Does Fast Adaptive Modulation Always Outperform Slow Adaptive Modulation?

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***Abstract*—Link adaptation techniques are considered for modern and future wireless communication systems to cope with quality of service fluctuations in fading channels. These techniques require knowledge on the state of the channel updated every coherence time of the process to be tracked, during which a portion of resources is devoted to channel estimation instead of data. In this paper we analyze fast and slow adaptive modulation systems with diversity and non-ideal channel estimation under energy constraints. The framework enables to address the following questions: (i) What is the impact of non-ideal channel estimation on fast and slow adaptive modulation systems? (ii) How to define a proper figure of merit considering both resources dedicated to data and those to channel estimation? (iii) Does fast adaptive always outperform slow adaptive techniques?**

***Index Terms*—Adaptive modulation, multichannel reception, channel estimation, fading channels, performance evaluation.**

I. Introduction

The diffusion of high speed digital wireless communications has increased the need of reliable high data rate communications in variable channel conditions. Adaptive modulation techniques allow to maximize the spectral efficiency (SE) in fading channels without compromising the performance in terms of bit error probability (BEP) and bit error outage (BEO) (see, e.g., [1]–[6]). The M-ary quadrature amplitude modulation (M-QAM) achieves high data rate and it is widely considered in adaptive modulation systems. In [3], for example, power and rate were both adapted to channel conditions for a M-QAM uncoded system. The gain derived from an adaptive rather than a fixed transmitted scheme is reported, together with the negligible channel capacity penalty that the system shows when varying only the data-rate rather than both rate and power. The fast adaptive modulation (FAM) technique tracks instantaneous channel variations due to small-scale fading; the receiver estimates the instantaneous signal-to-noise ratio (SNR) and send a feedback to the transmitter with the optimal constellation size and transmitting power to be used, [1], [4]. Those parameters are tuned to exploit good channel conditions by increasing the transmitted throughput but, at the same time, to preserve the performance in case of bad channel conditions. In [5], a slow adaptive modulation (SAM) technique has been proposed, where modulation parameters are adapted tracking the channel variations averaged over the small-scale fading (i.e., tracking large-scale fading); it has been analyzed for systems employing diversity with ideal channel estimation. It is worth noting that FAM leads to best performance at the cost of a frequent channel estimation or prediction and high feedback rate. With respect to FAM, the SAM technique requires a reduced feedback rate and has lower complexity.

Typical performance metrics for an adaptive communication system are the BEP, the BEO (i.e., the BEP-based outage probability [7]), and the SE. For a given target BEP, the SE and BEO achieved by SAM are close to that of FAM and show a significant improvement with respect to a fixed modulation scheme.

For both FAM and SAM techniques, an important role is played by the channel estimation. The effects of outdated channel estimation are investigated for adaptive modulation systems in [3]. Adaptive modulation systems with non-ideal channel estimation for single- and multi-carrier systems with FAM are analyzed in [3], [6], [8]–[10]. Channel estimation techniques typically utilize resources that would be devoted to data transmission (e.g., pilot symbols can be inserted during the transmission of data symbols) thus sacrificing the SE. Hence, it is important to define proper figure of merit able to capture the trade-off between quality of service and resource utilization depending on the amount of energy devoted to data and pilots.

In this paper, we analyze slow and fast adaptive M-QAM systems with diversity[1] and non-ideal channel estimation. The contribution is three-fold: (i) to define the achieved SE (ASE) which enables to take into account the portion of transmitted frame dedicated to data and pilot symbols; (ii) to analyze SAM with diversity in the presence of non-ideal CSI under energy constraints; (iii) to compare FAM and SAM under various conditions and constraints.

The paper is organized as follows: in Section II the system model and assumptions are presented, and in Section III the performance is derived and a new metric which considers also the resources utilized for channel estimation is defined. In Section IV the performance is evaluated under constraints in terms of BEP, BEO, and the new definition of SE. In Section V numerical results are given with indications on how they can be utilized by a system designer. Finally, our conclusions are given in Section VI.

II. System Model and Assumptions

We consider an adaptive modulation system (see Fig. 1) with M-QAM squared constellation signaling in composite

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[1]For the performance of subset diversity systems, see [11] and [12].

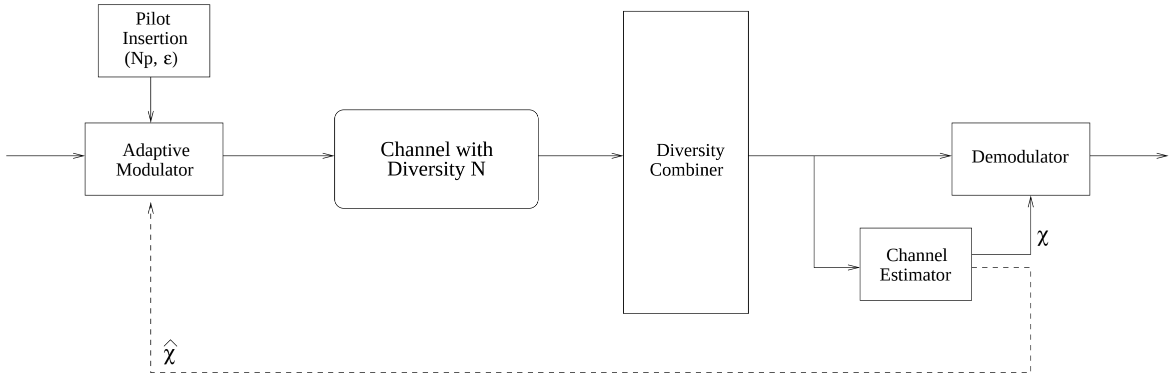


Figure 1. Block scheme of the considered adaptive communication system (χ assumes different meanings depending on the adaptive modulation technique).

