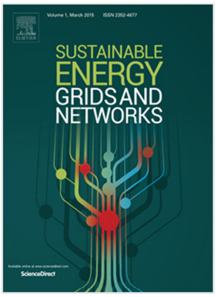

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A multi-objective framework for cost-unavailability optimisation of residential distributed energy system design

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Abstract

Future energy systems are expected to include distributed energy systems (DES) and microgrids (MG) at the distribution level. These energy efficient environments enable participating consumers to locally generate and share both electrical and thermal energy. Apart from the potential for a more cost-efficient energy system design, improved system availability is also increasingly put forward as a major advantage of MGs. This paper proposes a mixed-integer linear programming (MILP) approach for the design of a neighbourhood-based energy system, considering the trade-off between total annualised cost and electrical system unavailability. System design is optimised to meet the yearly neighbourhood energy demands by selecting technologies and interactions from a pool of dispatchable and renewable poly-generation and storage alternatives. The availability implementation

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employs a Markov chain approach combined with logic-gate integer programming. The Pareto trade-off sets of on- and off-grid MG modes are obtained using a weighted-sum approach. The developed model is subsequently applied to an Australian case-study. The sought after trade-off "knee" points for each Pareto curve are hereby identified. Additionally, through comparing on- and off-grid design trade-offs, the need for component redundancy for systems with islanding capabilities is analysed.

Keywords: dependability, distributed generation, logic-gate integer programming, markov state space analysis, microgrid, mixed-integer linear programming

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1. Introduction

1.1. Background

Residential distributed energy systems (DES) are gaining increasing interest as a solution for challenges affecting traditional top-down energy systems [1–3]. Conventionally, electricity is generated in large centralised power plants to be transmitted and distributed to consumers in the grid [2, 3]. This conventional system faces challenges with regard to growing global energy needs, emissions and the need for alternative energy resources [4]. DES have the potential to increase system efficiency and reduce emissions through strategic energy-integrated design. Residential DES refer to a residential area that has the option to install distributed generation units (DG), storage units and local energy sharing of heating, cooling and electricity [5, 6]. DG units refer to small-scale units located close to end-consumers at the distribution level in the grid [5, 6]. A small system where energy can be locally generated through DG units and shared among participants organised through a central control unit is defined as a microgrid (MG) − if predominantly electricity based −, or, a DES more generally. MGs introduce various potential benefits to end-consumers of which increased electrical system dependability is often highlighted [1, 7, 8].

In order for MGs to emerge on a wide-spread scale, a cost effective, efficient and dependable energy system design is required. This paper presents a generic optimisation-based decision-making approach to assess the relative benefit in terms of cost and electrical system availability of a small residential energy system. This potentially conflicting trade-off is especially interesting in low voltage MG systems since local energy generation and integration can offer increased electrical availability within low voltage distribution systems that are responsible for over 90 % of end-consumer interruptions [9].

1.2. Availability as an attribute of dependability

Distributed energy resource planning problems are inherently multi-objective (MO) since they involve many stakeholder interests, often conflicting, that need to be considered and traded off [10]. Apart from system cost, system dependability is of major importance in DES. A dependable system allows trusting the services it is supposed to deliver [11]. An analysis of the dependability of a system entails the research of a wide range of aspects [11–

19]. The two most employed attributes to measure system dependability are availability and reliability, which serve different purposes highlighted by their definitions [11–19];

Availability is the probability that a system is employable at a certain time t, i.e the readiness for correct service. Availability measures the dependability of repairable systems. Unavailability is its complement.

Reliability is the probability that a system works correctly over a certain time interval Δt, provided it worked correctly at the start of this interval. Reliability is mostly employed for irreparable or continuously operating systems. The complement of reliability is unreliability.

Availability is chosen as measure since residential DES are: (i) non-critical in operation in contrast with, e.g. continuous critical processes [20], (ii) readily maintainable and repairable within reasonable time frames [21], and (iii) expected to work at a certain time t, i.e. consumers expect the light to go on when flicking a switch. Availability refers here to the probability of a unit to provide (full) power to the load at any time t [21].

1.3. Determining availability

Electrical system availability is typically expressed through so-called "nines" [22]. Central grid availability, for example, can range from 3-nines (99.9 %) to 6-nines (99.9999 %). This indicates the hours throughout a year the component or system is available. Availability thus directly relates to the system up and down times, determined by failures and outages [19]. Failure rates are time dependent, typically presented through a "bathtub" curve with three failure periods over a component's life: early, useful and end of life [14, 15, 19, 22]. This paper assumes components to be in their useful life. The latter implies a constant probability of component failure or repair over a certain time period, i.e. constant component failure, ϕ, and repair, μ, rates or frequencies [19, 23].

System availability used to be assessed through deterministic criteria [9, 24]. Deterministic approaches, however, are not able to grasp stochastic failure behaviour. Therefore, probabilistic techniques have gained increasing interest. Two major groups can be distinguished. *Analytical approaches* assess state probabilities and indices through mathematical relations based on statistical component failure data [2, 3, 9]. The most common techniques are the Series-Parallel Reduction/Block Diagram Method, Event Tree Analysis, State Space Markov methods and Tie/Cut Set methods [14, 19, 25, 26]. In terms of specific probabilistic system indices, System Average Interruption Duration Index (SAIDI) is most commonly used for electrical system availability. SAIDI is a measure for the percentage duration of an outage and is often employed as a measure for conventional network availability [2, 3, 9, 13, 24, 26, 27]. *Simulation based techniques*, in contrast, determine expected values of indices and system state probabilities by averaging the results obtained through individual simulations. Simulation techniques are especially appropriate for large and complex systems.

Since DES are small, their components are quantifiable through probabilistic data and their energy sharing capabilities prevent reduction to series or parallel component connections, a Markov State Space analytical approach is taken to determine steady-state system availabilities (see Section 4.3).

1.4. Multi-objective optimisation of distributed energy systems

1.4.1. Key concepts

Multi-objective (MO) methods try to obtain a solution for a problem with several aims. In contrast with single-objective optimisation, no single solution can be found but rather a set of optimal solutions. The concept of dominance determines the relative importance of this obtained set of solutions [10]. The aim of MO optimisation is to construct a trade-off curve of non-dominated solutions between objectives (Pareto curve) or a set of solutions on this curve (Pareto set) [28]. Finding a Pareto set to a problem can be through either "classical approaches", such as the weighted-sum and the $\epsilon-$constraint methods, or through approaches based on evolutionary algorithms [10]. The aim is to find the "best" trade-off between criteria, which is a subjective decision. The "best" point will often be at a "knee-point" where a bigger return on an objective is achieved before the "knee" than after [28].

MO decision-making is increasingly adopted for DES design [10]. The considered objectives in DES planning problems can be classified under three themes, i.e. financial, environmental and technical [10]. DES planning problems are inherently non-convex combinatorial problems with complexity in terms of decision variables and equations. Linearising DES behaviour and simplifying assumptions enables the use of linear and mixed-integer linear programming (MILP), allowing for efficient optimisation of complex problems with a high degree of variables [10, 29]. MILP models are thus frequently used for complex DES problems. Within DES MO optimisation, cost minimisation is the most common objective, increasingly combined with an environmental criteria [10, 30–36]. If at all considered, technical criteria mostly relate to energy loss minimisation [31, 32, 34, 35].

1.4.2. Availability in DES optimisation

Dependability evaluation of DES has not received as much attention in literature as other potential DES benefits [21, 24, 32]. Modelling and optimisation of DES design dependability as a technical criterion can be divided in three major categories: *a posteriori* assessment, as indirect design objective/constraint, or, as direct design objective.

Most research regards a posteriori determination of the availability or reliability of a known system without optimising the system. Several alternative system topologies are selected and then compared regarding cost, reliability and/or availability when a trade-off or "optimal design" has to be selected. A model for electrical and thermal reliabilities of a known dispatchable combined heat and power (CHP) system, for example, was introduced by Haghifam *et al.* [27], employing a frequency-balance discrete State Space Markov process. A similar analysis was conducted for a building cooling, heating and power system by Wang *et al.* [37].

Within optimisation models, availability or reliability is often indirectly addressed as a design constraint. Ren *et al.* [38], for example, presented a linear model for the optimal operation strategy of a DES minimising energy cost and CO_2 emissions. Equipment availability was here integrated through an availability factor, placing an upper bound on energy generation. A planning strategy for DG units within electrical power systems was presented by Zangeneh *et al.* [39], employing a normal boundary intersection algorithm with four cost-related objectives. The cost of energy not supplied and availability factors were here the dependability measures.

Availability and reliability are technical objectives but have not been used explicitly, including system and component states, within superstructure DES design optimisation. They have, in contrast, been used as objectives in the context of selecting the optimal number of redundant identical components in generic networks. Fiori de Castro *et al.* [23], for example, suggested a genetic algorithm to maximise availability of a series engineering system configuration. An evolutionary optimisation approach to maximise redundancy availability in a generic parallel/series system was, additionally, suggested by Ratle *et al.* [40]. Within the application of DES, research is limited. Frangopoulos and Dimopoulos [41] analysed the effect of reliability for the optimal synthesis, design and operation in the selection of a number of generic cogeneration units through a genetic algorithm. Each system state probability, obtained through a Markov State Space approach, served to analyse expected cost and energy values. A MO planning tool with financial and technical objectives, such as reliability, was developed by Yassami *et al.* [42]. Reliability was, however, integrated as a cost through customer damage functions. Singh and Goswami [43], in their turn, proposed a MO genetic algorithm for optimal planning of DG units in terms of siting and sizing. The overall objective was formulated as a multi-objective performance index employing weighted indices for reliability of service, efficiency and power quality. Only overall system reliability performance indices were used, such as SAIDI. A strategic technology-policy framework for distributed energy resource allocation under a technical, financial and environmental objective was presented by Mallikarjun and Lewis [33] using Data Envelope Analysis and Goal Programming. A reliability factor was employed as technical objective. Lastly, a recent body of research looks at component sizing for the detailed electrical design of hybrid energy systems while minimising both cost and an energy supply reliability measure, such as the loss of power supply probability or expected energy not served, through genetic algorithms or particle swarm optimisation [44–48].

