (R2, Rn) and Synergy R2 (R2, R5) for each stage of growth in Manhattan.

2. Syntactic analysis of Barcelona's growth

In this section, we present a similar set of analytical investigations to that applied on Manhattan's case. The scope is to examine whether a different case study would present different trends of persistence and change in the statistical distribution of accessibility values. Similar to Manhattan's case, the choice of growth phases to be examined was limited to the data available (see appendix 1). Axial maps were drawn manually following the procedure explained in earlier sections. The *axial* representation was analysed to obtain *axial* Intelligibility and Synergy. A more refined segmental representation was also used to obtain angular segment *Integration*. Statistical distributions were plotted to display how aggregate properties of the urban system change in time.

The segmental angular measures appeared to plot analogous patterns in Barcelona to those explored in Manhattan (see Figure 2). Similarities appeared where organic or deformed local structures preserved the highest values. Over the period of growth, the old city continued to preserve the highest values of angular segment *Integration* (R 500m). Equally, emergent suburban town centres in the urban fringe maintained similar values of local *Integration*. Similar to Manhattan's case, segment integration maps radius (2000m) rendered how highly integrated centres expand and shift leaving lower values on the peripheries. In Barcelona, higher *Integration* values shifted from the old city to centre at the heart of the uniform grid. The lines representing major roads in the street network appeared to be particularly integrated connecting all parts of the urban grid with the local old city centre and the suburban town centres. As the spatial system grew and deformed, the distinct features of the two different grid patterns (organic and uniform) faded and a connective structure arose tracing the rural freeways that preceded the uniform grid.

The patterns of aggregate distributions were consistent with our observations in Manhattan. In most stages, the distribution plots for angular segment *Integration* radius 500 and radius 2000 appeared to fit into a lognormal curve (see Figure 2). The period from 1698 to 1855 presented a multi modal distribution pattern that hardly fits in the former lognormal curve. This is due to distinct differences in density, angularity and segment line length between fully urbanised agglomerations around the old city centre and the rural freeway network on the peripheries. This phenomenon may help delineate compact urban form statistically. Distinct transitional periods that characterise transformations in the street network might be recognised when comparing the shapes of statistical distributions. In two cases out of three, we found the distribution of local *Integration* turning from a relatively unimodal to multimodal. The case is less clear in Manhattan. With that comes also a change on the distribution moments including the median and mean. *Intelligibility* and *Synergy* coefficients came as to confirm these transitional periods revealing changes on the level of the structural unity between the parts and the whole. Again, similar to Manhattan's case; the changes on the structure were associated with a mass addition of segment elements to the urban networks.

On the whole, the analyses exposed several particularities that had to do with the inclusion of less urbanised areas in the analysis. Where such differences were clear, they had a significant impact on the shape of statistical distribution in Barcelona's case. The impact of stubs on the segment representation and the effect of highway infrastructure were found to disrupt the overall distribution patterns. This finding might help outline the characteristics of compact urban form; defined by how aggregate angular configurations of street structures fit into lognormal distributions.

The effect of anomalies on the representation is not likely to dismiss the more prevailing consistencies, particularly in how local centres conserve high local *Integration* values and in how global *Integration* spreads to central areas as the system grows. Consistencies are also evident in how transitional periods are characterised by an association with mass addition of segment street elements. Such associations often synchronised with changes on the fitness of the lognormal distribution function; where the *D* value –here describing the goodness of fit- dropped to half or else doubled. This all needs to be considered given different local radii. Here a radius of 2000m proved to be a stronger indicator of the transitions in the growing street network. The more the angular depth fits a lognormal distribution, the less likely the system would call for changes. On the contrary, the more erratic are the *Integration* values around the lognormal distribution curve the more likely it is for the system to change. In this way, the system's inclination to change is strongly related to how good it fits a lognormal distribution function.

The way in which the urban system settled back into a *lognormal* distribution after the transitional periods is again suggestive of an autonomous behaviour that cities exhibit in their growth patterns. Such hypothesis needs to be supported by a more extensive investigation into the local mechanisms that govern growth. Before going for a higher resolution, it is imperative to think about the time dimensionality and how it couples spatial transitions. In Manhattan, we noted a time factor, or what we might term as a radius of time that associates the radius of space, in that local *Integration* maintains a slower pace of change when compared to global radii. To further expose this duality we need to plot spatial changes against the time axis.

