Optimal Harvest-Use-Store Design for Delay-Constrained Energy Harvesting Wireless Communications

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Abstract

Recent advances in energy harvesting (EH) technology have motivated the adoption of rechargeable mobile devices for communications. In this paper, we consider a point-to-point (P2P) wireless communication system in which an EH transmitter with a *non-ideal* rechargeable battery is required to send a given fixed number of bits to the receiver before they expire according to a preset delay constraint. Due to the possible energy loss in the storage process, the harvest-use-and-store (HUS) architecture is adopted. We characterize the properties of the optimal solutions, for additive white Gaussian channels (AWGNs) and then block-fading channels, that maximize the energy efficiency (i.e., battery residual) subject to a given rate requirement. Interestingly, it is shown that the optimal solution has a water-filling interpretation with double thresholds and that both thresholds are monotonic. Based on this, we investigate the optimal double-threshold based allocation policy and devise an algorithm to achieve the solution. Numerical results are provided to validate the theoretical analysis and to compare the optimal solutions with existing schemes.

Index Terms

Energy harvesting, residual battery level, storage inefficiency, harvest-use-store, delay-constrained.

I. Introduction

Energy harvesting (EH) is recognized as an alternative energy-supplier to battery-restrained communication networks, such as wireless sensor networks, in order to extend the network lifetime by harvesting ambient energy (e.g., solar, vibration, etc.) [1]. As a revolutionary enhancement to battery-limited devices, an EH transmitter can theoretically operate over an unlimited time. With sporadic energy arrival in limited amounts, nevertheless, it is of great importance to optimize the transmission policy using the available information regarding the energy arrival processes to maximize delivery.

Recent efforts on optimizing data transmission with an EH transmitter have drawn great attention [2]–[9]. In particular, a staircase water-filling result was presented by [2] with an information-theoretic approach considering two types of side information to maximize the throughput. Later in [3], a similar directional water-filling algorithm was introduced for the problems of both throughput maximization with a deadline T, and transmission time minimization with a data delivery constraint. A save-then-transmit protocol is proposed in [4] to minimize the outage probability of energy harvesting transmitters by figuring out the optimal time fraction for energy harvesting in a phase. Other communication scenarios adopting EH which include broadcast channel [5], [6], multiple access channel [7], dual-hop networks [8], [9] have also been investigated.

On the other hand, battery imperfections are a key concern of EH. In [10], the influence of constant leakage rate and battery degradation over time were introduced into the battery model. It is suggested that if the total energy in an epoch is low, then energy should be depleted earlier to reduce leakage. For degradation issue, the optimal policy is shortest path within narrowing tunnel. In [11], the authors characterized the degradation process using some probabilistic technique (Markov chains), in which several degradation stages are identified and the battery state will random "walk" from one to the next with some (small) probability in every slot. Later, [12] studied the data maximization problem under finite battery constraints. More recently in [13], we proposed a harvest-use-store (HUS) policy to cope with storage loss in the battery, which schedules a given sequence of harvested energy in order to increase the energy usage efficiency and the throughput.

All these results including our work in [13] largely fall into two categories: 1) Maximizing the amount of information sent from the transmitter, and 2) minimizing the completion time for delivering a certain number of bits. However, under random energy arrivals, both approaches cannot guarantee that if a packet must be transmitted by the deadline, the expected energy consumed is minimized (i.e., the transmitter will not run out of energy before the next quality-of-service (QoS) request). This will be modeled delay-constrained case, such as VoIP, where packets arrive regularly and each must be received within a short delay window. In such a setting perhaps the most important design objective is to minimize the resources (in our case, energy) needed to meet the delay requirements. Therefore, it is of interest to maximize the energy efficiency (i.e., minimize the transmission energy or maximize the battery residual) subject to a specified amount of information successfully sent in order to ensure that the transmitter has enough energy for the next QoS request.

The maximization problem of energy efficiency using the instantaneous channel states information (CSI) at the transmitter was considered in [14], [15]. Also, Chong and Jorswieck [16] studied such problem when the transmit power is adapted and updated at each time slot based on the CSI over a period of time. To guarantee a satisfactory average throughput, [17] derived a closed-form power allocation solution for maximizing the energy efficiency for a single-carrier point-to-point (P2P) system. For delay-sensitive applications, however, there could be stringent end-to-end (e2e) delay requirement. In [18], strict delay constraints were considered and the optimal scheduling policy was presented assuming a continuous Markov process. In addition, [19] devised the optimal transmission policy with the constraint of a fixed given amount of energy, while in [20], the results were generalized to cope with multiple deadlines, instead of a single deadline at the end of the time horizon.

However, in EH systems, the intermittent nature of energy captured from a natural energy source leads to highly random energy availability at the transmitter. When taking into account this random nature of EH, the energy efficiency maximization problem will become much more complicated. Also, maximizing energy efficiency means to keep the final energy storage maximized in order to achieve reliable and efficient energy scheduling for the next scheduling period. Motivated by this, we investigate the problem of minimizing the energy required for transmitting a given amount of information over a P2P link with a finite delay constraint, using only the harvested energy under random energy arrivals. In particular, our focus is on the storage lossy EH system and therefore, we adopt the HUS strategy [13] which puts a higher priority to usage than storage, contrary to the harvest-store-use (HSU) strategy that suffers from severe energy loss due to lossy storage. During each time, the transmitter determines how many bits to transmit based on the current channel quality, energy harvested quality and the number of bits yet to be served. The scheduler must balance the desire to be opportunistic, i.e., wait to serve many of the bits when the channel and the harvested energy is in a good state, or use the energy it right now to avoid energy loss from storage.

This paper aims to answer the fundamental question which has not been answered before including [13]: How should the transmission be scheduled so that the residual battery level at the end is maximized instead of using up all the energy to maximize the throughput? Our contributions are as follows:

- Given full information about the energy arrivals, in the case of additive white Gaussian noise (AWGN) channels, we prove that the optimal transmission is a "non-idling" transmission, which means that the transmission lasts all the blocks without any break.
- We find that there will be a region between a given data constraint and the residual energy at the end of the transmission. Thus, we study a prescribed data threshold and battery residual energy (PDTBRE) region to characterize all the possible prescribed data threshold (in bits for wireless information transfer (WIT)) and battery residual energy (in joules for residual energy) pairs. Two boundary points of this PDTBRE region can be regarded as a special form of [13] and non-data transmission.
- In order to obtain the remaining boundary point of the region (i.e., the corresponding optimal power allocation solution of the maximum energy efficiency), a double-threshold structure is characterized, which illustrates that the power allocated is related to the battery mode (i.e., charging, discharging and neutral) and is monotonic.
- Apart from the above-mentioned properties, for the block-fading case, it is found that the optimal solution has a water-filling interpretation with double thresholds. Rather than having a single water level, there are multiple water levels that are nondecreasing over time.
- We notice that the optimal policy has a relationship with the one in [13] and provide an algorithm to achieve this.

The rest of the paper is organized as follows. In Section II, we introduce the system model and formulate the problem. In Section III, we analyze the optimal policy. Section IV then gives an optimal offline policy by investigating the properties of the solution, and then proposing a double-threshold based optimal allocation policy. Extension to block-fading channels is presented in Section V. Numerical results are provided in Section VI and we conclude the paper in Section VII.

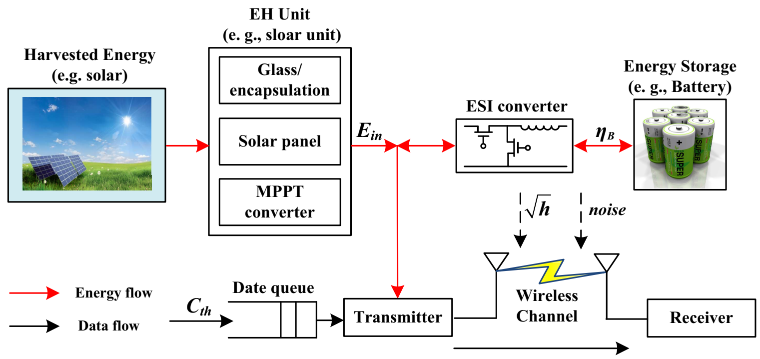

Fig. 1. A HUS wireless communication diagram of a transmitter powered by a energy harvester. Energy is replenished by an energy harvester but is drawn for transmission.

II. System Model

We consider a P2P single-antenna wireless communication system, which is fit with an EH transmitter equipped with an imperfect rechargeable battery as shown in Fig. 1. A packet of C_{th} bits which arrives in the data queue at the start of the time horizon must be transmitted within a strict delay constraint T through a fading channel with AWGN. We assume that no other packet is scheduled during this period of time T and within T, there are N energy arrivals. It is further considered that this energy comes from the ambient environment and is harvested by the energy unit. The transmitter operates in the HUS mode realized by the energy storage interface (ESI) converter which is a bidirectional power electronic circuit that stores excess energy in the battery and extracts the energy from the battery to the transmitter when needed. Perfect CSI is assumed available at the transmitter side.

In this system, our focus is on a finite horizon of N-block transmission which starts from block 1 and ends at block T, as illustrated in Fig. 2. We denote the energy arriving at the beginning of block n as E_n, and the time interval between two consecutive energy arrivals as l_n. Note that the transmission time in block n may not be equal to l_n. Therefore, we define $t_n \leq l_n$ to distinguish the transmission time from the energy arrival time interval. Similar to [21], it is assumed that both the harvested energy increments and their arrival times can be exactly known at the transmitter prior to transmission.

In this paper, the channel is modeled as a block-constant process, or widely known as block-fading channel. In other words, the channel state remains constant over each block but randomly

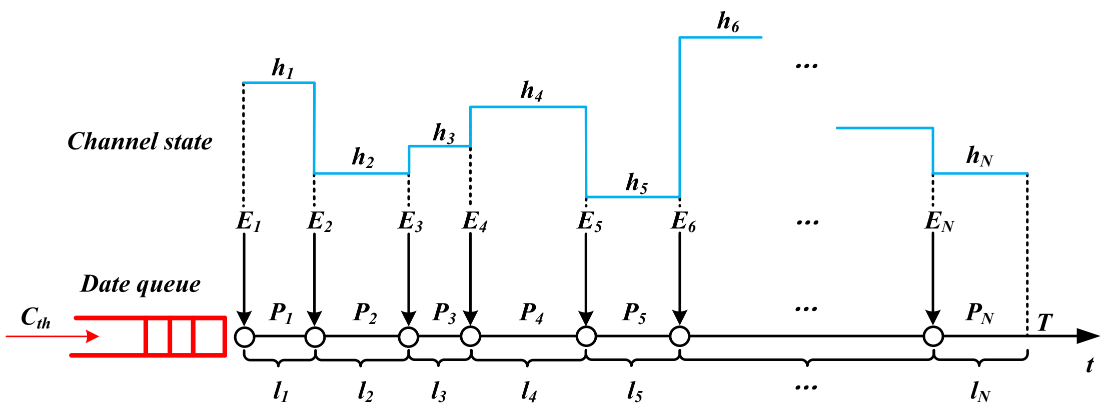

Fig. 2. The transmission model with random energy arrivals. Energies arrive at the beginning of the block which are denoted as ○. The transmission starts from block 1.

changes from one block to another. Based on this, the received signal in block n, y_n, can be written as

$$y_n = \sqrt{h_n} x_n + z_n, \tag{1}$$

where $\sqrt{h_n}$ denotes the channel gain for block n, x_n is the input signal for block n, which is considered to be Gaussian distributed with zero-mean and variance of p_n (i.e., transmission power in block n), and z_n is the zero-mean unit-variance AWGN for block n.

The HUS strategy is adopted and we characterize it by the following three battery modes:

(a) **Charging:** When $E_n > p_n t_n$, the transmitter uses $p_n t_n$ amount of energy from the energy unit, and the battery will store the excess energy $D_n \triangleq E_n - p_n t_n$.

(b) **Discharging:** When $E_n < p_n t_n$, the transmitter uses all the current harvested energy harvested, and the battery will replenish the difference $-D_n = p_n t_n - E_n$.

(c) **Neutral:** When $E_n = p_n t_n$, in this case, the transmitter uses up all the harvested energy for transmission without any operation to the battery.

We compare the energy usage efficiency between the HUS and HSU harvesting architectures in Table I. In the table, we can see that with a storage efficiency $0 < \eta_B < 1$, a portion of energy stored in the battery will be lost in all the cases for HSU. Instead, using HUS, only a fraction of the harvested energy suffers from this loss, that is, when it is in the state of charging. From this comparison, we can easily find that HUS is more efficient than HSU all the time. To characterize the HUS harvesting architecture, we define $[D_i]^+ \triangleq \max(0, D_i)$ to describe the

Harvesting Architecture	Waste Energy		
	Charging	Discharging	Neutral
HUS	$(1-\eta_B)(E_i - p_i l_i)$	0	0
HSU	$(1-\eta_B)E_i$	$(1-\eta_B)E_i$	$(1-\eta_B)E_i$

TABLE I

COMPARISON OF ENERGY USAGE EFFICIENCY

battery level at the end of block n (i.e., the residual battery level) as in [13], which is denoted by

$$B_n = \eta_B \sum_{i=1}^{n} [D_i]^+ - \sum_{i=1}^{n} [-D_i]^+, \tag{2}$$

where $\eta_B \sum_{i=1}^{n} [D_i]^+$ and $\sum_{i=1}^{n} [-D_i]^+$ represent, respectively, the energy stored into and taken out from the battery at the end of block n. For simplicity and ease of analysis, we have the following assumptions.

1) We only consider the energy consumption for information transmission, ignoring other types of energy consumption.
2) The battery capacity is always large enough.
3) The initial energy in the battery is zero, i.e., $B_0 = 0$.

Our model is applicable for a practically deterministic traffic (e.g., in VoIP, the next packet generally arrives before the previous ones expire), which allows us to focus on the central issue of meeting deadlines based upon energy and CSI. The purpose of the scheduler is to determine the energy to be served during each block such that the energy consumption is minimized and the bits are served by the deadline T.

