Coupling of a Granular Chain to an Acoustic Medium: Sensitivity Analyses of the Propagation of Ultrasonic Pulse Trains

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***Abstract*— Effects which arise as a result of Hertzian contact between adjacent spheres of a granular chain can potentially change the nature of a signal as it propagates down the chain. The possibility thus exists of generating signals with a different harmonic content to the signal input into one end of the chain. This transduction mechanism has the potential to be of use in both diagnostic and therapeutic ultrasound applications. Due to metrological challenges which arise when characterizing this transduction process, numerical models play a fundamental role in assisting the design of these novel devices. Previously, a finite element model was presented, which predicts the acoustic pressure generated by a sinusoidally excited granular chain coupled into an acoustic medium. The study described here exploits this model to carry out sensitivity analyses of the system to key input parameters, including excitation frequency and amplitude, sphere diameter and the number of spheres present in the chain. Granular chains were excited at one end using tone burst displacement signals with fundamental frequencies of 73 kHz and 100 kHz. The final sphere of the chain was assumed to be in contact with a cylindrical vitreous carbon layer, coupled to a half-space of water. Using the finite element method, it was possible to predict the acoustic pressure in the fluid, for a specific dynamic excitation of the first sphere of the granular chain. The sensitivity analyses demonstrated that, under tone burst excitation conditions, a train of impulses could be propagated into an acoustic medium. The sensitivity analyses also show that, due to inherent nonlinearities present in this type of system, the time and frequency domain characteristics of the signals are highly sensitive to input conditions.**

Keywords—Finite element analysis; Granular chain; Nonlinear systems; Ultrasonic transducers.

I. Introduction

Granular materials can be thought of as a conglomeration of discrete solid macroscopic particles characterized by a loss of energy whenever the particles interact. Granular crystals are defined as ordered aggregates of elastic particles in contact with each other and can be thought of as a type of nonlinear periodic phononic structure. Granular crystals display nonlinear characteristics which result from the nonlinear relationship of the force at the contact and the displacement between neighboring element centers (described by Herzian contact laws – a consequence of linear mechanics) and an asymmetric potential which arises between neighboring elements from the inability of granular crystals to support tensile loads [1]. An unusual feature of granular crystals that results from these nonlinearities is the negligible linear range for interaction forces between neighboring elements (in vicinity of zero pre-compression force). This results in a non-existent linear sound speed in the uncompressed material. This leads to a phenomenon known as "sonic vacuum" where the traditional wave equation does not support a characteristic speed of sound [2]. Granular crystals support a wide array of nonlinear phenomena: compact solitary waves, nonlinear normal modes, anomalous reflections and energy trapping Granular crystals remain one of the most studied examples of nonlinear lattices.

The study of the generation of solitary waves in a granular chain has recently been extended to biomedical applications [3,4,5]. In [3], high-amplitude focused acoustic pulses were generated using a one-dimensional array of granular chains. An investigation was conducted where the amplitude, size, and location of the focus could be controlled by varying the static pre-compression of the chains. Furthermore, granular chains have been used to assess the structural integrity of orthopedic implants [4]. In a recent study [5], displacements of the order of 1 μm were produced by a resonant 73 kHz ultrasonic source to drive a granular chain consisting of six 1 mm diameter chrome steel spheres. The final sphere of the chain was in contact with a fixed support. Travelling solitary wave impulses were observed, which were due to both nonlinearity between adjacent spheres and reflections within the chain. The axial velocity of the final sphere of the chain was measured using a laser vibrometer. The acquired waveforms showed a train of impulses possessing both high amplitude and wide bandwidth, and featuring spectral content up to 200 kHz. This work was subsequently expanded upon to study the response of granular

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chains to a narrow band ultrasonic source, as a function of the static pre-compression of the chain, and of its properties [6]. A transduction mechanism based on the nonlinear dynamics of granular chains may in fact possess distinct features that could make it attractive to both therapeutic high-intensity focused ultrasound applications and diagnostic applications [5,6]. A recent study showed that coupling a sinusoidally excited six-bead granular chain to water could generate acoustic pressures in the form of wideband impulses, featuring spectral content close to the biomedical ultrasound frequency range [7]. Due to the complexities associated with varying the experimental parameters and also due to the metrological challenges involved in acquiring traceable acoustic pressure measurements, theoretical models capable of simulating the nonlinear transduction process have a vital role to play in understanding and optimizing the mechanisms involved. The use of the finite element method to analyze the dynamic behavior of a granular chain was investigated in [8], providing good agreement with the discrete mechanics solution proposed and with experimental results described in [9] and [10]. Finite element analysis (FEA) was subsequently employed in [11] to model the dynamics of granular chains with signals relevant to biomedical ultrasound. This yielded good agreement with the discrete mechanics solution and demonstrated that the multiple collisions which occur between the beads of the chain could be accurately modeled using FEA. The coupling of this granular chain to an acoustic medium to predict the acoustic pressures generated by the sinusoidal excitation of a six-bead granular chain was described in [12].