1.5. Contributions of this work

Within superstructure DES optimisation, residential energy integrated systems in terms of electricity (through MG operation) as well as heating and cooling (through optimised pipeline networks) are a relative new area of research. This paper builds on the previously developed superstructure model by the authors, which involved an MILP total annualised cost minimisation of residential DES design [30, 49, 50]. This paper adds a second, technical objective. The aim is to identify the “best” neighbourhood electrical design as a trade-off between (i) total annualised cost to meet the total energy demand in the neighbourhood, and (ii) the average house electrical system unavailability, for on- and off-grid modes of the system. The specific contributions of this work are:

- developing a bi-objective economic-technical framework for fully energy integrated DES design whereby the technical objective is modelled as neighbourhood electrical system unavailability while explicitly taking into account different house- and neighbourhood-based electrical system configurations and the state of their components;
- combining logic-gate operation and discrete absorbing Markov chains with integer programming within a superstructure MILP framework to model availability;
- and a South Australian based case-study, a State with a high potential for residential DES due to favourable climatic conditions and remote load centres.

The methodology is detailed in Section 2. Section 3 illustrates the model equations and constraints. The researched case-study and required input data are presented in Section 4. The results are discussed in Section 5 to conclude in Section 6.

2. Methodology

2.1. Problem description

The energy system design of a small neighbourhood is optimised in terms of selection of technologies and interactions, their locations and capacities. A superstructure approach is employed; each component is a black-box with power/energy in- and out-flows. Although thermal energy supply is considered (see Appendix A.1), electrical supply is the focus here. The black-box diagram of the electrical supply alternatives and interactions of each neighbourhood house are given in Figure 1.

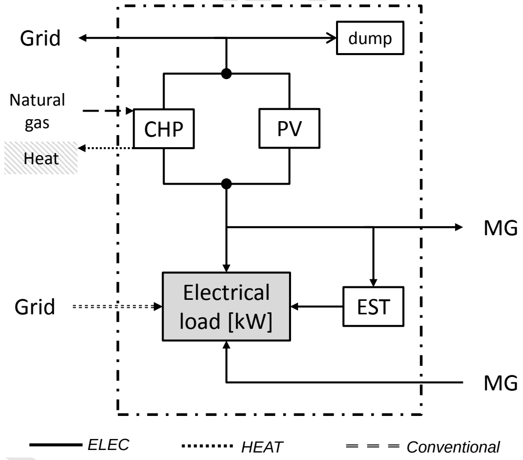


Figure 1: Black-box diagram of the considered generation and supply alternatives of each household in the neighbourhood to meet electricity demands, adapted from [30]. Note that the CHP unit is the coupling between the electrical and thermal supply. CHP=combined heat and power unit, PV=photovoltaic unit, EST=electrical storage unit, MG=microgrid, dump=dump load.

The conceptual diagram, Figure 2, illustrates the steps in determining the unavailability related equations. First, the total unavailability values of the electrical components are obtained. Second, potential component combinations, available to supply the electrical load of an individual house, are determined, i.e. potential household electrical system configurations. A discrete homogeneous Markov chain is subsequently constructed for each system configuration to obtain its steady-state unavailability. These system unavailabilities are model inputs. System configurations are then implemented through the use of logic-gate operations and binary integer programming. The model optimises average house electrical system unavailability as a combination of implemented system configurations of the different neighbourhood houses. The optimised neighbourhood design will thus implement one of the considered system configurations in each house. Additionally, for a technology to be considered available, it might require a minimum installed capacity, which introduces capacity constraints.

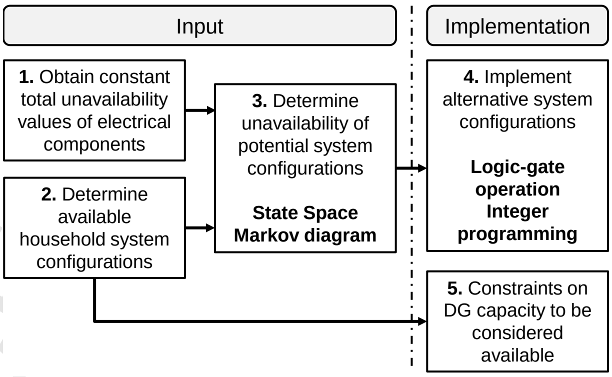


Figure 2: Conceptual diagram of unavailability implementation.

2.2. Model requirements and assumptions

The model is developed as an MILP in GAMS and solved with CPLEX to global optimality [51]. A typical day (24 hours) in each season over a yearly planning horizon is adopted. The *inputs* are:

- area specific climatological data (sunshine);
- specifications of the considered technologies;
- investment and operation and maintenance (OM) costs as well as utility energy tariffs;
- regulation in terms of governmental support schemes and upper bounds on installed DG unit capacities; and,
- spatial distributions of hourly average electricity, heating and cooling demands for each house in the neighbourhood.

The *outputs* are (i) total annualised cost; (ii) average house electrical system unavailability; and (iii) optimal system design in terms of selection, siting and sizing of units and interactions.

Technologies are assumed to have constant energy conversion efficiencies and no ramp-up and ramp-down times [30]. Furthermore, MG operation is assumed to be installed in a neighbourhood with an existing electrical infrastructure. The assumptions with respect to availability are [52]:

- only electrical systems are under availability optimisation;
- all components are in their useful life;

- steady-state availability assessment is made, no dynamic processes, such as relay switching, are considered. Instead, potential system configurations are determined;
- no fault occurs within repair intervals;
- no common-mode failures are considered;
- neither cold standby nor switching are included; and,
- components are independent in terms of failure and repair.

3. Cost − unavailability model

3.1. Optimisation problem

Neighbourhood energy system design is optimised by minimising the scaled total annualised energy cost, $C^{TOT,S}$ [kAUD y^{-1}], and the average house electrical system unavailability in the neighbourhood $UA^{TOT,S}$:

$$\min_{x,y,z} \begin{cases} C^{TOT,S} \\ UA^{TOT,S} \end{cases} \tag{1}$$

where x represent the technology options, y their capacity ranges and z their location in the neighbourhood. The objective function is constructed as a weighted-sum with factor $\lambda_c \in [0, 1]$ of the scaled objectives:

$$\min_{x,y,z}[\lambda_c \cdot C^{TOT,S} + (1 - \lambda_c) \cdot UA^{TOT,S}] \tag{2}$$

The cost, $C^{TOT,S}$, and unavailability, $UA^{TOT,S}$, objectives are detailed in Sections 3.1.1 and 3.1.2, respectively. The objective function is bound by:

- technology design and operational constraints of the DG units (Appendix A, Equations A.7 to A.13), the thermal energy technologies (Appendix A, Equations A.14 to A.17) and the electrical and thermal storage technologies (Appendix A, Equations A.18 to A.26). Technology decision variables are the selection of technology type $tech$ in house i through binary variable $B_{tech,i}$, installed technology capacity in house i ($DG^{MAX}_{tech,i} \in [L_{tech}, U_{tech}]$), and the hourly operational energy streams to and from each installed unit;
- hot and cold pipeline design and operational constraints (Appendix A, Equations A.27 to A.35). The pipeline decision variables include pipeline existence from house i to house j determined by binary variable $YP_{i,j}$, the order of houses in a network determined through integer variable OH_i, and energy transfers from houses i to houses j in hour h of season s, $QH_{i,j,s,h}$;
- electricity interaction constraints between the neighbourhood houses and the central grid (Appendix A, Equations A.36 to A.37);
- microgrid electricity sharing constraints between neighbourhood houses (Appendix A, Equations A.38 to A.44) where microgrid sharing is enabled through binary variable Z and hourly electricity transfer by DG units in houses i in season s is determined ($PE^{CIRC}_{techDG,i,s,h}$);
- hourly neighbourhood energy balance constraints to ensure that the

energy generated in the neighbourhood balances the neighbourhood demand of electricity (Appendix A, Equation A.45), heating (Appendix A, Equation A.46) and cooling (Appendix A, Equation A.47);

- capacity constraints to determine the supply availability of each component as 100% available or 100% unavailable (Equations 6 to 9 and Appendix A, Equations A.48 to A.53);
- and house electrical system configuration constraints, Equations in Table 1.

Below the objectives and availability constraints are summarised.

3.1.1. Cost

Total annualised cost, C^{TOT} [AUD y^{-1}], combines annualised investment, $C^{INV}_{i,tech}$, fixed and variable OM, $C^{OM}_{i,tech}$, and annual fuel costs, $C^{FUEL}_{i,tech}$, of technologies *tech* installed in houses i. Also, the annual cost of purchasing electricity from the central grid by house i, $C^{GRID}_{BUY,i}$, the carbon tax imposed on each household, C^{CT}_{i}, and potential household incomes through governmental subsidies, such as feed-in tariffs, through residential electricity export, $C^{GRID}_{SELL,i}$, are included:

$$C^{TOT} = \sum_{i,tech} (C^{INV}_{i,tech} + C^{OM}_{i,tech} + C^{FUEL}_{i,tech}) + \sum_{i} (C^{GRID}_{BUY,i} + C^{CT}_{i} - C^{GRID}_{SELL,i}) \quad (3)$$

The total cost is scaled ($C^{TOT,S}$) to kAUD. The terms of the cost objective function are detailed in Appendix A, Equations A.1 to A.6.

3.1.2. Unavailability

Average house electrical system unavailability $UA^{TOT,S}$ in a neighbourhood is determined by the sum of the system unavailability of each house i, UA_i, divided by the total number of houses in the neighbourhood, n_h:

$$UA^{TOT,S} = \frac{\sum_i UA_i}{n_h} \tag{4}$$

Individual household electrical system unavailability is determined through a parallel connection (sum) of unavailability values of potential system configurations [14, 15, 19]. Each potential household electrical system configuration, con, is represented by a binary decision variable, $B_{con,i}$, and a constant system unavailability, ua_{con}. The latter is obtained through a Markov chain (see Section 4.3). Note that potential household system configurations are mutually exclusive, since only one configuration can be adopted in each house. Mutual exclusivity is ensured through the AND-NOT configuration implementation, see Section 3.3 and Table 1. Neighbourhood average system unavailability hence optimises the combination of household system configurations. A logarithmic transformation of obtained unavailability inputs is employed to bring objectives within similar range and to indirectly measure unavailability as *availability* through a number of “nines”:

$$UA_i = \sum_{con} B_{con,i} \cdot log_{10}(ua_{con}) \qquad \forall i \tag{5}$$

3.2. Capacity constraints

Potential house electrical system configurations are each a combination of available electricity generating technologies to a house, i.e. a CHP unit, a PV unit, a battery, a MG connection fed by CHP units in other houses, and a potential grid connection. Each component is characterised by a total unavailability, which is a combined measure of the component supply unavailability (function of its nominal capacity), the unavailability level of its required resource input (e.g. renewable energy or natural gas unavailability) and the unavailability of its technical component, see Section 4.1 [27].