A. Problem Statement

The maximum reliable transmission rate in block n is then given by the mutual information $\mathcal{I}(p_n)$ in bits per symbol. In general, we assume that $\mathcal{I}(p_n)$ is concave and increasing in p_n. Consider a block fading channel with average signal power constraint p_n and noise power 1. As is well known in [22], the information-theoretic optimal channel coding, which employs randomly generated codes, achieves the channel rate

$$\mathcal{I}(p_n) = \frac{1}{2}\log(1+\mathrm{SNR}), \tag{3}$$

where the signal-to-noise ratio (SNR) is given by $\mathrm{SNR} = h_n p_n$. With the assumptions above, in block n, $t_n \log(1+\mathrm{SNR})/2$ bits of data will be sent to the destination during that interval t_n at the cost of $p_n t_n$, which describes the throughput in block n as

$$C_n = \frac{t_n}{2}\log(1+h_n p_n). \tag{4}$$

Our aim is to schedule the given EH sequence to maximize the battery level at the end subject to the precondition that the sum data transmission is greater than a prescribed data threshold C_{th}, which will provide QoS guarantee for next time transmission. Hence, our residual battery level maximization problem over N transmission blocks can be expressed as

$$(\mathbf{P1}) \begin{cases} \max\limits_{\{p_n,t_n\}} \quad B_N \\ \text{s.t.} \quad B_n = \eta_B \sum\limits_{i=1}^{n} [D_i]^+ - \sum\limits_{i=1}^{n} [-D_i]^+ \geq 0, \forall n \in \{1,2,\ldots,N\}, \\ \qquad \sum\limits_{n=1}^{N} \frac{t_n}{2}\log(1+h_n p_n) \geq C_{th}, \end{cases} \tag{5}$$

where the battery level must not be negative at the end of each block to supply sufficient energy for data transmission (i.e., the energy causality constraints to ensure that the energy harvested in future blocks cannot be used in the current one). Also, combined with the non-linearity and non-differentiability of the constraint conditions $[\cdot]^+$, $(\mathbf{P1})$ is difficult to solve. The goal of this paper is different from our previous paper tackling the problem that maximizes the throughput which will use up all the energy. For example, in sensor networks, sometimes we only need fixed data (e.g., temperature, humidity) instead of maximum throughput whose extra parts are useless and energy wasting. As such, this paper formulates the problem in order to save energy while transmitting reliable data. To start with our problem, we consider the optimization problem for two channel environments: AWGN channel and block-fading channel. We will derive the optimal power allocation policies for these two channel models.

III. Optimal Policy for AWGN Channel

Here, we first use an example to explain the complexity of our problem. Then we provide a theorem and a region definition to make our objective clearer and easier to analyze. For an

AWGN channel, we can assume, without loss of generality, the channel gain to be unity so that $h_1 = \cdots = h_N = 1$. Inserting this into $(\mathbf{P1})$, we obtain

$$(\mathbf{P2}) \begin{cases} \max\limits_{\{p_n,t_n\}} & B_N \\ \text{s.t.} & B_n = \eta_B \sum\limits_{i=1}^{n} [D_i]^+ - \sum\limits_{i=1}^{n} [-D_i]^+ \geq 0, \forall n \in \{1,2,\ldots,N\}, \\ & \sum\limits_{n=1}^{N} \frac{t_n}{2} \log(1+p_n) \geq C_{th}. \end{cases} \tag{6}$$

Note that $(\mathbf{P2})$ is a convex optimization problem and thus has a unique optimal solution. At the optimum, equality on the rate constraint will hold, since otherwise we can always decrease the data rate further by decreasing p_n without violating any other constraints. In the following, we will use a simple two-block problem to illustrate the difficulties involved in solving the convex optimization problem, dealing with the nonlinear function $[D_i]^+$ in three possible cases, i.e., $D_i \overset{>}{\underset{=}{\leq}} 0$. If we write them all out, then we will find that these problems are similar in structure. Even though each problem is differentiable and convex, when there are N blocks in the optimization, it will result in a total of 3^N possible cases to handle, and apparently the complexity for solving it increases exponentially, which makes it intractable for large N.

In order to solve this problem, the first step is to determine how to schedule the transmission time. After briefly introducing the basic step, we then find that to be energy efficient, the transmission policy must have the following property.

Theorem 1. *The optimal transmission strategy for the AWGN channel is a "non-idling" transmission, i.e.,* $t_n = l_n,\ \forall n \in \{1,2,\ldots,N\}$ *and* $\sum_n t_n = T$.

Proof: Recall that the achievable rate for block n can be found from (4). To determine the energy consumption ε_n in block n, we consider that $\mathcal{C}$ bits will be sent in this block. Substituting this into (4), we obtain

$$\varepsilon_n = t_n p_n = t_n \left(2^{\frac{2\mathcal{C}}{t_n}} - 1\right). \tag{7}$$

Differentiating ε_n with respect to t_n, we have

$$\frac{d\varepsilon}{dt_n} = 2^{\frac{2\mathcal{C}}{t_n}} - 1 + t_n \left[2^{\frac{2\mathcal{C}}{t_n}} \ln 2 \left(-\frac{2\mathcal{C}}{t_n^2}\right)\right]. \tag{8}$$

Now, define

$$f(x) \triangleq 2^x - 1 + \frac{2\mathcal{C}}{x}\left[2^x \ln 2\left(-\frac{x}{2\mathcal{C}}\right)\right], \tag{9}$$

where $x = 2\mathcal{C}/t_n$. If $f(x) < 0$, ε is monotonically decreasing. As such, the problem is simplified to prove that $f'(x) < 0$ and $f(0) \leq 0$. It is easy to see that it is monotonically decreasing and convex in t_n. The assertion that ε_n decreases with t_n implies that it would be suboptimal to have $t_n < l_n \; \forall n \in \{1, 2, \ldots N\}$ since we could simply increase the transmission times of one or more blocks and reduce ε_n accordingly. Hence, we only consider "non-idling" transmission schedules where $t_n = l_n, \; \forall n \in \{1, 2, \ldots N\}$ and $\sum_n t_n = T$, in order to store most energy in the battery, which completes the proof. ■

Also, the threshold C_{th} should not be too large so that the problem is feasible. If C_{th} is too large, then the harvesting energy will not be enough to fulfil the required transmission. Therefore, there should be a region between a given prescribed data constraint C_{th} and the residual energy at the end of transmission B_N. It motivates us to investigate the PDTBRE region to characterize all the possible data threshold (in bits for WIT) and battery residual energy (in joules for residual energy) pairs under the battery causality constraints. Mathematically, i.e.,

$$R_{PDTBRE} \triangleq \left\{ (C_{th}, B_N) : \begin{array}{c} C_{th} \leq \sum_{n=1}^{N} \frac{l_n}{2} \log(1 + p_n), \\ B_N \leq \sum_{i=1}^{N} \eta_B [E_i - p_i l_i]^+ - \sum_{i=1}^{N} [p_i l_i - E_i]^+, \\ B_n \geq 0, \;\text{for}\; n \in \{1, 2, \ldots N\} \end{array} \right\}. \tag{10}$$

In Fig. 3, an example of the PDTBRE region is provided for a P2P EH system operating with the HUS mode (see Section IV for the algorithm to calculate the boundary of the region). The amount of energy and the corresponding durations are specified by the harvested energy sequence

$$\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2] \tag{11}$$

and the durations sequence

$$\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]. \tag{12}$$

The tradeoff between the data threshold and the optimal residual energy is characterized by the boundary of the PDTBRE region. It is important to characterize all the boundary PDTBRE pairs

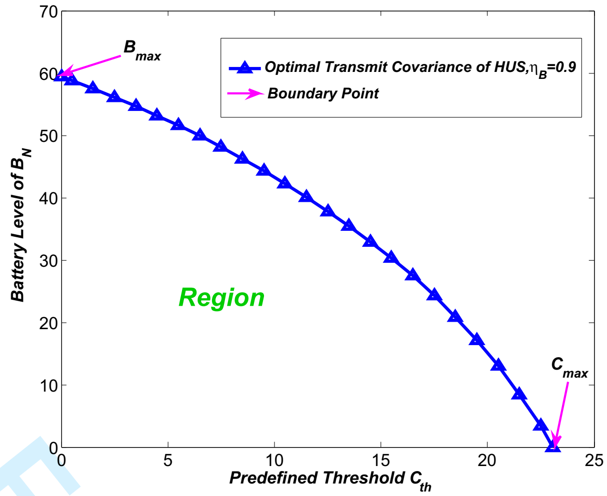

Fig. 3. The tradeoff of the prescribed data threshold and the battery residual energy for a P2P EH system with HUS. Block length $N = 15$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$, and the storage efficiency $\eta_B = 0.9$.

of R_{PDTBRE}, which yields the intervals:

$$0 \leq C_{th} \leq C_{\max}, \qquad 0 \leq B_N \leq B_{\max}. \tag{13}$$

Note that it is easy to identify two boundary points of this PDTBRE region denoted by $(0, B_{\max})$ or $(C_{\max}, 0)$, respectively. For the former boundary point, there is no transmission request during the finite blocks, which corresponds to the case that all the harvesting energy is stored in the battery. On the other hand, for the latter boundary point, the maximum WIT performance can be solved by the optimal HUS strategy in [13], which shows that for optimality, the battery level at the end should be zero. Also, we see that the optimal power of $C_{th} < C_{max}$ is less than or equal to the optimal solution of $C_{th} = C_{max}$, which gives insight to obtain our **Algorithm 1** in the next section for our optimal solution. The optimization now becomes to characterize the part of the boundary of the PDTBRE region remained over the intervals (13).

IV. Solvability and Properties for AWGN Channel

Based on the above analysis, we proceed to solve the problem $(\mathbf{P2})$. The convex optimization problem can be solved by the Lagrangian technique. As such, we denote the Lagrangian function for any $\lambda_n \geq 0$ $(\forall n \in \{1, \ldots N\})$, $\theta \geq 0$ by

$$\mathcal{L} = \eta_B \sum_{i=1}^{N} [D_i]^+ - \sum_{i=1}^{N} [-D_i]^+ + \sum_{n=1}^{N} \lambda_n \left(\eta_B \sum_{i=1}^{n} [D_i]^+ - \sum_{i=1}^{n} [-D_i]^+ \right) \tag{14}$$

$$+ \theta \left(\sum_{n=1}^{N} \frac{l_n}{2} \log(1+p_n) - C_{th} \right), \tag{15}$$

whose associated complimentary slackness conditions are

$$\lambda_n \left(\eta_B \sum_{i=1}^{n} [D_i]^+ - \sum_{i=1}^{n} [-D_i]^+ \right) = 0; \quad \forall n \in \{1, \ldots N\}, \tag{16}$$

$$\theta \left(\sum_{n=1}^{N} \frac{l_n}{2} \log(1+p_n) - C_{th} \right) = 0, \tag{17}$$

where λ_n and θ are the nonnegative Lagrangian multipliers. Note that the equality constraint of (18) is obvious since $\sum_{n=1}^{N} \frac{l_n}{2} \log(1+p_n) - C_{th}$ must be zero; otherwise, we can always increase the residual energy by decreasing p_i without violating any other constraints in (17). By differentiating $\mathcal{L}$ with respect to p_k and carrying on some mathematical manipulation similar to [13], we obtain the optimal power allocation (18).

$$p_k = \theta \left[\ln 2 \left(\sum_{n=k}^{N} \lambda_n + 1 \right) \alpha_k \right]^{-1} - 1, \quad k = 1, 2, \ldots, N \tag{18}$$

where $\alpha_k = (\eta_B + 1) + (\eta_B - 1)\,\mathrm{sgn}\,(E_k - l_k p_k)$. To tackle the signum function in α, we will first study the properties of the optimum solution. In order to do this, a key definition of our paper is first described as follows.

Definition 1. *A block is called a valley block or simply a* ***valley****, when its battery level at the end of the block is zero. On the other hand, the blocks between any two closest valleys constitute a* ***hill segment****.*

A hill segment starting from block j and ending at $m > j$, which means that the battery levels $B_{j-1} = 0$, $B_m = 0$ and the battery levels of all the blocks between them are nonzero; namely, $B_k > 0, \forall k \in [j, \ldots, m-1]$, is denoted as $\mathrm{HS}(j, m)$. Specifically, the last hill segment begins from the last valley block to the last block, although the battery level of the last block may not be zero for our optimization. To continue, we first state the properties of the solution, and then give the main result of this paper based on the properties.

Property 1. *Within a hill segment,* $\mathrm{HS}(j, m)$*, all the energy-charging blocks have the same power allocation,* P^C*, whereas the energy-discharging blocks have the similar property, which* *has a lower power allocation, P^D, where*

$$P^C = \theta\Big[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\eta_B\Big]^{-1} - 1, \quad P^D = \theta\Big[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\Big]^{-1} - 1 \tag{19}$$

In particular, all the energy-neutral blocks have the power allocation equal to the average harvesting power E_k/l_k, for $k \in [j, m]$. Here, we always have $P^C \geq E_k/l_k \geq P^D$.

Proof: Suppose that there is a hill segment starting from block j and ending at $m > j$, $\mathrm{HS}(j, m)$. To prove this property, we test the signum function for three mutually exclusive cases.

Case 1: Charging only with $E_k - l_k p_k$ at block k

Thus, we know $E_k > l_k p_k$, and based on (18), the optimal power is given by

$$p_k = \theta\Big[2\ln 2\left(\sum\nolimits_{n=k}^{N}\lambda_n + 1\right)\eta_B\Big]^{-1} - 1. \tag{20}$$

Since $B_k > 0$, we assert $\lambda_k = 0\ \forall k \in [j, m-1]$, in accordance with the slackness conditions in (16). Thus, we can obtain that the power will remain constant (i.e., p_k will always be equal to P^C), unless the battery is depleted. Therefore, we have

$$p_k = \theta\Big[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\eta_B\Big]^{-1} - 1 = P^C. \tag{21}$$

Specifically, the battery is charging with E_k, that is, $p_k = 0$. This will not happen because the optimal transmission strategy is "non-idling" which means that the power p_k must be greater than 0 to ensure the optimality.