Rayleigh fading and log-normal shadowing over N-branches multichannel reception with maximal ratio combining (MRC). Independent, identically distributed (i.i.d.) fading and same shadowing level over all branches, (i.e., microdiversity) is considered.[2] We denote h_i the small-scale fading gain on the i-th branch which is distributed as a complex Gaussian random variable (RV) with mean zero and variance $\sigma_{\rm h}^2$ per dimension for all branches.

When a discrete variable-rate modulation scheme is considered, a set of $J+1$ constellation signaling $\{M_0, M_1, \ldots, M_J\}$ can be adopted. As an example, for digital video broadcasting applications, $M \in \{4, 16, 64, 256\}$, then $M_j = 4^{j+1}$ and $J = 3$. The constellation size is chosen opportunistically depending on the value of a quantity χ which is, respectively, the instantaneous SNR γ in the case of FAM and the mean SNR $\overline{\gamma}$, averaged over small-scale fading, for SAM. Given a target BEP $P_{\rm b}^{\star}$ the required SNR for the modulation M_j is $\chi_j^{\star}$ such that $P_{\rm b}(\chi_j^{\star}) = P_{\rm b}^{\star}$. The opportunistic modulation is chosen by comparing the estimated SNR value in the feedback with the SNRs required for each modulation to satisfy the target BEP. When the SNR value falls in the region $\left[\chi_j^{\star}, \chi_{j+1}^{\star}\right)$ the j-th constellation size M_j is adopted.

We consider a pilot symbols assisted modulation (PSAM) scheme (see, e.g., [13]–[16]) where the transmitted frame is composed by $N_{\rm s}$ data symbols (each with mean energy $E_{\rm s}$) and $N_{\rm p}$ pilot symbols for channel estimation (each with mean energy $E_{\rm p} = \varepsilon E_{\rm s}$).[3] The mean SNR on each diversity branch is $\overline{\gamma} = \mathbb{E}\left\{|h|^2\right\} E_{\rm s}/N_0$, where $N_0/2$ is the two-sided spectral density of the additive white Gaussian noise (AWGN) and $\mathbb{E}\{\cdot\}$ denotes the statistical expectation (here evaluated over the small-scale fading). The shadowing is assumed log-normal distributed where $\overline{\gamma}_{\rm dB} = 10\log_{10}\overline{\gamma}$ is a Gaussian RV having mean $\mu_{\rm dB}$ and variance $\sigma_{\rm dB}^2$. The channel estimator is maximum-likelihood and fading channels over branches are such that $\mathbf{h} = [h_1 h_2 \ldots h_N]$ is constant over a frame. The estimated fading gain on the k-th branch is

$$\widehat{h}_k = h_k + e_k \tag{1}$$

where e_k is a zero mean complex Gaussian RV with variance per dimension $\sigma_{\rm e}^2$. In [14] the $\sigma_{\rm e}^2$ is derived as a function of the energy of pilot symbols and noise spectral density as given by

$$\sigma_{\rm e}^2 = \frac{N_0}{2N_{\rm p}E_{\rm p}}\,. \tag{2}$$

For M-QAM adaptive modulation systems, the non-ideal channel estimation affects both the transmitter side (in the choice of the opportunistic modulation) and the receiver side (in diversity combining and bit reconstruction). To adapt the constellation size to the most updated channel estimation, tight delay constraints should be met in the evaluation of the channel state information (CSI) used at the transmitter side. It follows that the channel estimate for the transmitter is typically less accurate [3] than that at the receiver. On the contrary, the CSI employed at the receiver side does not have such a tight delay constraint. Then, an interpolation between past and future pilot symbols can be harnessed at the receiver which results in a more reliable CSI than the one at the transmitter side. In the following we assume non-ideal CSI at the transmitter side and an ideal CSI at the receiver side.[4]

III. Performance Metrics with Non-Ideal Channel Estimation

For a given target BEP, typical performance metrics for adaptive modulation systems are the SE and the BEO. We evaluate them for multichannel communications with non-ideal channel estimation at the transmitter. For each possible constellation signaling, the SNR value required to reach the target BEP is evaluated and compared to the estimated SNR value $\widehat{\chi}$ (i.e., in FAM (SAM) systems, $\widehat{\gamma}_{\rm T}$ ($\breve{\gamma}$) is compared to the required instantaneous (mean) SNR that satisfies a target instantaneous (mean) BEP). The mean SE and the BEO in systems affected by channel estimation errors can be evaluated from the probability density function (PDF) or the cumulative distribution function (CDF) of $\widehat{\chi}$, that we now evaluate for FAM and SAM systems.