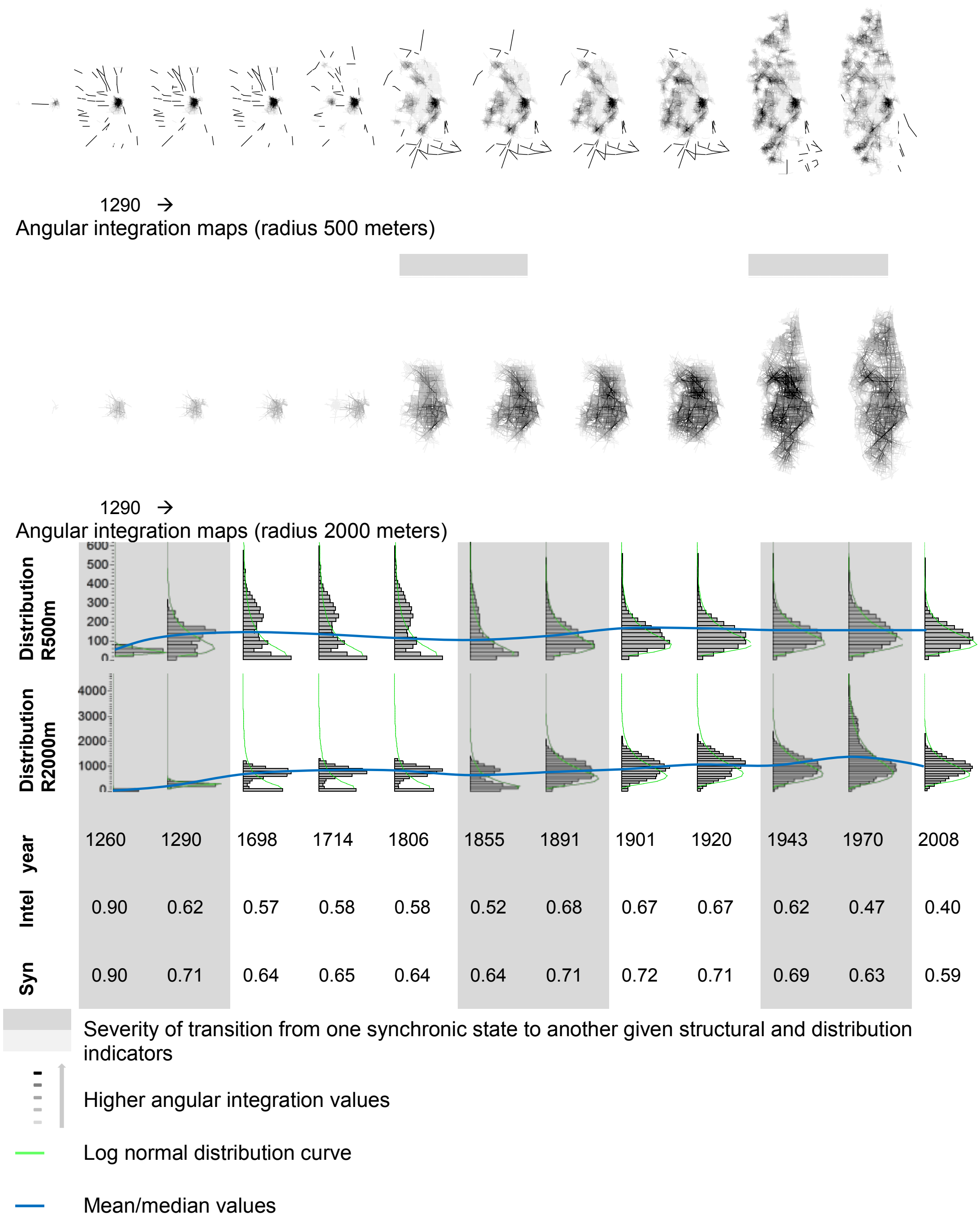


Figure 2 Comparing angular *Integration* maps (radius 500 and radius 2000 meters), their distribution plots, and the *axial Intelligibility* R^2 (R2, R*n*) and *Synergy* R^2 (R2, R5) for each stage of growth in Barcelona.

Generic Trends of Growth in street networks

Measuring on the overall growth patterns of the spatial structure in Manhattan and Barcelona, growth could be outlined by exponential trends that plot the increase in the number of street segments (see Figure 3). Overlaid on top of that exponential trend is a trend that marks the changes on the fractal dimension of the urban system as it grows. The range is relative to the image size of the two street structures. The fractal dimension is calculated using the box-counting algorithm, by iteratively overlaying boxes with different sizes and calculating the black pixels within (see appendix 2). The output of this procedure is a regression line. The slope of this regression line is the fractal dimension of the street structure.