Case 2: Discharging only at block k

In this case, we have $E_k < l_k p_k$, and based on (18), the optimal power is given by

$$p_k = \theta\Big[2\ln 2\left(\sum\nolimits_{n=k}^{N}\lambda_n + 1\right)\Big]^{-1} - 1. \tag{22}$$

Since $B_k > 0$, we assert $\lambda_k = 0\ \forall k \in [j, m-1]$, in accordance with the slackness conditions in (16). Also, the power will remain constant (i.e., p_k will always equal to P^D) with a lower level comparing to Case 1, unless the battery is depleted. Thus,

$$p_k = \theta\Big[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\Big]^{-1} - 1 = P^D. \tag{23}$$

To guarantee block m is a valley (i.e., $B_m = 0$), block m must be discharging with the amount B_{m-1}, which means $\lambda_m \neq 0$. As a result, we have

$$p_m = \theta\Big[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\Big]^{-1} - 1 = P^D. \tag{24}$$

Case 3: No charging or discharging (Neutral) at block k

This is the case where the node forwards all harvester power to the transmitter. Thus, $p_k = E_k/l_k$. In this case, substituting $\mathrm{sgn}(0) = 0$ in (18) gives

$$p_k = \theta\Big[\ln 2\left(\sum\nolimits_{n=k}^{N} \lambda_n + 1\right)(\eta_B + 1)\Big]^{-1} - 1 = E_k/l_k. \quad (25)$$

Since $0 \le \eta_B \le 1$, we can easily see that $P^D \le E_k/l_k \le P^C$, implying that the transmission power p_k in neutral block is restricted to be within the interval $\left[P^D, P^C\right]$. Also, the power will be equal to the average harvesting power E_k/l_k. ■

This property is a result of the fact that the mutual information is a concave function, suggesting that the energy should be allocated to ensure equal SNR over all the blocks for a maximum data rate. That is, as seen in Jensen's inequality, denoting $\ell = l_1 + \cdots + l_N$, we have

$$\frac{1}{2}\sum_{i=1}^{N} l_i \log_1(1 + p_i) \le \frac{\ell}{2}\log\left(1 + \frac{l_1 p_1 + \cdots + l_N p_N}{\ell}\right). \quad (26)$$

The equality condition is attained when $p_1 = \cdots = p_N$. In other words, the best way to obtain the optimal strategy is to allocate equal power to each block. Moreover, due to the presence of storage loss, the equal power is "split" into (i.e., replaced by) two thresholds for the optimal strategy. Although this property is similar to the one in [13], it should be emphasized that they are not the same. First, the expressions are different because there is a new numerator θ and a new term 1 in the denominator. Second, C_{th} can influence the value of θ, and in turn change the values of P_C and P_D. Last, we can see that $\lambda_N = 0$ unless $C_{th} = C_{max}$, since B_N is not zero. The WIT performance constraint demonstrates Property 1 described before. Next we show what the energy causality constraints reveal.

Property 2. *The optimal power in both the charging and discharging cases is monotonically non-decreasing from one hill segment to the next.*

Proof: Assuming that there are M valley blocks, denoted as $V_1, V_2, \ldots, V_M$, and then respectively denoting $P_{V_i}^C$ and$P_{V_i}^D$ as the optimal power of charging and discharging blocks within the hill segment $\mathrm{HS}\left(V_{i-1}+1, V_i\right)$, we have

$$P^C = \theta\Big[2\ln 2\left(\sum\nolimits_{n=V_i}^{N} \lambda_n + 1\right)\eta_B\Big]^{-1} - 1, \quad P^D = \theta\Big[2\ln 2\left(\sum\nolimits_{n=V_i}^{N} \lambda_n + 1\right)\Big]^{-1} - 1. \quad (27)$$

Since $\lambda_n \ge 0$, $V_i > V_{i-1}$, and thus, P^C and P^D are non-decreasing from one hill segment to the next. ■

Now, we give an intuition of the properties to shed some light on how the EH constraints lead to a different optimal power allocation. Property 1 illustrates that if all the harvested energy is already available at the beginning, i.e., sum-power constraint, then a uniform power allocation is optimal for the AWGN channel as shown above that the mutual information is a concave function in the form of logarithm. Thus, the energy will be transferred from the current block to future ones to have optimal benefit. However, this policy is modified by the causality constraints as shown in (5), which gives rise to Property 2 of the optimal solution.

Apparently, based on the properties, a threshold structure for the optimal solution can be proposed as below.

Theorem 2. *Within a hill segment (e.g., $\mathrm{HS}(j,m)$), the optimal transmission strategy has a double threshold structure, which can be described as*

$$p_k^* = \begin{cases} P^C, & \text{if } E_k \geq l_k P^C, \\ P^D, & \text{if } E_k \leq l_k P^D, \\ E_k/l_k, & \text{otherwise}, \end{cases} \tag{28}$$

where it is noted that

$$\eta_B(P^C + 1) = P^D + 1. \tag{29}$$

This indicates that during the hill segment $\mathrm{HS}(j,m)$, block $k \in [j, \ldots, m]$, where the power harvested is greater than the power consumption $E_k/l_k > P^C$ (i.e., charging), should be allocated with power P^C. Similarly, block $k \in [j, \ldots, m]$, where the power harvested is less than the power consumption $E_k/l_k < P^D$ (i.e., discharging), should be allocated with power P^D. Specifically, block $k \in [j, \ldots, m]$, where the harvested power is between P_C and P_D, $P^D < E_k/l_k < P^C$ (i.e., neutral), should be allocated with power E_k/l_k.

Now, the main problem is simplified to find the thresholds P_D and P_C in each hill segment. Knowing that it is non-decreasing and constant, and that the optimal power allocation when $C_{th} < C_{max}$ is less than or equal to the power allocation when $C_{th} = C_{max}$, we propose **Algorithm 1** to solve our problem. We search for the optimal p_n in a sequential way. In one loop, we first compute the maximum P_D and P_C within the current hill segment resulting in the maximum throughput using dynamic programming in [13], and then check if these two

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Algorithm 1 Proposed offline power allocation algorithm in HUS mode for EH wireless systems

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Initialization:

Block size N, using algorithm 1 in [13] to calculate C_{max}, $B_0 = 0$, $k = 1$, $m = 1$;

Iteration:

```
1: Begin from block k, find the thresholds P_D and P_C that deplete the battery at some block
   j by a one-dimensional search or by dynamic programming in [13];
2: n = k;
3: while n ≤ N do
4:    Compute p_n using equation (28);
5:    n ← n + 1;
6: end while
7: if Σ_{n=k}^{N} (l_n/2) log(1 + p_n) = C_th then
8:    Step out of the iteration;
9: else if Σ_{n=k}^{N} (l_n/2) log(1 + p_n) < C_th then
10:   m ← k;
11:   C_temp = 0;
12:   while m ≤ j do
13:      C_temp ← C_temp + (l_m/2) log(1 + p_m);
14:      m ← m + 1;
15:   end while
16:   C_th ← C_th − C_temp
17:   k ← j;
18:   Go to Step 1;
19: else
20:   Find the new thresholds P_D and P_C that satisfy the constraint Σ_{n=1}^{N} (l_n/2) log(1 + p_n) =
      C_th by a one-dimensional search, where p_n, n ∈ {k, k+1, ..., N} is calculated by equation
      (28);
21:   Step out of the iteration;
22: end if
```

Output:

The power allocation p_n for each block and the maximum residual battery level.

---

thresholds give the optimal solution. If the bits sent using the maximum threshold pair is greater than the prescribed data one, this means that the optimal threshold pair should be less than the maximum threshold pair, and we can use a one-dimensional search to find the optimal pair that completes the residual bits. If the bits sent are less than the prescribed data one, we record the data sent (i.e., C_{temp}) using the maximum threshold pair in the current hill segment and go to next loop with the new prescribed data (i.e., $C_{th} \leftarrow C_{th} - C_{temp}$). Specifically, if the bits sent are just equal to the prescribed data one, this loop is the last one and then the algorithm will end.

An example of a 15-block transmission that illustrates Theorem 2 is depicted in Fig. 4, where we consider an EH system with HUS mode for which, the storage efficiency is $\eta_B = 0.9$. The

Hill Segment	Charging Blocks	Power Value	Discharging Blocks	Power Value	Neutral Blocks	Power Value
HS(1,5)	1	10/9	4, 5	0.9	2, 3	1
HS(6,12)	6, 9	3.10	7, 8, 11, 12	2.69	10	3
HS(13,15)	13, 14	3.38	15	2.94	-	-

TABLE II

DETAILS OF POWER ALLOCATION FOR EACH HILL SEGMENT

harvesting energy amounts and arrivals are same as the case in Fig. 3. Using the optimal policy in [13], we obtain the maximum throughput C_{max} which is about 23.06. Thus, any prescribed data threshold should be within $(0, C_{max})$. Thus, we set $C_{th} = 22.5$. As shown in the figure, the power allocation of charging, discharging and neutral blocks are denoted by the blue, green and yellow bars respectively, while the corresponding battery level is illustrated by the red-star dotted line, the slope of which indicates the battery state:

$$\textbf{State} = \begin{cases} \text{Charging}, & \text{slope} > 0, \\ \text{Discharging}, & \text{slope} < 0, \\ \text{Neutral}, & \text{slope} = 0. \end{cases} \tag{30}$$

Note that from the figure, $B_0 = 0$ and $B_5 = 0$, indicating that blocks $1 \sim 5$ constitute the first hill segment. During this hill segment, block 1 with a positive slope corresponds to a charging process, having the allocated power equal to $10/9$. Blocks 4 and 5 on the other hand possess a negative slope representing a discharging process, with the same allocated power 0.9. The battery levels of blocks 2 and 3 remain constant in the neutral process, with 1 unit of power, which shows that the neutral power is between charing and discharging one. Similar observations can be seen in the remaining hill segments $\text{HS}\,(6, 12)$ and $\text{HS}\,(13, 15)$, as shown in TABLE II. We also observe the phenomenon that the optimal power belonging to different battery modes is non-decreasing. Importantly, the battery level at the last is not zero, i.e., the maximum residual energy we want is not zero and is now equal to 3.44.

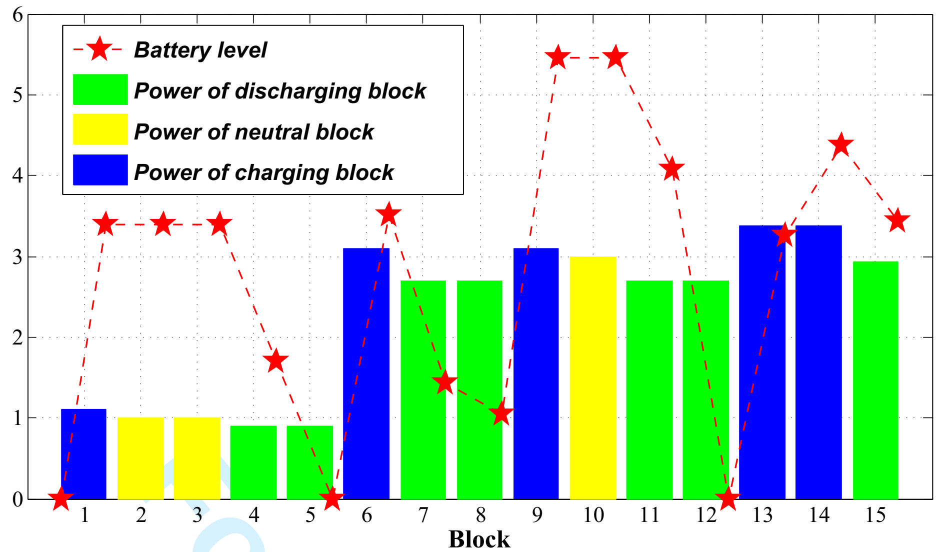

Fig. 4. Structure of optimal power allocation and battery level for each block. The storage efficiency $\eta_B = 0.9$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$.

V. Optimal Offline Policy for Block Fading Channel

In this section, we extend the results to the block fading channels. The fading gain is modeled as a block-constant process, with the CSI, $h_n, \forall n \in \{1, 2, \ldots N\}$ perfectly known to the transmitter side. We tackle this problem as in the AWGN channel, i.e., the optimization of $(\mathbf{P1})$. The difference is however that for deep fading, the optimal power may be zero to avoid data rate loss during blocks with poor channel conditions. Similar to the AWGN case, we can argue that the objective function is concave with respect to the power sequence and that the constraint set is convex. Therefore, the problem has a unique maximizer. Like Theorem 1 in Section III, here, under the case of block fading channels, we will have the same property of "non-idling" transmission.

Theorem 3. *The optimal transmission strategy for block fading channel is a "non-idling" transmission, i.e.,* $t_n = l_n, \ \forall n \in \{1, 2, \ldots, N\}$ *and* $\sum_n t_n = T$.

Proof: Similar to the proof of Theorem 1. ■

To solve the similar PDTBRE region problem, we apply the classical water-filling technique [22] to write the Lagrange function, and take the Karush-Kuhn-Tucker (KKT) conditions for the optimality of the power allocation as

$$\frac{\partial \mathcal{L}}{\partial p_k} \begin{cases} = 0, & \text{if } p_k > 0, \\ \leq 0, & \text{if } p_k = 0, \end{cases} \tag{31}$$

which guarantees the constraint $p_k \geq 0$ is satisfied. Conducting algebraic manipulation similar to Section IV, we obtain the optimal power sequence p_k^* in terms of the Lagrange multipliers, as shown by (32),

$$p_k = \left[\theta \left[\ln 2 \left(\sum_{n=k}^{N} \lambda_n + 1\right) \alpha_k\right]^{-1} - {h_k}^{-1}\right]^+, \quad k = 1, 2, \ldots, N \tag{32}$$

Identifying the three cases for which the signum function is explicitly expressible, yields similar properties for the water level to that in Section IV.

