Interplay of degree correlations and cluster synchronization

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We study the evolution of coupled chaotic dynamics on networks and investigate the role of degree-degree correlation in the networks' cluster synchronizability. We find that an increase in the disassortativity can lead to an increase or a decrease in the cluster synchronizability depending on the degree distribution and average connectivity of the network. Networks with heterogeneous degree distribution exhibit significant changes in cluster synchronizability as well as in the phenomena behind cluster synchronization as compared to those of homogeneous networks. Interestingly, cluster synchronizability of a network may be very different from global synchronizability due to the presence of the driven phenomenon behind the cluster formation. Furthermore, we show how degeneracy at the zero eigenvalues provides an understanding of the occurrence of the driven phenomenon behind the synchronization in disassortative networks. The results demonstrate the importance of degree-degree correlations in determining cluster synchronization behavior of complex networks and hence have potential applications in understanding and predicting dynamical behavior of complex systems ranging from brain to social systems.

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I. INTRODUCTION

Many social [1], technological, and biological systems [2] can be well described by complex networks where nodes represent individuals or organizations and links represent interactions among them [3]. Synchronization is one of the most fascinating phenomena exhibited by dynamical units arising due to the interactions among them [4–8]. Birds flapping wings in similar way, chirping crickets, and coherent firing of neurons in the brain are a few examples of the synchronization in real-world systems [9]. In general, synchronization can lead to more complicated patterns, including clusters where a group of nodes synchronize among themselves without getting synchronized with the nodes outside the group [10–13]. Furthermore, the importance of degree-degree correlations has been realized for providing a better understanding and predictions of the behavior of complex systems represented by networks. Social systems where people with the same age, nationality, education level, religion, or language prefer to interact with each other lead to assortative networks, whereas some other systems exhibiting a preference of higher degree nodes to link with low degree nodes indicate disassortative mixing, e.g., technological and biological networks [14–17].

The different types of degree correlations lead to different network architecture and hence it is important to understand how the degree correlations of networks relate to or affect the dynamical properties, particularly cluster synchronizability (CS), of the interacting units on these networks. Previous studies have shown that disassortativity increases the global synchronizability (GS) of scale-free (SF) networks [18]. Recent studies have demonstrated that degree-degree correlation can lead to an abrupt transition from an incoherent to a synchronized state [19]. Most of the studies pertaining to degree correlations and their relations with dynamical evolution have considered GS, and despite cluster synchronization being more realistic and frequently observed in a range of diverse systems [20], the impact of degree correlations on CS has not been investigated.

In this article we investigate the dependence of CS of a network on degree-degree correlations. Our analysis reveals that degree-degree correlations are very crucial for predicting the dynamical behavior of coupled units on these networks. We demonstrate how CS of a network can be very different from GS measured by the ratio of largest eigenvalue to the first nonzero eigenvalue of the corresponding Laplacian matrix of the network [21]. Furthermore, we study an impact of the change in the network parameter on CS of the assortative and disassortative networks and demonstrate that degeneracy at the zero eigenvalue ($N_{\lambda 0}$) of the adjacency matrix combined with the driven phenomenon provides an understanding of the abnormal behavior of cluster synchronization with an increase in the average degree of the network. Nodes of a cluster can synchronize in two different ways, (a) self-organized (SO) and (b) driven (D), which are defined as follows [10]. If nodes in a cluster synchronize due to intracluster couplings, we refer to this as SO synchronization. However, if nodes of a cluster synchronize due to intercluster couplings, it is referred to as D synchronization.

