Beyond a positive stance: integrating technology is demanding on teachers' mathematical knowledge for teaching

Nicola Bretscher

UCL Institute of Education, London, UK; n.bretscher@ucl.ac.uk

Research on technology in mathematics education highlights the importance of teachers having a positive stance towards technology for successful integration into classroom practice. However, such research has paid relatively little attention to teachers' knowledge of specific mathematical concepts in relation to technology. This paper examines the innovative use of technology by a teacher, Robert, as a critical case study, to argue that the significance of mathematical knowledge for teaching using technology should not be overlooked nor underestimated.

Keywords: Computer uses in (secondary mathematics) education, Knowledge base in teaching, TPACK, situated abstraction.

Introduction

Seeking to understand teachers' integration of technology, research on technology in mathematics education (e.g. Zbiek et al., 2007) has documented the important role teachers' beliefs and conceptions play in their integration of technology into classroom practice. For example, Zbiek et al (2007) identify the constructs of pedagogical fidelity and privileging as useful in understanding the extent and nature of technology integration in a teacher's classroom practice. Pedagogical fidelity is described as the degree to which teachers' beliefs about the way a digital technology allows students to act mathematically coincides with their beliefs about the nature of mathematical learning (Zbiek et al., 2007). Privileging is a notion developed by Kendal and Stacey (2001) to describe how teachers, consciously or unconsciously, frequently use or place a priority on certain things in their practice, for example, types of representation, skills or concepts and by-hand or by-technology methods (Zbiek et al., 2007). Both these constructs relate to teachers' conceptions of mathematics as a discipline (Thompson, 1992), their beliefs about the nature of teaching and learning mathematics and how these interact with their beliefs about technology.

Such studies have in common a focus on teachers' global conceptions of mathematics as a discipline and on teachers' beliefs about the nature of teaching and learning mathematics with technology. They do not tend to focus on teachers' knowledge of specific mathematical concepts in relation to technology. This is an important omission since the documented shifts in teachers' views suggest a move towards models of teaching aimed at developing conceptual understanding. Such models may require a great deal of knowledge for successful implementation and inconsistencies between teachers' professed beliefs and practices may be the result of lacking sufficient knowledge and skills necessary to implement them (Thompson, 1992).

Whilst highlighting the role of teachers' conceptions in technology integration is important, this paper argues that the significance of mathematical knowledge for teaching using technology should not be overlooked nor underestimated. For example, Bowers and Stephens (2011, p. 290) assert that the set of (teachable) knowledge and skills for teaching mathematics using technology may be empty, emphasising instead that teacher educators should seek to nurture a favourable conception of "technology as a critical tool for identifying mathematical relationships". Whilst it may be that teacher educators should seek to nurture favourable conceptions towards using ICT in their trainees, this paper argues the knowledge required to put such conceptions into practice should not be neglected.

Theoretical Framework

The central Technology, Pedagogy and Content Knowledge (TPACK) construct of Mishra and Koehler's (2006) framework is useful in highlighting mathematical knowledge for teaching using technology, by emphasising technology as a knowledge domain alongside pedagogy and content knowledge (Bretscher, 2015). Whilst space does not allow for a full description of the framework, the central TPACK construct serves to highlight the situated nature of such knowledge. In particular, in this paper, mathematical knowledge for teaching using technology is viewed as a *situated abstraction* (Noss & Hoyles, 1996), that is, 'abstract' mathematical knowledge simultaneously situated in the context of teaching with technology.

Borrowing from Shulman (1986), mathematical knowledge for teaching using technology is assumed not only to be a matter of knowing how – being competent in teaching mathematics using technology - but also of knowing what and why. That is, although much of teachers' knowledge may be tacit, craft knowledge (Ruthven, 2007), at least some of their know-how is underpinned by articulated knowledge that provides for "a rational, reasoned approach to decision-making" (Rowland et al., 2005, p.260) in relation to teaching mathematics using technology. In other words, mathematical knowledge for teaching using technology, as defined in this study, is when know-how or knowledge-in-action is underpinned by and coincides with the teacher's articulated knowledge. This intersection between articulated knowledge and knowledge-in-action is important because it is this type of knowledge that initial or in-service teacher education programmes focus on developing.

Method: Robert as a critical case

Four teachers were selected from a group of English mathematics teachers who took part in a survey of secondary school mathematics teachers' use of ICT (n=183) and who further agreed to be contacted as case study teachers (Bretscher, 2011; 2014). The four case study teachers were chosen along two dimensions of variation likely to be associated with mathematical knowledge for teaching using technology, based on their responses to survey items. Firstly, the case study teachers were chosen to be two of the most student-centred and two of the most teacher-centred in their approach to mathematics teaching in general (not limited to ICT use) of those who volunteered. Secondly, two teachers were chosen to be from schools with a high level of support for ICT and two with a low level of ICT support. In addition, the four case study teachers had described themselves as being confident with ICT. As technology enthusiasts, the case study teachers were likely to display mathematical knowledge for teaching using technology; the variation in case selection aimed to highlight such knowledge – making it more 'visible'.