Property 3. *Within a hill segment,* HS(j, m)*, all the energy-charging blocks have the same water level, equal to* W^C*, whereas the energy-discharging blocks have the similar property, which has a lower water level* W^D*, where*

$$W^C = \theta\Big[2 \ln 2 \left(\sum_{n=m}^{N} \lambda_n + 1\right)\eta_B\Big]^{-1}, \quad W^D = \theta\Big[2 \ln 2 \left(\sum_{n=m}^{N} \lambda_n + 1\right)\Big]^{-1}. \tag{33}$$

In particular, all the energy-neutral blocks have the water level equal to $E_k/l_k + {h_k}^{-1}$*, for* $k \in [j, m]$*. Here, we always have* $W^C \geq E_k/l_k + {h_k}^{-1} \geq W^D$*.*

Proof: Similar to Property 1. ■

Property 4. *The optimal water level of charging and discharging cases is monotonically non-decreasing respectively from one hill segment to the next.*

Proof: Similar to Property 2. ■

From the above properties, we find that the water levels possess the same properties as the optimal power policy for the AWGN channel. It turns out that the conventional water-filling algorithm is no longer optimal. Instead the type of water-filling where the water level is a hammered bottle surface. A change in water level occurs only when the battery level crosses a zero and the water level is monotonically non-decreasing over hill segments.

Theorem 4. *Within a hill segment (e.g., HS(j,m)), the optimal transmission strategy has a double threshold structure, which can be described as*

$$p_k^* = \begin{cases} \left[W^C - {h_k}^{-1}\right]^+, & \text{if } E_k > l_k \left[W^C - {h_k}^{-1}\right]^+, \\ \left[W^D - {h_k}^{-1}\right]^+, & \text{if } E_k < l_k \left[W^D - {h_k}^{-1}\right]^+, \\ E_k/l_k, & \text{otherwise}, \end{cases} \tag{34}$$

where

$$\eta_B W^C = W^D. \tag{35}$$

Modifying Algorithm 1 by changing P^C and P^D with W^C and W^D, we can obtain the algorithm that determines the thresholds W^C and W^D. Fig. 5 shows an example of water-level properties with the new algorithm. We use the same setting for energy arrival and amount as that in the AWGN channel case with $N = 15$ blocks. The channel level, defined as the reciprocal of channel gain, serves as the bottom of a vessel, which is generated from a $\chi^2(2)$ population that corresponds to Rayleigh fading in magnitude. Water finds its level when filled in a vessel with multiple openings until dripping the water to the last drop. Power allocation is the water amount from the current vessel to the current water level. We observe that within a hill segment, the blocks with the red label “Cha” have the equal water level, and the same phenomenon can be found in blocks with the red label “DisC”. Specifically, blocks with the red label “Neu” have the water level between that of “Cha” and that of “DisC”. Note that no transmit power is allocated to blocks 3, 5 and 9 to prevent performance loss from the channel impairments. This is due to the fact that the corresponding channel is so bad that $1/h$ exceeds the water level. The details are shown in TABLE III.

VI. Numerical results

In this section, we present the numerical results to demonstrate the performance of our offline policy, and to compare the PDTBRE region performance with other EH architectures.

The region performance versus storage efficiency is shown in Fig. 6. From the results, it can be seen that the battery level decreases with the prescribed data threshold, until achieving zero when $C_{th} = C_{max}$, for all cases. As the storage efficiency decreases, the region becomes small

Hill Segment	Charging Blocks	Water Level	Discharging Blocks	Water Level	Neutral Blocks	Water Level
HS(1,2)	1	3.105	2	2.484	None	None
HS(3,7)	3, 5, 6	5.248	7	4.199	4	4.626
HS(8,15)	8, 9, 13, 14	6.278	12, 15	5.022	10, 11	5.580, 5.605

TABLE III

DETAILS OF THE WATER LEVEL FOR EACH HILL SEGMENT

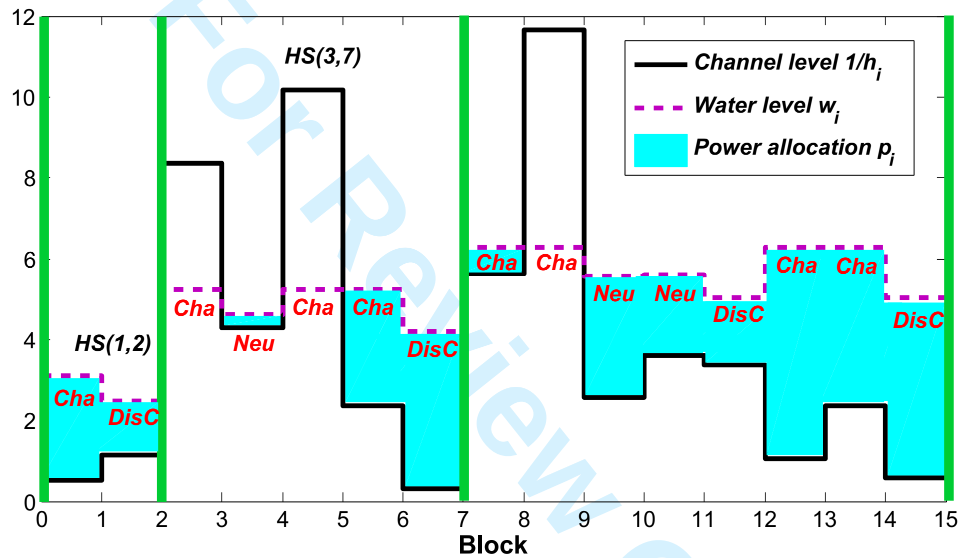

Fig. 5. Water level, channel level, battery level, power allocation of each block. The storage efficiency $\eta_B = 0.8$, $C_{th} = 16.5$, $C_{max} = 17.5811$. "Cha", "Neu", and "DisC" denote the charging, neutral and discharging block respectively.

indicating that on the precondition of same data transmission, lower storage efficiency will lead to lower residual battery level.

We compare the PDTBRE region of our policy to the HSU policy in Fig. 7, where the setting for energy arrival and amount is the same as that for Fig. 3. We determine the HSU results by using the optimal power policy in [10] and taking into account the storage efficiency. The performance for the two types of storage efficiency is captured. It is observed from the figure that HUS mode always outperforms its counterparts, regardless of the storage efficiency. For lower storage efficiency $\eta_B = 0.7$, the gap between the two policies will increase. Also, when there

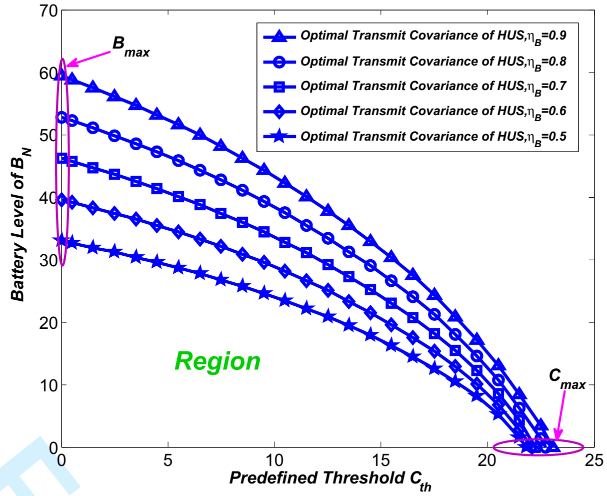

Fig. 6. The region performance versus storage efficiency under AWGN channel. Block length $N = 15$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$.

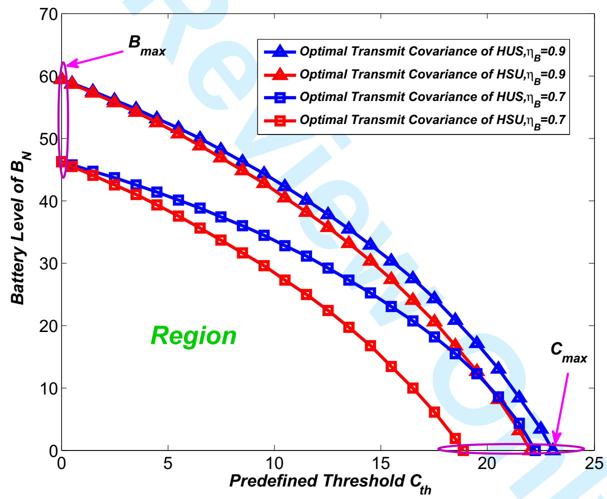

Fig. 7. The comparison of the region performance with HUS and HSU modes under AWGN channel. Block length $N = 15$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$, and the storage efficiency $\eta_B = 0.9$ or $\eta_B = 0.7$.

is no data to transmit, the residual battery level of these two policies will be equal obviously. From the figure, we note that the HUS policy reflects its energy efficiency from the perspective of maximizing the residual battery level.

We then compare the performance of the HUS in AWGN and block fading channel in Rayleigh fading of unit power, with results shown in Fig. 8. Each optimal transmit covariance point of

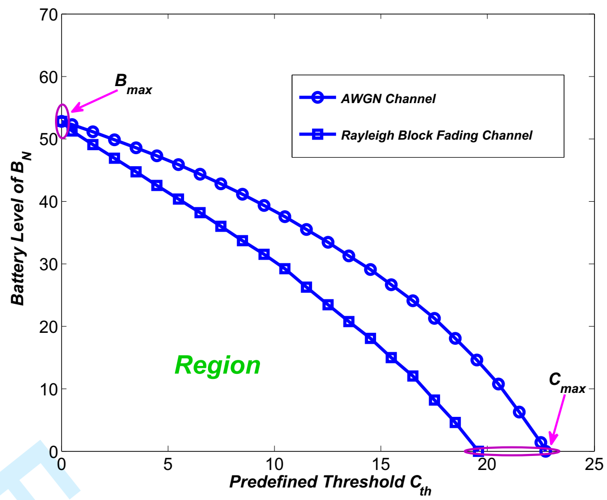

Fig. 8. The comparison of the region performance for AWGN channel and Rayleigh block fading channel. Block length $N = 15$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$, and the storage efficiency $\eta_B = 0.8$.

the block fading channel is obtained by averaging over 1000 random channel gain data. It is observed that the HUS performs better in AWGN channel than in block fading environment. It is also observed that if guaranteeing the same residual battery level, AWGN channel will transmit more data comparing to block fading channel on average.

VII. Conclusion

This paper studied the problem of maximizing the residual energy of the battery for the EH wireless communication with HUS mode. We provided an analysis of the optimal solution and investigated the properties of the optimal solution. It was shown that the optimal policy has a double-threshold structure, where the thresholds were proved to be non-decreasing that allow them to be determined using a simple search algorithm, i.e., Algorithm 1, and based on that, we proposed an optimal offline policy. The results were then extended to block fading channels, which reveals that traditional water filling is no longer optimal. The optimal water levels were shown to have the similar properties with the optimal power in AWGN channel. Numerical results showed the PDTBRE region performance of our offline solution, and also showed superiority over other offline strategies with different EH architectures.

References

[1] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, "Power management in energy harvesting sensor networks", *ACM Trans. Embed. Comput. System*, vol. 6, no. 4, Sep. 2007.

[2] C. Ho and R. Zhang, "Optimal energy allocation for wireless communications powered by energy harvesters," in *Proc. IEEE ISIT*, 2010, pp. 2368-2372.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: optimal policies", *IEEE J. Sel. Areas Commun*, vol. 29, pp. 1732-1743, Sep. 2011.

[4] S. Luo, R. Zhang, and T. J. Lim, "Optimal save-then-transmit protocol for energy harvesting wireless transmitters," *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1196-1207, Mar. 2013.

[5] M. Antepli, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link", *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1721-1731, Sep. 2011.

[6] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an energy harvesting rechargeable transmitter", *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 571-583, Feb. 2012.

[7] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with rechargeable nodes", in *Proc. IEEE ICC*, Kyoto, Japan, Jun. 2011.

[8] O. Orhan and E. Erkip, "Throughput maximization for energy harvesting two-hop networks," *Proc. IEEE ISIT*, Jul. 2013

[9] O. Orhan and E. Erkip, "Optimal transmission policies for energy harvesting two-hop networks," *Proc. CISS*, Princeton, NJ, Mar. 2012

[10] B. Devillers and D. Gunduz, "A general framework for the optimization of energy harvesting communication systems with battery imperfections," *Journal of Commun. and Netw., Spec. Issue on Energy Harvesting in Wireless Netw.*, vol. 14, no. 2, pp. 130-139, Apr. 2012.

[11] N. Michelusi, L. Badia, R. Carli, L. Corradini and M. Zorzi, "Energy Management Policies for Harvesting-Based Wireless Sensor Devices with Battery Degradation," *IEEE Trans. Commun.*, vol. 61, no. 12, pp. 4934-4947, Dec. 2013.

[12] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1180-1189, Mar. 2012.

[13] F. Yuan, Q. Zhang, S. Jin and H. Zhu, "Optimal Harvest-Use-Store Strategy for Energy Harvesting Wireless Systems," *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 698-710, Feb. 2015

[14] G. Miao, N. Himayat, G. Li, and S. Talwar, "Distributed interference- aware energy-efficient power optimization," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1323-1333, Apr. 2011.

[15] G. Miao, N. Himayat, and G. Li, "Energy-efficient link adaptation in frequency-selective channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 545-554, Feb. 2010.

[16] Z. Chong and E. Jorswieck, "Energy-efficient power control for MIMO time-varying channels," in *Proc. of IEEE Online Green Communications Conference (GreenCom)*, 2011.

[17] C. Li, S. Song, J. Zhang, and K. Letaief, "Maximizing energy efficiency in wireless networks with a minimum average throughput requirement," in *Proc. 2012 IEEE Wireless Commun. and Networking Conf.*, pp. 1130-1134, Apr.2012.

[18] M. Zafer and E. Modiano, "Delay constrained energy efficient data transmission over a wireless fading channel," in *Proc. Workshop on Inf. Theory and Appl.*, La Jolla, CA, pp. 289-298, Jan./Feb. 2007.