II. THEORETICAL FRAMEWORK

We consider a network of N nodes and N_c connections. Let each node of the network be assigned a dynamical variable x^i ($i = 1,2,\dots,N$). The dynamical evolution is defined by the

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well known coupled map model, given as [22]

$$x_i(t+1) = (1-\varepsilon)f(x_i(t)) + \frac{\varepsilon}{k_i}\sum_{j=1}^{N} A_{ij}\, g(x_j(t)). \quad (1)$$

The function $f(x) = \mu x(1-x)$ is the logistic map that governs the dynamics of individual nodes and $g(x)$ defines the nature of coupling between the nodes. In our study we take $g(x) = f(x)$ and consider $\mu = 4$ for which the dynamics exhibits a chaotic evolution [23]. Logistic maps display a rich dynamical behavior depending on the values of μ. Due to its simplicity, yet ability to display complex chaotic behavior, coupled logistic maps have been widely investigated to understand various complex phenomena manifested by a diverse range of real-world systems [24]. Furthermore, the coupled map model with diffusive coupling has been considered to understand the effects of spatial heterogeneity on population dynamics [25], and it was shown that a linear coupling is not biologically realistic.

We would like to note that for the linear coupling [$g(x) = x$], coupled logistic maps lead to a very high CS after a critical value of ε irrespective of the network architecture [10]. An important point which makes $g(x) = x$ coupling rather trivial is that this form of coupling leads to the periodic evolution of the coupled maps after a critical coupling strength for a wide range of network architectures. A is the adjacency matrix and A_{ij} is 1 if the nodes i and j are connected and zero if they are not. $k_i = \sum_{j=1}^{N} A_{ij}$ is the degree of the ith node and ε is the overall coupling constant in the range $0 \leqslant \varepsilon \leqslant 1$. For each node, Eq. (1) is evolved starting with a set of random initial conditions, and the values of x_i corresponding to different nodes are noted after an initial transient time (t_0). As nodes having the exact synchronization [$x_i(t) = x_j(t)$, $\forall t > t_0$] are much less in number, we study phase synchronization of the nodes. Two nodes of a network are said to be phase synchronized if the local maxima or local minima of the corresponding variables occur at the same time [10,26]. Global phase synchronizability corresponds to the state when a network forms only one cluster including all the nodes, whereas in cluster synchronization a group of nodes synchronize among each other while they may not synchronize with the rest of the nodes in the network. As ε increases with the evolution, the network splits into several phase synchronized clusters. We note that for each value of ε we evolve Eq. (1) with a random set of initial conditions, and results for different ε values are independent of each other. The CS of a network for a particular coupling strength is given by f_{clus}, which is defined as a ratio of the number of nodes forming the clusters to the total number of nodes in the network. Furthermore, in order to have a quantitative picture of the SO and D synchronizations, we use f_{intra} and f_{inter}, respectively, for the intra- and intercluster couplings: $f_{\text{intra}} = N_{\text{intra}}/N_c$ and $f_{\text{inter}} = N_{\text{inter}}/N_c$, where N_c is the total number of connections in the network, and N_{intra} and N_{inter} are the number of intra- and intercluster couplings, respectively. To quantify the degree-degree correlation of a network, we consider the Pearson degree-degree correlation coefficient (r) given as [14]

$$r = \frac{\left[N_c^{-1}\sum_{i=1}^{N_c} j_i k_i\right] - \left[N_c^{-1}\sum_{i=1}^{N_c} \frac{(j_i+k_i)^2}{2}\right]}{\left[N_c^{-1}\sum_{i=1}^{N_c} \frac{(j_i)^2+(k_i)^2}{2}\right] - \left[N_c^{-1}\sum_{i=1}^{M} \frac{(j_i+k_i)^2}{2}\right]}, \quad (2)$$

where j_i and k_i are degrees of nodes at both ends of the ith connection, and N_c represents the total number of connections in the network. Here $-1 \leqslant r \leqslant 1$ and positive values of r correspond to assortative networks while negative values correspond to disassortative networks.

