THE EFFECT OF HETEROGENEITY ON FINANCIAL CONTAGION DUE TO OVERLAPPING PORTFOLIOS

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We consider a model of financial contagion in a bipartite network of assets and banks recently introduced in the literature, and we study the effect of power law distributions of degree and balance-sheet size on the stability of the system. Relative to the benchmark case of banks with homogeneous degrees and balance-sheet sizes, we find that if banks have a power-law degree distribution the system becomes less robust with respect to the initial failure of a random bank, and that targeted shocks to the most specialised banks (i.e. banks with low degrees) or biggest banks increases the probability of observing a cascade of defaults. In contrast, we find that a power-law degree distribution for assets increases stability with respect to random shocks, but not with respect to targeted shocks. We also study how allocations of capital buffers between banks affects the system’s stability, and we find that assigning capital to banks in relation to their level of diversification reduces the probability of observing cascades of defaults relative to size based allocations. Finally, we propose a non-capital based policy that improves the resilience of the system by introducing disassortative mixing between banks and assets.

Keywords: Contagion; Systemic risk; Network Models.
1. Introduction

Financial institutions are increasingly diversifying their balance sheet across several asset classes in order to reduce the idiosyncratic component of their portfolio risk. This has led to increased global connectivity in the portfolio holdings across several institutions [4, 31]. However, recent studies including [36, 22, 35, 14, 3, 15] have shown that while increased interconnectivity can help diversify risk across the system, it also serves as a contagion propagating and amplification mechanism whenever a crisis is underway. This was partly the reason American International Group (AIG) was bailed out during the financial crisis as many of the biggest financial institutions had become exposed to it via derivative contracts ([40] provides more details). Financial institutions are connected directly via inter-institutional lending (e.g. interbank and repo transactions) and also indirectly through similar asset investments such as connections arising from overlapping portfolios. However, the former has drawn the most attention in the literature. Significant effort has been for instance devoted to studying the role of counterparty and roll-over risks in propagating contagion [22, 35, 21, 5, 3, 14, 2], and to understanding the impact of different interbank network topologies on the resilience of the financial system [11, 30, 4, 33, 23]. Other studies have focused on the effect that agents’ strategic choices have on the systemic stability (see for instance [6]), or on the characterization of feedback loops between the macroeconomy and the financial system [1, 25].

Recently, academics and policymakers have begun paying close attention to the risk posed by indirect connections associated with overlapping portfolios [28, 15, 24]. These connections provide a contagion channel for the propagation of mark-to-market portfolio losses to one or more financial institutions due to depression in asset prices resulting from fire sales by a distressed institution holding the same assets. In some cases, these losses may be sufficient to cause additional institutions to become distressed thereby resulting in more rounds of asset fire sales and further depression in asset prices. The 2007 quant crisis, for instance, was caused by a similar scenario in which the fire sales liquidation of the portfolio of one equity hedge fund depressed prices of assets held by other funds causing them to embark on additional rounds of selling which depressed asset prices even further and resulted in large portfolio losses (see [32] for an elaborate discussion). The existing literature on overlapping portfolios have only considered bank interlinkages in the context of a single asset [18, 36, 22, 3]. However, [15] have recently generalised the fire sales model introduced in [18] to the case of many assets. They characterised the stability of the financial system in terms of its structural properties including average degree, market crowding, leverage and market impact using a bipartite financial network model in which the contagion channel is formed through local portfolio overlaps between banks with homogeneous degrees.

The analysis of Ref. [15] has been carried out for the case of Erdős-Rényi networks and banks with the same size, but in fact empirical studies [20, 11, 34, 19] show that real financial networks of common portfolio holdings and balance sheet
size distributions are more heterogeneous. Specifically, they provide evidence of power laws in these distributions. Therefore, we consider the model of Ref. [15] and analyze the effect of power law distributions of banks’ size and degree on the stability of the system. We refer to banks with low degrees as specialised while those with high degrees are said to be diversified. In this way, we are able to distinguish between the systemic risk contribution of different categories of banks ranging from very specialised to very diversified. In this vein, our work builds on previous analysis by [30, 33, 14] on the impact of heterogeneity on the interbank network. While these previous works considered the effect of heterogeneity on the stability of interbank lending networks, here we focus on indirect connections due to overlapping portfolios. Furthermore, we study the effectiveness of various regulatory capital policy models guided by the intuition developed from the systemic risk contribution of the different types of banks. Finally, we consider the possibility of improving systemic stability by introducing structural correlation into the network without imposing new capital requirements.