[19] A. Fu, E. Modiano, and J. N. Tsitsiklis, "Optimal transmission sched- uling over a fading channel with energy and deadline constraints," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 630-641, Mar. 2006.

[20] D. Shuman and M. Liu, “Energy-efficient transmission scheduling with strict underflow constraints,” *IEEE Trans. Inf. Theory.*, vol. 57, no. 3, pp. 1344-1367, Mar. 2011.

[21] J. Yang and S. Ulukus, “Optimal packet scheduling in an energy harvesting communication system”, *IEEE Trans. Commun*, vol. 60, no. 1, pp. 220-230, Jan. 2012.

[22] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New York, NY, USA: Wiley, 2006.

Optimal Harvest-Use-Store Design for Delay-Constrained Energy Harvesting Wireless Communications

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***Abstract*—Recent advances in energy harvesting (EH) technology have motivated the adoption of rechargeable mobile devices for communications. In this paper, we consider a point-to-point (P2P) wireless communication system in which an EH transmitter with a *non-ideal* rechargeable battery is required to send a given fixed number of bits to the receiver before they expire according to a preset delay constraint. Due to the possible energy loss in the storage process, the harvest-use-and-store (HUS) architecture is adopted. We characterize the properties of the optimal solutions, for additive white Gaussian channels (AWGNs) and then block-fading channels, that maximize the energy efficiency (i.e., battery residual) subject to a given rate requirement. Interestingly, it is shown that the optimal solution has a water-filling interpretation with double thresholds and that both thresholds are monotonic. Based on this, we investigate the optimal double-threshold based allocation policy and devise an algorithm to achieve the solution. Numerical results are provided to validate the theoretical analysis and to compare the optimal solutions with existing schemes.**

***Index Terms*—Energy harvesting, residual battery level, storage inefficiency, harvest-use-store, delay-constrained.**

I. Introduction

ENERGY harvesting (EH) is recognized as an alternative energy-supplier to battery-restrained communication networks, such as wireless sensor networks, in order to extend the network lifetime by harvesting ambient energy (e.g., solar, vibration, etc.) [1]. As a revolutionary enhancement to battery-limited devices, an EH transmitter can theoretically operate over an unlimited time. With sporadic energy arrival in limited amounts, nevertheless, it is of great importance to optimize the transmission policy using the available information regarding the energy arrival processes to maximize delivery.

Recent efforts on optimizing data transmission with an EH transmitter have drawn great attention [2]–[9]. In particular, a staircase water-filling result was presented by [2] with an information-theoretic approach considering two types of side information to maximize the throughput. Later in [3], a similar directional water-filling algorithm was introduced for the problems of both throughput maximization with a deadline T, and transmission time minimization with a data delivery constraint. A save-then-transmit protocol is proposed in [4] to minimize the outage probability of energy harvesting transmitters by figuring out the optimal time fraction for energy harvesting in a phase. Other communication scenarios adopting EH which include broadcast channel [5], [6], multiple access channel [7], dual-hop networks [8], [9] have also been investigated.

On the other hand, battery imperfections are a key concern of EH. In [10], the influence of constant leakage rate and battery degradation over time were introduced into the battery model. It is suggested that if the total energy in an epoch is low, then energy should be depleted earlier to reduce leakage. For degradation issue, the optimal policy is shortest path within narrowing tunnel. In [11], the authors characterized the degradation process using some probabilistic technique (Markov chains), in which several degradation stages are identified and the battery state will random “walk” from one to the next with some (small) probability in every slot. Later, [12] studied the data maximization problem under finite battery constraints. More recently in [13], we proposed a harvest-use-store (HUS) policy to cope with storage loss in the battery, which schedules a given sequence of harvested energy in order to increase the energy usage efficiency and the throughput.

All these results including our work in [13] largely fall into two categories: 1) Maximizing the amount of information sent from the transmitter, and 2) minimizing the completion time for delivering a certain number of bits. However, under random energy arrivals, both approaches cannot guarantee that if a packet must be transmitted by the deadline, the expected energy consumed is minimized (i.e., the transmitter will not run out of energy before the next quality-of-service (QoS) request). This will be modeled delay-constrained case, such as VoIP, where packets arrive regularly and each must be received within a short delay window. In such a setting perhaps the most important design objective is to minimize the resources (in our case, energy) needed to meet the delay requirements. Therefore, it is of interest to maximize the energy efficiency (i.e., minimize the transmission energy or maximize the battery residual) subject to a specified amount of information successfully sent in order to ensure that the transmitter has enough energy for the next QoS request.

The maximization problem of energy efficiency using the instantaneous channel states information (CSI) at the transmitter was considered in [14], [15]. Also, Chong and Jorswieck [16] studied such problem when the transmit power is adapted and updated at each time slot based on the CSI over a period of time. To guarantee a satisfactory average throughput, [17] derived a closed-form power allocation solution for maximizing the energy efficiency for a single-carrier point-to-point (P2P) system. For delay-sensitive applications, however, there could be stringent end-to-end (e2e) delay requirement. In [18], strict delay constraints were considered and the optimal scheduling policy was presented assuming a continuous Markov process. In addition, [19] devised the optimal transmission policy with the constraint of a fixed given amount of energy, while in [20], the results were generalized to cope with multiple deadlines, instead of a single deadline at the end of the time horizon.

However, in EH systems, the intermittent nature of energy captured from a natural energy source leads to highly random energy availability at the transmitter. When taking into account this random nature of EH, the energy efficiency maximization problem will become much more complicated. Also, maximizing energy efficiency means to keep the final energy storage maximized in order to achieve reliable and efficient energy scheduling for the next scheduling period. Motivated by this, we investigate the problem of minimizing the energy required for transmitting a given amount of information over a P2P link with a finite delay constraint, using only the harvested energy under random energy arrivals. In particular, our focus is on the storage lossy EH system and therefore, we adopt the HUS strategy [13] which puts a higher priority to usage than storage, contrary to the harvest-store-use (HSU) strategy that suffers from severe energy loss due to lossy storage. During each time, the transmitter determines how many bits to transmit based on the current channel quality, energy harvested quality and the number of bits yet to be served. The scheduler must balance the desire to be opportunistic, i.e., wait to serve many of the bits when the channel and the harvested energy is in a good state, or use the energy it right now to avoid energy loss from storage.

This paper aims to answer the fundamental question which has not been answered before including [13]: How should the transmission be scheduled so that the residual battery level at the end is maximized instead of using up all the energy to maximize the throughput? Our contributions are as follows:

- Given full information about the energy arrivals, in the case of additive white Gaussian noise (AWGN) channels, we prove that the optimal transmission is a "non-idling" transmission, which means that the transmission lasts all the blocks without any break.
- We find that there will be a region between a given data constraint and the residual energy at the end of the transmission. Thus, we study a prescribed data threshold and battery residual energy (PDTBRE) region to characterize all the possible prescribed data threshold (in bits for wireless information transfer (WIT)) and battery residual energy (in joules for residual energy) pairs. Two boundary points of this PDTBRE region can be regarded as a special form of [13] and non-data transmission.
- In order to obtain the remaining boundary point of the region (i.e., the corresponding optimal power allocation solution of the maximum energy efficiency), a double-threshold structure is characterized, which illustrates that the power allocated is related to the battery mode (i.e., charging, discharging and neutral) and is monotonic.
- Apart from the above-mentioned properties, for the block-fading case, it is found that the optimal solution has a water-filling interpretation with double thresholds. Rather than having a single water level, there are multiple water levels that are nondecreasing over time.

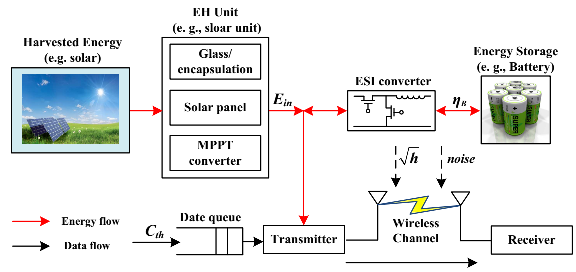

Fig. 1. A HUS wireless communication diagram of a transmitter powered by a energy harvester. Energy is replenished by an energy harvester but is drawn for transmission.

- We notice that the optimal policy has a relationship with the one in [13] and provide an algorithm to achieve this.

The rest of the paper is organized as follows. In Section II, we introduce the system model and formulate the problem. In Section III, we analyze the optimal policy. Section IV then gives an optimal offline policy by investigating the properties of the solution, and then proposing a double-threshold based optimal allocation policy. Extension to block-fading channels is presented in Section V. Numerical results are provided in Section VI and we conclude the paper in Section VII.

II. System Model

We consider a P2P single-antenna wireless communication system, which is fit with an EH transmitter equipped with an imperfect rechargeable battery as shown in Fig. 1. A packet of C_{th} bits which arrives in the data queue at the start of the time horizon must be transmitted within a strict delay constraint T through a fading channel with AWGN. We assume that no other packet is scheduled during this period of time T and within T, there are N energy arrivals. It is further considered that this energy comes from the ambient environment and is harvested by the energy unit. The transmitter operates in the HUS mode realized by the energy storage interface (ESI) converter which is a bidirectional power electronic circuit that stores excess energy in the battery and extracts the energy from the battery to the transmitter when needed. Perfect CSI is assumed available at the transmitter side.

In this system, our focus is on a finite horizon of N-block transmission which starts from block 1 and ends at block T, as illustrated in Fig. 2. We denote the energy arriving at the beginning of block n as E_n, and the time interval between two consecutive energy arrivals as l_n. Note that the transmission time in block n may not be equal to l_n. Therefore, we define $t_n \leq l_n$ to distinguish the transmission time from the energy arrival time interval. Similar to [21], it is assumed that both the harvested energy increments and their arrival times can be exactly known at the transmitter prior to transmission.

In this paper, the channel is modeled as a block-constant process, or widely known as block-fading channel. In other words, the channel state remains constant over each block but randomly changes from one block to another. Based on this, the received signal in block n, y_n, can be written as

$$y_n = \sqrt{h_n} x_n + z_n, \tag{1}$$

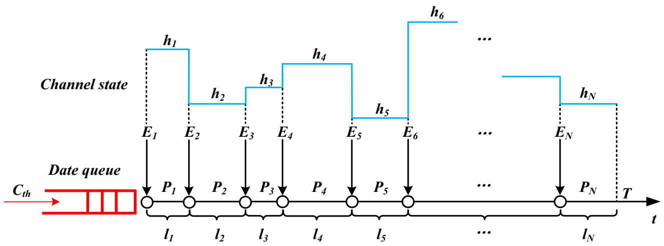

Fig. 2. The transmission model with random energy arrivals. Energies arrive at the beginning of the block which are denoted as $\circ$. The transmission starts from block 1.

Harvesting Architecture	Waste Energy		
	Charging	Discharging	Neutral
HUS	$(1-\eta_B)(E_i - p_i l_i)$	0	0
HSU	$(1-\eta_B) E_i$	$(1-\eta_B) E_i$	$(1-\eta_B) E_i$

TABLE I
COMPARISON OF ENERGY USAGE EFFICIENCY

where $\sqrt{h_n}$ denotes the channel gain for block n, x_n is the input signal for block n, which is considered to be Gaussian distributed with zero-mean and variance of p_n (i.e., transmission power in block n), and z_n is the zero-mean unit-variance AWGN for block n.

The HUS strategy is adopted and we characterize it by the following three battery modes:

(a) **Charging:** When $E_n > p_n t_n$, the transmitter uses $p_n t_n$ amount of energy from the energy unit, and the battery will store the excess energy $D_n \triangleq E_n - p_n t_n$.

(b) **Discharging:** When $E_n < p_n t_n$, the transmitter uses all the current harvested energy harvested, and the battery will replenish the difference $-D_n = p_n t_n - E_n$.

(c) **Neutral:** When $E_n = p_n t_n$, in this case, the transmitter uses up all the harvested energy for transmission without any operation to the battery.

We compare the energy usage efficiency between the HUS and HSU harvesting architectures in Table I. In the table, we can see that with a storage efficiency $0 < \eta_B < 1$, a portion of energy stored in the battery will be lost in all the cases for HSU. Instead, using HUS, only a fraction of the harvested energy suffers from this loss, that is, when it is in the state of charging. From this comparison, we can easily find that HUS is more efficient than HSU all the time. To characterize the HUS harvesting architecture, we define $[D_i]^+ \triangleq \max(0, D_i)$ to describe the battery level at the end of block n (i.e., the residual battery level) as in [13], which is denoted by

$$B_n = \eta_B \sum_{i=1}^{n} [D_i]^+ - \sum_{i=1}^{n} [-D_i]^+, \tag{2}$$

where $\eta_B \sum_{i=1}^{n} [D_i]^+$ and $\sum_{i=1}^{n} [-D_i]^+$ represent, respectively, the energy stored into and taken out from the battery at the end of block n. For simplicity and ease of analysis, we have the following assumptions.

1) We only consider the energy consumption for information transmission, ignoring other types of energy consumption.
2) The battery capacity is always large enough.
3) The initial energy in the battery is zero, i.e., $B_0 = 0$.

Our model is applicable for a practically deterministic traffic (e.g., in VoIP, the next packet generally arrives before the previous ones expire), which allows us to focus on the central issue of meeting deadlines based upon energy and CSI. The purpose of the scheduler is to determine the energy to be served during each block such that the energy consumption is minimized and the bits are served by the deadline T.