The electrical supply probability of a component (part of its total availability) is thus a function of its installed capacity. A first analysis is conducted in this work with one availability−capacity step rather than a gradual relation between both (see Section 5.2). Installed units consequently require a minimum nominal capacity to be considered available to supply the load of their accommodating house. A lower capacity is allowed but the corresponding unit is then considered unavailable. In the first instance, two discrete nominal capacity levels are thus allowed for each installed technology in a house, unavailable and 100 % available for its accommodating house. The latter is a capacity of a single unit, able to fully meet the peak load of its accommodating house in each hour. The former combines all unavailable and reduced available capacity values of this unit for its accommodating house into one unavailable level.

For an installed PV unit or battery to supply their accommodating house, their installed capacity ($DG^{MAX}_{tech,i}$) should be greater or equal than a threshold capacity, $T^{av}_{tech,i}$ [m^2 or kWh]. Their capacity can thus fall within one of two categories, characterised by binary variables $B_{tech,i}$ (installed and unavailable) and $B^{av}_{tech,i}$ (installed and 100% available), respectively. Total installed technology capacity should additionally fall within bounds [L_{tech};U_{tech}]. With *tech*, PV units or batteries:

$$\begin{aligned} & L_{tech} \cdot B_{tech,i} + T^{av}_{tech,i} \cdot B^{av}_{tech,i} \leq DG^{MAX}_{tech,i} \\ & DG^{MAX}_{tech,i} \leq T^{av}_{tech,i} \cdot B_{tech,i} + U_{tech} \cdot B^{av}_{tech,i} \qquad \forall i \end{aligned} \tag{6}$$

$$B_{tech,i} + B^{av}_{tech,i} \leq 1 \qquad \forall i \tag{7}$$

Note that PV units are only considered available to supply the load of their accommodating house, not to supply the whole neighbourhood through MG sharing. CHP units, in contrast, can perform the different tasks of (i) meeting the electricity load of their accommodating house, and (ii) meeting the electricity demand of the whole neighbourhood through MG sharing. Their installed electrical capacity, $DG^{MAX}_{CHP,i}$ [kW], should fall within bounds [L_{CHP}, U_{CHP}]. Depending on the task of the CHP unit, its capacity should be at least equal to threshold capacities $T^{av}_{CHP,i}$ [kW] (available for its accommodating house) or $T^{av}_{CHPmg,i}$ [kW] (available for the MG), respectively. This characterises three CHP capacity categories, unavailable ($B_{CHP,i}$), 100% available for house i ($B^{av}_{CHP,i}$) and 100% available for MG operation ($B^{av}_{MG,tech,i}$). These categories are represented by three binary variables CHP_i^A, CHP_i^B and CHP_i^C to impose alternative upper and lower bounds on installed CHP capacity:

$$DG_{CHP,i}^{MAX} \leq T_{CHP,i}^{av} \cdot CHP_i^A + T_{CHPmg,i}^{av} \cdot CHP_i^B + U_{CHP} \cdot CHP_i^C \qquad \forall i \quad (8)$$

$$L_{CHP} \cdot CHP_i^A + T_{CHP,i}^{av} \cdot CHP_i^B + T_{CHPmg,i}^{av} \cdot CHP_i^C \leq DG_{CHP,i}^{MAX} \qquad \forall i \quad (9)$$

These three mutually exclusive binary variables each represent a combination of the three CHP capacity categories (AND ($\wedge$) – NOT ($\overline{B}$) gate), see Appendix A.

3.3. Electrical system configurations: logic-gate operation

Potential household system configurations are each characterised by a binary variable of which its value is determined through an AND–NOT relation between all the binary variables (enabled, or, disabled (NOT)) of individually considered available components to each house. Different component combinations can in this way be represented by a series of ones and zeros, which enables (“switching on”) and disables (“switching off”) the implementation of components to represent different household system configurations. System configurations are thus feasible combinations of the five individually available components to each house i, i.e. a grid connection, a CHP unit, a PV unit, a battery and an operational MG with a number of MG-available CHP units in houses j (with $i \neq j$) $\in [0, n_h - 1]$. Each house can thus have one of 2^5 possible component combinations, i.e. system configurations, including the option of no installed components. Only certain combinations are, however, feasible, see Table 1. An appropriately sized battery, for example, is only considered available together with an appropriately sized (available) PV unit or CHP unit in the same house. Note that authorised islanding is assumed. Without authorised islanding, installed DG units have to be switched off in case of central system outages, limiting DES redundancy advantages.

Binary variables of some of the considered components and logic-gate operation linearisation are clarified in Appendix A. Each household (i) system configuration can then be modelled as an AND–NOT gate of combinations of considered individual components, see Table 1. The house configuration with an available CHP ($B^{av}_{CHP,i}$) and available grid connection (GC_i), for example, is then:

$$XC_i = GC_i \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}} \qquad \forall i \qquad (10)$$

AND–NOT expressions of the considered system configurations are given in Table 1. An AND-gate represents a product of binary variables and has been linearised using the procedure presented in [53, 54]. A NOT-gate inverts its binary input.

4. Case-study: A South Australian neighbourhood

Australia has potential for DES to reduce investment in long transmission lines to cover the extended distances between load centres. Moreover,

Table 1: AND–NOT expressions of potential electrical system configurations for house i. Each expression is equal to a binary variable $B_{con,i}$. GR=grid, CHP=combined heat and power unit, PV=photovoltaic unit, MG=microgrid, EST=battery.

Technology combination	AND–NOT expression $\forall i, k$
GR	$GC_i \wedge \overline{B^{av}_{CHP,i}} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}}$
CHP	$\overline{GC_i} \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}}$
PV	$\overline{GC_i} \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}}$
MG	$\overline{GC_i} \wedge \overline{B^{av}_{CHP,i}} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
GR and CHP	$GC_i \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}}$
GR and PV	$GC_i \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}}$
GR and MG	$GC_i \wedge \overline{B^{av}_{CHP,i}} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
CHP and PV	$\overline{GC_i} \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}}$
CHP and EST	$\overline{GC_i} \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge B^{av}_{EST,i} \wedge \sum_k \overline{MGA_{i,k}}$
CHP and MG	$\overline{GC_i} \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
PV and EST	$\overline{GC_i} \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge \sum_k \overline{MGA_{i,k}}$
PV and MG	$\overline{GC_i} \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
GR, CHP and PV	$GC_i \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge \sum_k \overline{MGA_{i,k}}$
GR, CHP and EST	$GC_i \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge B^{av}_{EST,i} \wedge \sum_k \overline{MGA_{i,k}}$
GR, CHP and MG	$GC_i \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
GR, PV and EST	$GC_i \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge \sum_k \overline{MGA_{i,k}}$
GR, PV and MG	$GC_i \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
CHP, PV and EST	$\overline{GC_i} \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge \sum_k \overline{MGA_{i,k}}$
CHP, PV and MG	$\overline{X_i} \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
CHP, EST and MG	$\overline{GC_i} \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge B^{av}_{EST,i} \wedge MGA_{i,k}$
PV, EST and MG	$\overline{GC_i} \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge MGA_{i,k}$
CHP, PV, EST and GR	$GC_i \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge \sum_k \overline{MGA_{i,k}}$
GR, CHP, PV and MG	$GC_i \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge \overline{B^{av}_{EST,i}} \wedge MGA_{i,k}$
GR, CHP, EST and MG	$GC_i \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{PV,i}} \wedge B^{av}_{EST,i} \wedge MGA_{i,k}$
GR, PV, EST and MG	$GC_i \wedge \overline{B^{av}_{CHP,i}} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge MGA_{i,k}$
CHP, PV, EST and MG	$\overline{GC_i} \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge MGA_{i,k}$
CHP, PV, EST, GR and MG	$GC_i \wedge B^{av}_{CHP,i} \wedge B^{av}_{PV,i} \wedge B^{av}_{EST,i} \wedge MGA_{i,k}$

Nonfeasible technology combinations:
EST, (GR and EST), (MG and EST), (GR, MG and EST), and,
no installed technologies.

South Australia has a high level of renewable energy resources, such as solar irradiance, worth exploiting on a residential scale. The researched small fictitious neighbourhood consists of five average houses. Its lay-out together with the total yearly energy demands of each house are given in Figure 3. Input data are detailed in [30]. The following Section details the availability related inputs.

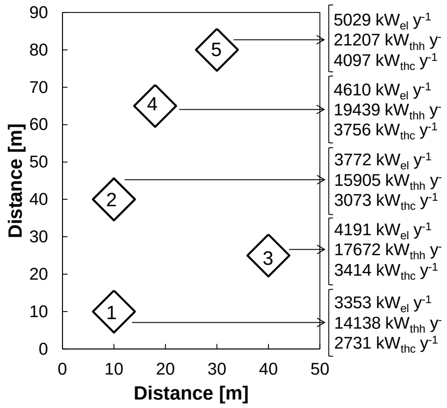


Figure 3: Distance [m] between each pair of households [55] as well as the yearly energy demands of each house in terms of electricity (el), heating (thh) and cooling (thc) [kWh y^{-1}], adapted from [30].

4.1. Total component availability

Each component — i.e. the CHP, PV and battery — has a total unavailability (UA_{tech}^{tot}), obtained as a series relation of its resource availability (A_{tech}^{res}), its component availability (A_{tech}^{com}) and its supply availability (A_{tech}^{sup}) [27]:

$$\begin{aligned} UA_{tech}^{tot} &= 1 - (A_{tech}^{res} \cdot A_{tech}^{com} \cdot A_{tech}^{sup}) \\ &= 1 - (1 - UA_{tech}^{res}) \cdot (1 - UA_{tech}^{com}) \cdot (1 - UA_{tech}^{sup}) \qquad (11) \end{aligned}$$

Resource availability of CHP units relates to natural gas supply availability. PV unit resource availability relates to the average hourly probability of available solar irradiation in each hour to meet the load in that hour. Battery resource availability is based on its state of charge [56]. The latter is determined by the availability of a PV and/or CHP unit in the same house that can charge an available battery in hour h to be able to sustain battery discharge during autonomy time. Battery autonomy time refers to the hours or days it can fully meet the load if fully charged [56]. For a PV or CHP unit to be able to charge the battery for full autonomy discharge, an installed capacity is assumed that not only allows them to meet their household peak load in hour h but also charge the battery in that hour h (worst case). Battery resource availability in house i is thus either the probability of an appropriately sized available CHP unit in house i, an appropriately sized available PV unit in house i, or, both appropriately sized available CHP and PV units in house i

Component unavailability refers to the unavailability of the component to perform, based on the state of its internal mechanical and electrical parts. Component *supply availability* relates to the probability that the component can supply the load in each hour throughout the year, dependent on its installed capacity or state of charge [56]. In this work, discrete supply availability steps are employed (see Section 5.2), i.e. 100 % available or unavailable, based on nominal capacity thresholds. Total component *availability* values are presented in Table 2.