The model used for our simulations belongs to the same class of contagion mechanisms used extensively in the literature of counterparty network models [36, 43, 22]. In a nutshell, the system is exogenously perturbed and the resulting impact is recursively propagated through the network until no new default is observed. This feedback mechanism is essentially driven by asset devaluations based on a market impact function that revalues an asset with respect to its traded volume [9, 10].

Our goal is to understand the impact of heterogeneity in the portfolio structure of banks on financial contagion due to overlapping portfolios. As such, we abstract from strategic processes used by banks in choosing a particular portfolio structure as in [44], who show using a microfounded model that in equilibrium the risk of joint liquidation motivates investors towards heterogeneous portfolio configurations. Moreover, a mechanistic approach keeps the model general enough for stress testing real financial systems by calibrating the model. A further assumption is that of passive portfolio management so as to keep the dynamics simple (i.e. banks do not deleverage or rebalance their portfolios during a crisis). In this sense, a bank’s portfolio remains fixed until it becomes liquidated whenever it defaults. This assumption can be justified from the fact that most financial markets are illiquid relative to the positions held by large institutions such that whenever a crisis is underway, banks usually have insufficient time to deleverage until they become insolvent (see [15] for an elaborate discussion).

Our stress tests reveal that heterogeneous bank degrees and sizes make the system more unstable relative to the homogeneous benchmark case with respect to random shocks but not with respect to targeted shocks. In contrast, heterogeneity in asset concentrations makes the system more resilient to random shocks but not with respect to targeted shocks. We then proceeded to study possible capital policy models guided by these results and find that a regulatory policy that assigns capital to the most specialised banks performs better than random assignments when the average degree is high. Moreover, diversification is a more significant factor than
size in improving the financial system’s resilience with capital based policies. The
insights we develop can be used to address one of the major drawbacks of the Basel
accords in ignoring the role of diversification for setting capital requirements [17].
An example is the risk weighted capital requirement framework which is heavily
criticised for providing banks with incentives to concentrate in low risk asset classes
such as interbank loans, sovereign debt etc. which not surprisingly turned out to be
at the centre of the 2007 financial crisis [7]. Finally, we investigated the possibility of
improving financial stability with a non-capital based policy that imposes a partic-
ular configuration in the bipartite network and find that disassortative mixing (i.e.
connecting the most specialised banks with the most concentrated assets) increases
the stability of the system.

The rest of this paper is organised as follows. In the next section, we outline
the main features of the model. In section 3, we explore the stability impact of
heterogeneous network topology and balance sheet sizes. section 4 provides insights
on the effectiveness of capital based policies and proposes a non-capital based policy
by introducing structural correlations into the bipartite network. Finally, a summary
of our findings is presented in section 5.

2. The Model

In this section we describe the model of Ref. [15], that we will then study in section
3 under different scenarios pertaining to the degree and size distribution of banks.

2.1. Network

We consider a bipartite network of a financial system consisting of \( N \) banks and \( M \)
assets as shown in Figure 1. A link from bank \( i \) to asset \( j \) implies that \( j \) constitutes
part of the portfolio of bank \( i \). We define \( k_i \) as the degree (i.e. the total number of
links) of bank \( i \). Hence, the average bank degree is defined as:

\[
\mu_b = \frac{1}{N} \sum_{i=1}^{N} k_i
\]

Similarly, we can define the average degree of the assets as:

\[
\mu_a = \frac{1}{M} \sum_{j=1}^{M} l_j
\]

where, \( l_j \) is the number of banks holding asset \( j \) in their portfolio.

2.2. Balance sheet structure

A bank’s portfolio in the network discussed above consists of investments in non-
liquid assets (e.g. shares in stocks) and liquid assets (e.g. cash). Figure 2 depicts the
general structure of a bank’s balance sheet. The initial total assets held by bank \( i \)
Fig. 1: A Heterogeneous bipartite financial network. Banks are depicted in red circles while assets are shown in blue. The links of the banks follows a power law distribution.