A. Problem Statement

The maximum reliable transmission rate in block n is then given by the mutual information $\mathcal{I}(p_n)$ in bits per symbol. In general, we assume that $\mathcal{I}(p_n)$ is concave and increasing in p_n. Consider a block fading channel with average signal power constraint p_n and noise power 1. As is well known in [22], the information-theoretic optimal channel coding, which employs randomly generated codes, achieves the channel rate

$$\mathcal{I}(p_n) = \frac{1}{2} \log(1 + \mathrm{SNR}), \tag{3}$$

where the signal-to-noise ratio (SNR) is given by $\mathrm{SNR} = h_n p_n$. With the assumptions above, in block n, $t_n \log(1 + \mathrm{SNR})/2$ bits of data will be sent to the destination during that interval t_n at the cost of $p_n t_n$, which describes the throughput in block n as

$$C_n = \frac{t_n}{2} \log(1 + h_n p_n). \tag{4}$$

Our aim is to schedule the given EH sequence to maximize the battery level at the end subject to the precondition that the sum data transmission is greater than a prescribed data threshold C_{th}, which will provide QoS guarantee for next time transmission. Hence, our residual battery level maximization problem over N transmission blocks can be expressed as

$$(\mathbf{P1}) \left\{ \begin{array}{ll} \max\limits_{\{p_n, t_n\}} & B_N \\ \text{s.t.} & B_n = \eta_B \sum\limits_{i=1}^{n} [D_i]^+ - \sum\limits_{i=1}^{n} [-D_i]^+ \geq 0, \\ & \forall n \in \{1, 2, \ldots, N\}, \\ & \sum\limits_{n=1}^{N} \frac{t_n}{2} \log(1 + h_n p_n) \geq C_{th}, \end{array} \right. \tag{5}$$

where the battery level must not be negative at the end of each block to supply sufficient energy for data transmission (i.e., the energy causality constraints to ensure that the energy harvested in future blocks cannot be used in the current one). Also, combined with the non-linearity and non-differentiability of the constraint conditions $[\cdot]^+$, $(\mathbf{P1})$ is difficult to solve. The goal of this paper is different from our previous paper tackling the problem that maximizes the throughput which will use up all the energy. For example, in sensor networks, sometimes we only need fixed data (e.g., temperature, humidity) instead of maximum throughput whose extra parts are useless and energy wasting. As such, this paper formulates the problem in order to save energy while transmitting reliable data. To start with our problem, we consider the optimization problem for two channel environments: AWGN channel and block-fading channel. We will derive the optimal power allocation policies for these two channel models.

III. Optimal Policy for AWGN Channel

Here, we first use an example to explain the complexity of our problem. Then we provide a theorem and a region definition to make our objective clearer and easier to analyze. For an AWGN channel, we can assume, without loss of generality, the channel gain to be unity so that $h_1 = \cdots = h_N = 1$. Inserting this into $(\mathbf{P1})$, we obtain

$$(\mathbf{P2}) \begin{cases} \max\limits_{\{p_n, t_n\}} & B_N \\ \text{s.t.} & B_n = \eta_B \sum_{i=1}^{n} [D_i]^+ - \sum_{i=1}^{n} [-D_i]^+ \geq 0, \\ & \forall n \in \{1, 2, \ldots, N\}, \\ & \sum_{n=1}^{N} \frac{t_n}{2} \log(1 + p_n) \geq C_{th}. \end{cases} \tag{6}$$

Note that $(\mathbf{P2})$ is a convex optimization problem and thus has a unique optimal solution. At the optimum, equality on the rate constraint will hold, since otherwise we can always decrease the data rate further by decreasing p_n without violating any other constraints. In the following, we will use a simple two-block problem to illustrate the difficulties involved in solving the convex optimization problem, dealing with the nonlinear function $[D_i]^+$ in three possible cases, i.e., $D_i \gtreqless 0$. If we write them all out, then we will find that these problems are similar in structure. Even though each problem is differentiable and convex, when there are N blocks in the optimization, it will result in a total of 3^N possible cases to handle, and apparently the complexity for solving it increases exponentially, which makes it intractable for large N.

In order to solve this problem, the first step is to determine how to schedule the transmission time. After briefly introducing the basic step, we then find that to be energy efficient, the transmission policy must have the following property.

Theorem 1. *The optimal transmission strategy for the AWGN channel is a "non-idling" transmission, i.e., $t_n = l_n$, $\forall n \in \{1, 2, \ldots, N\}$ and $\sum_n t_n = T$.*

Proof: Recall that the achievable rate for block n can be found from (4). To determine the energy consumption ε_n in block n, we consider that $\mathcal{C}$ bits will be sent in this block. Substituting this into (4), we obtain

$$\varepsilon_n = t_n p_n = t_n \left(2^{\frac{2\mathcal{C}}{t_n}} - 1 \right). \tag{7}$$

Differentiating ε_n with respect to t_n, we have

$$\frac{d\varepsilon}{dt_n} = 2^{\frac{2\mathcal{C}}{t_n}} - 1 + t_n \left[2^{\frac{2\mathcal{C}}{t_n}} \ln 2 \left(-\frac{2\mathcal{C}}{t_n^2} \right) \right]. \tag{8}$$

Now, define

$$f(x) \triangleq 2^x - 1 + \frac{2\mathcal{C}}{x} \left[2^x \ln 2 \left(-\frac{x}{2\mathcal{C}} \right) \right], \tag{9}$$

where $x = 2\mathcal{C}/t_n$. If $f(x) < 0$, ε is monotonically decreasing. As such, the problem is simplified to prove that $f'(x) < 0$ and $f(0) \leq 0$. It is easy to see that it is monotonically decreasing and convex in t_n. The assertion that ε_n decreases with t_n implies that it would be suboptimal to have $t_n < l_n$ $\forall n \in \{1, 2, \ldots N\}$ since we could simply increase the transmission times of one or more blocks and reduce ε_n accordingly. Hence, we only consider "non-idling" transmission schedules where $t_n = l_n$, $\forall n \in \{1, 2, \ldots N\}$ and $\sum_n t_n = T$, in order to store most energy in the battery, which completes the proof. ∎

Also, the threshold C_{th} should not be too large so that the problem is feasible. If C_{th} is too large, then the harvesting energy will not be enough to fulfil the required transmission. Therefore, there should be a region between a given prescribed data constraint C_{th} and the residual energy at the end of transmission B_N. It motivates us to investigate the PDTBRE region to characterize all the possible data threshold (in bits for WIT) and battery residual energy (in joules for residual energy) pairs under the battery causality constraints. Mathematically, i.e.,

$$R_{PDTBRE} \triangleq \left\{ (C_{th}, B_N) : \begin{array}{c} C_{th} \leq \sum_{n=1}^{N} \frac{l_n}{2} \log(1 + p_n), \\ B_N \leq \sum_{i=1}^{N} \eta_B [E_i - p_i l_i]^+ \\ - \sum_{i=1}^{N} [p_i l_i - E_i]^+, \\ B_n \geq 0, \text{ for } n \in \{1, 2, \ldots N\} \end{array} \right\}. \tag{10}$$

In Fig. 3, an example of the PDTBRE region is provided for a P2P EH system operating with the HUS mode (see Section IV for the algorithm to calculate the boundary of the region). The amount of energy and the corresponding durations are specified by the harvested energy sequence

$$\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2] \tag{11}$$

and the durations sequence

$$\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]. \tag{12}$$

The tradeoff between the data threshold and the optimal residual energy is characterized by the boundary of the PDTBRE region. It is important to characterize all the boundary PDTBRE pairs of R_{PDTBRE}, which yields the intervals:

$$0 \leq C_{th} \leq C_{\max}, \qquad 0 \leq B_N \leq B_{\max}. \tag{13}$$

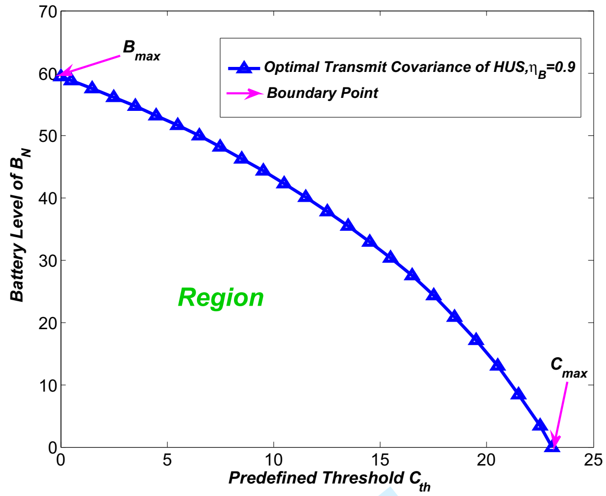

Fig. 3. The tradeoff of the prescribed data threshold and the battery residual energy for a P2P EH system with HUS. Block length $N = 15$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$, and the storage efficiency $\eta_B = 0.9$.

Note that it is easy to identify two boundary points of this PDTBRE region denoted by $(0, B_{\max})$ or $(C_{\max}, 0)$, respectively. For the former boundary point, there is no transmission request during the finite blocks, which corresponds to the case that all the harvesting energy is stored in the battery. On the other hand, for the latter boundary point, the maximum WIT performance can be solved by the optimal HUS strategy in [13], which shows that for optimality, the battery level at the end should be zero. Also, we see that the optimal power of $C_{th} < C_{max}$ is less than or equal to the optimal solution of $C_{th} = C_{max}$, which gives insight to obtain our **Algorithm 1** in the next section for our optimal solution. The optimization now becomes to characterize the part of the boundary of the PDTBRE region remained over the intervals (13).

IV. Solvability and Properties for AWGN Channel

Based on the above analysis, we proceed to solve the problem ($\mathbf{P2}$). The convex optimization problem can be solved by the Lagrangian technique. As such, we denote the Lagrangian function for any $\lambda_n \geq 0$ ($\forall n \in \{1, \ldots N\}$), $\theta \geq 0$ by

$$\mathcal{L} = \eta_B \sum_{i=1}^{N} [D_i]^+ - \sum_{i=1}^{N} [-D_i]^+ + \sum_{n=1}^{N} \lambda_n \left(\eta_B \sum_{i=1}^{n} [D_i]^+ - \sum_{i=1}^{n} [-D_i]^+ \right) + \theta \left(\sum_{n=1}^{N} \frac{l_n}{2} \log\left(1 + p_n\right) - C_{th} \right), \quad (14)$$

whose associated complimentary slackness conditions are

$$\lambda_n \left(\eta_B \sum_{i=1}^{n} [D_i]^+ - \sum_{i=1}^{n} [-D_i]^+ \right) = 0; \quad \forall n \in \{1, \ldots N\}, \quad (15)$$

$$\theta \left(\sum_{n=1}^{N} \frac{l_n}{2} \log\left(1 + p_n\right) - C_{th} \right) = 0, \quad (16)$$

where λ_n and θ are the nonnegative Lagrangian multipliers. Note that the equality constraint of (18) is obvious since $\sum_{n=1}^{N} \frac{l_n}{2} \log\left(1 + p_n\right) - C_{th}$ must be zero; otherwise, we can always increase the residual energy by decreasing p_i without violating any other constraints in (17). By differentiating $\mathcal{L}$ with respect to p_k and carrying on some mathematical manipulation similar to [13], we obtain the optimal power allocation (17).

$$p_k = \theta \left[\ln 2 \left(\sum\nolimits_{n=k}^{N} \lambda_n + 1 \right) \alpha_k \right]^{-1} - 1, \quad k = 1, 2, \ldots, N \quad (17)$$

where $\alpha_k = (\eta_B + 1) + (\eta_B - 1) \operatorname{sgn}\left(E_k - l_k p_k\right)$. To tackle the signum function in α, we will first study the properties of the optimum solution. In order to do this, a key definition of our paper is first described as follows.

Definition 1. *A block is called a valley block or simply a* ***valley****, when its battery level at the end of the block is zero. On the other hand, the blocks between any two closest valleys constitute a* ***hill segment****.*

A hill segment starting from block j and ending at $m > j$, which means that the battery levels $B_{j-1} = 0$, $B_m = 0$ and the battery levels of all the blocks between them are nonzero; namely, $B_k > 0, \forall k \in [j, \ldots, m-1]$, is denoted as $\mathrm{HS}(j, m)$. Specifically, the last hill segment begins from the last valley block to the last block, although the battery level of the last block may not be zero for our optimization. To continue, we first state the properties of the solution, and then give the main result of this paper based on the properties.

Property 1. *Within a hill segment,* $\mathrm{HS}(j, m)$*, all the energy-charging blocks have the same power allocation,* P^C*, whereas the energy-discharging blocks have the similar property, which has a lower power allocation,* P^D*, where*

$$P^C = \theta \left[2 \ln 2 \left(\sum\nolimits_{n=m}^{N} \lambda_n + 1 \right) \eta_B \right]^{-1} - 1, \quad (18)$$

$$P^D = \theta \left[2 \ln 2 \left(\sum\nolimits_{n=m}^{N} \lambda_n + 1 \right) \right]^{-1} - 1 \quad (19)$$

In particular, all the energy-neutral blocks have the power allocation equal to the average harvesting power E_k/l_k*, for* $k \in [j, m]$*. Here, we always have* $P^C \geq E_k/l_k \geq P^D$*.*

Proof: Suppose that there is a hill segment starting from block j and ending at $m > j$, $\mathrm{HS}(j, m)$. To prove this property, we test the signum function for three mutually exclusive cases.

Case 1: Charging only with $E_k - l_k p_k$ *at block* k

Thus, we know $E_k > l_k p_k$, and based on (17), the optimal power is given by

$$p_k = \theta \left[2 \ln 2 \left(\sum\nolimits_{n=k}^{N} \lambda_n + 1 \right) \eta_B \right]^{-1} - 1. \quad (20)$$

Since $B_k > 0$, we assert $\lambda_k = 0 \; \forall k \in [j, m-1]$, in accordance with the slackness conditions in (15). Thus, we can obtain that the power will remain constant (i.e., p_k will always be equal to P^C), unless the battery is depleted. Therefore, we have

$$p_k = \theta\left[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\eta_B\right]^{-1} - 1 = P^C. \quad (21)$$

Specifically, the battery is charging with E_k, that is, $p_k = 0$. This will not happen because the optimal transmission strategy is "non-idling" which means that the power p_k must be greater than 0 to ensure the optimality.