Table 2: Component availabilities [%]. CHP=combined heat and power unit, EST=battery, MGCC=MG central control unit, PV=photovoltaic unit. Solar availability is determined by the average probability that the available sun in an hour can meet the load in that hour with data from [30].

	Components		
Availability [%]	CHP	PV	EST
Technical	96.0000 % [27]	99.9990 % [57]	99.9967 [57]
Supply	100 %	100 %	100 %
Resource	99.9975% [21]	22.2489 %	95.9976 % (CHP) or 22.2487 % (PV) or 96.8881 % (PV and CHP)
Total a_{tech}	95.9976 %	22.2487 %	95.9944 % (CHP) or 22.2479 % (PV) or 96.8849 % (PV and CHP)

Apart from technologies, each house can also have available electrical supply through a grid connection or a connection with the MG fed by a certain number of CHP units. Central grid unavailability in South Australia is 0.060%, i.e. availability of 3 nines (SAIDI) [58, 59]. $(UA^{tot}_{CHP})^k$ is the total unavailability of k CHPs available for MG operation to a house. A MG is however only available together with an available control unit with 0.0200% component unavailability (UA^{tot}_{MGCC}) [21]. Total MG unavailability ($UA^{tot}_{MG,k}$) then becomes:

$$UA^{tot}_{MG,k} = 1 - (1 - UA^{tot}_{MGCC}) \cdot (1 - (UA^{tot}_{CHP})^k) \qquad \forall k \tag{12}$$

4.2. Threshold capacities

To be available to supply the household electrical load, component threshold capacities are set to the peak hourly accommodating household electricity load for available PV and CHP units, and to the peak hourly neighbourhood electricity load for a MG-available CHP unit. The battery threshold capacity is based on being able to supply the average hourly electricity demand of its accommodating house for a certain autonomy time (on-grid: 2 hours, off-grid: 2 days [60, 61]). Note that electrical threshold demands include electricity demand for both appliances and cooling through air-conditioning units (maximum electricity demand of each house).

4.3. House system configurations

For each potential house electrical system configuration, a Markov State Space diagram is constructed to determine its system (un)availability [27]. The State Space diagram and system (un)availability of a house configuration with a CHP unit and a grid connection is illustrated in Figure 4 for illustration. The mathematical Markov model describes a time related random process in which a system moves between defined states through step-wise transitions [11, 17, 25]. After a certain number of time steps, the steady-state state probabilities are obtained. System states are based on the status of the system components. State transitions therefore occur due to component failure and repair rates. Since discrete system states and constant steady-state state transitions − based on constant failure and repair rates − are employed, a homogeneous discrete Markov chain is constructed. In Figure 4, for example, each of the states represents a combination of up or down conditions of the system components. The probability of the system being in state s, P_s, can be obtained by multiplying the steady-state (asymptotic) total (un)availabilities of the different components in the state as indicated in Equation 13. The system availability, a_{xc}, and unavailability, ua_{xc}, can then be found as the OR-gate (sum) of the probability of being in available and unavailable system states respectively. If both components are individually able to meet the operational requirement of the system, states 1, 2 and 3 are considered available.

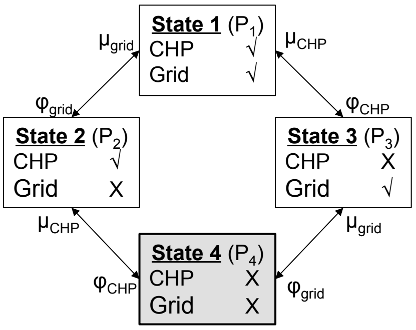


Figure 4: State Space diagram of the system available state grid-CHP (XC). The state transitions occur through constant failure, ϕ, and repair, μ, rates.

$$\begin{gathered} P_1 = a_{CHP} \cdot a_{grid} \qquad P_2 = a_{CHP} \cdot ua_{grid} \\ P_3 = ua_{CHP} \cdot a_{grid} \qquad P_4 = ua_{CHP} \cdot ua_{grid} \\ a_{xc} = P_1 + P_2 + P_3 = 1 - P_4 = 1 - ua_{xc} \end{gathered} \tag{13}$$

4.4. Case-studies

The model presented in Section 3 is solved to obtain Pareto sets for two modes: on-grid (no restrictions on presented model) and off-grid (through pre-restricting the import and export capability of each house). This allows to assess the “knee-point” designs for both configurations and the need for component redundancy for systems with islanding capability.

5. Results and discussion

5.1. *Cost-availability trade-off*

The presented model is solved to global optimality for λ_c decreasing from 1 to 0 in discrete steps to obtain an appropriate Pareto set covering a range of optimal solutions. Figure 5 compares both trade-offs.

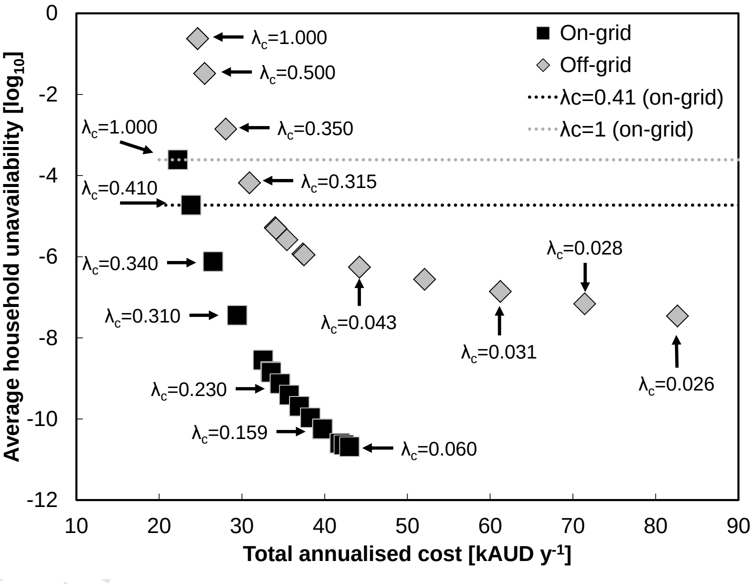
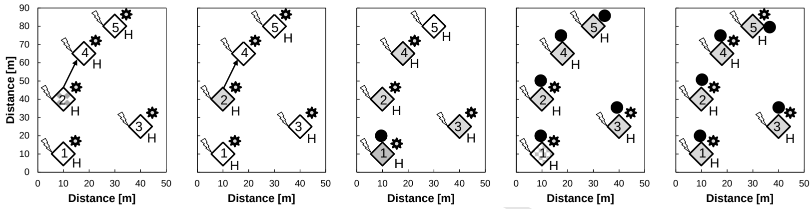


Figure 5: Pareto set of the trade-off between the average household electrical unavailability in the neighbourhood [log] (nines) with total annualised cost [kAUD y^{-1}] for on- and off-grid modes of the neighbourhood.

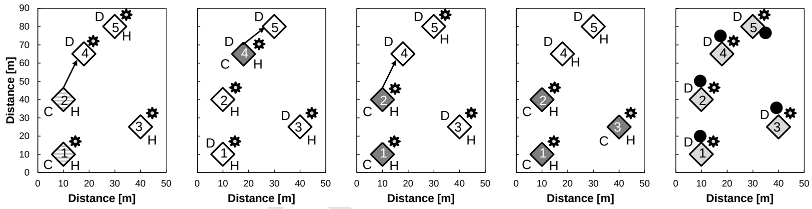
Average house system availability increases from about 3 to 10 nines (on-grid) and about 0 to 7 nines (off-grid) when traded off with cost. On-grid designs dominate off-grid designs. Both trade-offs present a significant drop in unavailability with relative small cost increase from the first ($\lambda_c = 1.000$) to the the second point thereafter (on-grid: $\lambda_c = 0.410$, off-grid: $\lambda_c = 0.500$). The relative availability increase is here in both cases of the order of one nine combined with a relatively small cost increase of 7.2 % and 3.4 %, for the on- and off-grid modes, respectively. The latter designs are most favourable in the trade-off decision-making, i.e. the sought-after "knee-points" (largest gradient).

For both modes, this represents a design change (see Figure 6). In the on-grid mode, the capacity of the installed CHP unit increases from available for the house in which it is installed to available for the MG ($\lambda_c = 0.410$). In the off-grid mode, two smaller household-available CHP units are replaced by a single MG-available unit ($\lambda_c = 0.500$).

In between the illustrated designs (see Figure 6 and Table 3), the transition is more gradual with an increasing number of MG-available CHP units or batteries. Available PV units are installed in all on-grid houses until $\lambda_c = 0.230$. From this point, batteries start to appear. The combination of PV and CHP charging sources for batteries, combined with an increasing number of batteries in the neighbourhood, leads from here on to a more gradual trade-off. In the discussed off-grid points, cost still dominates, which makes it cheaper to dump excess electricity rather than invest in batteries. Additionally, there is a larger focus on dispatchable generation through CHP units. This leads to a heat generation surplus, which is mostly used for household cooling generation with absorption chillers and limited heat transfer to other houses. Discrete jumps between Pareto points occur due to the discrete relationship between unavailability and unit capacity (see Section 5.2).