Fig. 2: A typical bank’s initial balance sheet structure. The bank holds a fixed amount of its asset in the form of cash and the value is assumed to remain fixed throughout the simulation for the purpose of simplicity.

are denoted by \( A_0^i \). A fraction \( \theta \) of total assets are liquid assets, which we denote as \( C_i = \theta A_0^i \), while the rest is assumed to be uniformly spread across the assets in the bank’s portfolio. The initial equity is equal to \( E_0^i = \gamma A_0^i \). In the following, for consistency with previous work \cite{15, 22}, we consider \( \theta = 20\% \) and \( \gamma = 4\% \). Moreover, reports in \cite{43} suggest that the capital structure of banks in advanced economies typically conforms with this configuration. We further assume that the remaining portion of the liabilities side of bank’s \( i \) balance sheet comprises customers’ deposits \( D_i \). We define the total asset of bank \( i \) at any time \( t \) as:

\[
A_t^i = \sum_{j=1}^{M} Q_{ij} p_j^i + C_i \tag{3}
\]
Where $Q_{ij}$ denotes the number of shares of stock $j$ held by bank $i$ and $C_i$ is assumed to remain fixed throughout the simulation. We define $p^t_j$ as the price of stock $j$ at time $t$ such that:

$$p^t_j = p^{t-1}_j f_j(x^t_j),$$

where $x^t_j$ denotes the quantity of asset $j$ sold at time $t$. In the model, a bank is declared insolvent whenever its initial capital endowment $E^0_i$ is completely eroded due to losses incurred from the depreciation of its asset values. Hence, the solvency condition for a bank $i$ is defined as:

$$A^0_i - \sum_{j=1}^{M} Q_{ij} p^t_j - C_i \leq E^0_i$$

### 2.3. Contagion mechanism

A simulation of the model follows the sequence enumerated below:

1. Exogenously shock the system at time step $t = 0$
2. Check banks for solvency condition as in Equation 5 at each successive time steps $t = 1, 2, ..$
3. Liquidate the portfolios of any newly bankrupt bank and re-compute asset prices. In order to keep the model simple, liquidated assets are assumed to be traded with parties outside the banking system.
4. Terminate the simulation when no new defaults occurs between successive time steps.

This dynamics is captured by the flowchart depicted in Figure 3.

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Fig. 3: Flowchart representation of the contagion mechanism. A Bank is only declared bankrupt whenever it becomes insolvent.

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### 2.3.1. Exogenous shocks

We consider two kinds of initial shocks: random and targeted shocks. In a random shock, a bank or asset is randomly selected and exogenously perturbed while a specific kind of bank or asset is perturbed in the case of a targeted shock.
2.3.2. Market impact

We assume a market impact function of the form \( f_j(x_j) = e^{-\alpha x_j} \) as in \([22, 3, 18]\) such that \( x_j \) is the liquidated fraction of asset \( j \). The price of asset \( j \) is then updated according to the rule: \( p_j \rightarrow p_j f_j(x_j) \). As in \([22, 36, 15]\), we set \( \alpha = 1.0536 \) such that the liquidation of 10% of an asset results in a 10% price drop in the asset’s value.

2.3.3. Systemic stability

We characterise the stability of the financial system in terms of the systemic risk posed by an exogenous shock. We define systemic risk as the probability that the number of defaults exceeds a threshold \( \phi \). We define \( \phi \) as 5% of the total number of banks in the system for consistency with previous work \([22, 15]\).

3. Effect of heterogeneity on contagion properties

In this section, we consider different scenarios to understand the effect of heavy-tailed distributions of assets and degree on contagion due to overlapping portfolios.

