Quantum Computing with an Always-On Heisenberg Interaction

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Many promising schemes for quantum computing (QC) involve switching "on" and "off" a physical coupling between qubits. This may prove extremely difficult to achieve experimentally. Here we show that systems with a *constant* Heisenberg coupling can be employed for QC if we actively "tune" the transition energies of individual qubits. Moreover, we can *collectively* tune the qubits to obtain an exceptionally simple scheme: computations are controlled via a single "switch" of only six settings. Our schemes are applicable to a wide range of physical implementations, from excitons and spins in quantum dots through to bulk magnets.

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Quantum computing could in principle be performed by a one-dimensional array of simple systems, such as single electron spins, coupled via the Heisenberg ("exchange") interaction [1–3]. Elegant schemes exist whereby this interaction alone generates all the *gates*, or elementary operations on qubits, required for computation [4,5]. It is also known that it can suffice to control the qubits collectively [6]. However, all these schemes require the experimentalist to control the magnitude of the Heisenberg interaction, effectively to be able to switch it on and off. A typical idea for achieving this is to somehow dynamically manipulate the wave function overlap between neighboring qubits. This appears feasible, but *highly* challenging. Recently Zhou *et al.* [7] have explored a possible means of avoiding this switching. They observe that the Heisenberg interaction can be effectively negated by inserting EPR spin pairs between the qubits in a (necessarily) two-dimensional architecture. The approach is conceptually rather beautiful, but from a practical point of view it is complex in terms of the physical arrangement of qubits, the initialization, and the steps involved in generating gates. Here we take an entirely different approach and demonstrate that an "always-on" interaction can suffice even in a generic one-dimensional array. Our gate procedure is very simple and can support additional features in suitable systems: the entire device can be controlled *without* local manipulation of any kind, and the Zeno effect can be harnessed to reduce errors.

For convenience of exposition, we will use the terms "spin" and "Zeeman energy" to refer to our generic two-state systems and their level splitting. Consider a linear chain of N spins, with a Hamiltonian $\hat{H} = \hat{H}_{\text{Zeeman}} + \hat{H}_{\text{int}}$ where

$$\hat{H}_{\text{Zeeman}} = \sum_{i=1}^{N} E_i(t)\hat{\sigma}_i^Z, \qquad \hat{H}_{\text{int}} = J\sum_{i=1}^{N-1} \underline{\hat{\sigma}}_i \cdot \underline{\hat{\sigma}}_{i+1}.$$

Here $\hbar = 1$ and subscript i denotes an operator acting in the subspace of the ith qubit. $\{\hat{\sigma}^X, \hat{\sigma}^Y, \hat{\sigma}^Z\}$ are the Pauli matrices, and $\underline{\hat{\sigma}} \equiv \underline{i}\hat{\sigma}^X + \underline{j}\hat{\sigma}^Y + \underline{k}\hat{\sigma}^Z$. Zeeman energies E_i may vary with time, but the interaction couples all nearest neighbors with a common magnitude and is *constant*. We exploit the well-known observation that when the Zeeman energies vary to the extent that $|E_i - E_{i+1}| \gg J$, then the interaction tends to an *effective* Ising form [8]: $\hat{H}_{\text{int}} \approx J\sum \hat{\sigma}_i^Z \hat{\sigma}_{i+1}^Z$.

The choice of scheme for performing quantum computing (QC) on such systems depends on the available experimental abilities:

(1) Ability for universal single-qubit gates.—Suppose that a mechanism exists whereby general rotations of individual spins can be performed (essentially the same physical starting point assumed by Zhou *et al* [7]). If such rotations are extremely fast, then there is an immediate solution [Fig. 1(a)]. Alternatively, Fig. 1(b) shows the approach when fast tuning of Zeeman energies is possible [9], but the additional manipulation(s) used to compose universal single-qubit gates are not rapid. We separate the qubits to negate their continuous residual Ising interaction; a two-qubit gate must then involve temporarily altering the pattern of Zeeman energies. Consider a section of the array with the initial Zeeman pattern *BABAB* and containing two qubits represented by the states of the *A* spins [as the five leftmost spins in Fig. 1(b)]. Referring to these spins by the numbers $1, \ldots, 5$, assume that the outer spins 1 and 5 are in state $|\uparrow\rangle$, and that the central spin 3 is $|\downarrow\rangle$. We will show that a gate can be achieved by tuning *only* the Zeeman energy of spin 3, which we will denote $\epsilon(t)$. Since the Zeeman energies of spins 1 and 2 will remain far out of resonance, the interaction between them will remain of the form $J\hat{\sigma}_1^Z\hat{\sigma}_2^Z$. Then spin 1 remains in state $|\uparrow\rangle$ throughout, effectively producing a shift of $+J$ to the Zeeman energy of spin 2. The same holds for spins 4 and 5, so we can describe the nontrivial

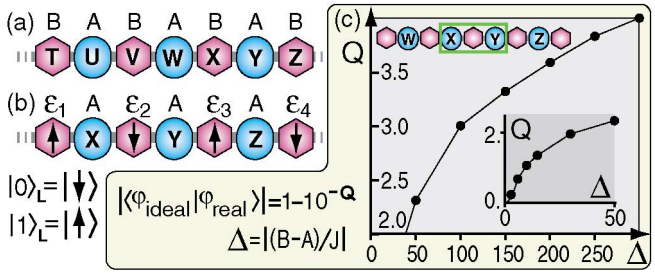

FIG. 1 (color online). Strategies for implementing QC on a 1D chain. Blocks represent individual spins and qubits are denoted by letters $T, U, \ldots, Z$. Fixed Zeeman energies are denoted by A and B, tunable Zeeman energies by ϵ_i. Here we assume an independent mechanism exists for single qubit gates. If these are much faster than $1/J$, then one simply adopts the trivial scheme shown in (a). Qubits are placed on adjacent cells and will suffer continuous phase gates with their neighbors, but techniques developed for NMR QC [10] can be employed to actively negate this via fast rotations. However, universal single qubit gates in solid state systems are usually slow [4] compared to $1/J$. In this case we would choose to place qubits only on alternate spins (b), with intervening spins in definite classical states. The Ising interaction is then entirely negated, and two-qubit operations are achieved by "tuning" a spin's Zeeman energy, as described in the text. Part (c) displays data from a numerical analysis of the process: a gate on X and Y is achieved by tuning ϵ_2 in the nine spin section shown. Plots show worst case defects over the 16 possible basis states. Phase noise (not shown) was always smaller.

dynamics of the five spins via a three spin Hamiltonian

$$\hat{H} = (A + J)(\hat{\sigma}_2^Z + \hat{\sigma}_4^Z) + \epsilon(t)\hat{\sigma}_3 + J(\underline{\hat{\sigma}}_2.\underline{\hat{\sigma}}_3 + \underline{\hat{\sigma}}_3.\underline{\hat{\sigma}}_4).$$

Suppose that at $t = 0$ we move abruptly from the passive state $\epsilon = B$ to the perfectly resonant case $\epsilon = (A + J)$. Then the two qubits X and Y will "spread" over all three cells. However, at a time $t_r = \hbar/(6J)$, spin 3 returns to its $|\downarrow\rangle$ state and an unitary transformation $\hat{G}$ is achieved between X and Y. In their computational basis $\{|00\rangle_{24}, |01\rangle_{24}, |10\rangle_{24}, |11\rangle_{24}\}$, we find [11]

$$\hat{G} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & W & i\sqrt{3}W & 0 \\ 0 & i\sqrt{3}W & W & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $W = \frac{1}{2}e^{i\pi/3}$. This is an entangling gate and it is simple to use established formalisms [12,13] to generate a CNOT using four applications of $\hat{G}$. Therefore tuning the Zeeman energy of "barrier" spins is adequate, in combination with single-qubit gates, to efficiently implement quantum algorithms. In this scheme and the following ones, barrier spin initialization can be achieved by relaxing to the spin-polarized ground state followed by selective spin rotations, either via local gates or frequency selective global pulses.