(a) On-grid, $\lambda_c = 1.000$ (b) On-grid, $\lambda_c = 0.410$ (c) On-grid, $\lambda_c = 0.230$ (d) On-grid, $\lambda_c = 0.159$ (e) On-grid, $\lambda_c = 0.060$



(f) Off-grid, $\lambda_c = 1.000$ (g) Off-grid, $\lambda_c = 0.500$ (h) Off-grid, $\lambda_c = 0.350$ (i) Off-grid, $\lambda_c = 0.315$ (j) Off-grid, $\lambda_c = 0.026$

Figure 6: Major electrical system design changes for on- and off-grid modes of the neighbourhood for several values of λ_c: diamonds=houses, grey hatched house=available CHP for house and airco, light grey diamond=available CHP for MG and airco, light grey horizontally striped=available CHP for household and AC, dark grey diamond=available CHP for MG and AC, lightning=grid connection, sun=PV, black circle=battery, black arrow=heating pipeline, H=heat storage, C=cold storage, D=dump load, AC=absorption chiller.

Since switching to islanding, i.e. disconnecting from the grid, is not taken into account, comparing both on- and off-grid design trade-offs provides an illustration of the need for component redundancy in the transition from on-grid to islanding. The dashed lines in Figure 5 highlight the availability levels of the first two *on-grid* designs points. To obtain a similar availability level

Table 3: Total neighbourhood unit capacity [kW or kW^{-1}] for on- and off-grid designs at various λ_c levels for cost-unavailability trade-off. PV=photovoltaic unit, CHP=combined-heat and power unit, AC=absorption chiller, B=boiler, airco=air-conditioning unit, EST=battery, HST=heat storage, CST=cold storage, D=dump load. MG operation is always adopted.

	PV	CHP	AC	B	airco	EST	HST	CST	D
On-grid									
$\lambda_c = 1.000$	10.5	2.1	–	26.1	10.8	–	5.4	–	–
$\lambda_c = 0.410$	10.5	8.0	–	25.0	10.8	–	4.9	–	–
$\lambda_c = 0.230$	10.0	34.1	–	7.6	10.8	2.1	20.4	–	–
$\lambda_c = 0.159$	13.5	45.4	–	–	10.8	13.5	11.1	–	–
$\lambda_c = 0.060$	25.0	52.2	–	–	10.8	13.5	5.5	–	–
Off-grid									
$\lambda_c = 1.000$	6.6	5.2	3.6	20.5	7.4	–	31.3	8.4	4.1
$\lambda_c = 0.500$	7.8	8.0	2.3	24.3	8.1	–	32.0	4.9	4.3
$\lambda_c = 0.350$	6.4	16.1	3.5	20.5	7.4	–	31.6	7.8	3.9
$\lambda_c = 0.315$	5.4	24.1	5.6	14.6	5.1	–	30.2	16.9	5.2
$\lambda_c = 0.043$	15.6	42.1	–	–	10.8	33.4	20.5	–	10.3
$\lambda_c = 0.026$	32.5	52.2	–	–	10.8	208.8	–	–	16.6

in the off-grid mode to the “best” on-grid configuration, i.e. an availability level around 4 nines (between dashed lines in Figure 5), three MG-availability CHP units would be required (off-grid: $\lambda_c = 0.315$), compared with one in the on-grid configuration. This requires a cost increase of about 30 % compared to on-grid to ensure component redundancy and system availability when allowing the system to island.

5.2. Discussion on capacity intervals

Different nominal capacity-availability implementations can be considered, either a single discrete step as adopted here (single step in Figure 7), more refined steps, or, a continuous relation. The current leads to discrete jumps in solutions throughout the Pareto sets. In practice, the probability that an installed component with certain capacity cannot supply the load in each hour throughout the year (supply unavailability) comprises a more gradual relation with decreasing installed capacity. The shape of this curve can be determined by a load model for each house (household level technologies) and for the neighbourhood (microgrid-available technologies), such as a load duration curve (LDC) of the hourly demand profiles. A LDC represents each hour by its peak demand [kW] [25, 62, 63]. These hourly peak demands are then rearranged in descending order. Combining this load relation with a certain installed generation capacity enables to assess the number of hours throughout the year a certain demand level will exceed a generation capacity level, i.e. loss of load indices. Figure 8 illustrates both a more realistic and a simplified linearised load duration curve, adapted from [25, 62, 63]. For a certain installed generation unit capacity level (CL), the hourly load can exceed installed capacity for a certain number of hours (t). Dividing this number of hours of load loss throughout the year with the total number of hours in a year results in the probability that the unit cannot supply the load (supply unavailability). From the point where the installed capacity is able to meet the load at each time t (plus a potential reserve margin RM), the supply availability becomes 100 %. Figure 7 translates the simplified linearised load duration curve into a relation between supply availability and unit capacity level. Note that implementing a relationship between supply availability and cost implies implementing supply availability as a variable, which might introduce non-linearities in the model through variable multiplications that can be linearised.

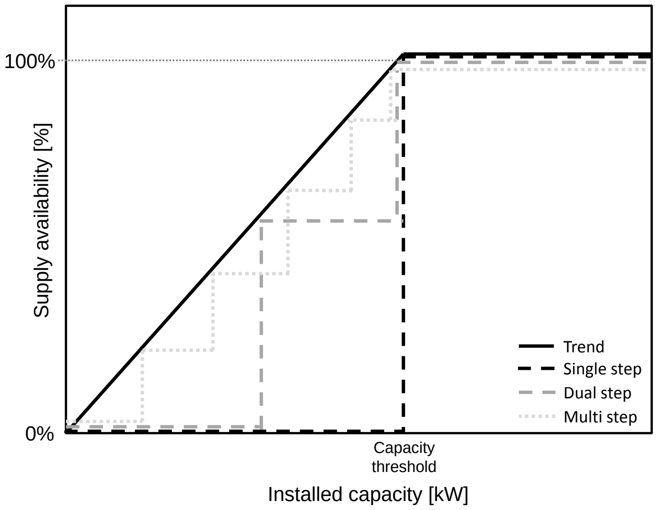
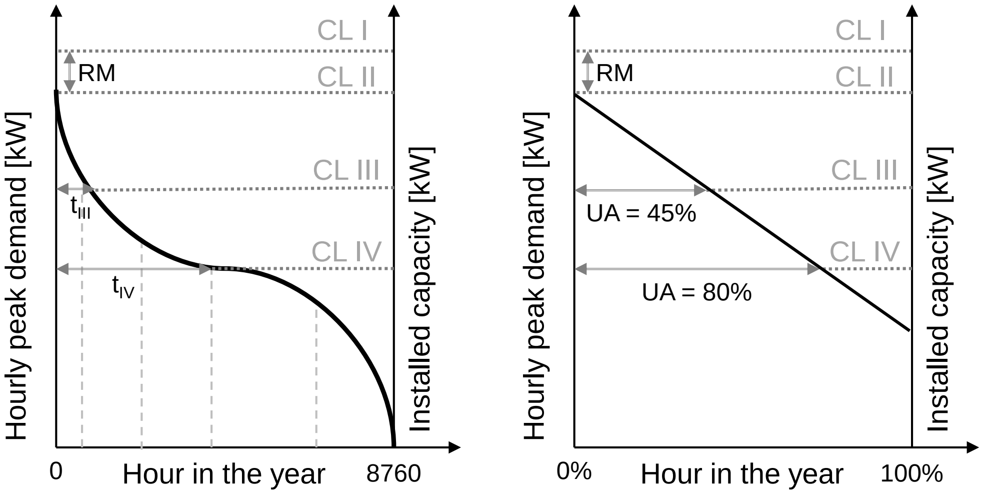


Figure 7: Illustration of capacity steps of the relation between installed capacity of technologies [kW] and electrical supply availability [%].



(a) Load duration curve

(b) Linearised load duration curve

Figure 8: Load duration curves to determine the probability that a generation unit of capacity level (CL) cannot supply the load during a certain amount of hours throughout the year (Δt), adapted from [25, 62, 63]. RM=reserve margin.

The developed availability approach does not yet take into account fully optimised capacity values since each point (often through threshold capacities) on the Pareto set in Figure 5 represents the dominant point of a range of designs where the capacity of the installed units increases (more expensive) but has not yet reached the next capacity/availability threshold. For example, the last point on each set in Figure 5 represents the situation where the maximum availability level is achieved. For $\lambda_c = 0$, however, cost is no longer an issue. The installed capacity of CHP units will therefore be maximised without an improvement in availability level. The last illustrated points on each set thus dominate any designs thereafter.

The obtained results present a relative ordering of designs. When changing capacity threshold values, the obtained trends in Figure 5 remain the same. It is only the relative spreading of the points that will either reduce (less additional cost for the next availability step) or increase (more additional cost for the next availability step) with decreasing or increasing capacity thresholds, respectively. The current capacity thresholds are a first step towards a more gradual relation between capacity and availability, similarly to that which already exists between capacity and cost.

6. Conclusion

A deterministic MILP-based decision-making strategy has been presented for the total annualised cost-electrical availability trade-off for designing a small residential distributed energy system. A neighbourhood distributed energy system design has been optimised, selecting from a pool of available energy generation and storage alternatives including a potential grid connection as well as energy integration through MG operation. A framework based on Markov chains and logic-gate integer programming has been implemented to analyse the on- and off-grid modes of an Australian based neighbourhood. The presented methodology is able to obtain a set of non-dominated Pareto solutions to identify the "best" system designs ("knee-points"). Additionally, through comparing on- and off-grid design trade-offs, the need for component redundancy for systems with islanding capabilities could be analysed. Future work will look at both refining the presented methodology in terms of the capacity–availability relation as well as to analyse the robustness of the model with regard to uncertainty of the availability input data.