Each case study teacher was observed teaching one lesson in a computer suite where pupils were given direct access to ICT. These observations provided opportunities to infer the case study teachers' knowledge-in-action in a situation involving the work of teaching mathematics with technology. Post-observation interviews then provided an opportunity to infer the case study teachers' articulated knowledge and hence, triangulated against their knowledge-in-action observed in the lesson, provide evidence indicating mathematical knowledge for teaching using technology.

Robert was selected as a case study teacher because he was one of the most student-centred teachers in the survey sample. In addition, his school appeared to be generally supportive of ICT use compared to the other schools surveyed. He stood out, even amongst the case study teachers, as being a critical case of a teacher likely to display mathematical knowledge for teaching using technology for two main reasons. Firstly, Robert showed a favourable conception of technology, as described in the following section, in relation to mathematics teaching and in line with Bowers and Stephen's (2011) description of viewing "technology as a critical tool for identifying mathematical relationships". Secondly, Robert's lesson appeared to be exceptional: he used GeoGebra software to affect his pupils' learning in an innovative way that would not be easy to achieve without digital technology, in comparison to the other lessons observed where software was used to replicate and enhance paper-and-pencil activities. He had 4-6 years of teaching experience, held a management position within the mathematics department and had completed a Masters in Education degree. Robert was also the most technologically proficient of the four case study teachers: his undergraduate degree was a Bachelor of Engineering in Computing.

Analysis and discussion

Robert's favourable conception of technology use in mathematics teaching

For the first part of his lesson, Robert had created a series of maze activities, embedded in GeoGebra files, designed to take advantage of students' intuitive, tacit understandings of reflection as a means of making these understandings explicit and thus leading towards a more formal understanding of reflection. Using the mouse to direct the movement of a point, coloured in blue, the pupils had to guide the blue point's reflection, shown in red, successfully through a maze (see Figure 1).

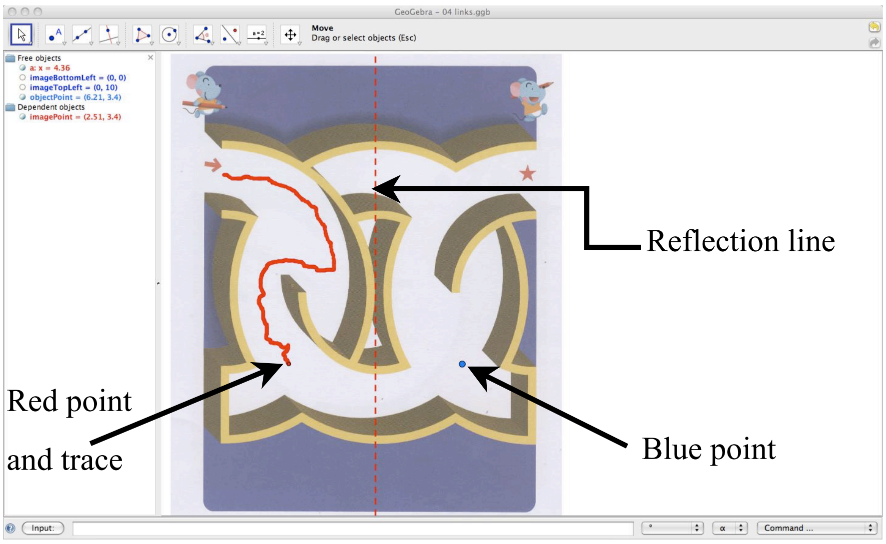

Figure 1: One of Robert's GeoGebra maze activities - by dragging the blue point, guide the reflected red point through the maze

The reflection line was super-imposed on the maze diagram and the path of the red point was traced. Robert hoped that the activity would encourage pupils to predict how the reflected red point would move in relation to movement of the blue point as a means of increasing their chances of completing the maze successfully. By predicting the movement of the red and blue points, he hoped his pupils intuitive understandings of reflection would be made more explicit.

In the post observation interview, Robert explained what inspired him to create the maze activities. He provided a critique of similar GeoGebra activities as lacking an impetus to focus attention on and articulate tacit understandings:

Robert: I had a look on the GeoGebra wiki and most things tended to be 'Here's a mirror line, here's a shape, if you drag this, what's happening?' just kind of ... and say what you see. And I could imagine them sitting there with that and basically just dragging the mouse a bit and seeing it happen and ... and then where does it go from there?