The performance of FAM systems depends on the estimated

[2]Since we consider microdiversity, in the following we will omit the branch subscript in the notation of the mean SNR, averaged over small-scale fading.

[3]The frame is structured to transmit $N_{\rm p}$ pilots within a coherence time of the channel.

[4]For the effects of delayed channel estimates (outdated channels) the reader may refer to, e.g., [3], [10], here we focus on updated but erroneous CSI feedback.

instantaneous SNR at the combiner output $\widehat{\gamma}_{\mathrm{T}}$. For MRC

$$\widehat{\chi} = \widehat{\gamma}_{\mathrm{T}} = \sum_{k=1}^{N} \widehat{\gamma}_k = \sum_{k=1}^{N} |\widehat{h}_k|^2 E_{\mathrm{s}}/N_0.$$

Both real and imaginary parts of $\widehat{h}_k$ are zero-mean Gaussian distributed with variance $\sigma_{\mathrm{h}}^2 + \sigma_{\mathrm{e}}^2$. Therefore, the PDF of the estimated instantaneous SNR $\widehat{\gamma}_{\mathrm{T}}$, conditioned to $\overline{\gamma}$, is a chi-square distribution:

$$f_{\widehat{\gamma}_{\mathrm{T}}|\overline{\gamma}}(\xi) = \frac{\xi^{N-1}}{\overline{\gamma}^N \sigma_t^{2N} \Gamma(N)} \exp\left[-\frac{\xi}{\overline{\gamma}\sigma_{\mathrm{t}}^2}\right] \tag{3}$$

for $\xi \geq 0$ and 0 otherwise, where $\sigma_{\mathrm{t}}^2 = 1/\rho^2$, and

$$\begin{aligned}
\rho &= \frac{\left(\mathbb{E}\left\{h_k \widehat{h}_k^*\right\} - \mathbb{E}\left\{h_k\right\} \mathbb{E}\left\{\widehat{h}_k^*\right\}\right)}{\sqrt{\mathbb{E}\left\{\left|h_k - \mathbb{E}\left\{h_k\right\}\right|^2\right\} \mathbb{E}\left\{\left|\widehat{h}_k - \mathbb{E}\left\{\widehat{h}_k\right\}\right|^2\right\}}} \\
&= \frac{\mathbb{E}\left\{h_k \widehat{h}_k^*\right\}}{\sqrt{\mathbb{E}\left\{\left|h_k\right|^2\right\} \mathbb{E}\left\{\left|\widehat{h}_k\right|^2\right\}}} \\
&= \sqrt{\frac{\sigma_{\mathrm{h}}^2}{\sigma_{\mathrm{h}}^2 + \sigma_{\mathrm{e}}^2}} = \frac{N_{\mathrm{p}}\varepsilon}{N_{\mathrm{p}}\varepsilon + \frac{1}{\overline{\gamma}}}
\end{aligned} \tag{4}$$

is the evelope of the complex correlation between h_k and $\widehat{h}_k$ [14]. Note that, in (4), $\varepsilon = E_{\mathrm{p}}/E_{\mathrm{s}}$ and the second equality follows from $\mathbb{E}\left\{h_k\right\} = \mathbb{E}\left\{\widehat{h}_k^*\right\} = \mathbb{E}\left\{e_k\right\} = 0$. From (3), it is immediate to derive the marginal PDF and CDF of the estimated instantaneous SNR.

For log-normal shadowing the PDF of the mean branch SNR is given by (with $\nu = 10/\ln 10$)

$$f_{\overline{\gamma}}(w) = \frac{\nu}{\sqrt{2\pi}\sigma_{\mathrm{dB}} w} \exp\left[-\frac{(10\log_{10} w - \mu_{\mathrm{dB}})^2}{2\sigma_{\mathrm{dB}}^2}\right] \tag{5}$$

for $w \geq 0$ and 0 otherwise.

The performance of SAM systems depends on the estimated mean SNR $\widehat{\chi} = \breve{\gamma}$ which is given by[5]

$$\breve{\gamma} = \mathbb{E}\{\widehat{\gamma}_k\} = 2(\sigma_{\mathrm{e}}^2 + \sigma_{\mathrm{h}}^2)\frac{E_{\mathrm{s}}}{N_0} = \sigma_{\mathrm{t}}^2\overline{\gamma} = \overline{\gamma} + \frac{1}{N_{\mathrm{p}}\varepsilon}. \tag{6}$$

From (5) and (6), the CDF of the estimated mean SNR results

$$F_{\breve{\gamma}}(x) = Q\left(\frac{\mu_{\mathrm{dB}} - 10\log_{10}(x - \frac{1}{N_{\mathrm{p}}\varepsilon})}{\sigma_{\mathrm{dB}}}\right) \tag{7}$$

for $x \geq 1/(N_{\mathrm{p}}\varepsilon)$ and 0 otherwise, where $Q(x) = \int_x^\infty e^{-t^2/2}dt$ is the Gaussian-Q function.