Acknowledgement

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Nomenclature

Abbreviations	
A	Availability
AC	Absorption chiller
C_{ELEC}^{LOAD}	Electricity load [kW]
C	Cold storage tank
CHP	Combined heat and power unit
D/dump	Dump load
DES	Distributed energy system
DG	Distributed generation unit
ELEC/E	Electricity
EST	Electrical storage unit

GRID/GR	Central electricity supply
h	House
H	Hot storage tank
HEAT/H	Heating
LDC	Load duration curve
MG	Microgrid
MGCC	Microgrid central control unit
MILP	Mixed-integer linear programming
MO	Multi-objective
OM	Operation and maintenance
PV	Photovoltaic unit
SAIDI	System Average Interruption Duration Index
U/UA	Unavailability
Parameters	
A_{tech}^{com}	Component availability of technology *tech* [%]
A_{tech}^{res}	Resource availability of technology *tech* [%]
A_{tech}^{sup}	Supply availability of technology *tech* [%]
a_{tech}	Availability of technology *tech* [%]
L_{tech}	Lower bound on installed capacity of technology *tech* [kW or kWh]
n_h	Number of houses in the neighbourhood
P_s	Probability of a system being in state *s* [%]
$T_{tech,i}^{av}$	Threshold capacity for 100 % availability of technology *tech* in house *i* [m^2 or kWh]
$T_{CHPmg,i}^{av}$	Threshold capacity for 100 % MG-availability of technology *tech* to house *i* [m^2 or kWh]
U_{tech}	Upper bound on installed capacity of technology *tech* [kW or kWh]
ua_{con}	Unavailability of household electrical system unavailability
ua_{tech}	Unavailability of technology *tech* [%]
UA_{tech}^{com}	Component unavailability of technology *tech* [%]
UA_{tech}^{res}	Resource unavailability of technology *tech* [%]
UA_{tech}^{sup}	Supply unavailability of technology *tech* [%]
UA_{tech}^{tot}	Total unavailability of technology *tech* [%]
λ_c	Unity weighting factor
μ	Constant component repair rate
ϕ	Constant component failure rate
Variables	
$B_{con,i}$	Binary variable that decides on the installation of an electrical system configuration *con* in house *i*
$B_{tech,i}$	Binary variable that decides on the installation of (unavailable) technology *tech* in house *i*

$B^{av}_{MG,tech,i}$	Binary variable that decides on the installation of MG-available technology *tech* in house i
$B^{av}_{tech,i}$	Binary variable that decides on the installation of available technology *tech* in house i
C^{CT}_{i}	Annualised carbon tax cost of houses i [AUD y^{-1}]
$C^{FUEL}_{i,tech}$	Annualised fuel cost of technologies *tech* in houses i [AUD y^{-1}]
$C^{GRID}_{BUY,i}$	Annualised grid electricity import cost of houses i [AUD y^{-1}]
$C^{GRID}_{SELL,i}$	Annualised grid electricity export income of houses i [AUD y^{-1}]
$C^{INV}_{i,tech}$	Annualised investment cost of technologies *tech* in houses i [AUD y^{-1}]
$C^{OM}_{i,tech}$	Annualised OM cost of technologies *tech* in houses i [AUD y^{-1}]
C^{TOT}	Total annualised cost [AUD y^{-1}]
$C^{TOT,S}$	Scaled total annualised cost [kAUD y^{-1}]
CHP^{A}_{i}	Binary variable that decides on the implementation of CHP capacity level
CHP^{B}_{i}	Binary variable that decides on the implementation of CHP capacity level
CHP^{C}_{i}	Binary variable that decides on the implementation of CHP capacity level
$DG^{MAX}_{tech,i}$	Total installed capacity of technology *tech* in house i [kW or kWh]
GC_{i}	Binary variable that decides on the implementation of grid connection to house i
$MGA_{i,k}$	Binary variable that decides on the implementation of an available MG with k MG-available CHP units to house i
UA_{i}	Electrical system unavailability of household i [nines]
$UA^{TOT,S}$	Scaled average household electrical system unavailability [nines]
XC_{i}	Binary variable that decides on the implementation of a household-available CHP and grid connection to house i

Appendix A. Mathematical Model

Appendix A.1. Problem description additional aspects

The full cost-model with assumptions and referenced input data can be found in [30] and is summarised here for completeness. The cost-approach continued the work of [55, 64]. Although the focus of this work is the electrical system, thermal systems are also optimised. Figure A.9 illustrates the thermal supply options for each house.

Appendix A.2. Terms of the objective function

The investment cost, C^{INV}, consists of technology (*tech*) unit costs, C^{C}_{tech}, and installed capacities, $DG^{MAX}_{tech,i}$. Installed capacities are either a constant

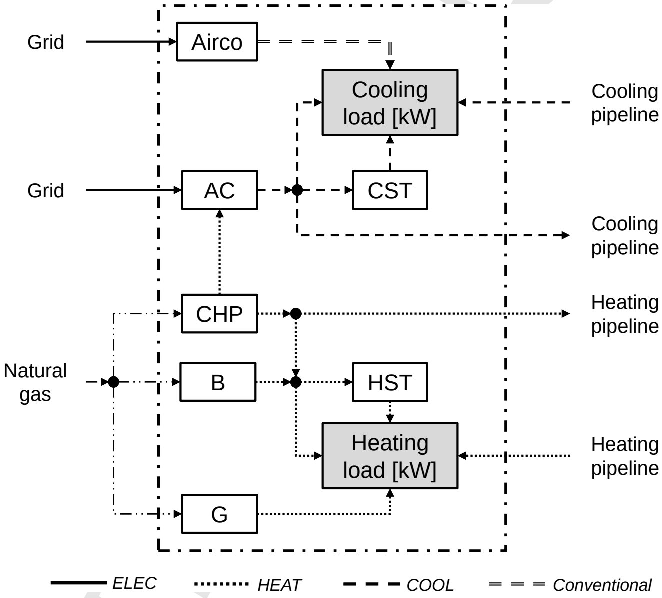

Figure A.9: Black-box diagram of the considered heating and cooling supply alternatives of each household in the neighbourhood, adapted from [30]. AC=absorption chiller, airco=air-conditioning unit, B=condensing boiler, CHP=combined-heat and power unit, G=gas heater, CST=cold thermal storage unit, HST=hot thermal storage unit, pipelines=optimised pipeline networks between pairs of households.

value combined with a binary selection variable $B_{tech,i}$ for each house i or a positive variable $DG^{MAX}_{tech,i}$. A capital recover factor, CRF_{tech}, is employed for annualisation. Thermal technologies ($techTH$), storage units ($techsto$), PV and CHP DG units ($DGtech$), pipelines ($PIPE$) and a MG central control unit ($MGCC$) are included:

$$\begin{aligned} C^{INV} = & \sum_{techTH} \sum_{i} CRF_{techTH} \cdot C^{C}_{techTH} \cdot DG^{MAX}_{techTH,i} \\ & + \sum_{techsto} \sum_{i} CRF_{techsto} \cdot C^{C}_{techsto} \cdot DG^{MAX}_{techsto,i} \\ & + \sum_{i} CRF_{DGtech} \cdot C^{C}_{DGtech} \cdot DG^{MAX}_{DGtech,i} \\ & + \sum_{i \neq j} \sum_{j} CRF_{PIPE} \cdot C^{C}_{PIPE} \cdot YP_{i,j} \cdot l_{i,j} \\ & + CRF_{MGCC} \cdot C^{C}_{MGCC} \cdot Z \end{aligned} \tag{A.1}$$

Pipelines are installed between house pairs (i,j) with length ($l_{i,j}$) and binary existence variable ($YP_{i,j}$). The MGCC is characterised by binary variable Z. The annual electricity import cost, C^{GRID}_{BUY}, consists of electricity purchased throughout the year ($PE^{GRID}_{i,s,h}$) in each hour h in each season s at electricity tariff (T_{ELEC}):

$$C^{GRID}_{BUY} = \sum_{i} \sum_{s} \sum_{h} hr \cdot d_s \cdot T_{ELEC} \cdot PE^{GRID}_{i,s,h} \tag{A.2}$$

Annual OM costs, C^{OM}, include fixed (C^{omf}_{tech}) and variable costs (C^{omv}_{tech}) based on generation ($PE^{GEN}_{tech,i,s,h}$) (with d_s the number of days in each season s). The fixed PV, battery (EST) and pipelines ($PIPE$) OM cost are included. Note that battery unit capacity is kWh compared to kW for other technologies. The PV surface variable is A^{PV}_{i} and rated capacity $Cprat$.

$$\begin{aligned}C^{OM} = &\sum_{tech}\sum_{i}\sum_{s}\sum_{h} hr \cdot d_s \cdot C^{omv}_{tech} \cdot PE^{GEN}_{tech,i,s,h} \\ &+ \sum_{i} C^{omf}_{PV} \cdot Cprat \cdot A^{PV}_{i} + \sum_{i} C^{omf}_{EST} \cdot DG^{MAX}_{EST,i} \\ &+ \sum_{i\neq j}\sum_{j} l_{i,j} \cdot C^{omf}_{PIPE} \cdot YP_{i,j} \qquad \text{(A.3)}\end{aligned}$$

Gas fuelled boilers, gas heaters ($techTH$) and CHPs have a fuel cost (C^{FUEL}) through heat ($PH^{GEN}_{techTH,i,s,h}$) or electricity ($PE^{GEN}_{CHP,i,s,h}$) generation at a gas tariff (T_{GAS}), with n^{TH}_{techTH} (n^{ELEC}_{CHP}) component thermal (electrical) efficiency:

$$\begin{aligned}C^{FUEL} = &\sum_{techTH}\sum_{i}\sum_{s}\sum_{h} hr \cdot d_s \cdot PH^{GEN}_{techTH,i,s,h} \cdot \frac{T_{GAS}}{n^{TH}_{techTH}} \\ &+ \sum_{i}\sum_{s}\sum_{h} hr \cdot d_s \cdot PE^{GEN}_{CHP,i,s,h} \cdot \frac{T_{GAS}}{n^{ELEC}_{CHP}} \qquad \text{(A.4)}\end{aligned}$$

The yearly carbon tax (CT) imposed on the neighbourhood, C^{CT}, depends on imported electricity ($PE^{GRID}_{i,s,h}$) and technology gas consumption, with CI_{ELEC} and CI_{GAS} the carbon intensities of grid electricity and natural gas respectively:

$$\begin{aligned}C^{CT} = &\sum_{i}\sum_{s}\sum_{h} CT \cdot hr \cdot d_s \cdot [CI_{ELEC} \cdot PE^{GRID}_{i,s,h} \\ &+ CI_{GAS} \cdot \sum_{techTH} \frac{PH_{techTH,i,s,h}}{n^{TH}_{techTH}} + CI_{GAS} \cdot \frac{PE_{CHP,i,s,h}}{n^{ELEC}_{CHP}}] \qquad \text{(A.5)}\end{aligned}$$

An annual income, C^{GRID}_{SELL}, can be created through local electricity export of on-site DG units $techDG$ ($PE^{SAL}_{techDG,i,s,h}$) at market feed-in tariffs (T^{SAL}_{techDG}):

$$C^{GRID}_{SELL} = \sum_{techDG}\sum_{i}\sum_{s}\sum_{h} hr \cdot d_s \cdot T^{SAL}_{techDG} \cdot PE^{SAL}_{techDG,i,s,h} \qquad \text{(A.6)}$$