(2) No single-qubit ability.—Suppose that we cannot perform general rotations on individual spins (we can *only* tune their Zeeman energies).We then adopt the architecture shown in Fig. 2(a). The passive state of the device now has a sequence of Zeeman energies $ABCABC\ldots$ with $C - B \gg J$ and $B - A \gg J$. (Other patterns such as $ABABAB\ldots$ may still suffice [14], but ABC is convenient for the purpose of exposition.) Qubit representation is as specified in Fig. 2(a), and the mechanism for single-qubit gates is illustrated in Fig. 2(b). Fortuitously this encoding constitutes a subspace that protects against long wavelength phase noise. The one- and two-qubit gates defined in Fig. 2 respect the constraint that there is exactly one $|\downarrow\rangle$ spin among the three associated with each qubit.

To perform a two-qubit gate, we allow a qubit to spread onto the barrier spin as shown in Fig. 2(c). Our complete process must of course return the barrier back to

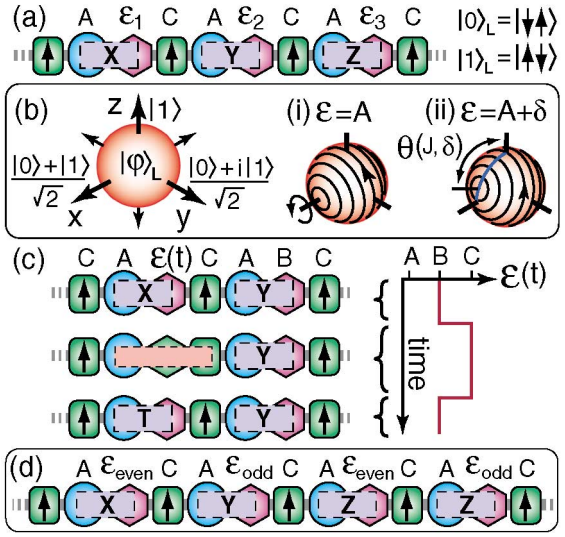

FIG. 2 (color online). Strategies for implementing QC when no independent mechanism for single qubit gates exists; these are now synthesised via Zeeman tuning alone. This requires an encoding (a) of two spins per qubit, with a third acting as a barrier. All gates can be implemented purely by tuning the Zeeman energy of the qubit that initially has energy $\epsilon = B$. For single qubit gates: If we abruptly tune this energy to $\epsilon = A$, perfectly matching the energy of its neighbor, then the logical qubit represented by this pair will experience [11] a continuous rotation given by $\exp(ikt\hat{\sigma}_x)$ as shown in (b)(i). Tuning to an energy $\epsilon = A + \delta$ will produce a rotation about an axis in the z-x plane, the axis being determined by the ratio J to δ, as depicted in (b)(ii). Such rotations can synthesize any one-qubit gate. For a two-qubit gate we employ the process shown in (c): we set the energy ϵ of our tunable spin to a value near C. This effectively allows qubit X to "spread" onto the barrier spin, where it experiences a conditional phase gate due to the proximity Y. Part (d) shows an architecture for the case where Zeeman energies cannot be tuned independently for nearby spins. QC can still be achieved, by collectively tuning one of the two subsets with energies ϵ_{even} and ϵ_{odd}.

the definite state $|\uparrow\rangle$. By modeling a four-spin section $A\epsilon CA$, we find that this can be achieved by choosing $\epsilon = C + J$: a suitable "revival" of the barrier spin then occurs at time $t_r = (\pi/\sqrt{5})(\hbar/J)$. It is easy to show that the resulting unitary transformation $\hat{K}$, along with two suitable single-qubit gates [14], generates the transformation

$$\hat{M} = (\hat{Q}1 \otimes \hat{Q}2) \cdot \hat{K} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\pi/\sqrt{5}} \end{pmatrix}$$

in the basis of qubits X and Y. We can then use established formalisms [12] to generate a CNOT gate using two applications of $\hat{M}$. We later note that this gate has some advantages over the one employed in our 1st architecture.