3.1. Heterogeneous bank degrees

We desire to investigate the stability impact of heterogeneity in the degree of banks. As such, we consider a heterogeneous bipartite financial networks where the degrees of banks are generated according to a power law distribution i.e. \( P(k) \propto k^{-\gamma} \) with \( \gamma = 2.5 \). Each bank then forms a link with a random asset until it reaches its generated degree such that no bank is linked to an asset more than once. This link formation approach implies that the number of links of the assets follows a Poisson distribution since every asset has the same probability of being selected. A bank’s degree can be interpreted as its level of diversification since it denotes the number of different investments of the bank. We have used the term specialised bank to mean a bank with focused investments in contrast to a bank holding a diversified portfolio. Our focus here lies in understanding the systemic risk contribution of different types of banks ranging from very specialised to very diversified banks without mixing in the influence of size. This approach mandates an assumption of the same balance sheet sizes across all banks.

In the left panel of Figure 4 we plot the probability of contagion as a function of \( \mu_b \) when a random bank fails. We compare the unstable region for the system with heterogeneous bank degrees relative to the homogeneous case. We find that the unstable region is wider in the heterogeneous system. The right panel of Figure 4 shows that this observation is independent of the kind of exogenous shock. In particular, we plot the contagion probability for the case when an asset is randomly devalued.

\(^a\)We have chosen this value for \( \gamma \) in order to generate power law distributions where the first moment is defined and the second is infinite i.e. \( 2 \leq \gamma \leq 3 \). Moreover, we choose this particular value for \( \gamma \) to ensure consistency with previous work \([14]\) on counterparty default contagion.
and still find that heterogeneity in banks’ degree results in greater instability. A similar finding is reported by [24], who show that heterogeneity increases aggregate vulnerability of the financial system to adverse shocks. The existence of a wider

![Contagion Probability](image)

Fig. 4: Left Panel: Contagion probability as a function of $\mu_b$ for the case when a random bank fails. Red circles: system with heterogeneous bank degrees. Blue squares: system with homogeneous bank degrees. Right Panel: Contagion probability as a function of $\mu_b$ for the case when a random asset is devalued. Contagion is worse in the heterogeneous system irrespective of the kind of exogenous shock. Result refer to 1000 simulations for $N = M = 1000$

unstable region in the heterogeneous system can be understood by observing that, contrary to the homogeneous case, the heterogeneous system is characterized by a few highly diversified banks and many specialized banks. Hence, the probability that a specialized bank is hit from the initial shock is relatively higher. Consequently, specialised banks induce higher devaluations on their assets since they hold large amounts of these assets.

However, this result is in contrast to general reports in the complex networks literature in which heterogeneous network topology has been shown to create more stability, for instance, [14] show that heterogeneity in a counterparty network creates a more robust system relative to the homogeneous case. The reason for this lies in the fact these previous works have considered a network of direct bilateral exposures. In such case the few hubs (i.e. the most connected nodes) become the most systemically relevant because they can impact a higher number of counterparties, whereas in our case the specialised nodes are the most systemically relevant because they concentrate their investments in specific assets, and have a higher impact of liquidation on these. This result may shed some light on why specialised institutions like mortgage banks, building and loan associations, specialist funds etc., who hold significant amounts of specific assets, should be considered systemically important. Moreover, it provides further support to the conjecture given by Andrew Haldane,
the Bank of England’s Chief Economist, in one of his speeches that the “rapid growth in specialist funds potentially carry risk implications, both for end-investors and for the financial system as a whole” [27]. Furthermore, [44] also suggests imposing higher diversity requirements on portfolio holdings of financial institutions with high liquidation risk relative to those with low risk.

In Figure 5, we show the impact of targeted shocks on the stability of the system. We plot the probability of contagion as a function of $\mu_b$ when the initial shock is aimed at specific banks. We find that the unstable region is widest when any of the top 5% most specialised banks is hit while targeted shocks on any of the top 5% diversified banks results in the smallest unstable region. This can be understood from the fact that banks hold lesser amounts of specific assets with increasing degrees since we assume here that all banks are endowed with the same asset sizes. Hence, targeting shocks at the most diversified banks would effectively close the fire-sale contagion channel quicker since only small amounts of assets would be sold, which implies lower price devaluation than the case when banks are randomly perturbed. However, the reverse is observed when shocks are directed at the most specialised banks since they hold significant amounts of specific assets and thereby carry higher liquidation risk. We refer to these banks as ”Too Specialised To Fail” (TSTF).