A. Bit Error Outage

The BEO is an important performance metric for digital communication systems defined as the probability that the BEP is greater than the target BEP [7], [17]–[19]

$$P_{\mathrm{o}}(P_{\mathrm{b}}^\star) = \mathbb{P}\left\{P_{\mathrm{b}}(\chi) > P_{\mathrm{b}}^\star\right\}. \tag{8}$$

The system is in outage when even the constellation size M_0, which is the more robust, does not satisfy the target BEP. For FAM and SAM, respectively, the exact BEP expression is given by (9) and (10), reported at the bottom of this page [20], [21] where

$$I_N(\overline{\gamma}) = \frac{1}{\pi}\int_0^{\pi/2} \left[\frac{\sin^2(\theta)}{\sin^2(\theta) + \frac{3(2i+1)^2}{2(M-1)}\overline{\gamma}}\right]^N \mathrm{d}\theta\,. \tag{11}$$

In the case of ideal channel estimation, the BEO is given by

$$P_{\mathrm{o}}(P_{\mathrm{b}}^\star) = F_\chi(\chi_0^\star) \tag{12}$$

where $\chi = \gamma_{\mathrm{T}}$ for FAM and $\chi = \overline{\gamma}$ for SAM [5].

In systems with non-ideal CSI at the transmitter, the estimated SNR $\widehat{\chi}$ can be an underestimate or an overestimate of the true value. The former case leads to a reduction of the SE and the BEO, while the latter leads to an increase of the SE and BEO. In particular, when $\widehat{\chi} > \chi$, although the true

[5] Note that $\breve{\gamma} = \overline{\gamma}$ for ideal channel estimation.

$$\begin{aligned}
P_{\mathrm{b}}(\gamma_{\mathrm{T}}) = \frac{2}{\sqrt{M}\log_2(\sqrt{M})} &\sum_{h=1}^{\log_2(\sqrt{M})} \sum_{i=0}^{(1-2^{-h})\sqrt{M}-1} (-1)^{\lfloor \frac{i2^{h-1}}{\sqrt{M}} \rfloor} \\
&\times \left(2^{h-1} - \left\lfloor \frac{i2^{h-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right) Q\left((2i+1)\sqrt{\frac{3\gamma_{\mathrm{T}}}{(M-1)}}\right)
\end{aligned} \tag{9}$$

$$\begin{aligned}
P_b(\overline{\gamma}) &= \int_0^\infty P_b(e|\gamma_{\mathrm{T}}) f_{\gamma_{\mathrm{T}}|\overline{\gamma}}(\xi) d\xi \\
&= \frac{2}{\sqrt{M}\log_2(\sqrt{M})} \sum_{h=1}^{\log_2(\sqrt{M})} \sum_{i=0}^{(1-2^{-h})\sqrt{M}-1} (-1)^{\lfloor \frac{i2^{h-1}}{\sqrt{M}} \rfloor} \\
&\times \left(2^{h-1} - \left\lfloor \frac{i2^{h-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right) \underbrace{\int_0^\infty Q\left((2i+1)\sqrt{\frac{3\gamma_{\mathrm{T}}}{(M-1)}}\right) f_{\gamma_{\mathrm{T}}|\overline{\gamma}}(\xi) d\xi}_{I_N(\overline{\gamma})}
\end{aligned} \tag{10}$$

SNR would fall within the range for the j-th modulation, the modulation M_{j+1} might be adopted. In this case, the BEP can be greater than the target BEP and the system is in outage.

Thus, the system is in outage when $\chi < \chi_0^\star$ or $\widehat{\chi} > \chi_i^\star$ and $\chi \le \chi_i^\star$ for all i. Therefore, for $\widehat{\chi} = \chi + \Delta\chi$, the outage occurs for $\chi_i^\star - \Delta\chi < \chi \le \chi_i^\star$ or $\chi < \chi_0^\star$ and the conditioned BEO $P_{\mathrm{o}|\Delta\chi}$ results

$$\begin{cases} F_\chi(\chi_0^\star) + \sum_{j=1}^{J}\left[F_\chi(\chi_j^\star) - F_\chi(\chi_j^\star - \Delta\chi)\right] & \text{with } \Delta\chi \ge 0 \\ F_\chi(\chi_0^\star) & \text{otherwise.} \end{cases} \tag{13}$$

For FAM systems, $\Delta\chi = \Delta\gamma_{\mathrm{T}}$ is a RV whose PDF $f_{\Delta\chi}(\cdot)$ is derived in Appendix and the unconditioned BEO is given by

$$P_{\mathrm{o}} = \int P_{\mathrm{o}|\Delta\chi} f_{\Delta\chi}(\xi) d\xi\,. \tag{14}$$

For SAM systems, we note from (6) that $\Delta\chi = 1/(N_{\mathrm{p}}\varepsilon)$ is deterministic and

$$P_{\mathrm{o}} = Q(a_0) + \sum_{j=1}^{J}\left[Q\left(a_j\right) - Q\left(b_j\right)\right], \tag{15}$$

where
$$a_j = \frac{\mu_{\mathrm{dB}} - \overline{\gamma}^*_{j,\mathrm{dB}}}{\sigma_{\mathrm{dB}}}$$
$$b_j = \frac{\mu_{\mathrm{dB}} - 10\log_{10}(\overline{\gamma}^*_j - 1/(N_{\mathrm{p}}\varepsilon))}{\sigma_{\mathrm{dB}}}\,.$$