The exponential growth parameters recorded different values for the two urban regions. This difference might be reasoned by the system's *sensitive dependence on initial conditions*; a phenomenon that characterises complex systems. The circumstances that limit growth at certain stages might also play a role in driving the exponential trend. Physical boundary limitation –for example- presents a constraint during the last stages of growth diverting the exponential trend in Manhattan to a linear trend that settles at a certain global maxima. At this point, we were able to distinguish a point of *inflection* that marked a change on the concavity of the curve. In Barcelona, the addition of new elements continued to follow an exponential trend provided that the structure has not reached its maximum geographical boundaries. On the exponential models, we marked transitional states that were derived from critical changes on the probability distributions of the street structures to expose any correspondences with the generic trends. We noted a remarkable correspondence between these transitions and a sudden rise or fall in the number of elements, along with the grid's fractal dimension. Such transitions were also traceable in the changes on the overall size of the urban system and in the patterns it rendered.

While both growth trends followed a nonlinear exponential model, they did that with different rates. Manhattan's exponential growth covered a period of 240 years starting from $year_o$ =1640, whereas Barcelona's growth spanned over a period of 750 years starting from $year_o$=1260. The number of elements (NoE) variable was modelled as a nonlinear function of *year*. To build the exponential model, a prediction formula was set. The prediction formula included parametric estimates (X_o, X_1). X_o is the prediction of the number of elements at year 1240 in Barcelona and year 1698 in Manhattan. It should be near the actual value of the number of street segments in the network representation of the historical data. The formula contained the parameters' initial values. The X_1 growth rate parameter was given the initial value of 0.006 for Barcelona, and 0.013 for Manhattan, which would match the estimate of the slope that is derived from fitting the natural log of NoE to *year* with a straight line. The initial values did fit reasonably well with the final parameter estimates of the exponential model. The prediction formula was defined as follows;

$$f(x) = X_o * \mathrm{Exp}(X_1 * (year - year_o)) \qquad (1)$$

where the parameters initial values are;

X_1 = 0.013, X_o = 159 for Manhattan

X_1 = 0.006, X_o = 90 for Barcelona

The nonlinear fitting process was tailored so that the *confidence limits*[8] retained a value for Alpha=0.05 and the convergence criterion was set to 0.00001. Using this method, solutions were found for a minimised Sum Square Errors (SSE) value. The fitness to an exponential trend appeared to be different for the two urban regions. The mean squared error (MSE) in Manhattan's growth trend was about 1978047.3 while in Barcelona's trend MSE approximated 7270391.8. The MSE can be significantly reduced for Manhattan, if we did not consider the last stage of growth (*year* = 2005) in the calculation. If we were to miss that year from the analysis, we would have reached out to an MSE value of 531727.42; four times less than the MSE value for the full period of growth. This test highlighted the role of physical boundaries in shaping the growth process. It is important to mention here that such constraint is only one of many. For this reason, it might be impossible to find an averaged solution that fits both urban scenarios. This is not only made difficult by the *sensitive dependence on initial conditions* but is also very much dependant on the circumstances and conditions that direct growth at each time interval and for each localised addition of street segment to the network. Given the difficulty in finding data that maps the progress of urban development at this spatiotemporal scale, it is maybe reasonable to pursue another form of *morphogenetic* mapping that only reflects on how structures change in-between different time frames following the methods developed in (Al Sayed et al, 2012). Such an approach would help inferring the rules and forces that operate on the local level that ultimately build into the growth patterns outlined in this section. Before opting for an investigation into the local transformations of urban structures, we need to cast some light on the localised processes that are thought to constitute the principal feedback dynamics in urban systems.

[8] The iterations for *confidence limits* do not find the profile-likelihood confidence intervals successfully.

Manhattan | **Barcelona**

Linear and nonlinear fit solutions

Manhattan	Barcelona
Fractal D R^2 = 0.53 Linear fit	**Fractal D** R^2 = 0.83 Linear fit

Nonlinear fit solution

Manhattan:

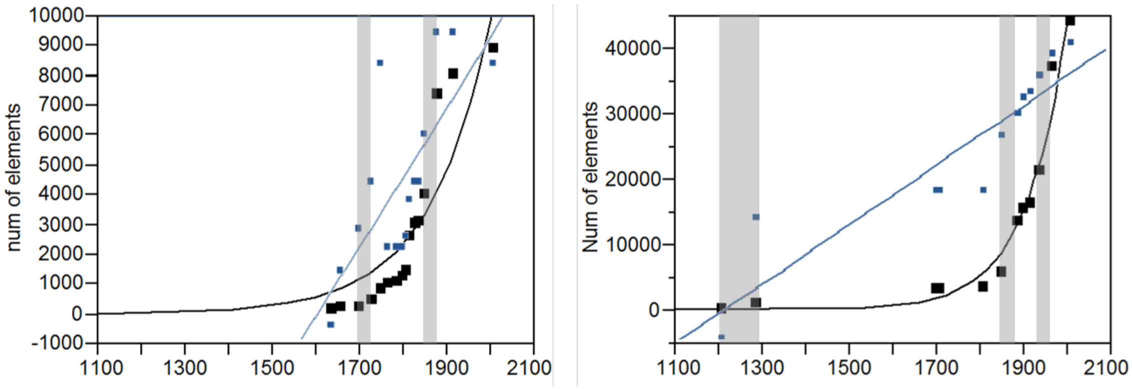
SSE	DFE	MSE	RMSE
27692661.637	14	1978047.3	1406.4307