III. CLUSTER SYNCHRONIZABILITY VERSUS DEGREE-DEGREE CORRELATION

First, we investigate the impact of degree-degree correlation on SF networks. The SF networks are constructed with the preferential attachment method [3]. The Erdös-Renýi (ER) random networks are generated by connecting every pair of nodes of the network with a probability $\langle k\rangle/N$. Note that ER networks have a binomial degree distribution for a small network size and, as size increases, the degree distribution becomes Poissonian [3]. The degree correlation coefficient (r) is varied following the procedure given in Refs. [27,28]. To vary r, two links of a network are chosen randomly and corresponding nodes are ordered with respect to their degrees. To generate assortative networks, we rewire two high degree nodes with the probability p and the remaining two nodes with the probability $1-p$. This procedure is repeated until all links are rewired. Networks of different r values are generated by varying the parameter p. Disassortative networks are generated in a similar way, except during the rewiring higher degree nodes are connected with lower degree ones. It may be possible that very high values of r lead to the disconnected components and, in order to avoid any change in the synchronization property of the nodes arising because they belong to different components and not because of change in the manner they are interacting, we introduce a few connections among all the disconnected components to make them connected. While introducing the random connections we ensure that the assortative property as well as $\langle k\rangle$ of the network remain unaffected.

We find that an increase in the degree-degree correlation coefficient leads to a decrement in CS of a heterogeneous network [Figs. 1(a), 1(b), and 1(c)]. This behavior is in line with GS of a network which is known to get suppressed with an increase in r [18]. Global synchronizability of a network can be defined using a ratio of the largest to the first nonzero eigenvalue of the corresponding Laplacian matrix as Λ_N/Λ_2, where $\Lambda_N > \Lambda_{N-1} > \cdots > \Lambda_2 > 0$ are eigenvalues of the Laplacian matrix ($L = \mathcal{D} - A$), with $\mathcal{D}$ being the matrix having degree of nodes at the diagonal elements and all other off-diagonal elements being zero. As indicated by the value of Λ_N/Λ_2, GS for SF [Figs. 2(d) and 2(e)] and ER networks [Fig. 2(f)] exhibits a continuous increase, except for very high disassortative values which we discuss later, as networks become more disassortative. Furthermore, in SF disassortative networks, at high ε regions, SO synchronization is the dominant mechanism behind cluster formation [Fig. 1(c)], indicating that the synchronization among the nodes of a cluster is mainly due to the coupling terms in Eq. (1). In

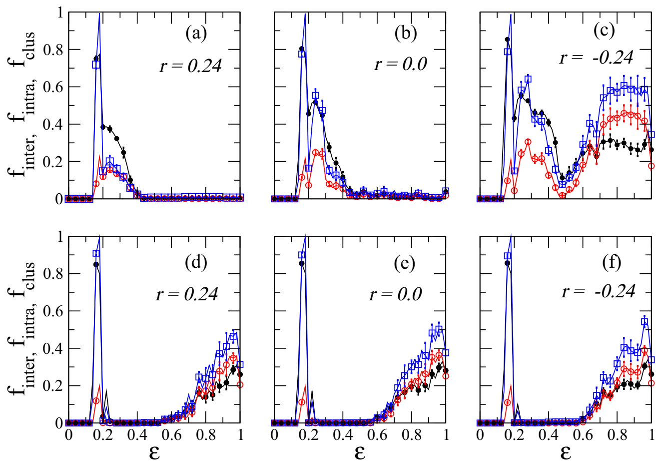

FIG. 1. $f_{\rm clus}$ (□), $f_{\rm inter}$ (•), and $f_{\rm intra}$ (○) as a function of ε for various values of r. (a)–(c) Increase in $f_{\rm clus}$ with a decrease in r for SF networks; (d)–(f) almost constant behavior for ER networks. Network size is 200 and $\langle k\rangle = 4$. Each panel is plotted for an average over 20 random realizations of the networks and the initial conditions.

disassortative SF networks, the intracluster couplings are more frequent as compared to the intercluster couplings arising due to the presence of a few high degree nodes (hubs) forming clusters accompanied by a large number of low degree nodes leading to a high value of $f_{\rm intra}$ [Fig. 1(c)].