Case 2: Discharging only at block k

In this case, we have $E_k < l_k p_k$, and based on (17), the optimal power is given by

$$p_k = \theta\left[2\ln 2\left(\sum\nolimits_{n=k}^{N}\lambda_n + 1\right)\right]^{-1} - 1. \quad (22)$$

Since $B_k > 0$, we assert $\lambda_k = 0\ \forall k \in [j, m-1]$, in accordance with the slackness conditions in (15). Also, the power will remain constant (i.e., p_k will always equal to P^D) with a lower level comparing to Case 1, unless the battery is depleted. Thus,

$$p_k = \theta\left[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\right]^{-1} - 1 = P^D. \quad (23)$$

To guarantee block m is a valley (i.e., $B_m = 0$), block m must be discharging with the amount B_{m-1}, which means $\lambda_m \neq 0$. As a result, we have

$$p_m = \theta\left[2\ln 2\left(\sum\nolimits_{n=m}^{N}\lambda_n + 1\right)\right]^{-1} - 1 = P^D. \quad (24)$$

Case 3: No charging or discharging (Neutral) at block k

This is the case where the node forwards all harvester power to the transmitter. Thus, $p_k = E_k/l_k$. In this case, substituting $\mathrm{sgn}(0) = 0$ in (17) gives

$$p_k = \theta\left[\ln 2\left(\sum\nolimits_{n=k}^{N}\lambda_n + 1\right)(\eta_B + 1)\right]^{-1} - 1 = E_k/l_k. \quad (25)$$

Since $0 \le \eta_B \le 1$, we can easily see that $P^D \le E_k/l_k \le P^C$, implying that the transmission power p_k in neutral block is restricted to be within the interval $\left[P^D, P^C\right]$. Also, the power will be equal to the average harvesting power E_k/l_k. ■

This property is a result of the fact that the mutual information is a concave function, suggesting that the energy should be allocated to ensure equal SNR over all the blocks for a maximum data rate. That is, as seen in Jensen's inequality, denoting $\ell = l_1 + \cdots + l_N$, we have

$$\frac{1}{2}\sum_{i=1}^{N} l_i \log_1(1+p_i) \le \frac{\ell}{2}\log\left(1 + \frac{l_1 p_1 + \cdots + l_N p_N}{\ell}\right). \quad (26)$$

The equality condition is attained when $p_1 = \cdots = p_N$. In other words, the best way to obtain the optimal strategy is to allocate equal power to each block. Moreover, due to the presence of storage loss, the equal power is "split" into (i.e., replaced by) two thresholds for the optimal strategy. Although this property is similar to the one in [13], it should be emphasized that they are not the same. First, the expressions are different because there is a new numerator θ and a new term 1 in the denominator. Second, C_{th} can influence the value of θ, and in turn change the values of P_C and P_D. Last, we can see that $\lambda_N = 0$ unless $C_{th} = C_{max}$, since B_N is not zero. The WIT performance constraint demonstrates Property 1 described before. Next we show what the energy causality constraints reveal.

Property 2. *The optimal power in both the charging and discharging cases is monotonically non-decreasing from one hill segment to the next.*

Proof: Assuming that there are M valley blocks, denoted as $V_1, V_2, \ldots, V_M$, and then respectively denoting $P^C_{V_i}$ and$P^D_{V_i}$ as the optimal power of charging and discharging blocks within the hill segment $\mathrm{HS}\left(V_{i-1}+1, V_i\right)$, we have

$$P^C = \theta\left[2\ln 2\left(\sum\nolimits_{n=V_i}^{N}\lambda_n + 1\right)\eta_B\right]^{-1} - 1, \quad (27)$$

$$P^D = \theta\left[2\ln 2\left(\sum\nolimits_{n=V_i}^{N}\lambda_n + 1\right)\right]^{-1} - 1. \quad (28)$$

Since $\lambda_n \ge 0$, $V_i > V_{i-1}$, and thus, P^C and P^D are non-decreasing from one hill segment to the next. ■

Now, we give an intuition of the properties to shed some light on how the EH constraints lead to a different optimal power allocation. Property 1 illustrates that if all the harvested energy is already available at the beginning, i.e., sum-power constraint, then a uniform power allocation is optimal for the AWGN channel as shown above that the mutual information is a concave function in the form of logarithm. Thus, the energy will be transferred from the current block to future ones to have optimal benefit. However, this policy is modified by the causality constraints as shown in (5), which gives rise to Property 2 of the optimal solution.

Apparently, based on the properties, a threshold structure for the optimal solution can be proposed as below.

Theorem 2. *Within a hill segment (e.g.,* $\mathrm{HS}(j,m)$*), the optimal transmission strategy has a double threshold structure, which can be described as*

$$p_k^* = \begin{cases} P^C, & \text{if } E_k \ge l_k P^C, \\ P^D, & \text{if } E_k \le l_k P^D, \\ E_k/l_k, & \text{otherwise}, \end{cases} \quad (29)$$

where it is noted that

$$\eta_B(P^C + 1) = P^D + 1. \quad (30)$$

This indicates that during the hill segment $\mathrm{HS}(j,m)$, block $k \in [j, \ldots, m]$, where the power harvested is greater than the power consumption $E_k/l_k > P^C$ (i.e., charging), should be allocated with power P^C. Similarly, block $k \in [j, \ldots, m]$, where the power harvested is less than the power consumption $E_k/l_k < P^D$ (i.e., discharging), should be allocated with power P^D. Specifically, block $k \in [j, \ldots, m]$, where the harvested power is between P_C and P_D, $P^D < E_k/l_k < P^C$ (i.e., neutral), should be allocated with power E_k/l_k.

Now, the main problem is simplified to find the thresholds P_D and P_C in each hill segment. Knowing that it is non-decreasing and constant, and that the optimal power allocation when $C_{th} < C_{max}$ is less than or equal to the power allocation when $C_{th} = C_{max}$, we propose **Algorithm 1** to solve our problem. We search for the optimal p_n in a sequential way. In one loop, we first compute the maximum P_D and P_C within the current hill segment resulting in the maximum throughput using dynamic programming in [13], and then check if these two thresholds give the optimal solution. If the bits sent using the maximum threshold pair is greater than the prescribed data one, this means that the optimal threshold pair should be less than the maximum threshold pair, and we can use a one-dimensional search to find the optimal pair that completes the residual bits. If the bits sent are less than the prescribed data one, we record the data sent (i.e., C_{temp}) using the maximum threshold pair in the current hill segment and go to next loop with the new prescribed data (i.e., $C_{th} \leftarrow C_{th} - C_{temp}$). Specifically, if the bits sent are just equal to the prescribed data one, this loop is the last one and then the algorithm will end.

An example of a 15-block transmission that illustrates Theorem 2 is depicted in Fig. 4, where we consider an EH system with HUS mode for which, the storage efficiency is $\eta_B = 0.9$. The harvesting energy amounts and arrivals are same as the case in Fig. 3. Using the optimal policy in [13], we obtain the maximum throughput C_{max} which is about 23.06. Thus, any prescribed data threshold should be within $(0, C_{max})$. Thus, we set $C_{th} = 22.5$. As shown in the figure, the power allocation of charging, discharging and neutral blocks are denoted by the blue, green and yellow bars respectively, while the corresponding battery level is illustrated by the red-star dotted line, the slope of which indicates the battery state:

$$\textbf{State} = \begin{cases} \text{Charging}, & \text{slope} > 0, \\ \text{Discharging}, & \text{slope} < 0, \\ \text{Neutral}, & \text{slope} = 0. \end{cases} \tag{31}$$

Note that from the figure, $B_0 = 0$ and $B_5 = 0$, indicating that blocks $1 \sim 5$ constitute the first hill segment. During this hill segment, block 1 with a positive slope corresponds to a charging process, having the allocated power equal to $10/9$. Blocks 4 and 5 on the other hand possess a negative slope representing a discharging process, with the same allocated power 0.9. The battery levels of blocks 2 and 3 remain constant in the neutral process, with 1 unit of power, which shows that the neutral power is between charing and discharging one. Similar observations can be seen in the remaining hill segments HS$(6, 12)$ and HS$(13, 15)$, as shown in TABLE II. We also observe the phenomenon that the optimal power belonging to different battery modes is non-decreasing. Importantly, the battery level at the last is not zero, i.e., the maximum residual energy we want is not zero and is now equal to 3.44.

Algorithm 1 Proposed offline power allocation algorithm in HUS mode for EH wireless systems

Initialization:

Block size N, using algorithm 1 in [13] to calculate C_{max}, $B_0 = 0$, $k = 1$, $m = 1$;

Iteration:

1: Begin from block k, find the thresholds P_D and P_C that deplete the battery at some block j by a one-dimensional search or by dynamic programming in [13];
2: $n = k$;
3: **while** $n \leq N$ **do**
4: Compute p_n using equation (29);
5: $n \leftarrow n + 1$;
6: **end while**
7: **if** $\sum_{n=k}^{N} (l_n/2) \log(1 + p_n) = C_{th}$ **then**
8: Step out of the iteration;
9: **else if** $\sum_{n=k}^{N} (l_n/2) \log(1 + p_n) < C_{th}$ **then**
10: $m \leftarrow k$;
11: $C_{temp} = 0$;
12: **while** $m \leq j$ **do**
13: $C_{temp} \leftarrow C_{temp} + (l_m/2) \log(1 + p_m)$;
14: $m \leftarrow m + 1$;
15: **end while**
16: $C_{th} \leftarrow C_{th} - C_{temp}$
17: $k \leftarrow j$;
18: Go to Step 1;
19: **else**
20: Find the new thresholds P_D and P_C that satisfy the constraint $\sum_{n=1}^{N} (l_n/2) \log(1 + p_n) = C_{th}$ by a one-dimensional search, where p_n, $n \in \{k, k+1, \ldots, N\}$ is calculated by equation (29);
21: Step out of the iteration;
22: **end if**

Output:

The power allocation p_n for each block and the maximum residual battery level.

V. Optimal Offline Policy for Block Fading Channel

In this section, we extend the results to the block fading channels. The fading gain is modeled as a block-constant process, with the CSI, $h_n, \forall n \in \{1, 2, \ldots N\}$ perfectly known to the transmitter side. We tackle this problem as in the AWGN channel, i.e., the optimization of $(\mathbf{P1})$. The difference is however that for deep fading, the optimal power may be zero to avoid data rate loss during blocks with poor channel conditions. Similar to the AWGN case, we can argue that the objective function is concave with respect to the power sequence and that the constraint set is convex. Therefore, the problem has a unique maximizer. Like Theorem 1 in Section III, here, under the case of block fading channels, we will have the same property of "non-idling" transmission.

Theorem 3. *The optimal transmission strategy for block fading channel is a "non-idling" transmission, i.e.,* $t_n = l_n$, $\forall n \in \{1, 2, \ldots, N\}$ *and* $\sum_n t_n = T$.

Hill Segment	Charging Blocks	Power Value	Discharging Blocks	Power Value	Neutral Blocks	Power Value
HS(1,5)	1	10/9	4, 5	0.9	2, 3	1
HS(6,12)	6, 9	3.10	7, 8, 11, 12	2.69	10	3
HS(13,15)	13, 14	3.38	15	2.94	-	-

TABLE II
DETAILS OF POWER ALLOCATION FOR EACH HILL SEGMENT

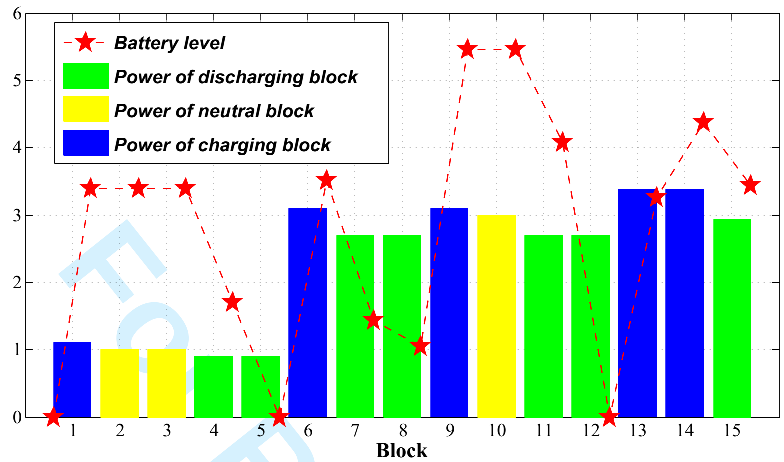

Fig. 4. Structure of optimal power allocation and battery level for each block. The storage efficiency $\eta_B = 0.9$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$.

Proof: Similar to the proof of Theorem 1. ■

To solve the similar PDTBRE region problem, we apply the classical water-filling technique [22] to write the Lagrange function, and take the Karush-Kuhn-Tucker (KKT) conditions for the optimality of the power allocation as

$$\frac{\partial \mathcal{L}}{\partial p_k} \begin{cases} = 0, & \text{if } p_k > 0, \\ \leq 0, & \text{if } p_k = 0, \end{cases} \tag{32}$$

which guarantees the constraint $p_k \geq 0$ is satisfied. Conducting algebraic manipulation similar to Section IV, we obtain the optimal power sequence p_k^* in terms of the Lagrange multipliers, as shown by (33),

$$p_k = \left[\theta \left[\ln 2 \left(\sum_{n=k}^{N} \lambda_n + 1 \right) \alpha_k \right]^{-1} - {h_k}^{-1} \right]^+, \quad k = 1, 2, \ldots, N \tag{33}$$

Identifying the three cases for which the signum function is explicitly expressible, yields similar properties for the water level to that in Section IV.

Property 3. *Within a hill segment,* HS(j, m)*, all the energy-charging blocks have the same water level, equal to* W^C*, whereas the energy-discharging blocks have the similar property, which has a lower water level* W^D*, where*

$$W^C = \theta \left[2 \ln 2 \left(\sum_{n=m}^{N} \lambda_n + 1 \right) \eta_B \right]^{-1}, \tag{34}$$

$$W^D = \theta \left[2 \ln 2 \left(\sum_{n=m}^{N} \lambda_n + 1 \right) \right]^{-1}. \tag{35}$$

In particular, all the energy-neutral blocks have the water level equal to $E_k/l_k + {h_k}^{-1}$*, for* $k \in [j, m]$*. Here, we always have* $W^C \geq E_k/l_k + {h_k}^{-1} \geq W^D$*.*

Proof: Similar to Property 1. ■

Property 4. *The optimal water level of charging and discharging cases is monotonically non-decreasing respectively from one hill segment to the next.*

Proof: Similar to Property 2. ■

From the above properties, we find that the water levels possess the same properties as the optimal power policy for the AWGN channel. It turns out that the conventional water-filling algorithm is no longer optimal. Instead the type of water-filling where the water level is a hammered bottle surface. A change in water level occurs only when the battery level crosses a zero and the water level is monotonically non-decreasing over hill segments.