Parameter	Estimate	ApproxStdErr
b1	0.007164405	0.00106851
b0	733.0202662	237.867098
b1	0.013	0
b0	159	0

Barcelona:

SSE	DFE	MSE	RMSE
72703917.749	10	7270391.8	2696.3664

Parameter	Estimate	ApproxStdErr
b1	0.0105310111	0.00088276
b0	10.25766467	6.91634685
b1	0.006	0
b0	90	0



year | *year*

—— Fractal Dimension trend

▬ Transitional states in the growth trend

Figure 3 A nonlinear exponential growth model for Manhattan and Barcelona marking transitional states corresponding to those outlined in the statistical distributions of integration values (figure 1 & 2).

Local mechanisms of preferential attachment and pruning in street networks

The previous section elucidated the general trends of growth that outlined the yearly increase in the number of street elements in Manhattan and Barcelona. Looking at the exponential models, it is imperative to regard the factors that might have affected and distorted the growth patterns. The increase in the number of street elements did not go without resistance. At different rates and on different scales, streets emerged or disappeared locally due to artificial and natural causes. Artificial interventions could be an effect of intentional planning actions that alter the street layout. Natural causes could be theoretically defined as a tendency in the urban system to follow an autonomous process of self-organisation in which the urban system adapts following artificial interventions to preserve the structural unity between the whole and the parts. It is implicit to our knowledge, how such processes build from the local to the global to render the hierarchical organisation that makes urban systems. Yet, it might be possible to set forth some assumptions on the mechanisms involved by looking at the elementary processes of addition and deletion, mergence and subdivision that lead to growth and differentiation in cities.

The two basic processes that govern trends of growth or shrinking in cities are based on a positive feedback loop that results from the addition of new elements and a reinforcing feedback loop that results from pruning certain elements (Al Sayed et al, 2012). In this way, urban growth is a product of the elementary processes of addition, pruning, mergence and subdivision. The latter processes branch from the main loops (Figure).

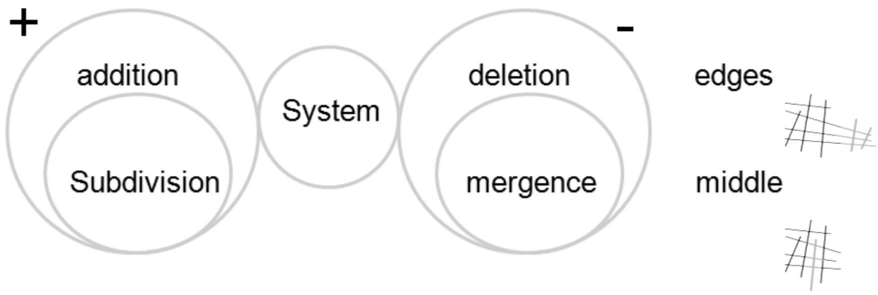


Figure 4 Positive and reinforcing feedback loops in the spatial networks of streets.

It is suggested that in periods of expansion, a positive feedback mechanism operates and takes the form of exponential addition of elements. New street structures are preferentially added where there is a potential increase in the street network accessibility. At each stage of growth the connectivity (degree) of street lines follows power law distributions, a phenomenon that is also observed in scale-free networks. The observed historical growth behaviour comes as to confirm this finding. In a process that resembles preferential attachment in information networks; the emergence of new patches of grid structures in 18th century Barcelona follows high values in the connectivity of straight street lines.