The study described in this paper features an extension of the prior model in order to carry out sensitivity analyses of the system to key input parameters. The granular chains were excited at one end using 30-cycle sinusoidal displacement signal with a Gaussian envelope, and with a fundamental frequency of either 73 kHz or 100 kHz. The final sphere of the chain was assumed to be in contact with a cylindrical vitreous carbon Sigradur® K layer of 0.25 mm thickness, coupled to a half-space of water. Using the finite element method, it the acoustic pressure at 1 mm from the fluid/structure interface was predicted, for a specific dynamic excitation of the first sphere of the granular chain. The beads were assumed to be made of chrome steel. The first sphere of the chain was excited via a stainless steel cylindrical piston. The displacement magnitude was varied between 1.65 μm and 4.95 μm. This is representative of the experimental conditions described in [5,6]. The FEA was carried out using a transient analysis in ANSYS Mechanical version 16.1 [13].

II. Materials and Methods

A. Finite element analysis

It is common to formulate the problem of frictionless contact between two solid bodies as a variational inequality. This presents a special type of minimization problem with inequality constraints. The Lagrange multiplier method, or the Normal Lagrange Formulation as it is described in ANSYS Mechanical [13], was used to solve the minimization problem. This method adds an extra degree of freedom (contact pressure) to satisfy contact compatibility. Consequently, instead of resolving contact force as contact stiffness and penetration, contact pressure is solved for explicitly as an extra degree of freedom. Whilst more computationally intensive than penalty methods, it has the advantage of enforcing near-zero penetration when modelling frictionless contact between two bodies.

Damping was implemented in the form of dashpots connecting adjacent solid bodies. One of the principle sources of damping arises from internal viscoelastic mechanisms which occur as the bodies are compressed together [14]. Previous work [5] suggests that good comparison with experiment is obtained by using a value of 0.3 $N{\cdot}s{\cdot}m^{-1}$ for the longitudinal damping factor when adjacent bodies are in contact. Damping was set to zero when bodies were not in contact with one another. This task involved extracting the displacements at the dashpots nodes at each time step of the analysis, and assigning a zero or a 0.3 $N{\cdot}s{\cdot}m^{-1}$ damping factor, depending on the relative positions of the dashpot nodes.

The propagation of acoustic waves inside the fluid was assumed to be governed by the linear, inviscid acoustic wave equation, so that the fluid could be defined in terms of its equilibrium density and speed of sound. Coupling at the fluid/structure interface assumed continuity of normal velocity. An absorbing boundary was placed around the acoustic finite element mesh in order to simulate the Sommerfeld radiating condition and propagation of acoustic waves into a half-space. Details of the underlying equations and physical principles may be found in [13].

A mesh of the structural section of the model is displayed in Fig. 1, with a description of the forcing and boundary conditions. This mesh features refinements around the contact regions to improve accuracy of the solution, as well as convergence. In all simulations, a mesh conversion analysis was carried out to ensure that the mesh density and time step were suitable for the generation of accurate results.

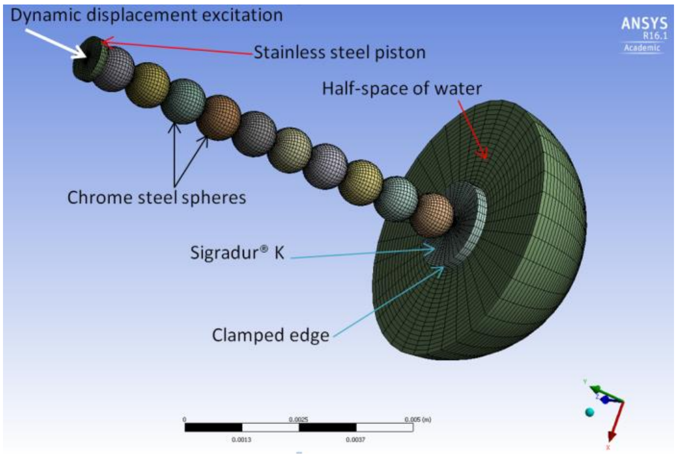

Fig. 1. FEA model mesh: 3D visualization of the axisymmetric model for a ten-sphere, 1 mm bead diameter granular chain, coupled to a fluid region.