Fig. 5: Contagion probability as a function of $\mu_b$ when banks have heterogeneous degrees. Blue squares: contagion probability when a random bank fails. Green diamonds: contagion probability when shocks are targeted at the most specialised banks. Red circles: contagion probability when shocks are targeted at only the most diversified banks. The region where contagion occurs is widest when specialised banks are targeted. Result refer to 1000 simulations for $N = M = 1000$. 
3.2. Heterogeneous asset concentration

In the previous section, the distribution of the banks’ degrees was heavy-tailed, but degrees of assets (i.e. the concentration of assets) followed a Poisson distribution. In this section, we turn our attention to the opposite case when the distribution of the number of banks holding each asset is heavy tailed and the degree distribution of banks is homogeneous. We follow the approach of the previous section and assume a power law distribution in the asset concentrations. An asset’s concentration can be interpreted as the preference of banks towards that asset. Our aim is to study how this preference structure affects the stability of the entire system.

Fig. 6: Left Panel: Contagion probability as a function of $\mu_a$ for homogeneous and heterogeneous distributions of asset concentrations. Blue squares: system with homogeneous asset concentrations. Red circles: system with heterogeneous asset concentrations. A random bank fails in both cases. Introducing heterogeneity into the distribution of asset concentrations results in a more robust system. Right Panel: Targeted shocks on a system with heterogeneous asset concentrations. Targeting concentrated assets amplifies contagion probability. Result refer to 1000 simulations for $N = M = 1000$

In the left panel of Figure 6, we plot the probability of contagion as a function of average asset degree for the case when a random bank fails. In contrast to the results observed for heterogeneous bank degrees, we find that introducing heterogeneity in the concentration of the assets produces a slightly more robust system relative to the homogeneous system. This can be understood from the fact that the probability that a highly concentrated asset is perturbed is relatively low since the scale free network comprises very few concentrated assets and many less concentrated (i.e. isolated) ones. This effectively reduces the unstable region since fewer banks are affected by contagion.

The right panel of Figure 6 shows the impact of targeting initial shocks at any
of the top 5% most concentrated (i.e. with highest degree) assets. As expected, targeting initial shocks at these highly concentrated assets has the effect of amplifying contagion since more banks are negatively affected by the initial asset devaluation. However, the width of the unstable region is essentially the same as in the homogeneous system. This is because as soon as banks reach a critical average degree the exogenous shock is not amplified by the system (irrespective of whether the shock consists in the initial default of a bank or the initial devaluation of an asset).

3.3. Heterogeneous bank sizes

In the previous sections, we assumed that all banks have the same balance sheet sizes in order to separate the influence of size from diversification. However, empirical evidence in the literature clearly suggest that banks also have largely heterogeneous sizes [8]. For instance, a recent data analysis by SNL Financial shows that the top 5 biggest banks have 44% of the total assets held by banks in the U.S. [39]. Our aim in this section is to study the impact of this kind of heterogeneity in the size distribution of banks on the stability of the financial system. To do this, we model the bank sizes according to a power law distribution i.e. $P(A) \propto A^{-\gamma}$ resulting in the creation of a few banks with significantly larger asset sizes than most banks whilst abstracting from the influence of diversification by assuming a Poisson degree distribution.

In the left panel of Figure 8, we plot the probability of contagion as a function of $\mu_b$ for the case of random bank shocks. We find that the probability of contagion as a function of $\mu_B$ decays much faster when banks have homogeneous sizes relative to the heterogeneous case. The following argument provides an intuition to why this is the case. In the heterogeneous system, the fire sales impact on asset prices is more severe whenever any of the large banks are hit as these banks hold significant amounts of their assets relative to the entire system since we have assumed a Poisson degree distribution. This effectively shifts the critical threshold for which contagion is no longer possible to the right.

The right panel shows the contagion probability as a function of $\mu_b$ for the case of initial shocks to specific banks. We observe that the system is significantly more unstable when exogenous shocks are targeted at any of the top 5% biggest banks but more stable when the shocks are targeted at any of the top 5% smallest banks. This follows from the fact that big banks hold comparatively larger amounts assets for each value of $\mu_b$ relative to other banks, which implies that targeting shocks at them would cause higher devaluations of the assets they hold, effectively fuelling the contagion mechanism that leads to a wider unstable region. We refer to these banks as ”Too Big To Fail” (TBTF).