Appendix A.3. Technology design and operational constraints

Appendix A.3.1. Distributed energy generation technologies

PV electricity ($PE_{PV,i,s,h}^{GEN}$) depends on solar irradiation ($It_{s,h}$) as well as a rated capacity and efficiency (n_{PV}^{ELEC}). Capacity (A_{UP}^{PV}) and daily export bounds are specified in the SA market as 10 kW and 45 kWh per day respectively [65].

$$PE_{PV,i,s,h}^{GEN} \leq \min(A_i^{PV} \cdot Cprat; A_i^{PV} \cdot It_{s,h} \cdot n_{PV}^{ELEC}) \qquad \forall i, s, h \tag{A.7}$$

Installed capacity $DG_{CHP,i}^{MAX} \in [L_{CHP}^{PE};U_{CHP}^{PE}]$ bounds CHP electricity ($PE_{CHP,i,s,h}^{GEN}$) with binary variable $B_{CHP,i}$:

$$L_{CHP}^{PE} \cdot B_{CHP,i} \leq PE_{CHP,i,s,h}^{GEN} \leq DG_{CHP,i}^{MAX} \qquad \forall i, s, h \tag{A.8}$$

$$DG_{CHP,i}^{MAX} \leq U_{CHP}^{PE} \cdot B_{CHP,i} \qquad \forall i \tag{A.9}$$

CHP waste heat is generated proportionally with electricity, i.e. Heat-to-Electricity ratio HER. This heat can be used either for heating ($PH_{CHP,i,s,h}^{HEAT}$) or cooling purposes ($PH_{CHP,i,s,h}^{COOL}$):

$$PE_{CHP,i,s,h}^{GEN} \cdot HER = PH_{CHP,i,s,h}^{HEAT} + PH_{CHP,i,s,h}^{COOL} \qquad \forall i, s, h \tag{A.10}$$

Heating purposes are: meeting the load of the accommodating house ($SELF$), storing heat (STO) or pipeline transfer ($PIPE$):

$$PH_{CHP,i,s,h}^{HEAT} = PH_{CHP,i,s,h}^{SELF} + PH_{CHP,i,s,h}^{STO} + PH_{CHP,i,s,h}^{PIPE} \qquad \forall i, s, h \tag{A.11}$$

Heat for cooling purposes is delivered to an absorption chiller ($PC_{AC,i,s,h}^{GEN}$), with a coefficient of performance COP_{AC}:

$$PC_{AC,i,s,h}^{GEN} = PH_{CHP,i,s,h}^{COOL} \cdot COP_{AC} \qquad \forall i, s, h \tag{A.12}$$

Total DG electricity, $PE^{GEN}_{techDG,i,s,h}$, can meet its house load ($SELF$), can be exported (SAL), can be MG circulated ($CIRC$) or can be stored in the battery (STO):

$$\begin{aligned} PE^{GEN}_{techDG,i,s,h} = {} & PE^{SELF}_{techDG,i,s,h} + PE^{SAL}_{techDG,i,s,h} \qquad \forall i, s, h \\ & + PE^{CIRC}_{techDG,i,s,h} + PE^{STO}_{techDG,i,s,h} \end{aligned} \tag{A.13}$$

Appendix A.3.2. Thermal energy technologies

Thermal technologies generate either heat (H) or cooling (C), $PH/C^{GEN}_{techT,i,s,h}$ limited by their installed capacity $DG^{MAX}_{techT,i} \in [L^{PH/C}_{techT};U^{PH/C}_{techT}]$ and characterised by a binary variable ($B_{techT,i}$):

$$L^{PH/C}_{techT} \cdot B_{techT,i} \leq PH/C^{GEN}_{techT,i,s,h} \leq DG^{MAX}_{techT,i} \qquad \forall i, s, h \tag{A.14}$$

$$DG^{MAX}_{techT,i} \leq U^{PH/C}_{techT} \cdot B_{techT,i} \qquad \forall i \tag{A.15}$$

Thermal power generated by boilers, $PH^{GEN}_{B,i,s,h}$, and absorption chillers (AC), $PC^{GEN}_{AC,i,s,h}$, can meet its house load ($SELF$) or can be stored (STO). ACs can also serve pipelines ($PIPE$).

$$PH^{GEN}_{B,i,s,h} = PH^{SELF}_{B,i,s,h} + PH^{STO}_{B,i,s,h} \qquad \forall i, s, h \tag{A.16}$$

$$PC^{GEN}_{AC,i,s,h} = PC^{SELF}_{AC,i,s,h} + PC^{STO}_{AC,i,s,h} + PC^{PIPE}_{AC,i,s,h} \qquad \forall i, s, h \tag{A.17}$$

A house can either have a gas heater or boiler. Heat storage can only be installed with a boiler or CHP. Furthermore, air-conditioning units can not be installed together with an AC.

Appendix A.3.3. Storage technologies

Storage is modelled based on a daily roll-over where the first hour is a function of the last hour of the day taking into account seasonal independence. Thermal stored power, $PS^{STO}_{i,s,h}$, is subject to a static loss percentage (ζ) plus an inflow ($PS^{IN}_{i,s,h}$) minus an outflow ($PS^{OUT}_{i,s,h}$) and limited by an installed capacity $DG^{MAX}_{STO,i} \in [L^{PH}_{STO};U^{PH}_{STO}]$:

$$PS^{STO}_{i,s,h} = (1-\zeta)\cdot PS^{STO}_{i,s,h-1} + PS^{IN}_{i,s,h} - PS^{OUT}_{i,s,h} \qquad \forall i,s,h \tag{A.18}$$

$$(1-\zeta)\cdot PS^{STO}_{i,s,h-1} + PS^{IN}_{i,s,h} \leq DG^{MAX}_{STO,i} \qquad \forall i,s,h \tag{A.19}$$

CHPs or boilers can store heat and ACs cooling:

$$PS^{IN}_{i,s,h} = (PH^{STO}_{B,i,s,h} + PH^{STO}_{CHP,i,s,h}) \text{ or } PC^{STO}_{AC,i,s,h} \qquad \forall i,s,h \tag{A.20}$$

Batteries are modelled similarly to thermal storage including additional charge (χ) and discharge ($\delta\chi$) rates, maximum charge ($max\chi$) and discharge rates ($max\delta\chi$), a depth of charge (DOC) and a binary decision variable, $B_{EST,i}$, with an installed capacity $DG^{MAX}_{EST,i} \in [L^{ES}_{EST};U^{ES}_{EST}]$:

$$\begin{aligned} ES^{STO}_{EST,i,s,h} = (1-\eta)\cdot ES^{STO}_{EST,i,s,h-1} + hr*(1-\chi)\cdot PS^{IN}_{EST,i,s,h} \\ - hr*\frac{PS^{OUT}_{EST,i,s,h}}{(1-\delta\chi)} \qquad \forall i,s,h \end{aligned} \tag{A.21}$$

The stored and retrieved energy is restricted by maximum charge and discharge rates related to the installed capacity:

$$hr*(1-\chi)\cdot PS^{IN}_{EST,i,s,h} \leq max\chi\cdot DG^{MAX}_{EST,i} \qquad \forall i,s,h \tag{A.22}$$

$$hr*\frac{PS^{OUT}_{EST,i,s,h}}{(1-\delta\chi)} \leq max\delta\chi\cdot DG^{MAX}_{EST,i} \qquad \forall i,s,h \tag{A.23}$$

$$L^{ES}_{EST}\cdot B_{EST,i} \leq DG^{MAX}_{EST,i} \leq U^{ES}_{EST}\cdot B_{EST,i} \qquad \forall i \tag{A.24}$$

$$(1-DOC)\cdot DG^{MAX}_{EST,i} \leq ES^{STO}_{EST,i,s,h} \qquad \forall i,s,h \tag{A.25}$$

Batteries are charged through contributions of the DG units:

$$PS^{IN}_{EST,i,s,h} = \sum_{techDG} PE^{STO}_{techDG,i,s,h} \qquad \forall i,s,h \tag{A.26}$$

Appendix A.3.4. Pipelines

No temperature differences are taken into account in the pipelines, consistent with electrical system detail excluding active and reactive power flows as well as voltage drops. Decision variable, $YP_{i,j}$, decides on the installation of a uni-directional pipe between houses i and j, with heat transfer $QH_{i,j,s,h}$ $\leq U^{PIPE}$. Heating and cooling pipelines are modelled similarly.

$$QH_{i,j,s,h} \leq U^{PIPE} \cdot YP_{i,j} \qquad \forall i,j,s,h \text{ and } i \neq j \tag{A.27}$$

$$YP_{i,j} + YP_{j,i} \leq 1 \qquad \forall i,j \text{ and } i \geq j \tag{A.28}$$

Positive integer variable, OH_i, represents the house visiting order in a pipeline network. Multiple non-closed uni-directional pipeline networks can be installed. Hence, houses i are connected to one network with strictly increasing order from the source house(s) to the sink house(s) (Equation A.29), with n_h the total amount of houses in the neighbourhood. Pipeline optimisation is based on the travelling salesman problem [64, 66]:

$$OH_j \geq OH_i + 1 - n_h \cdot (1 - YP_{i,j}) \qquad \forall i,j \text{ and } i \neq j \tag{A.29}$$

Pipeline heat ($PH^{PIPE}_{CHP,i,s,h}$) can only be supplied by CHPs and can either be transferred between houses ($QH_{i,j,s,h}$) or can meet part of the heat load of receiving houses, $QH^{LOAD}_{i,s,h}$. Thermal balances are given for all i, j, s, h where $i \neq j$:

$$PH^{PIPE}_{CHP,i,s,h} + \sum_j QH_{j,i,s,h} - QH^{LOSS}_{i,s,h} = QH^{LOAD}_{i,s,h} + \sum_j QH_{i,j,s,h} \tag{A.30}$$

$$PH^{PIPE}_{CHP,i,s,h} - \sum_i QH^{LOSS}_{i,s,h} = \sum_i QH^{LOAD}_{i,s,h} \tag{A.31}$$