He also described a pedagogic strategy of predict-then-test that he aimed to use in the lesson to make pupils' understandings of mathematical relationships explicit:

Robert: just you know introduce that 'pause' of what do we think is going to happen and then let's test that it's going to happen

and how he intended to formalise these understandings during the lesson by introducing mathematical vocabulary:

Robert: So one of the things I wanted to talk about was that if you're moving that point parallel to the mirror line, the point moves in the same direction, whereas as soon as you're moving it in a direction that's not parallel, the point doesn't move in the same way.

Summarising at the end of the lesson, he did introduce mathematical vocabulary during class discussion, in a similar way to the intention described above, describing the movement of the red and blue points. Thus Robert's design of the maze activities, his use of them in the lesson and his comments about the lesson in the post-observation interview demonstrate the strong emphasis he placed on the use of technology to explore the mathematical relations behind the mathematical phenomenon of reflection, consistent with Bowers and Stephens' (2011) description of a favourable conception of technology.

Robert's mathematical knowledge for teaching using technology

Using the series of maze activities successfully to meet the aims of the lesson depended on transforming students' strategies for completing the mazes into more formal understandings of reflection that could be used as strategies for constructing the image given an object and line of reflection. As indicated above in excerpts from the post-observation interview, Robert recognised his interventions with individual pupils and directing whole class discussion as being critical to effecting this transformation.

The maze activities potentially addressed two complementary strategies for using geometric properties to construct the image given the object and line of reflection: 1) using the local geometry of the object together with the properties of reflection, namely, preservation of length and of direction parallel to the line of reflection and reversal of direction in the axis perpendicular to the line of reflection, to construct the image; and 2) using the geometric property that the line of reflection is the perpendicular bisector of line segments connecting corresponding points on the object and image.

The first strategy was addressed through the maze activities by the necessity of considering how to drag the blue point, i.e. in what direction and how far, to guide the reflected red point through the maze. In particular, the main challenge in completing the maze is derived from the reversal of direction caused by the reflection. Less obvious perhaps is that length is preserved: dragging the blue point causes the red point to move the same distance. The second strategy was addressed in later maze activities by the addition of the line segment connecting the blue and red points as a possible aid to maze completion.

Robert was not satisfied with his interventions during the lesson. In the post-observation interview, he pointed to technical difficulties, his desire to let the students enjoy the maze activities and his rush to move onto the second activity as contributing to the result that he did not spend as much time as intended on discussing the geometric implications of the pupils' maze-solving strategies. Timing was certainly a factor and technical difficulties meant that he was unable to direct a whole class discussion juxtaposing the identical mazes with and without the line segment joining the red and blue points. As a result, Robert was unable to address the second strategy outlined above involving recognition of the line of reflection as the perpendicular bisector of the line segment joining the red and blue points. However, he did have two opportunities during the lesson to elicit the geometric properties of reflection that underpin the first strategy through whole class discussion.

The first opportunity came when Robert brought the class back together after some time engaging with the maze activities. He displayed one of the early maze activities with a vertical line of reflection and asked pupils to give instructions to a pupil-volunteer to direct their movement of the blue point. Robert summarised their responses, drawing attention to the relative direction of movement of the red and blue points i.e. that when the blue point was dragged up or down the red point moved in the same way but that dragging the blue point left or right caused the red point to move in the opposite direction. Whilst drawing their attention to the direction of movement, Robert did not mention that dragging the blue point causes the red point to move the same distance, thus he did not draw his pupils' attention to the geometric property that length is preserved under reflection.

Robert then displayed a maze with a horizontal line of reflection and, employing his predict-then-test strategy, asked the pupils to predict whether the relative direction of movement would be the same or different. The pupils correctly predicted it would change: now, dragging the blue point left or right would result in the red point moving in the same way but dragging the blue point up or down would cause the red point to move in the opposite direction. Contrasting these diagrams made the point that the relative direction of movement of the red and blue points was connected to the orientation of the line of reflection. At this juncture, Robert could have introduced the mathematical terms parallel and perpendicular to specify the nature of the connection between the relative direction of movement and the orientation of the line of reflection, thus generalising to state the effect of reflection on direction. He could also have noted that in both maze diagrams, independent of the orientation of the line of reflection, dragging the blue point causes the red point to move the same distance, hence length is preserved under reflection.