In summary, the findings of the stress tests conducted in section 3 are the following:

(1) Introducing heterogeneity in the degrees of banks exacerbates the fragility of the system to random shocks in contrast to [14, 22] who show that a scalefree coun-
Fig. 7: Left Panel: contagion probability as a function of $\mu_b$ for homogeneous and heterogeneous distribution of banks’ sizes. Blue squares: system with similar balance sheet sizes. Red circles: system with heterogeneous balance sheet sizes. The system is subject to random bank failures in both cases. Contagion probability is wider in the heterogeneous system relative to the homogeneous case. Right Panel: Targeted shocks on a system with heterogeneous distribution of banks’ balance sheet sizes. Blue squares: contagion probability when a random bank is perturbed. Red circles: contagion probability when shocks are targeted at the biggest banks. Green diamonds: contagion probability when shocks are targeted at the smallest banks. Targeting shocks at the biggest bank results in the widest unstable region. Result refer to 1000 simulations for $N = M = 1000$.

(2) Heterogeneity in asset concentrations improves the resilience of the system to random shocks in contrast to heterogeneous bank degrees. Moreover, targeting highly concentrated assets increases the probability of contagion, however the average degree threshold where contagion dies out is effectively unchanged.

(3) Cascading default stops at $r$ when banks have homogeneous sizes relative to the heterogeneous case and is greater when exogenous shocks are targeted at the biggest banks.

4. Policy Impact Analysis

The 2007-2009 financial crisis has precipitated calls for higher regulatory capital requirements for banks. Although higher capital requirements can improve financial
stability, they however carry some implicit costs\(^\text{\textcopyright}\), namely reduced profitability for banks and higher lending cost which may have a negative impact on social welfare \cite{29, 12, 13}. Hence, it is important that new regulatory capital requirements are assigned to banks in the way that gives the most stable configuration. To this end, we investigate how the intuition developed from the stress tests in \textbf{section 3} can influence capital based regulatory policies. We then propose an alternative non-capital based policy by studying the structure of the bipartite network.

\subsection*{4.1. Capital based policy}

Here, we compare the performance of possible capital policy models following the intuition developed in \textbf{section 3}. In each model, the same amount of capital \(\chi\) is injected into the system. The difference in the policies lies in the way \(\chi\) is distributed amongst the banks. In each analysis, we test the response of the system to the initial default of a random bank.

\subsection*{4.1.1. Targeted versus random}

The stress tests done in \textbf{section 3} suggests that ”Too Specialised To Fail” and ”Too Big To Fail” banks are systemically important. Hence, it becomes interesting to ask if assigning capital requirements to only this group of banks can improve financial stability relative to targeting a random group of banks. We consider two kinds of targeted policies. In one, we assign the capital equally to only the top 5\% most specialised banks and refer to this policy as \(T_S\) (Targeted Specialised) while in the second, which we call \(T_B\) (Targeted Big), only the top 5\% biggest banks are required to hold more capital. We model a random policy for the purpose of comparison. In the random policy, 5\% of the banks are randomly selected and assigned additional capital requirements equally.

\(T_S\) : We now investigate the stability impact of the \(T_S\) policy relative to the random policy as such we abstract away from the influence of size by assuming similar balance sheet sizes across all banks. We show this comparison in left panel of \textbf{Figure 8} by computing the ratio \(R\) of the contagion probability of both policies as a function of \(\mu_b\) such that \(R = 1\) implies similar performance, \(R > 1\) means the \(T_S\) policy supersedes the random policy and \(R < 1\) implies that the \(T_S\) policy outperforms the random policy. We focus our analysis on only those regions where contagion occurs in both systems to avoid divisions by zero. The plot suggests that a policy that focuses on the most specialised banks results in greater stability relative to a random policy in the region with high values of \(\mu_b\), which is significant from a policy perspective because real world financial networks are more likely to be in

\(^\text{\textcopyright}^\text{\textcopyright}\)This is based on the assumption that Modigliani-Miller theorem does not hold, which essentially implies that a bank’s capital structure does not affect profit or social welfare in an idealised world without frictions such as interest payments on debts, taxes, bankruptcy and agency costs \cite{20}.
this region.