Thermal losses, $QH^{LOSS}_{i,s,h}$, depend on transferred heat, the distance and a fixed loss percentage. Each house can in each hour either receive or send hot water to a pipeline, determined by binary variables $Y^{rec}_{i,s,h}$ and $Y^{snd}_{i,s,h}$ respectively:

$$Y^{rec}_{i,s,h} + Y^{snd}_{i,s,h} \leq 1 \qquad \forall i, s, h \tag{A.32}$$

An installed gas heater (binary variable $B_{G,i}$) excludes a pipeline connection. Furthermore, either CHP (binary variable $B_{CHP,i}$) or a gas heater or boiler (binary variable $B_{techTH,i}$) can be installed in a house. An installed CHP will be dimensioned to meet the heat load of that house plus potential pipeline transfer. These houses are thus assumed to either send or pass through heat to or from the pipeline network, not receive.

$$B_{CHP,i} + Y^{rec}_{i,s,h} \leq 1 \qquad \forall i, s, h \tag{A.33}$$

A maximum utilisation rate, U^{snd}, and the total heat load of the house, $C^{LOAD}_{HEAT,i,s,h}$, bound the heat sent and received from the pipe network respectively:

$$PH^{PIPE}_{CHP,i,s,h} \leq U^{snd} \cdot Y^{snd}_{i,s,h} \qquad \forall i, s, h \tag{A.34}$$

$$QH^{LOAD}_{i,s,h} \leq C^{LOAD}_{HEAT,i,s,h} \cdot Y^{rec}_{i,s,h} \qquad \forall i, s, h \tag{A.35}$$

Appendix A.4. Operational constraints

Appendix A.4.1. Grid interactions

Each house can import, $PE^{GRID}_{i,s,h}$, or export, $PE^{SAL}_{techDG,i,s,h}$, electricity from and to the central grid $\leq U^{ELEC}_{rec/snd}$, characterised by binary decision variables $X^{rec}_{i,s,h}$ and $X^{snd}_{i,s,h}$ respectively.

$$\sum_{techDG} PE^{SAL}_{techDG,i,s,h} \leq U^{ELEC}_{snd} \cdot X^{snd}_{i,s,h} \qquad \forall i, s, h \tag{A.36}$$

$$PE^{GRID}_{i,s,h} \leq U^{ELEC}_{rec} \cdot X^{rec}_{i,s,h} \qquad \forall i, s, h \tag{A.37}$$

Appendix A.4.2. Microgrid operation

The neighbourhood with installed MG (Z) will interact as a whole with the grid:

$$X^{snd}_{i,s,h} + X^{rec}_{i,s,h} \leq 1 \qquad \forall i, s, h \tag{A.38}$$

$$X^{snd/rec}_{i,s,h} - X^{snd/rec}_{i-1,s,h} \leq 1 - Z \qquad \forall i, s, h \text{ and } i > 1 \tag{A.39}$$

$$X^{snd/rec}_{i-1,s,h} - X^{snd/rec}_{i,s,h} \leq 1 - Z \qquad \forall i, s, h \text{ and } i > 1 \tag{A.40}$$

MG transfer between a pair of houses in hour h in season s is characterised by binary selection variable $MGC_{i,j,s,h}$.

$$MGC_{i,j,s,h} + MGC_{j,i,s,h} \leq Z \qquad \forall i, j, s, h \text{ and } i \neq j \tag{A.41}$$

The MG electricity balances should be respected. Here, MG electricity transfer loss depends on the transferred electricity and a constant distance related loss percentage.

$$PE^{snd}_{i,j,s,h} - PE^{LOSS}_{i,j,s,h} = PE^{rec}_{i,j,s,h} \qquad \forall i, j, s, h \text{ and } i \neq j \tag{A.42}$$

$$\sum_{techDG} \sum_{i} PE^{CIRC}_{techDG,i,s,h} - \sum_{i} \sum_{j} PE^{LOSS}_{i,j,s,h} = \sum_{i} PE^{MG}_{rec,i,s,h} \qquad \forall i, j, s, h \text{ and } i \neq j \tag{A.43}$$

Total DG electricity for MG circulation cannot exceed U^{MG}:

$$\sum_{techDG} \sum_{i} \sum_{s} \sum_{h} PE^{CIRC}_{techDG,i,s,h} \leq U^{MG} \cdot Z \qquad \forall i, j, s, h \text{ and } i \neq j \tag{A.44}$$

Appendix A.4.3. Energy balances

Electricity demands, $C^{LOAD}_{ELEC,i,s,h}$, combined with potential dump loads ($Pdl_{i,s,h}$), absorption chillers (electricity-to-cooling ratio: AC_{ELEC}) and air-conditioning units (coefficient of performance COP_{airco}) should be balanced by the consideration and combined use of the grid ($PE^{GRID}_{i,s,h}$), MG sharing ($PE^{MG}_{rec,i,s,h}$), DG generation ($PE^{SELF}_{techDG,i,s,h}$) and batteries ($PS^{OUT}_{EST,i,s,h}$):

$$\begin{aligned} C^{LOAD}_{ELEC,i,s,h} + Pdl_{i,s,h} + PC^{GEN}_{AC,i,s,h} \cdot AC_{ELEC} + \frac{PC^{GEN}_{airco,i,s,h}}{COP_{airco}} \\ = PE^{GRID}_{i,s,h} + PE^{MG}_{rec,i,s,h} + \sum_{techDG} PE^{SELF}_{techDG,i,s,h} \\ + PS^{OUT}_{EST,i,s,h} \qquad \forall i, s, h \end{aligned} \tag{A.45}$$

Heating demands, $C^{LOAD}_{HEAT,i,s,h}$ are met through either gas heaters ($PH^{GEN}_{G,i,s,h}$), boilers ($PH^{SELF}_{B,i,s,h}$) or CHPs ($PH^{SELF}_{CHP,i,s,h}$). Cooling loads, $C^{LOAD}_{COOL,i,s,h}$, are supplied by air-conditioning units ($PC^{GEN}_{airco,i,s,h}$) or absorption chillers ($PC^{SELF}_{AC,i,s,h}$). Additionally, pipeline heating and cooling transfer ($QH/C^{LOAD}_{i,s,h}$) or storage units ($PS^{OUT}_{STO,i,s,h}$) can occur for all i, s, h:

$$\begin{aligned} C^{LOAD}_{HEAT,i,s,h} = PH^{GEN}_{G,i,s,h} + PH^{SELF}_{B,i,s,h} + PH^{SELF}_{CHP,i,s,h} \\ + QH^{LOAD}_{i,s,h} + PS^{OUT}_{HST,i,s,h} \end{aligned} \tag{A.46}$$

$$C^{LOAD}_{COOL,i,s,h} = PC^{GEN}_{airco,i,s,h} + PC^{SELF}_{AC,i,s,h} + QC^{LOAD}_{i,s,h} + PS^{OUT}_{CST,i,s,h} \tag{A.47}$$

Appendix A.5. Availability-based capacity constraints

CHP units can perform three tasks represented by three mutually exclusive binary variables that are each The three mutually exclusive CHP binary variables ($CHP^{A/B/C}_{i}$) each representing a combination of the three CHP availability-capacity categories (AND ($\wedge$) - NOT ($\overline{B}$) gate): $B_{CHP,i}$, $B^{av}_{CHP,i}$, $B^{av}_{MG,CHP,i}$:

$$CHP^{A}_{i} = B_{CHP,i} \wedge \overline{B^{av}_{CHP,i}} \wedge \overline{B^{av}_{MG,CHP,i}} \qquad \forall i \tag{A.48}$$

$$CHP^{B}_{i} = B_{CHP,i} \wedge B^{av}_{CHP,i} \wedge \overline{B^{av}_{MG,CHP,i}} \qquad \forall i \tag{A.49}$$

$$CHP^{C}_{i} = B_{CHP,i} \wedge B^{av}_{CHP,i} \wedge B^{av}_{MG,CHP,i} \qquad \forall i \tag{A.50}$$

An AND-gate represents a product of binary variables and has been linearised using the procedure presented in [53, 54]. A NOT-gate inverts its binary input. Equation A.48 has, for example, been linearised as:

$$\begin{aligned}
&CHP_i^A \geq B_{CHP,i} + (1 - B_{CHP,i}^{av}) + (1 - B_{CHP,i}^{MG}) - 2 \qquad \forall i \\
&CHP_i^A \leq B_{CHP,i} \\
&CHP_i^A \leq (1 - B_{CHP,i}^{av}) \\
&CHP_i^A \leq (1 - B_{CHP,i}^{MG})
\end{aligned} \tag{A.51}$$

Also, the three variables are constrained by CHP existence:

$$CHP_i^A + CHP_i^B + CHP_i^C \leq B_{CHP,i} \qquad \forall i \tag{A.52}$$

Furthermore, the hierarchical relation between the binary variables that characterise CHP existence, 100 % availability and 100 % microgrid availability is:

$$B_{MG,CHP,i}^{av} \leq B_{CHP,i}^{av} \leq B_{CHP,i} \qquad \forall i \tag{A.53}$$

Appendix A.6. Potential electrical system configurations

Binary variables of some of the considered components are clarified here. A house has an available grid connection, GC_i, if it imports electricity from the grid, $X_{i,s,h}^{rec}$, in at least one hour, h, throughout the year:

$$X_{i,s,h}^{rec} \leq GC_i \leq \sum_{s,h} X_{i,s,h}^{rec} \qquad \forall i, s, h \tag{A.54}$$

The number of MG-available CHP units to house i ($k \in [0; n_{chp,i}]$) adopted in houses j in the neighbourhood ($B_{MG,CHP,j}^{av}$) can vary from zero to $n_{chp,i}$ ($n_{chp,i} = n_h - 1$). $Y_{i,k}^{chp}$ is a binary variable that decides whether a certain number of CHP units (k), installed in houses j in the neighbourhood, is available to a house i through MG operation:

$$\sum_{j \neq i} B^{av}_{MG,CHP,j} = \sum_{k=0}^{n_{chp,i}} k \cdot Y^{chp}_{i,k} \qquad \text{and} \qquad \sum_{k} Y^{chp}_{i,k} \leq 1 \qquad \forall i \tag{A.55}$$

For a CHP unit to be available for microgrid operation, both a CHP unit of available capacity and a microgrid central control unit (binary variable Z) must be available (binary variable $MGA_{i,k}$). This leads to the following AND-relation:

$$MGA_{i,k} = Z \wedge Y^{chp}_{i,k} \qquad \forall i, k \tag{A.56}$$

and resulting linearisation:

$$MGA_{i,k} \geq Z + Y^{chp}_{i,k} - 1 \ , \qquad MGA_{i,k} \leq Z \ , \qquad MGA_{i,k} \leq Y^{chp}_{i,k}$$

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