As expected, by increasing $N_{\mathrm{p}}\varepsilon$ the channel estimation accuracy increases and the BEO approaches the one for ideal CSI. Then, we compare the case of non-ideal channel estimation with the ideal one, by evaluating the penalty on the target median SNR for a given BEO which is defined as

$$\Delta\mu_{\mathrm{dB}} \triangleq \widehat{\mu}_{\mathrm{dB},0} - \mu_{\mathrm{dB},0} \tag{16}$$

where $\widehat{\mu}_{\mathrm{dB},0}$ and $\mu_{\mathrm{dB},0}$ are the median SNR values that reach the target BEO when the lowest constellation size is considered, respectively for non-ideal and ideal CSI.

B. Achieved Spectral Efficiency

The available SE is given by $\log_2 M_J$ which would be reached if channel conditions are such that the system is always in service with the greater constellation size. In the presence of non-ideal channel estimation and outage events, the ASE might be lower than the available SE and its characterization is important for system design. The mean SE [bps/Hz] with ideal channel estimation is given by [5]

$$\eta = \sum_{j=0}^{J-1}\widetilde{M}_j\left[F_\chi(\chi_{j+1}^\star) - F_\chi(\chi_j)\right] + \widetilde{M}_J\left[1 - F_\chi(\chi_J^\star)\right], \tag{17}$$

where $\widetilde{M}_j = \log_2 M_j$.

In the case of non-ideal channel estimation at the transmitter, the SNR χ is replaced by the estimated one $\widehat{\chi}$. The insertion of pilot symbols occupies part of the resources that could otherwise be utilized for data symbols. In each frame of length N_{tot} symbols, N_{p} pilot symbols and $N_{\mathrm{s}} = N_{\mathrm{tot}} - N_{\mathrm{p}}$ data symbols are transmitted. By denoting the fraction dedicated to pilots and data as $n_{\mathrm{p}} = N_{\mathrm{p}}/N_{\mathrm{tot}}$ and $n_{\mathrm{s}} = N_{\mathrm{s}}/N_{\mathrm{tot}}$, respectively, we define the ASE as

$$\eta_{\,\mathrm{A}} \triangleq \eta\; n_{\mathrm{s}} = \eta\; \frac{(N_{\mathrm{tot}} - N_{\mathrm{p}})}{N_{\mathrm{tot}}}\,. \tag{18}$$

We define the mean spectral efficiency penalty as the ratio of the SE evaluated in the ideal and that in the non-ideal CSI case

$$\Delta\eta \triangleq \frac{\eta^{(\mathrm{ideal})}}{\eta_{\mathrm{A}}} \tag{19}$$

where $\eta^{(\mathrm{ideal})}$ is the spectral efficiency achieved without constraints and with ideal channel estimation, thus in that case $\Delta\eta = 1$. We recall that in SAM systems, the coherence time of tracked channel variation is greater than the one in FAM, and for a given channel estimation quality (i.e., given N_{p} and ε) the portion of the frame dedicated to data symbols is greater than the one for FAM, thus $n_{\mathrm{s}}^{\mathrm{FAM}} \le n_{\mathrm{s}}^{\mathrm{SAM}}$. Conversely, for a given value of n_{p}, the given n_{s} results in more accurate channel estimation for SAM than for FAM.

IV. Performance Analysis Under Constraints

In adaptive modulation systems with non-ideal CSI, the performance strictly depends on the pilot scheme adopted. In the following, we analyze the effects of the imposed constraints and the pilot scheme design on the system performance.

In both FAM and SAM systems, for each adopted pilot scheme, the following constraints are imposed

$$\begin{cases} N_{\mathrm{p}} + N_{\mathrm{s}} = N_{\mathrm{tot}} \\ N_{\mathrm{p}}E_{\mathrm{p}} + N_{\mathrm{s}}E_{\mathrm{s}} = E_{\mathrm{tot}}. \end{cases} \tag{20}$$

Then, the energy dedicated to data depends on the adopted pilot scheme as

$$E_{\mathrm{s}} = \frac{E}{\frac{N_{\mathrm{p}}}{N_{\mathrm{tot}}}(\varepsilon - 1) + 1} \tag{21}$$

where $E \triangleq E_{\mathrm{tot}}/N_{\mathrm{tot}}$. Note that the increasing of $N_{\mathrm{p}}\varepsilon$ leads to a better channel estimation and to a lowering of E_{s}, the two situations have opposite effects on the system performance. Consequently, even the exact SNR χ is a function of the pilot scheme.