Preferential attachment was previously suggested to be a possible model for growth in street networks taking the topological Space Syntax representation (Volchenkov and Blanchard, 2008). This suggestion was made after observing that control values (degree of choice) fall into power law, an observation that was also compatible with Porta et. al.'s findings (2006). The model was presented in the form of cautious suggestion since no evidence was explicitly presented for why such model –famous in information networks- would fit the case of physically constrained urban systems. Here we found historical traces for such processes in both the geometric and topological representations of Barcelona's urban growth, less clearly in Manhattan. The observed phenomena we reported indicated that a process of preferential attachment did not exactly follow the model initially proposed in (Barabási-Albert, 1999). In the famous Barabási-Albert model description for preferential attachment, nodes that are highly connected (high degree) will have more likelihood to attach to more nodes. In other words; nodes that are rich with connections will be more likely to increase their connections. The presence of physical constraints in urban networks, might have contributed to what our observations here, that is a form of preferential attachment where nodes that are highly connected developed a whole neighbourhood cluster in their localities (Al Sayed et al, 2012).

This Barabási-Albert model will probably need to be adjusted to fit with the nature of street networks. Considering the topological axial representation of streets and how it developed in time (Figure 5), we might note some patterns arising from the use of simple network measures like connectivity (degree). These observations were reconstructed from historical map data to track how the system changed with the addition of new elements in Barcelona. The emergence of new grid structures seemed to coincide with high values of connectivity. At each stage in the system, the connectivity values exhibited Power law distributions (Figure 6). These observations would stand as an empirical validation for the applicability of the Barabasi-Albert model on urban networks. However, it is important to emphasise here that spatial networks in cities are not really scale-free; since they have geometric properties that are particularly crucial for them to operate. Moreover, there is a cap on how many connections an axial line may connect to given its relative length and the intensity of the grid structure that neighbours it. Some additional constraints -possibly axial line length- may be applied to distinguish those elements in vacant land.

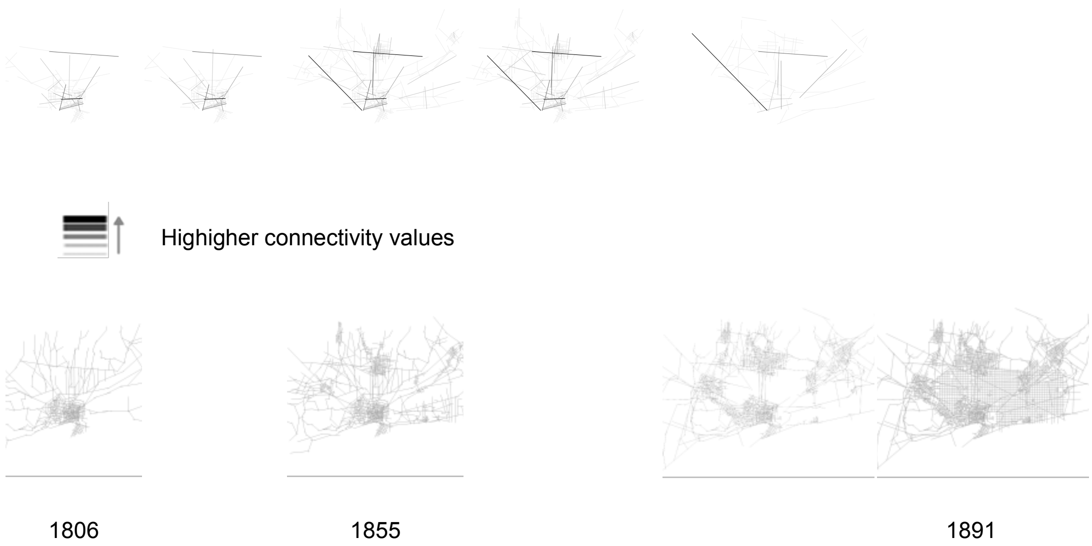


Figure 5 A reconstruction of Barcelona's growth process considering the historical maps dated (at the bottom). The Connectivity values (at the top) rendered again the phenomenon of preferential attachment in how high values precede the emergence of new grid patches.