The right panel of Figure 8 provides an insight to why the $T_S$ policy outperforms the random policy. It shows the probability that a bank $i$ with degree $k_i$ defaults before the occurrence of contagion. The plot suggests that the specialised banks are the most likely to default before contagion occurs. As such, it is reasonable to conjecture that focusing the capital policy on these banks is more likely to increase the resilience of the system.

Fig. 8: Left panel: Stability impact of $T_S$ policy relative to the random policy for a system with heterogeneous bank degrees. Dotted line: comparison basis i.e. $R=1$. The $T_S$ policy produces more stability relative to the random policy for high values of $\mu_b$. Right panel: Probability that a bank $i$ with degree $k_i$ defaults before contagion occurs. The most specialised banks have a greater chance of defaulting before contagion occurs.

$T_B$: We now abstract from heterogeneous degrees and consider only heterogeneous sizes in order to study the stability impact of the $T_B$ policy relative to the random policy. We show this comparison in left panel of Figure 9 by computing the ratio $R$ of the contagion probability of both policies as a function of $\mu_b$ such that $R = 1$ implies similar performance, $R > 1$ means the $T_B$ policy supersedes the random policy and $R < 1$ implies that the $T_B$ policy outperforms the random policy. The plot markers oscillate around 1 suggesting that a policy that focuses only on the biggest banks is not effective.

In order to understand why the $T_B$ policy does not perform better than the random policy, we plot the probability that a bank $i$ with size $A_i$ defaults before the occurrence of contagion in the right panel of Figure 9 and find that big banks have a smaller chance of failing before contagion occurs. This implies that allocating capital requirements to only these banks is likely to be ineffective in the context of this model.
Fig. 9: Left panel: Stability impact of \( T_B \) policy relative to the random policy for a system with heterogeneous bank sizes. Dotted line: comparison basis i.e. \( R = 1 \). The \( T_B \) policy appears to be ineffective relative to the random policy. Right panel: Probability that a bank \( i \) with size \( A_i \) (shown in log-scale) defaults before contagion occurs. The biggest banks have a greater chance of defaulting before the occurrence of contagion.

4.1.2. Diversification versus size

In the previous section, we simplified the model in order to separate the impact of diversification and size. However, it is also interesting to ask which of the two factors namely diversification and size is the more significant factor for capital requirement policies. In order to facilitate this comparison, we introduce heterogeneity into the degrees and sizes of the banks. The diversification based policy we consider assigns capital requirements to banks based on their degrees such that banks with higher degrees are required to hold lesser capital i.e.

\[
\epsilon_i = \frac{1}{k_i} \sum_i \frac{1}{k_i} \chi
\]  

(6)

Where, \( k_i \) denotes the degree of bank \( i \). While the size based policy allocates capital requirements to banks based on the size of their balance sheets such that big banks are required to hold more capital i.e.

\[
\epsilon_i = \frac{A_i}{\sum_i A_i} \chi
\]  

(7)

Where, \( A_i \) denotes the size of bank \( i \). Consequently, we compute the initial capital \( E_0^i \) of bank \( i \) as \( E_0^i = E_0^i + \epsilon_i \). In Figure 10, we compare the stability impact of a diversification based policy relative to a size based policy by computing the ratio \( R \) of their respective contagion probabilities as a function of \( \mu_b \) such that \( R = 1 \) implies similar performance, \( R > 1 \) means the diversification based policy supersedes the size based policy and \( R < 1 \) implies that the size based policy outperforms the diversification based policy. The figure suggests that assigning capital based on a
bank’s degree supersedes assignment based on size further confirming recent findings reported by [16].

Fig. 10: Stability impact of policy based on diversification relative to policy based on size as a function of $\mu_b$ for a system with heterogeneous sizes and degrees. Using banks’ diversification levels as a proxy for assigning capital requirements is superior to using bank sizes.