Furthermore, for different values of r, networks having a heterogeneous degree distribution demonstrate a stronger impact of changing r on CS as compared to those networks having a homogeneous degree distribution. For example, in ER networks, change in CS is negligible for different values of r [Figs. 1(d), 1(e), and 1(f)]. Additionally, contributions of the SO and D mechanisms to cluster synchronization are almost equal for high ε. Since connections between the set comprising a large number of low degree nodes and the set comprising of a few high degree nodes turn out to be a prime factor contributing to the D synchronization in SF disassortative networks, and this kind of structure is not possible for ER networks due to the absence of hub nodes, ER networks do not manifest a particular contribution from the D mechanism. Similarly, in networks having a heterogeneous degree distribution, such as SF networks, assortativity reshuffling connects similar degree nodes with each others, i.e., low-low and high-high within a network, and as a consequence different layers of different connection densities are formed. The number of connections within these layers is large compared to the number of connections between the layers yielding community structures in the network [27]. In SF networks of size N and average degree $\langle k\rangle$ approximately $N/2$ nodes have degree $\langle k\rangle/2$. Consequently, the reshuffling, which connects these low degree nodes among themselves, leads to a drastic change in the structure of the network. ER networks, due to a rather homogeneous degree, lack the presence of a large number of very low degree nodes, and therefore changes in the degree-degree correlations do not induce much structural change as was brought for SF networks and also visible from a change in the diameter (denoted as D). For ER networks, rewiring from a very disassortative architecture to a very assortative one does not bring a significant change in the diameter [Fig. 2(f)], whereas SF networks depict a significant change due to the rewiring, leading to change in the degree-degree correlations [Fig. 2(d)].

A. Disassortativity and driven synchronization

In general, disassortative networks are better synchronizable than the corresponding assortative networks (Fig. 1). Here, correspondence means the networks have the same size, average degree, and degree heterogeneity. Degree heterogeneity is an important factor in comparing the CS of two networks. For instance, SF assortative networks may be poorly synchronizable compared to ER networks having the same assortative value [Figs. 1(a), 1(d), 2(a), and 2(c)]. For networks with the same degree sequence and with varying degree-degree correlations, CS and the diameter display similar behavior with a smaller diameter [Fig. 2(d)] favoring CS [Fig. 2(a)]. This observation is in line with the phenomena exhibited by GS [Fig. 2(d)]. However, most importantly, our analysis reveals that CS and GS of a network may have very different behavior depending upon the value of r. For instance, similar to the assortativity, a high disassortativity also destroys the GS of SF networks [Figs. 2(d) and 2(e)] depending on the average degree, whereas, for these very large negative r values, CS may increase or decrease depending on the average connectivity of the network. For instance, disassortative ($r < -0.4$) SF networks with lower average connectivity exhibit a decrease in CS as well as GS [Fig. 2(a) and 2(d)], whereas, for a higher average degree, GS keeps exhibiting the same behavior, i.e., a suppression for $r < -0.4$ [Fig. 2(e)], whereas CS shows an enhancement [Fig. 2(b)].

In order to understand this anomalous behavior, we perform a deeper analysis of the dynamical evolution and the connection architecture of the nodes forming clusters and find that two different phenomena are responsible for cluster