A. FAM Systems

From the above constraints, for FAM systems, the SNR $\chi = \gamma_{\mathrm{T}}$ can be expressed as

$$\begin{aligned}\gamma_{\mathrm{T}} &= \sum_{k=1}^{N}\frac{|h_k|^2}{N_0}\frac{E}{\frac{N_{\mathrm{p}}}{N_{\mathrm{tot}}}(\varepsilon - 1) + 1} \\ &= \frac{\upsilon_{\mathrm{T}}}{\frac{N_{\mathrm{p}}}{N_{\mathrm{tot}}}(\varepsilon - 1) + 1}\end{aligned} \tag{22}$$

where $\upsilon_{\mathrm{T}} \triangleq \sum_{k=1}^{N}|h_k|^2 E/N_0$. By substituting (22) in the BEP expressions $P_{\mathrm{b}}(\chi)$ reported in (9) and (10), the P_{b} results a function of two parameters: $N_{\mathrm{p}}\varepsilon$ which characterizes the pilot scheme design and υ_T which represents the SNR per generic (pilot or data) symbol. Thus, the instantaneous BEP is

$$P_\mathrm{b}\left(\upsilon_\mathrm{T}, N_\mathrm{p}\varepsilon\right) = P_\mathrm{b}\left(\gamma_\mathrm{T}\right)|_{\gamma_\mathrm{T}=\frac{\upsilon_\mathrm{T}}{\frac{N_\mathrm{p}}{N_\mathrm{tot}}(\varepsilon-1)+1}}\,. \tag{23}$$

Unlikely the SNR γ_T, υ_T does not depend on the pilot scheme, but only on the mean energy over the frame, N_0, and channel conditions. Therefore, we will compare systems with different pilot schemes for a given υ_T. When constraints are imposed, υ_T is the SNR variable in the BEP expression based on which the constellation size is chosen (for $\varepsilon = 1$, then $\upsilon_\mathrm{T} \to \gamma_\mathrm{T}$). It means that the j-th SNR threshold ($M = M_j$) is defined as

$$\upsilon^\star_{\mathrm{T},j} \quad \text{s.t.} \quad P_\mathrm{b}\left(\upsilon_\mathrm{T}, N_\mathrm{p}\varepsilon\right) = P^\star_\mathrm{b}\,. \tag{24}$$

Due to the imposed constraints, a double effect of the pilot assisted channel estimation is present: i) parameters N_p and ε affect the data energy, leading to an increase of the thresholds levels $\upsilon^\star_{\mathrm{T},j}$; ii) the accuracy of the estimation in the feedback channel depends on N_p and ε. In particular, the feedback estimated SNR is

$$\widehat{\upsilon}_\mathrm{T} = \sum_{k=1}^{N} |\widehat{h}_k|^2 \frac{E}{N_0}\,.$$

When ideal systems (ideal CSI without constraints) are considered, $\widehat{\upsilon}_\mathrm{T} \to \gamma_\mathrm{T}$.

In FAM systems with energy and symbols constraints, the BEO can be evaluated from (8) and (13) for ideal and non-ideal CSI, with $\chi = \upsilon_\mathrm{T}$, $\chi^\star_j = \upsilon^\star_{\mathrm{T},j}$, $\Delta\chi = \Delta\upsilon_\mathrm{T}$, and $F_\chi(\xi) = F_{\upsilon_\mathrm{T}}(\xi')$, where $F_{\upsilon_\mathrm{T}}(\xi')$ is the CDF of υ_T. Note that the instantaneous SNR υ_T is still Rayleigh distributed. The performance in terms of ASE can still be evaluated from (17) and (18), with $F_\chi(\xi) = F_{\widehat{\upsilon}_\mathrm{T}}(\xi')$. The CDF of $\widehat{\upsilon}_\mathrm{T}$ can be derived from the marginal PDF of $f_{\widehat{\upsilon}_\mathrm{T}}(\xi) = \int f_{\widehat{\upsilon}_\mathrm{T}|\overline{\upsilon}}(\xi) f_{\overline{\upsilon}}(w)dw$, where the conditional PDF is

$$f_{\widehat{\upsilon}_\mathrm{T}|\overline{\upsilon}}(\xi) = \frac{\xi^{N-1}}{\overline{\upsilon}^N \sigma_t^{2N}\Gamma(N)} \exp\left[-\frac{\xi}{\overline{\upsilon}\sigma_\mathrm{t}^2}\right], \quad \xi \geq 0$$

and 0 otherwise.

B. SAM Systems

In SAM systems, the same considerations of the FAM case still hold, but based on quantities averaged over the small-scale fading. The mean SNR is given by

$$\begin{aligned}\overline{\gamma} &= \frac{\mathbb{E}\{|h|^2\}}{N_0} \frac{E}{\frac{N_\mathrm{p}}{N_\mathrm{tot}}(\varepsilon-1)+1} \\ &= \frac{\overline{\upsilon}}{\frac{N_\mathrm{p}}{N_\mathrm{tot}}(\varepsilon-1)+1}\end{aligned} \tag{25}$$

where $\overline{\upsilon} \triangleq \mathbb{E}\{|h|^2\}E/N_0$. In the degenerative case of $\varepsilon = 1$, we have $\overline{\upsilon} \to \overline{\gamma}$. The mean BEP expression is given by

$$P_\mathrm{b}\left(\overline{\upsilon}, N_\mathrm{p}\varepsilon\right) = P_\mathrm{b}\left(\overline{\gamma}\right)|_{\overline{\gamma}=\frac{\overline{\upsilon}}{\frac{N_\mathrm{p}}{N_\mathrm{tot}}(\varepsilon-1)+1}} \tag{26}$$

and the j-th SNR threshold ($M = M_j$) is

$$\overline{\upsilon}^\star_j \quad \text{s.t.} \quad P_\mathrm{b}\left(\overline{\upsilon}, N_\mathrm{p}\varepsilon\right) = P^\star_\mathrm{b}\,.$$