B. Sensitivity analysis

Four granular chain configurations were investigated:

(1) Six-sphere granular chain with 1 mm diameter beads.

(2) Six-sphere granular chain with 0.5 mm diameter beads.
(3) Six-sphere granular chain with 2 mm diameter beads.
(4) Ten-sphere granular chain with 1 mm diameter beads.

A displacement excitation was applied to the steel piston in contact with the first sphere of the granular chain. Two fundamental excitation frequencies were considered for all four configurations: 73 kHz and 100 kHz. The displacement signals consisted of 30 cycles of a sinusoidal waveform with a Gaussian envelope. A sample displacement excitation is shown in Fig. 2, corresponding to a 73 kHz fundamental frequency and featuring a peak displacement of 3.3 μm.

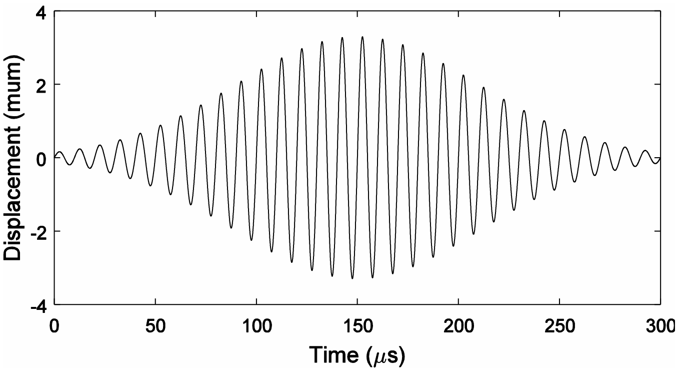

Fig. 2. Normal displacement applied to the outer surface of the stainless steel piston in the FEA model.

For all configurations, 11 displacement excitation waveforms were considered with a peak value varying linearly between 1.65 μm and 4.95 μm.

III. Finite Element Modeling Results

The input properties used for each structural material is displayed in table I.

TABLE I. MATERIAL PROPERTIES

Material	Young's modulus (GPa)	Poisson's ratio	Density (kg·m^{-3})
Stainless steel	200	0.35	7800
Chrome steel	201	0.35	7833
Sigradur® K	35	0.15	1540

The fluid region was assigned the properties of water, i.e. a speed of sound of 1500 m·s^{-1} and a density of 1000 kg·m^{-3}.

For all configurations, the velocity at the center of the final sphere of the chain was obtained as a function of time, along the *y*-direction (i.e. the axis of the chain). The resulting acoustic pressure radiated by the front face of the Sigradur® K cylinder was extracted at the post-processing stage, at 1 mm into the medium and along the axis of symmetry of the chain. Finally, the FFT magnitude of each acoustic pressure signal was evaluated and normalized to its maximum value. Three sets of results are discussed in this Section, which all bear a specific relevance to biomedical ultrasound applications. The results in Fig. 3 correspond to a 73 kHz 4.95 μm peak excitation of a six-sphere chain of 1 mm diameter beads.

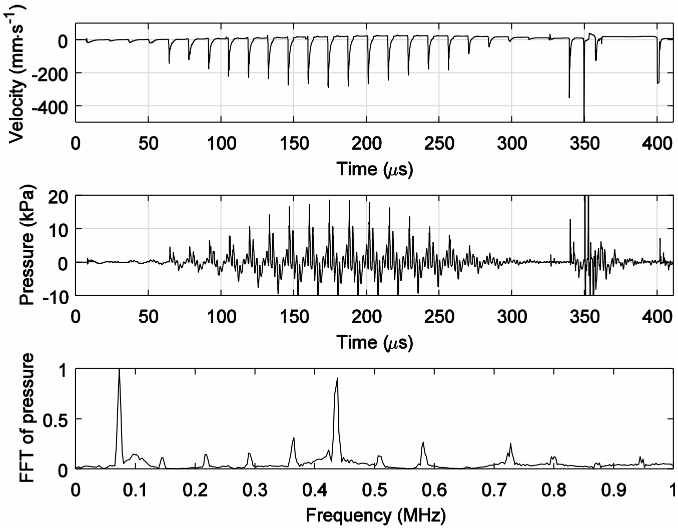

Fig. 3. Six-sphere granular chain, 1 mm bead diameter, 73 kHz 4.95 μm peak excitation. Top to bottom: velocity of last sphere of chain, acoustic pressure 1 mm from fluid/structure interface and normalized FFT of acoustic pressure.