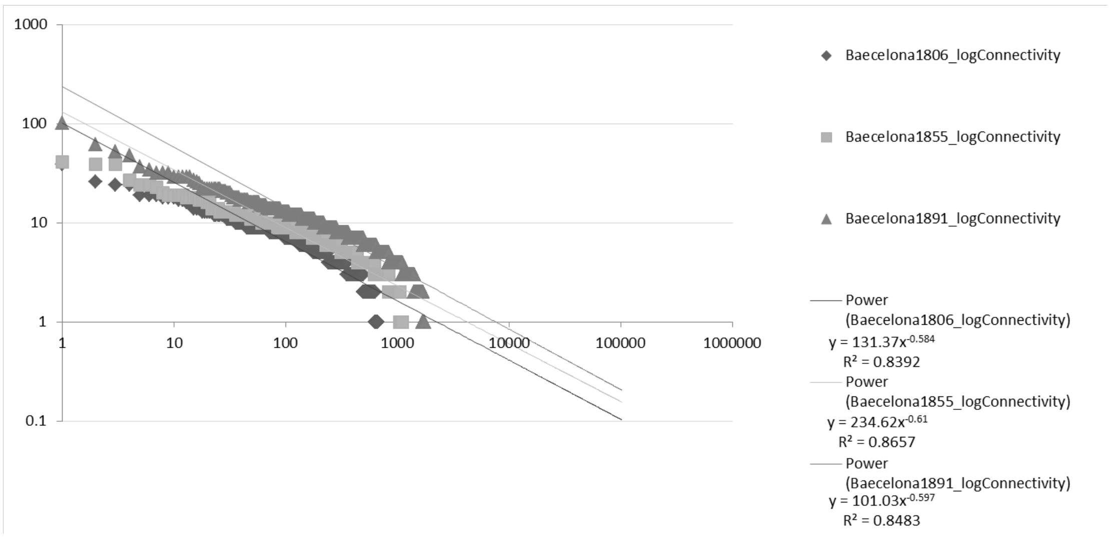
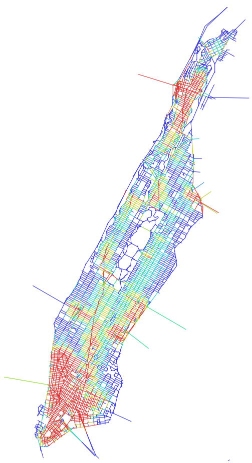


Figure 6 Log-Log plot of Connectivity (degree) values for different developmental stages of Barcelona's axial structure fitting with Power law distributions.

Another observed regularity in street network growth is the disappearance of weakly connected street structures (Pruning). Once the urban system reached its maximum boundary, another process of reinforcing feedback took place. This mechanism was mostly evident in Manhattan, where the filling of the central park rendered this exact area as poorly connected indicating a process of pruning of weak local structures (Figure 7). The process of pruning was less clear in Barcelona, where the filling of the citadel area shows a slight increase in local integration values in the area adjacent to the old city centre. The mechanism of pruning took place at a certain stage of growth and contributed to the demarcation of a structure deforming the homogeneity of the grid. It is important to consider here the historical circumstances that led to the formation of a void within a densely occupied urban structure in both cases. Some are explained in (Al Sayed, 2014). Ultimately, the abstract physical form of the urban grid is a materialisation of many less easily delineated social and economic conditions that historically shape urban regions (Griffiths, 2009; Vaughan et al, 2013).

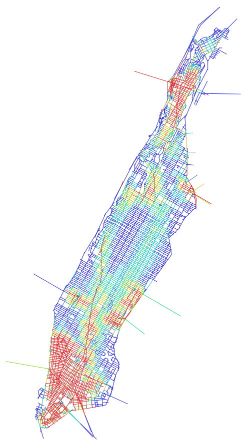
Manhattan 2008

Integration R500m - Range (60, 100)



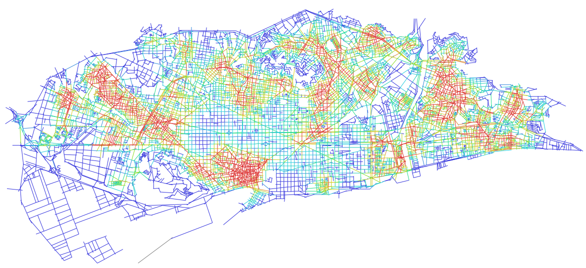
Manhattan – Central Park area filled with grid

Integration R500m - Range (60, 100)



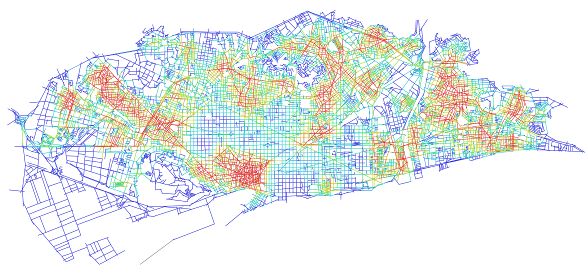
Barcelona 2008

Integration R500m - Range (60, 200)



Barcelona – citadel area filled with grid

Integration R500m - Range (60, 200)



Higher values of accessibility

Figure 7 A pruning process of elements with low integration values (radius 500m) in Manhattan and Barcelona. The models on the right are produced by filling the Central Park void in Manhattan and the Citadel area in Barcelona with a grid that has an analogous structure to its surrounding.