In the case of non-ideal channel estimation, the mean SNR is

$$\breve{\upsilon} \triangleq \mathbb{E}\{|\widehat{h}|^2\}\frac{E}{N_0} = \overline{\upsilon} + \underbrace{\frac{1}{N_\mathrm{tot}} + \frac{1-n_\mathrm{p}}{N_\mathrm{p}\varepsilon}}_{\Delta\overline{\upsilon}}\,. \tag{27}$$

When ideal systems (ideal CSI without constraints) are considered, $\breve{\upsilon} \to \overline{\gamma}$.

For ideal CSI systems, the BEO is still evaluated from (8) and (12), with $\chi = \overline{\upsilon}$, $\chi^\star_j = \overline{\upsilon}^\star_j$ and $F_\chi(\xi) = F_{\overline{\upsilon}}(\xi')$, where $F_{\overline{\upsilon}}(\xi')$ is the CDF of $\overline{\upsilon}$. For log-normal shadowing, $\overline{\upsilon}_\mathrm{dB}$ is Gaussian with mean μ'_dB and variance σ^2_dB, and $\overline{\upsilon}$ is a log-normal RV with CDF given by[6]

$$F_{\overline{\upsilon}}(w) = Q\left(\frac{\mu'_\mathrm{dB} - 10\log_{10}(w)}{\sigma_\mathrm{dB}}\right)\,. \tag{28}$$

For non-ideal CSI systems, the BEO can be derived from (13) and (27)

$$P_\mathrm{o} = Q(a_0) + \sum_{j=1}^{J}\left[Q\left(a_j\right) - Q\left(b_j\right)\right], \tag{29}$$

where

$$a_j = \frac{\mu'_\mathrm{dB} - \overline{\upsilon}^\star_{j,\mathrm{dB}}}{\sigma_\mathrm{dB}}$$

$$b_j = \frac{\mu'_\mathrm{dB} - 10\log_{10}\left(\overline{\upsilon}^\star_j - \frac{1}{N_\mathrm{tot}} - \frac{1-n_\mathrm{p}}{N_\mathrm{p}\varepsilon}\right)}{\sigma_\mathrm{dB}}\,.$$

The performance in terms of ASE can still be evaluated from (17) and (18), with $\chi = \overline{\upsilon}$, $\chi^\star_j = \overline{\upsilon}^\star_j$ and $F_\chi(\xi) = F_{\breve{\upsilon}}(x)$ given by

$$F_{\breve{\upsilon}}(x) = Q\left(\frac{\mu'_\mathrm{dB} - 10\log_{10}(x - \frac{1}{N_\mathrm{tot}} - \frac{1-n_\mathrm{p}}{N_\mathrm{p}\varepsilon})}{\sigma_\mathrm{dB}}\right)$$

for $x \geq (1/N_\mathrm{tot}) + (1-n_\mathrm{p})/N_\mathrm{p}\varepsilon$ and 0 otherwise.

V. Numerical Results

We now present numerical results in terms of ASE and BEO for both FAM and SAM systems with Gray code mapping. We assume coherent detection of M-QAM with N-branches MRC. Composite Rayleigh fading and log-normal shadowing channels is considered with both ideal and non-ideal channel estimation.[7] For non-ideal channel estimation, the ASE is evaluated by (18), with a target BEP of 10^{-2} and a maximum BEO of 5% (typical values for uncoded systems). We denote by K the ratio between the frame lengths with SAM and FAM (i.e., the ratio of the coherence time), for small- and large-scale fading, as

$$K = \frac{N^\mathrm{SAM}_\mathrm{tot}}{N^\mathrm{FAM}_\mathrm{tot}}\,.$$

We assume $N^\mathrm{FAM}_\mathrm{tot} = 100$ symbols and $K = 1000$ [22]–[24].[8] For a mobile terminal, the coherence time of the fast fading is inversely proportional to the maximum Doppler frequency:

[6] When $\varepsilon = 1$, $\mu'_\mathrm{dB} = \mu_\mathrm{dB}$, while for $\varepsilon \neq 1$, the median SNR penalty becomes $\Delta\mu_\mathrm{dB} \triangleq \widehat{\mu}'_\mathrm{dB,0} - \mu'_\mathrm{dB,0}$.

[7] Without loss of generality, we assume $\sigma^2_\mathrm{h} = 1/2$.

[8] For realistic shadowing and fading channels [23], [25]–[28], K can be greater. Thus, the gain of the SAM compared to the FAM can be even higher than what shown in this section.