This set of results clearly shows that a pulse train is propagated into the acoustic medium, with a peak acoustic pressure of 20 kPa 1 mm from the fluid/structure interface. The acoustic signal has multiple harmonics and features spectral content up to 0.94 MHz, at -20 dB relative to the fundamental frequency.

Fig. 4 shows results corresponding to a 73 kHz 3.63 μm peak excitation of a six-sphere chain of 0.5 mm diameter beads. The acoustic pressure waveform also shows that a train of impulse is being propagated into the acoustic medium.

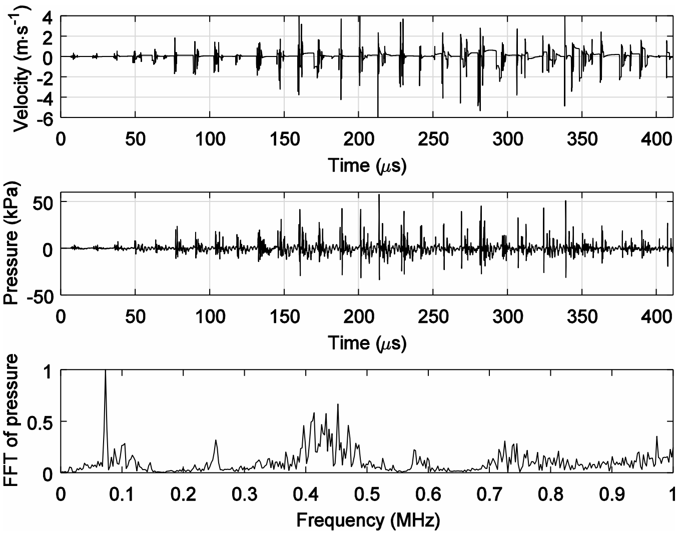

Fig. 4. Six-sphere granular chain, 0.5 mm bead diameter, 73 kHz 3.63 μm peak excitation. Top to bottom: velocity of last sphere of chain, acoustic pressure 1 mm from fluid/structure interface and normalized FFT of acoustic pressure.

However, in addition to higher order harmonics, it can be seen that broadband noise occurs. Rather than this noise being numerical in nature, it is thought that it is due to the propensity of such systems to exhibit chaotic behavior.

In Fig. 5, results corresponding to a 100 kHz 4.29 μm peak excitation of a six-sphere chain of 0.5 mm diameter beads. This specific set of results show that a high-amplitude acoustic pressure time domain waveform is obtained (200 kPa peak positive pressure), and also shows that a train of impulse is being propagated into the acoustic medium.

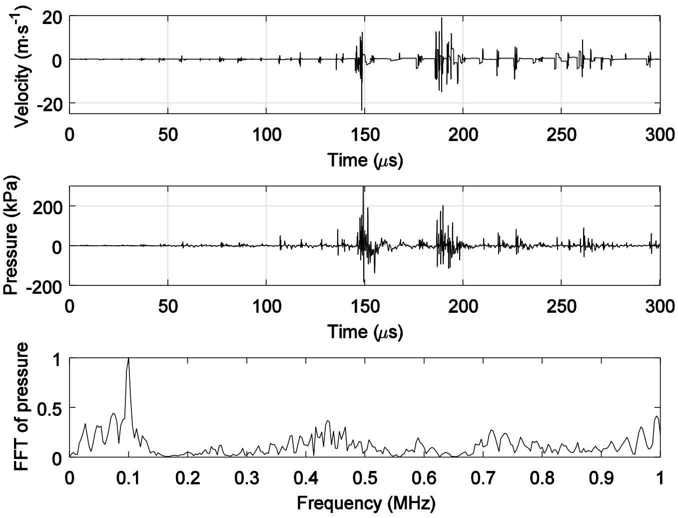

Fig. 5. Six-sphere granular chain, 0.5 mm bead diameter, 100 kHz 4.29 μm peak excitation. Top to bottom: velocity of last sphere of chain, acoustic pressure 1 mm from fluid/structure interface and normalized FFT of acoustic pressure.

IV. CONCLUSION

A finite element model has been developed for the analysis of acoustic signals resulting from the coupling of a dynamically excited granular chain, into an inviscid fluid. A sensitivity analysis was carried out in which the excitation frequency and amplitude, the number of spheres in the chain and the sphere diameter were varied. Selected results presented in this paper demonstrate that when undergoing sinusoidal excitations, granular chains can propagate acoustic signals with time and frequency domain content relevant to biomedical ultrasound applications. This conclusion is substantiated by the experimental work carried out in [7].

Nevertheless, due to inherent nonlinearities present in such systems, the acoustic output pressure for a given input displacement can be highly sensitive to the small variations initial conditions.

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