4.2. Non-capital based policy

From a policy maker’s perspective, it is interesting to ask if there is a network structure that improves systemic stability without imposing new capital requirements (see [42] for example)? We address this question by introducing some structural correlation into the bipartite network. In the subsequent paragraphs, we use the term “assortative network” for a bipartite network in which the most diversified banks hold the most widely held (i.e. concentrated) assets and “disassortative network” for one in which the most specialised banks hold the most widely held assets while the most diversified banks hold the least held assets. The correlated networks are generated based on the algorithm proposed in [37]. The procedure essentially involves minimising a network cost function until a stationary state using Monte Carlo simulations. This cost function is defined as:

$$H(G) = -\frac{J}{2} \sum_{i,j=1}^{N} a_{ij} k_i k_j$$

Where, $k_i = \sum_j a_{ij}$ and $J$ denotes a control parameter for tuning the level of assortativity i.e. $J < 0 (J > 0)$ gives a disassortative (assortative) network respectively.
while $J = 0$ produces an uncorrelated network.

![Graph](image)

**Fig. 11:** Left Panel: Contagion probability as a function of $\mu_b$ for different network correlation configurations subject to the initial failure of a random bank. Blue squares: Uncorrelated network. Red circles: Assortative network. Green diamonds: disassortative network. The disassortative network gives the most stable configuration, while the assortative network results in the most unstable system. Right Panel: Contagion probability as a function of $\mu_b$ for different network correlation configurations. Again, the disassortative network gives the most stable configuration.

In the left panel of *Figure 11*, we study the resilience of the system as a function of $\mu_b$ for the different network configurations for the case when a random bank fails. The right panel shows the same plot but for the case when a random asset is devalued. In both cases, we find that the disassortative network produces the most stable configuration. This is so because in a disassortative network, assets with high degree are held by the most fragile banks (i.e., banks with low degrees, that are less diversified). This implies that fire sales impact on the asset prices resulting from the default of any of these fragile banks would be minimal. However, in the assortative network, assets with low degrees are held by these fragile banks, which implies that the fire sales resulting from their default would be much more severe thus leading to a wider unstable region. This result raises a question of whether it is possible to implement a structure of incentives that makes the bipartite network disassortative? For instance, such a scheme is proposed by [38] for reducing the build up of systemic risk in the financial system.

### 5. Conclusion

Previous studies on overlapping portfolios have relied on the assumption of homogeneity in the degrees and sizes of banks, however, empirical findings show that real
financial networks deviate from this assumption \cite{23, 11, 8, 34, 19}. In particular, they provide evidence that bank degrees and sizes follow power law distributions. In our work, we considered the model recently introduced in \cite{15} and studied the effect of these features. This approach makes it possible to study the aggregate risk contribution of different types of banks with varying degrees and sizes. We found that separately introducing heterogeneity into the degrees and sizes of the banks widens the unstable region relative to the homogeneous case with respect to the initial failure of a random bank but not with respect to targeted shocks. In contrast, heterogeneity in asset concentrations makes the system more resilient to random shocks but not with respect to targeted shocks.

Based on these intuitions, we proceeded to study possible capital policy models. Our findings suggest that a regulatory capital policy that assigns capital requirements to the most specialised banks performs better than random capital assignments when the network connectivity is high. However, focusing capital requirements on only the biggest bank does not appear to be effective relative to random assignments within the context of the model. Furthermore, we investigated the relevance of using diversification or size in building the capital based policies and find that the diversification based policy outperforms the size based policy with increasing network connectivity.

We then proposed a non-capital based policy that improves financial stability by introducing structural correlation into the bipartite network. Our results suggest that disassortative mixing (i.e. connecting the most specialised banks with the most concentrated assets) improves the resilience of the system. This can be understood from the fact that the fire sales impact of the specialised banks is significantly reduced due to the smaller quantity of traded shares relative to the entire volume of the assets.

In an ongoing work, we plan to break away from the mechanistic stress test models used in this paper and consider a more realistic agent based model in which negative externalities from overlapping portfolios endogenously evolve. This way we can implement measures to disincentive banks from structuring their portfolios in a manner that increases the fragility of the system.

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References


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Opeoluwa Banwo, Fabio Caccioli, Paul Harrald, Francesca Medda


[43] Upper, C., Simulation methods to assess the danger of contagion in interbank markets,
THE EFFECT OF HETEROGENEITY ON FINANCIAL CONTAGION DUE TO OVERLAPPING PORTFOLIOS