Conclusion

This paper presents empirical models of street network growth that are explanatory of some of the mechanisms of growth and change in Manhattan and Barcelona. Looking at the generic trends elucidated in this paper, there seems to be what suggests that spatial systems are inclined to follow a simple exponential model in their growth that pertain to the invariant geometric transformations in the network structure. The exponential trend was also marked in population size (Bretagnolle *et. al.*, 2002; Pumain *et. al.*, 2006). The mapping method was instrumental in looking for patterns of growth and differentiation in the urban regions under study. In general, these patterns seem to be drawn by certain forces governing how the physical processes of addition, subdivision, mergence and disappearance operate on the level of segment element or axial line. The addition and subdivision are products of a process of preferential attachment that is similar to the mechanism that govern growth in information networks, in that nodes (here axial lines) that are highly connected are more likely to attach to new nodes in subsequent stages of growth. The mergence and disappearance of streets are products of a process of pruning, where weak local structures disappear in later stages of growth to reinforce structural differentiation within the urban grid. The physical processes appear to be associated with changes on angular *Integration* values in the street network. The local and global *Integration* values seem to increase to a certain level to settle at certain average values once the system has grown beyond the radii of measurement. While integrated centres are preserved, changes seem to travel in certain directions. Where such processes are interrupted by a massive addition of segment street elements, they seem to revert back to their initial rates. These interruptions might be recognised as periods of transitions. The transitions might also be concurrent to innovation cycles where information, economic and industrial developments take place on a large scale transforming the landscape of a city (Pumain and Moriconi, 1997; Pumain *et. al.*, 2006). They appear to have a destabilisation effect on the structural unity of the system and the aggregate distributions of *Integration* values that consequently reduced its fitness level to a lognormal distribution.

The ambition to find a model that unlocks the process of growth should not be taken without raising concerns over the specific initial conditions that would inevitably determine the course of growth in some cases. The geographic and topological affordances of both regions had clear impact on growth behaviour. In Barcelona, the inclusion of peri-urban areas in the distribution plots distorted the aggregate distribution patterns that would otherwise settle into a lognormal distribution. In Manhattan, the effect of geography was visible in all sets of analyses. The physical boundary constraint was marked as an inflection point on the exponential trend that a system follows as new segment street elements are added. What is characterised by a change on concavity in the exponential curve is also marked on the averaged *Integration* trends, and the probability distributions of values of change on angular *Integration*. The system diverts at that point from growth to differentiation. The differentiation process is probably augmented by the system's tendency to react to the imposition of the uniform grid, where weakly integrated local structures are pruned to give rise to a heterogeneous structure.

The findings at hands come as to support the hypothesis that urban growth is not entirely a random process, and that there seems to be some inherent organisation that governs spatial structures. We might even go further with that to claim that street networks exhibit some form of autonomy, in the way they adapt back to their original patterns of growth after certain periods of transition. Through the act of some implicit self-organisation mechanism, the two urban regions observed in this paper appeared to retrieve the characteristics of their macrostate following large-scale planning interventions. To expose the mechanisms that contribute to this behaviour, there is a need to go further with our analytical investigation to observe how elements change locally and how they settle into certain hierarchies.

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Appendix 1

Historical street maps of Manhattan and Barcelona

New York City 1642 [Map overlays streets as of 1880] From Report on the Social Statistics of Cities, Compiled by George E. Waring, Jr., United States. Census Office, Part I, 1886.

New York City 1661 [Map overlays streets as of 1880] From Report on the Social Statistics of Cities, Compiled by George E. Waring, Jr., United States. Census Office, Part I, 1886.

New York City 1695 From Manual of the Corporation of the City of New York for 1852 by D.T. Valentine, 1852

New York City 1728 [Map overlays streets as of 1880] From Report on the Social Statistics of Cities, Compiled by George E. Waring, Jr., United States. Census Office, Part I, 1886.

New York City 1755 [Map overlays streets as of 1880] From Report on the Social Statistics of Cities, Compiled by George E. Waring, Jr., United States. Census Office, Part I, 1886.

New York City 1767 From A History of the American People, Woodrow Wilson, Harper and Brothers Publishers, New York and London, (c)1902, Vol II

New York City 1775 A plan of the city of New-York & its environs : to Greenwich, on the North or Hudsons River, and to Crown Point, on the East or Sound River, showing the several streets, public buildings, docks, fort & battery, with the true form & course of the commanding grounds, with and without the town : From John Montresor, engineer ; P. Andrews, sculp

New York City 1782 [Map overlays streets as of 1880] From Report on the Social Statistics of Cities, Compiled by George E. Waring, Jr., United States. Census Office, Part I, 1886.

New York City 1808 From D. Longworth's map of 1808 for D.T. Valentine's Manual for 1852, by G. Hayward, lithr.