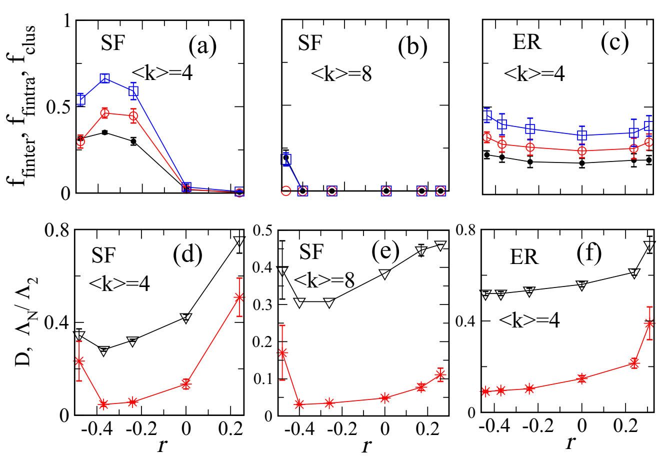

FIG. 2. (a), (b) Dependence of CS on r for different values of $\langle k\rangle$. (c) For ER networks CS remains almost independent of r. Various structural and dynamical measures such as $f_{\rm inter}$, $f_{\rm intra}$, $f_{\rm clus}$, Laplacian eigenvalue ratio, and diameter are represented by •, ○, □, ∗, and ▽, respectively. Here $N = 200$ for all the networks, and each data point is obtained for an average over ten random realizations of the networks. Each variable in (d)–(f) is normalized by its maximum value.

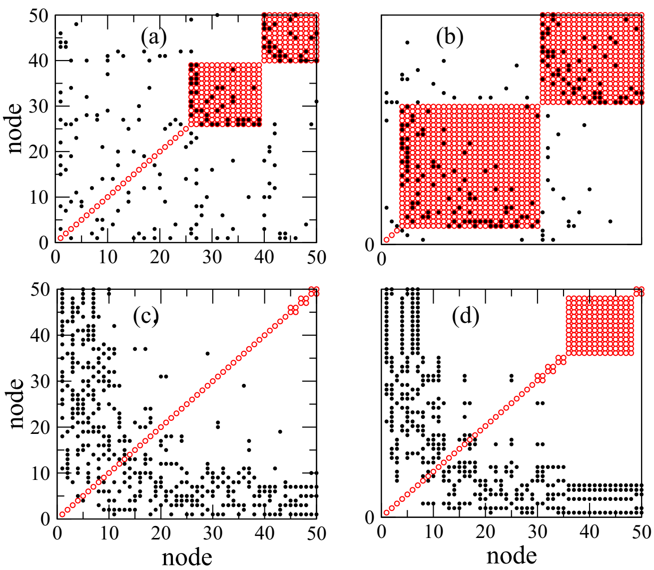

FIG. 3. Node-node diagram for SF network of $N = 50$ depicting an increase in SO synchronization governing the enhancement in CS as r changes from (a) -0.36 to (b) -0.46 at $\langle k\rangle = 4$ and $\varepsilon = 0.8$. (c), (d) Increase in the D synchronization is the prime mechanism behind enhancement in cluster synchronization when r changes from -0.7 to -0.8 at $\langle k\rangle = 8$ and $\varepsilon = 0.8$. Laplacian eigenvalue ratios are (a) 59, (b) 40, (c) 29, and (d) 382, depicting changes in the GS. Squares along the diagonal represent clusters and black dots are the connections among nodes. Nodes are renumbered such that those belonging to the same cluster come consequently.

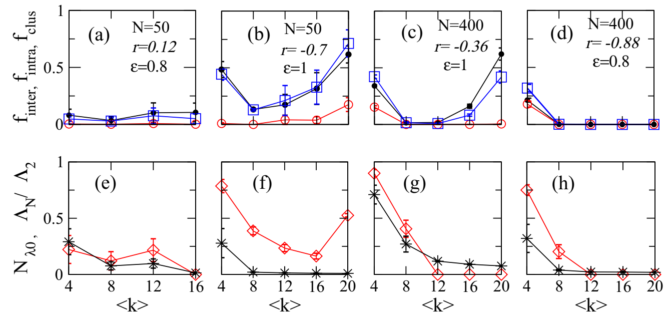

FIG. 4. Various dynamical and structural properties are plotted as a function of average degree $\langle k\rangle$. (a)–(c) $f_{\rm clus}$ ($\square$), $f_{\rm inter}$ ($\bullet$), and $f_{\rm intra}$ ($\circ$) plotted for SF networks of various size and degree-degree correlations. (e)–(g) Λ_N/Λ_2 ($*$) plotted as a measure of the GS as well as degeneracy at the zero eigenvalue of the adjacency matrix [N_{λ_0} ($\diamond$)] for the same network parameters as the corresponding (a)–(c). (d), (h) Behavior for ER networks. All the plots are for an average over ten random realizations of the networks. Variables in (e)–(h) are normalized by their maximum values.