Theorem 4. *Within a hill segment (e.g., HS*(j, m)*), the optimal transmission strategy has a double threshold structure, which can be described as*

$$p_k^* = \begin{cases} \left[W^C - {h_k}^{-1} \right]^+, & \textit{if } E_k > l_k \left[W^C - {h_k}^{-1} \right]^+, \\ \left[W^D - {h_k}^{-1} \right]^+, & \textit{if } E_k < l_k \left[W^D - {h_k}^{-1} \right]^+, \\ E_k/l_k, & \textit{otherwise}, \end{cases} \tag{36}$$

where

$$\eta_B W^C = W^D. \tag{37}$$

Modifying Algorithm 1 by changing P^C and P^D with W^C and W^D, we can obtain the algorithm that determines the thresholds W^C and W^D. Fig. 5 shows an example of water-level properties with the new algorithm. We use the same setting for energy arrival and amount as that in the AWGN channel case with $N = 15$ blocks. The channel level, defined as the reciprocal of channel gain, serves as the bottom of a vessel, which is generated from a $\chi^2(2)$ population that corresponds to Rayleigh fading in magnitude. Water finds its level when filled in a vessel with multiple openings until dripping the water to the last drop. Power allocation is the water amount from the current vessel to the current water level. We observe that within a hill segment, the blocks with the red label "Cha" have the equal water level, and the same phenomenon can be found in blocks with the red label "DisC". Specifically, blocks with the red label "Neu" have the water level between that of "Cha" and that of "DisC". Note that no transmit power is allocated to blocks 3, 5 and 9 to prevent performance loss from the channel impairments. This is due to the fact that the corresponding channel is so bad that $1/h$ exceeds the water level. The details are shown in TABLE III.

VI. Numerical results

In this section, we present the numerical results to demonstrate the performance of our offline policy, and to compare the PDTBRE region performance with other EH architectures.

The region performance versus storage efficiency is shown in Fig. 6. From the results, it can be seen that the battery level decreases with the prescribed data threshold, until achieving zero when $C_{th} = C_{max}$, for all cases. As the storage efficiency decreases, the region becomes small indicating that on the precondition of same data transmission, lower storage efficiency will lead to lower residual battery level.

We compare the PDTBRE region of our policy to the HSU policy in Fig. 7, where the setting for energy arrival and amount is the same as that for Fig. 3. We determine the HSU results by using the optimal power policy in [10] and taking into account the storage efficiency. The performance for the two types of storage efficiency is captured. It is observed from the figure that HUS mode always outperforms its counterparts, regardless of the storage efficiency. For lower storage efficiency $\eta_B = 0.7$, the gap between the two policies will increase. Also, when there is no data to transmit, the residual battery level of these two policies will be equal obviously. From the figure, we note that the HUS policy reflects its energy efficiency from the perspective of maximizing the residual battery level.

We then compare the performance of the HUS in AWGN and block fading channel in Rayleigh fading of unit power, with results shown in Fig. 8. Each optimal transmit covariance point of the block fading channel is obtained by averaging over 1000 random channel gain data. It is observed that the HUS performs better in AWGN channel than in block fading environment. It is also observed that if guaranteeing the same residual battery level, AWGN channel will transmit more data comparing to block fading channel on average.

VII. Conclusion

This paper studied the problem of maximizing the residual energy of the battery for the EH wireless communication with HUS mode. We provided an analysis of the optimal solution and investigated the properties of the optimal solution. It was shown that the optimal policy has a double-threshold structure, where the thresholds were proved to be non-decreasing that allow them to be determined using a simple search algorithm, i.e., Algorithm 1, and based on that, we proposed an optimal offline policy. The results were then extended to block fading channels, which reveals that traditional water filling is no longer optimal. The optimal water levels were shown to have the similar properties with the optimal power in AWGN channel. Numerical results showed the PDTBRE region performance of our offline solution, and also showed superiority over other offline strategies with different EH architectures.

References

[1] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, "Power management in energy harvesting sensor networks", *ACM Trans. Embed. Comput. System*, vol. 6, no. 4, Sep. 2007.

[2] C. Ho and R. Zhang, "Optimal energy allocation for wireless communications powered by energy harvesters," in *Proc. IEEE ISIT*, 2010, pp. 2368-2372.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: optimal policies", *IEEE J. Sel. Areas Commun*, vol. 29, pp. 1732-1743, Sep. 2011.

[4] S. Luo, R. Zhang, and T. J. Lim, "Optimal save-then-transmit protocol for energy harvesting wireless transmitters," *IEEE Trans. Wireless Commun.*, vol. 12, no. 3, pp. 1196-1207, Mar. 2013.

[5] M. Antepli, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link", *IEEE J. Sel. Areas Commun.*, vol. 29, no. 8, pp. 1721-1731, Sep. 2011.

[6] J. Yang, O. Ozel, and S. Ulukus, "Broadcasting with an energy harvesting rechargeable transmitter", *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 571-583, Feb. 2012.

[7] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with rechargeable nodes", in *Proc. IEEE ICC*, Kyoto, Japan, Jun. 2011.

[8] O. Orhan and E. Erkip, "Throughput maximization for energy harvesting two-hop networks," *Proc. IEEE ISIT*, Jul. 2013

[9] O. Orhan and E. Erkip, "Optimal transmission policies for energy harvesting two-hop networks," *Proc. CISS*, Princeton, NJ, Mar. 2012

[10] B. Devillers and D. Gunduz, "A general framework for the optimization of energy harvesting communication systems with battery imperfections," *Journal of Commun. and Netw., Spec. Issue on Energy Harvesting in Wireless Netw.*, vol. 14, no. 2, pp. 130-139, Apr. 2012.

[11] N. Michelusi, L. Badia, R. Carli, L. Corradini and M. Zorzi, "Energy Management Policies for Harvesting-Based Wireless Sensor Devices with Battery Degradation," *IEEE Trans. Commun.*, vol. 61, no. 12, pp. 4934-4947, Dec. 2013.

[12] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1180-1189, Mar. 2012.

[13] F. Yuan, Q. Zhang, S. Jin and H. Zhu, "Optimal Harvest-Use-Store Strategy for Energy Harvesting Wireless Systems," *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 698-710, Feb. 2015

[14] G. Miao, N. Himayat, G. Li, and S. Talwar, "Distributed interference-aware energy-efficient power optimization," *IEEE Trans. Wireless Commun.*, vol. 10, no. 4, pp. 1323-1333, Apr. 2011.

[15] G. Miao, N. Himayat, and G. Li, "Energy-efficient link adaptation in frequency-selective channels," *IEEE Trans. Commun.*, vol. 58, no. 2, pp. 545-554, Feb. 2010.

[16] Z. Chong and E. Jorswieck, "Energy-efficient power control for MIMO time-varying channels," in *Proc. of IEEE Online Green Communications Conference (GreenCom)*, 2011.

[17] C. Li, S. Song, J. Zhang, and K. Letaief, "Maximizing energy efficiency in wireless networks with a minimum average throughput requirement," in *Proc. 2012 IEEE Wireless Commun. and Networking Conf.*, pp. 1130-1134, Apr.2012.

Hill Segment	Charging Blocks	Water Level	Discharging Blocks	Water Level	Neutral Blocks	Water Level
HS(1,2)	1	3.105	2	2.484	None	None
HS(3,7)	3, 5, 6	5.248	7	4.199	4	4.626
HS(8,15)	8, 9, 13, 14	6.278	12, 15	5.022	10, 11	5.580, 5.605

TABLE III
DETAILS OF THE WATER LEVEL FOR EACH HILL SEGMENT

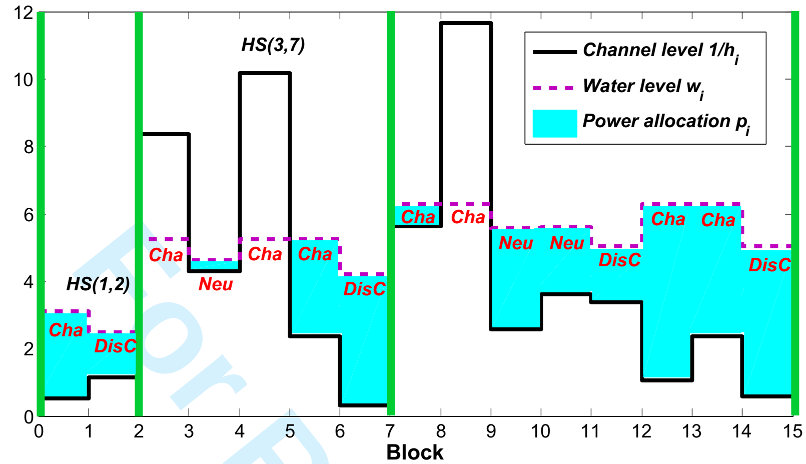

Fig. 5. Water level, channel level, battery level, power allocation of each block. The storage efficiency $\eta_B = 0.8$, $C_{th} = 16.5$, $C_{max} = 17.5811$. "Cha", "Neu", and "DisC" denote the charging, neutral and discharging block respectively.

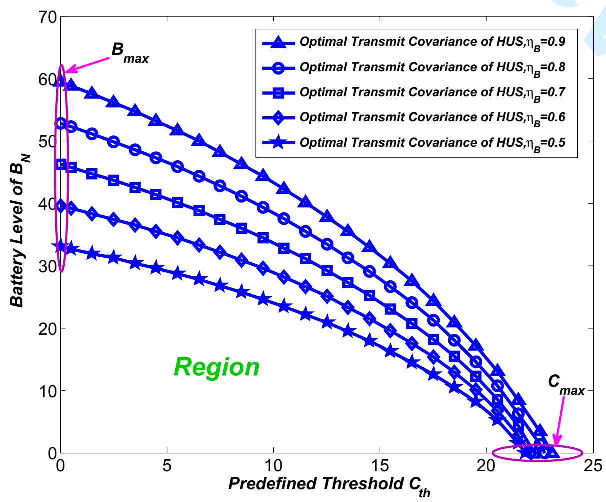

Fig. 6. The region performance versus storage efficiency under AWGN channel. The energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$, block length $N = 15$

[18] M. Zafer and E. Modiano, "Delay constrained energy efficient data transmission over a wireless fading channel," in *Proc. Workshop on Inf. Theory and Appl.*, La Jolla, CA, pp. 289-298, Jan./Feb. 2007.
[19] A. Fu, E. Modiano, and J. N. Tsitsiklis, "Optimal transmission scheduling over a fading channel with energy and deadline constraints," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 630-641, Mar. 2006.
[20] D. Shuman and M. Liu, "Energy-efficient transmission scheduling with strict underflow constraints," *IEEE Trans. Inf. Theory.*, vol. 57, no. 3, pp. 1344-1367, Mar. 2011.
[21] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system", *IEEE Trans. Commun*, vol. 60, no. 1, pp. 220-230, Jan. 2012.
[22] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. New York, NY, USA: Wiley, 2006.

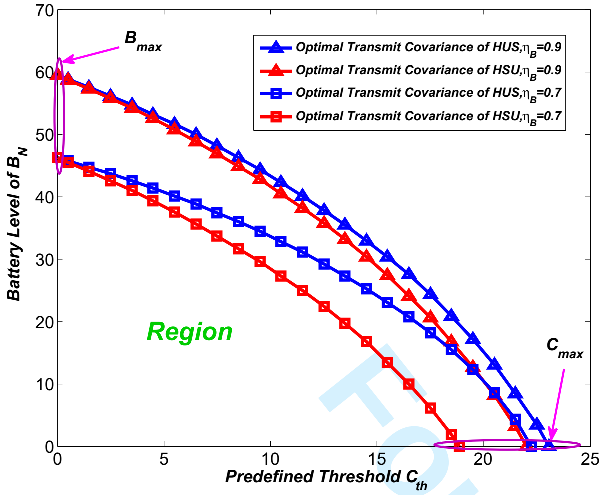

Fig. 7. The comparison of the region performance with HUS and HSU modes under AWGN channel. Block length $N = 15$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$, and the storage efficiency $\eta_B = 0.9$ or $\eta_B = 0.7$.

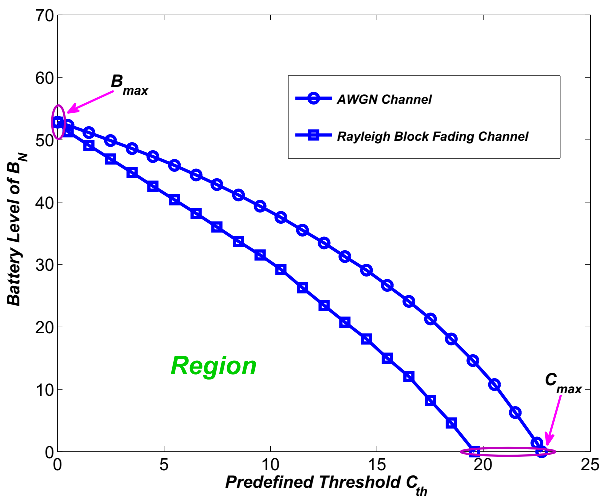

Fig. 8. The comparison of the region performance for AWGN channel and Rayleigh block fading channel. Block length $N = 15$, the energy amount $\mathbf{E} = [6, 2, 2, 1, 1, 7, 6, 5, 8, 3, 4, 4, 7, 8, 2]$, the block duration $\mathbf{L} = [2, 2, 2, 3, 3, 1, 3, 2, 1, 1, 2, 3, 1, 2, 1]$, and the storage efficiency $\eta_B = 0.8$.