New York City 1817 From Thos. H. Poppleton, city surveyor ; P. Maverick sc. Newark.
Topographical map of the city and county of New-York, and the adjacent country 1836: Published by J.H. Colton & Co., No. 4 Spruce St., 1836 (New-York : Engraved & printed by S. Stiles & Co.)

New York City 1842 "New-York" From Tanner, H.S. The American Traveller; or Guide Through the United States. Eighth Edition. New York, 1842.

New York City 1850 From Mitchell Sr., S. A., A New Universal Atlas Containing Maps of the various Empires, Kingdoms, States and Republics Of The World.

New York City 1880 [Map overlays streets as of 1880] "The Original Topography of Manhattan Island from the Battery to 155th Street" From Report on the Social Statistics of Cities, Compiled by George E. Waring, Jr., United States. Census Office, Part I, 1886.

New York City 1891 From Bromley, G. W. and Bromley, W. S., Atlas of the city of New York / Manhattan island from actual surveys and official plans. Philadelphia: G. W. Bromley & Co., 1891. [web: rumsey]

New York City (Lower Manhattan) 1920 "Chief Points of Interest in Lower Manhattan" From Automobile Blue Book 1920

New York City (Upper Manhattan) 1920 "Chief Points of Interest in Upper Manhattan" From Automobile Blue Book 1920

Barcelona 1200-1300 From Bensch, S. P. (1995) Barcelona and Its Rulers, 1096-1291. Cambridge \ New York \ Melbourne: Cambridge University Press. pp. 27

Barcelona 1260 From Planta de la ciutat romana amb l'hipòtesi de situació dels elements distribuïdors d'aigua, From segons C. Miró i H. Orengo

Barcelona 1494-1495 From Hieronymus Münzer account

Barcelona 1698 French map from 1698 of the city of Barcelona with indications for a siege. Title: Plan du Siège de la ville de Barcelonne : Avec la Carte de la côte de la Mer depuis le Cap de Cervera jusqu'aux environs de Llobregat.

Barcelona 1706 Plan de la Ville de Barcelone et Chateau de Mont Iuy (Plan of the City of Barcelona and Castle of Montjuic) From Anna Beeck

Barcelona 1711 From Nicholas De Fer, published in 1711 in Paris.

Barcelona 1745 From I. BASIRE 1745. prepared for Mr Tindal's continuation of Mr Rapin's History of England.

Barcelona 1751 From Bodenehr's Curioses Staats und Kriegs Theatrum.

Barcelona 1806 The walled city of Barcelona and the Citadel 1806. Plan of the City and Port of Barcelona. From: "Voyage de l'Espagne" by Alexandre de Laborde. - Paris, 1806-1820 arrow oriented with the north to the northeast of the map.

Barcelona 1851 From G Heck

Barcelona 1860 published by J & C Walker for the British Admiralty.

Barcelona 1862 From Coronel, Teniente-Coronel de Ingenieros D. Francisco Coello. Aided by D. Pascual Madoz author of the statistics and historical notes Madrid 1862.

Barcelona 1890 Plano de Barcelona y sus Alrededores en 1890. From D. J. M. Serra. by Gerona; S. by the Mediterranean Sea; S.W. by Tarragona; and W. and N.W. by Lrida.

Barcelona 1914 From Plànol general de Barcelona. La febre tifoide a Barcelona: gràfic de l'epidèmia de l'any 1914

Barcelona 1929 From Wagner & Debes, Leipzig,

Barcelona 1943 From [http://www.lib.utexas.edu/maps/ams/spain_city_plans/txu-pclmaps-oclc-6478271-barcelona.jpg]

Barcelona 1970 From the David Williams collection

Barcelona Field Studies Centre. Source: De Barcino a Barcelona 1992 Barcelona Contemporary Culture Centre [online] Available from <http://geographyfieldwork.com/Poll2.htm> [Date accessed: 14 October 2012]

Appendix 2

Box Counting method

The fractal *D* dimension might be obtained using the following method. A square mesh of various sizes *s* is laid over an image (containing the object that we want to compute its fractal dimension). The number of mesh boxes *N(s)* that contain part of the image are counted. The fractal (box) dimension *D* is given by the slope of the linear portion of a log(*N(s)*) vs log(1/*s*) graph.

$$\log\big(N(s)\big) = D\log(\frac{1}{s}) \qquad (2)$$

Since there is no preferred origin for the boxes with respect to the pixels in the image, multiple measures *N(s)* can be computed for different mesh origins. The graphed value of *N(s)* is usually the average of *N(s)* from different mesh origins.

The range of box sizes used here are; 2,3,4,6,8,12,16,32,64.