synchronization depending upon the nature of the GS. When CS follows a behavior which is similar to that of GS, the SO mechanism plays the prime role in the enhancement or suppression of synchronization. For example, in SF networks of $N = 200$ and $\langle k\rangle = 4$, CS and GS manifest an increase [Figs. 2(a) and 2(d)] as r is varied from $r \approx 0.2$ to $r \approx -0.2$, and this enhancement is due to the SO mechanism reflected in a continuous increase in the value of $f_{\rm intra}$ [Fig. 2(a)] as well as from the node-node diagrams [Figs. 3(a) and 3(b)]. Another example is sparse SF networks ($N = 200$ nodes and $\langle k\rangle = 4$), which exhibit a decrease in GS and CS for a very high disassortative value [Fig. 2(a)], and again this decrease in the CS is due to a decrease in the SO mechanism [Fig. 2(a)]. However, for $\langle k\rangle = 8$, there is a decrease in GS, but CS increases as disassortativity increases $r \approx -0.4$ onwards [Figs. 2(b) and 2(e)]. For this case, the increase in CS is due to an increase in the D mechanism with the SO mechanism remaining the same [Fig. 2(b)]. Figure 3 presents one such clear picture of the D mechanism being the reason behind the enhancement in CS as r is varied. As discussed above, ER networks do not exhibit this intriguing behavior and CS is almost independent of r [Fig. 2(c)].

An interesting cluster synchronization behavior and the phenomenon governing this is revealed for disassortative networks as we increase the average degree by keeping all other properties, i.e., degree-degree correlation, network size, as well as coupling strength, the same. We find that, as the average degree increases, CS of disassortative networks first decreases, then remains almost the same for a certain regime of $\langle k\rangle$, and finally exhibits an increase with a further increase in the average degree [Figs. 4(b) and 4(c)], whereas assortative networks do not manifest any significant changes in CS until a certain average degree [Fig. 4(a)], after which there is a transition to a global synchronized state spanning all the nodes. As $\langle k\rangle$ increases, an increase in CS ($f_{\rm clus}$) as well as in GS (measured in terms of Λ_N/Λ_2) has already been reported; here we elaborate more on the interesting behavior depicted by networks having very high disassortative values. Global synchronizability of disassortative networks reflects a continuous increase (Λ_N/Λ_2 decreases) with an increase in the average degree [Figs. 4(f) and 4(g)], whereas Figs. 4(b) and 4(c) depict a decrease followed by an increase in CS as connections in the network increase. In order to understand this decrease and increase in CS, as well as the contrasting behavior in GS, we investigate the mechanism behind cluster synchronization.

As already reported in the previous sections, these changes in CS here are mainly due to the D mechanism, except for those coupling values where a global synchronized cluster spanning all the nodes is formed. As noted earlier, we focus here on the parameter regimes which yield a cluster state instead of the global synchronized state. Therefore, we explore further the network architecture which facilitates an enhancement in cluster synchronization before a suppression as the average degree increases. This is more intriguing as the enhancement is not due to the SO mechanism, which one would expect if an increase in the number of interactions among the nodes is causing the synchrony, but due to the D mechanism. In fact, Fig. 4(c) depicts a very clear picture of the D mechanism behind synchronization for larger $\langle k\rangle$ values, and Fig. 5 provides a schematic diagram for the structural changes in a network that affect synchronizability as $\langle k\rangle$ increases. For smaller $\langle k\rangle$, a very negative degree-degree correlation leads to a structure having large lower degree nodes connected to the same set of hub nodes [Fig. 5(a)], facilitating D synchronization of these lower degree nodes. As the average degree increases, the lowest possible degree