Robert did not introduce the mathematical terms parallel and perpendicular at this point nor did he note the geometric property that length is preserved under reflection. Instead, apparently on impulse, he offered his pupils a new challenge: to find out whether turning the mouse back to front would help them to complete the mazes, presumably by double-reversing the direction of movement. This challenge risked distracting from the aims of the lesson, since turning the mouse back to front involves a rotation of 180 degrees and not a reflection. Later in the post-observation interview, Robert dismissed it as "just a silly question to get a few of them thinking". However, in asking this question, he missed an opportunity to capitalise on his pupils' correct predictions to generalise their maze-solving strategies towards a shared, formal understanding of the geometric properties of reflection. In particular, Robert's challenge highlights the situated nature of mathematical knowledge for teaching using technology in terms of weighing up the pedagogical value of interpreting how the mouse movement relates (or not) to the geometric properties of reflection.

The second opportunity occurred at the end of the lesson. Due to the shutdown of the computer system, the students were unable to begin the second GeoGebra activity Robert had prepared. After spending some time wrestling with the technology, Robert gave up and gathered the pupils to summarise the lesson. In this moment of contingency, Robert was inspired to ask his pupils to imagine the join between two rectangular tables, where they met along their longest edge, was a mirror. One of the pupils sitting at the table was holding a ball: this became the de facto 'blue point'. Robert discussed moving the 'blue point' close to the mirror, through the mirror (which he noted you can't do in reality), and finally parallel to the mirror. He did not have another chance to discuss what happens when the 'blue point' moves perpendicular to the mirror nor to discuss the preservation of length under reflection because, at that point, the bell rang for the next lesson.

Although his second opportunity to elicit the geometric properties of reflection was cut short, in the post-observation interview, when asked what he wished to do had there been more time, Robert did not articulate that he meant to discuss what happened when the blue point moved perpendicular to the line of reflection and to note that distances remained the same under reflection. These missed opportunities, together with the post-observation interview, suggest that Robert had not planned precisely what and how he would use mathematical terminology in his interventions to support his pupils' interpretation of controlling the red and blue points via the mouse, thereby transforming his pupils' maze-solving strategies into more formal understandings of reflection to connect with the aims of the lesson. In addition, when asked what he would have done differently in preparing the lesson, he focused solely on planning to prevent the technical difficulties arising rather than suggesting he could have been more precise in his use of mathematical terminology. Although Robert did not have much time to deliberate over the lesson (as the author has) and it is understandable that the technical difficulties that were so disruptive were uppermost in his mind, this suggests his experience during the lesson did not prompt Robert to recognise the need to plan his interventions more precisely to connect his series of maze activities with the mathematical aims of the lesson. In particular, Robert appeared to lack a frame of reference to help him identify what his mathematical difficulties were in using technology to make his pupils' tacit understandings explicit and, as a result, why his interventions appeared unsatisfactory. However, such a frame of reference can be seen as part of mathematical knowledge for teaching using technology, since in this study such knowledge is assumed not only to be a matter of knowing how – being competent in teaching mathematics using technology - but also of knowing what and why (Shulman, 1986, p.13).

Conclusion

Despite his favourable conception of technology, using the maze activities in practice was not trivial and Robert did not entirely succeed in making explicit the mathematical relationships the pupils were exploring using the GeoGebra software. His difficulties, in supporting his pupils' mathematical interpretation of controlling the red and blue points via the mouse to elicit the properties of reflection, appear at once *mathematical* and yet simultaneously *situated* in the context of teaching using technology. In particular, the strength of Robert's maze activities lay in the real difficulty of controlling the direction of movement of the reflected red point via the mouse. This difficulty focused attention on how the direction of movement changes under reflection, which Robert drew to his pupils' attention through his interventions, albeit without making use of precise mathematical terminology. However, dragging the blue point using the mouse results in the red point moving the same distance unproblematically. Thus the maze activities did not draw attention to preservation of length in the same way, underlining the need for teacher intervention to highlight this property of reflection. The strain placed on his mathematical knowledge for teaching using technology was most evident perhaps when Robert included a challenge relating to rotation, finding out what happens when the mouse is turned back to front, which distracted from his stated lesson aims regarding reflection. This challenge again highlights the situated nature of mathematical knowledge for teaching using technology in terms of weighing up the pedagogical value of interpreting how the mouse movement relates to the geometric properties of reflection.

This suggests that a positive stance towards technology, in terms of global aspects of teacher knowledge (e.g. Bowers & Stephens, 2011; Zbiek et al., 2007), may not be sufficient to ensure a teacher's use of technology enhances mathematical instruction. The missed opportunities to transform pupils' maze-solving strategies into more formal statements of the geometric properties of reflection, using precise mathematical terminology to make connections between the maze activities and the aims of the lesson, suggest that mathematical knowledge for teaching using technology has a significant role to play in successful technology integration. Thus, whilst highlighting the role of teachers' conceptions in technology integration is important, this paper has argued that the significance of mathematical knowledge for teaching using technology should not be overlooked nor underestimated.

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