(3) No ability for local gates.—Until now we have assumed that the experimentalist can tune the spins *independently* from one to another. Even this requirement can be dispensed with, using a variant of the method defined in Ref. [6] (a descendant of Lloyd's global control scheme [15]). Consider the architecture of the previous section in which one in every three spins is "tunable." Now notionally divide those spins into two groups, the "odd" and "even" groups, in an alternating pattern [Fig. 2(d)]. Introduce the dramatic simplification that all spins within the odd group have the same energy ϵ_{odd}, and similarly for the even group. Now suppose that we permit ourselves to tune ϵ_{even}, ϵ_{odd} through a sequence of values *always* respecting the constraint $|\epsilon_{\text{odd}} - C| \gg J$. By the results of the previous section, we know that this will allow us to perform any single-qubit gates on the corresponding *odd* and *even* qubits, and to produce our phase gate $\hat{M}$ between each *even* qubit and the *odd* qubit to its right. This process, together with the complementary process (under constraint $|\epsilon_{\text{even}} - C| \gg J$), meets the fundamental conditions in Ref. [6]. In this way we can immediately translate the protocol defined there to the present scheme. Universal QC (including error correction [16]) on our entire multiqubit device is thus governed via *global* experimental parameters: ϵ_{odd} and ϵ_{even}. Moreover, these parameters need only assume certain fixed values, given that the duration is a continuous variable: for one-qubit gates, A and $A + J$ (say), and for a two-qubit gate, $C + J$. It follows that only six specific pairs of values for ϵ_{even},ϵ_{odd} suffice (Fig. 3 left).

We have used the term "global" even though the process does require differentiation on the local scale between the regular sets of even and odd spins (e.g., via a single electrode running the length of the device, patterned at the local scale [6]). But even this requirement can be dispensed with in suitable systems, to yield *pure* global control. The necessary physical property is illustrated on the right side of Fig. 3: the Zeeman energies of certain classes of spin must intersect as some external

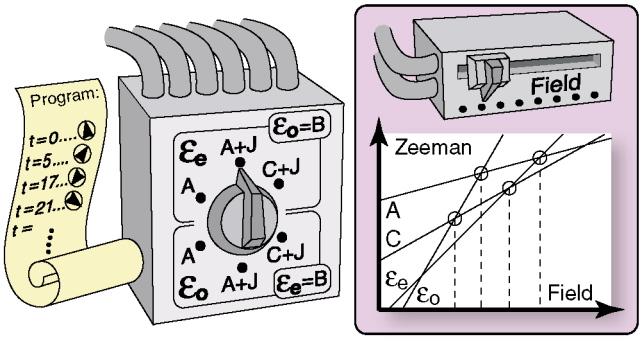

FIG. 3 (color online). Left: Cartoon emphasizing that, in our 3rd architecture, the entire computation on all qubits can be implemented simply by switching between six settings. In practice such switching would be of course be performed by a conventional computer (not manually), and moreover one would require additional setting(s) for measurement. Right: in certain physical systems, the switch can be equivalent to adjusting just a *single* global parameter.

parameter (typically a field strength) is swept. Since multiple Zeeman energies then change simultaneously, unwanted phase shifts would occur during two-qubit operations, but these can be compensated for in subsequent steps.

To meet the ultimate goal of full scale QC, one must suppress all operational error rates sufficiently for them to be handled by general error correction protocols [16]. We now review the potential error sources.

(i) Imperfectly localized tuning of Zeeman energies: In reality nearby spins may be effected to some degree. Fortunately all our schemes are very robust against this effect. The two-qubit gates rely on being able to bring two spins into resonance, which is possible even if the second spin is experiencing a small tuning effect. At worst one would simply generate an easily corrected phase shift.