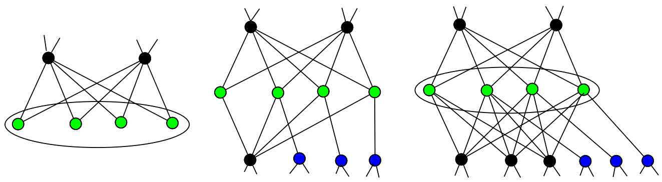

FIG. 5. Schematic diagram presenting structure in SF networks for various average degrees (a) $\langle k\rangle = 4$, (b) $\langle k\rangle = 8$, and (c) $\langle k\rangle = 20$. The nodes for which synchronization behavior is investigated are represented as green circles. Synchronized nodes are enclosed in an ellipse. Black and blue circles denote common and uncommon neighbors of the green circles, respectively.

a node can have increases and consequently the lower degree nodes get connected to few other nodes outside the common set of the driver nodes, resulting in destroying the synchrony among them [Fig. 5(b)]. As connectivity increases further, a set of lower degree nodes, with an increased number of connections, has an enhanced number of common higher degree nodes [Fig. 5(c)], which is sufficient to drive them in synchrony, yielding D clusters. Note that this formation of a structure of lower degree nodes connected to a set of common higher degree nodes is possible due to a large value of negative degree-degree correlations. As can be seen from Fig. 4(d), homogeneous networks do not manifest the intriguing behavior depicted by the heterogeneous networks and CS displays a continuous decrease with an increase in the average degree. The initial decrease in $f_{\rm clus}$ can be understood with the same analogy as for the SF networks but further increase in $f_{\rm clus}$ as observed for SF networks is absent. It should be noted that, in random networks, degrees are distributed around $\langle k\rangle$ with a small spread, which is in sharp contrast to the SF networks having the same average degree where a large number of nodes ($N/2$) have very small degree and a few nodes have a very high degree. For example, at $\langle k\rangle = 20$, SF networks have about $N/2$ nodes with degree 10 whereas in random networks with the same average degree there are hardly or very few degree 10 nodes. In SF disassortative networks, low degree nodes manage to get a set of common nodes, which is lacking in ER networks due to the homogeneous degree distribution. For this reason, the ER networks do not manifest an enhancement in the D synchronization and hence in the cluster synchronization. In the following we demonstrate how degeneracy in the zero eigenvalue further supports the D synchronization behind the mechanism governing synchronization in disassortative networks.

B. Driven synchronization and zero degeneracy

Degeneracy at the zero eigenvalues ($N_{\lambda 0}$) of the network's adjacency matrix provides further insight into the D mechanism behind this abnormal behavior. Degeneracy at the zero eigenvalue in a network spectrum arises due to the following reasons: (i) complete duplication of the nodes, represented as

$$R_i = R_j,$$

where R_i, R_j are the ith and jth rows of the adjacency matrix; (ii) partial duplication of the nodes, for example,

$$R_i = aR_j + bR_k,$$

where a and b are any real numbers; or (iii) if there are isolated nodes in a network [29]. Since we consider here only connected networks, condition (iii) does not exist and any degeneracy in the zero eigenvalue arises only due to the first and second conditions. It is rather easy to see that an increase in the number of complete duplicate nodes leads to an increase in D synchronization as there is a cancellation of coupling terms in the difference variable of a pair of the nodes having the same set of neighbors:

$$[x_i(t+1) - x_j(t+1)]^2 \\ = \{(1-\varepsilon)^2[f(x_i(t)) - f(x_j(t))]^2\}.$$

As follows from Ref. [10], the Lyapunov function analysis provides insight into the stability of synchronization between these two nodes having the same set of neighbors. Though such a simple explanation for the synchrony of partial duplicate nodes is not possible, the results demonstrating one-to-one correlations between the number of zero eigenvalues and CS of a network suggest an easy synchronization of the partial duplicate nodes, which are connected with one or more common nodes acting as a driving force to synchronize these partial duplicate nodes. Figures 4(b) and 4(f) depict that $f_{\rm inter}$ and $f_{\rm clus}$ follow $N_{\lambda 0}$ as $\langle k\rangle$ increases and a contribution of the SO mechanism in the increase in $f_{\rm clus}$ is almost negligible. This behavior of first decrease in $f_{\rm clus}$ followed by a subsequent increase with an increase in the average degree is only evident for disassortative networks and is observed for larger networks as well [Fig. 4(c)]. Although the changes in the values of $f_{\rm clus}$ with $\langle k\rangle$ are always accompanied by the similar behavior of the dominant D mechanism, larger networks depicts a continuous decrease in N_{λ_0} with an increase in $\langle k\rangle$. The reason behind a decrease in N_{λ_0} with an increase in the average degree is not surprising as it leads to a lesser probability of complete and partial duplications of the nodes [29], but interestingly very high disassortativity facilitates such a kind of structure even for larger $\langle k\rangle$ values. As the network size increases, the value of maximum disassortativity that a network can attain decreases and consequently leads to a lesser possibility of formation of complete and partial duplicates which was mostly led by lower degree nodes being connected with the same set of higher degree nodes. However, an increase in $f_{\rm clus}$ as well as the D phenomenon for the synchronization suggest that at higher $\langle k\rangle$ values a pair of nodes can still synchronize through the D mechanism even if the coupling terms in the difference variables canceled out partially, a phenomenon which is clearly seen for smaller networks where $N_{\lambda 0}$ again increases.

One very simple example of the D mechanism governing the contradictory behavior of CS and GS is the star network type, which exhibits a very high cluster synchronization. However, GS, measured in terms of the Laplacian eigenvalue ratio, of star networks is extremely low compared to that of globally connected networks of the same size, as the Laplacian eigenvalue ratios for globally connected networks and star networks of size N are 1 and N, respectively. However, the values of f_{clus} are 1 and $1-(1/N)$, respectively, indicating almost equal CS for both networks.

The article presents the following phenomena: for disassortative networks, (i) contrasting behavior of global and CS, (ii) dependence of CS on $\langle k \rangle$, and (iii) relation between the D mechanism and the zero eigenvalues, and for neutral as well as assortative networks having small r values (iv) there is no relation between CS and $\langle k \rangle$. For the parameter values of networks for which these results have been demonstrated, the space of the graphs is connected.

IV. CONCLUSION

In summary, disassortative networks exhibit remarkably a much higher CS than assortative networks. Interestingly, for very large values of the degree-degree correlation on the negative side, i.e., for very high disassortavity, with an increase in the average degree CS may increase or decrease depending upon the degree distribution as well as the value of the average degree. This behavior is in contrast to GS of the networks, which always indicates an enhancement as the average degree of a network increases. It turns out that, in the presence of the D mechanism, CS of a network can be high even though GS indicated by the Laplacian eigenvalue ratio is poor, whereas the SO synchronization remains the prime mechanism behind an increase or decrease in the cluster synchronization whenever it follows the same trend as GS of the networks. Furthermore, change in the degeneracy at the zero eigenvalue which relates to partial as well as complete duplication of nodes in a network provides a further understanding of the D mechanism behind the abnormal behavior of CS.

Our results suggest that the degree-degree correlations have tremendous impact on the CS of underlying networks. We provide understanding of the contrasting behavior of GS and CS of a network. These findings explain how a network, despite being poorly globally synchronizable, can display a very good cluster synchronization. There are several real-world systems, for instance, brain networks, where excessive (global) synchronization is not required [30]; however, cluster synchronization is necessary in order to perform some functions. Further on, these insights pertaining to the relation between structural and dynamical properties can be used to build up artificial networks [31] possessing such synchronization behavior.

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