(ii) Imperfect gate operations: All QC proposals inevitably demand exquisite control of their physical gate processes (to within error correction thresholds). For our schemes this means precise timing of the spin resonance periods. An advantage of our approach is that there is a simple tactic to make this goal more achievable. We can "put to work" our redundant barrier spins via the quantum Zeno effect [17]. If we repeatedly collapse the state of the barrier spins to their $|\uparrow\rangle$, $|\downarrow\rangle$ basis, on a time scale short compared to the rate at which they would accumulate errors, then we can actually *suppress* that accumulation. Note that we use the term "collapse" rather than "measurement" to emphasize that the phenomenon does not require one to detect the outcome. For maximum efficiency the process should be performed simultaneously for all barrier spins. The process fails if a spin ever collapses to the "wrong" state, but the *total* probability of such an event vanishes with increasing frequency of collapse. Therefore the ideal would be to collapse the barrier spin wave functions after *each* gate operation (although never during an operation of course). We

emphasize that this exploitation of the Zeno effect is not a requirement of our schemes. We are merely observing that *if* the phenomenon is supported by a physical system, then we can make good use of it. In cases where it is not possible, then it may be desirable to choose the type of two-qubit gate employed in schemes 2 and 3, an imperfect operation there would not generate three-qubit correlations.

(iii) Irregularity in physical separations, interaction strengths, or susceptibility to Zeeman tuning: these could be tackled by "calibrating" the system and tailoring the set of Zeeman shifts to each spin. Again the form of two-qubit gate in architecture 2 is robust, since this does not rely on a symmetry of two interactions. However, the nonlocal addressing in scheme 3 cannot accommodate inhomogeneities in this way, therefore it is only suited to very regular structures (e.g., periodic molecular systems, or atomically accurate quantum dot arrays).

(iv) Finite value of Δ: the importance of this energy ratio is shown in Fig. 1(c). For large values of Δ the gate is nearly perfect, but the fidelity falls with Δ and below 10 it rapidly becomes unusable, except perhaps for initial "proof in principle" experiments. Therefore ideal physical implementations will be those in which strong tuning of the Zeeman energy is possible.

We will now highlight a few realizations. It is natural to first consider "true" spin systems, e.g., single electron arrays. These are often discussed as potential quantum computers; typical proposals involve a mechanism for switching the interaction and a second independent mechanism for performing single-qubit gates. Our schemes allow one to dispense with the former and retain only the latter. To exploit the Zeno effect, one could employ the Pauli blockade phenomenon: a suitable optical pulse can *conditionally* create an exciton (a bound electron-hole pair) in the region of a preexisting electron (the qubit) *depending* on its state. A previous QC proposal makes sophisticated use of this idea [18] but here we exploit it very crudely: merely by allowing the exciton to dissipatively decay (or to relax), one would indirectly collapse the state of the electron spin.

Our schemes are also relevant to a different class of system that operates (and decoheres) on a far more rapid time scale: pure exciton computing. In typical exciton QC schemes the up/down pseudospin states are the presence/absence of an exciton on a quantum dot (QD), thus our Zeeman energy would correspond to the exciton creation energy. This could be tuned either by shifting the exciton localization between regions of different band gap (somewhat analogously to Ref. [3]) or via the quantum confined Stark effect. The latter is expected [19] to be very strong in double-dot structures: of order 100 meV for achievable fields. Both the dc and ac Stark effects are relevant: the latter could permit "all-optical" control. Moreover, the Stark effect is seen in many other quantum systems (including molecular structures) and could allow them to be similarly exploited. Furthermore, since the creation energy is nonzero at zero field, the Stark effect could in principle support the "pure" global switching illustrated in the right side of Fig. 3. Exciton systems can also provide sufficiently rapid wave function collapse for our Zeno exploitation, e.g., via a laser tuned to generate an *excited* exciton state with rapid (picosecond) intraband relaxation.

Thus it appears that several of the phenomena associated with excitonic systems may be well suited to our purposes. Looking beyond such systems, we speculate that the minimal demands of our 3rd architecture may introduce the possibility of QC to new classes of system. For example, in 1D Heisenberg magnets such as $KCuF_3$ [20], the effect of coupling between two chains can be replaced by an effective inhomogeneous magnetic field on one of the chains [20]. Zeeman tuning might then be accomplished by controlling the distance and alignment of one 1D spin chain with respect to another.

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