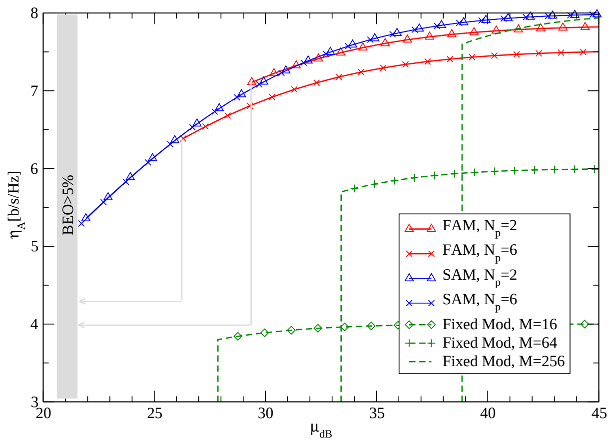


Figure 2. ASE for SAM and FAM systems with non-ideal channel estimation: $M_{\max} = 256$, dual-branches MRC, maximum BEO 5%, $P_b^\star = 10^{-2}$, $\varepsilon = 1$, $N_{\mathrm{tot}}^{\mathrm{FAM}} = 100$, $K = 1000$, and $\sigma_{\mathrm{dB}} = 8$. Comparison with fixed modulation systems ($M = 16, 64$ and 256).

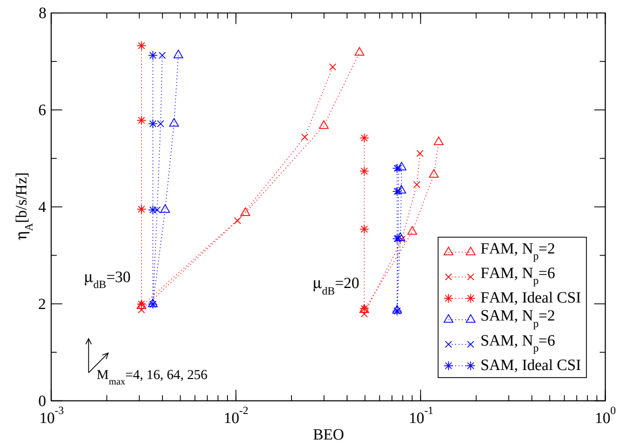


Figure 3. ASE vs. BEO for FAM and SAM systems with $M_{\max} = 4, 16, 64,$ and 256, dual-branches MRC, $\varepsilon = 1$, $\sigma_{\mathrm{dB}} = 8$, $\mu_{\mathrm{dB}} = 20$, and 30, $N_{\mathrm{tot}}^{\mathrm{FAM}} = 100$, and $K = 1000$. Both ideal and non-ideal CSI (with $N_p = 1, 2,$ and 6)

with a carrier frequency of 900 MHz, the coherence time is about 72 ms and 4 ms for a mobile speed of 3 km/h and 50 km/h, respectively. On the other hand, the coherence time of the shadowing is proportional to the coherence distance (e.g., 100-200 m in a suburban area and tens of meters in an urban area [29]). Assuming a coherence distance of 100 m, this results in a coherence time of about 120×10^3 ms and 7.2×10^3 ms at 3 km/h and 50 km/h, respectively. This would lead to $K = 1600 \div 1800$. Assuming a symbol period of $66\mu s$ [30], $N_{\mathrm{tot}}^{\mathrm{FAM}} = 60 \div 1000$ symbols in a coherence time. It is worth noting that, the comparison between FAM and SAM can be made assuming a constant number of pilot symbols N_{p} or a constant portion of pilot symbols within each frame n_{p}. In the former case, the channel estimation quality will be the same for both FAM and SAM, while the portion dedicated to the data symbols will be greater in SAM systems, it means that $n_{\mathrm{s}}^{\mathrm{SAM}} = (1 - n_{\mathrm{p}}^{\mathrm{SAM}}) = (1 - (n_{\mathrm{p}}^{\mathrm{FAM}}/K)) > (1 - n_{\mathrm{p}}^{\mathrm{FAM}}) = n_{\mathrm{s}}^{\mathrm{FAM}}$. Otherwise, by assuming the same portion of pilot symbol n_{p} for both SAM and FAM, $n_{\mathrm{s}}^{\mathrm{FAM}} = n_{\mathrm{s}}^{\mathrm{SAM}}$ while $N_{\mathrm{p}}^{\mathrm{SAM}} > N_{\mathrm{p}}^{\mathrm{FAM}}$, leading to a better channel estimation for SAM. In the following results, we consider a constant N_{p} value.

In Fig. 2 the ASE of FAM, SAM, and fixed modulation systems is reported as a function of the mean SNR with $N = 2$, $M_{\max} = 256$, $\varepsilon = 1$ ($i.e., E_{\mathrm{p}} = E_{\mathrm{s}}$), and $\sigma_{\mathrm{dB}} = 8$.[9] For non-ideal channel estimation, SAM can outperform FAM, for both $N_{\mathrm{p}} = 2$ and 6, which confirm the importance of the analysis for design in practical systems. In particular it can be noticed that the lowest median SNR over which the BEO requirement is satisfied becomes advantageous for SAM as the channel estimation accuracy increases (i.e., N_{p} increases). Then, by increasing N_{p} within the frame, the crossing point beyond which SAM outperforms FAM is reduced. Note also that, for both $N_{\mathrm{p}} = 2$ and 6, the SAM achieves almost the same performance. The reason is that the portion of pilot

[9]For fixed modulation systems, that does not require a feedback, ideal CSI is assumed.

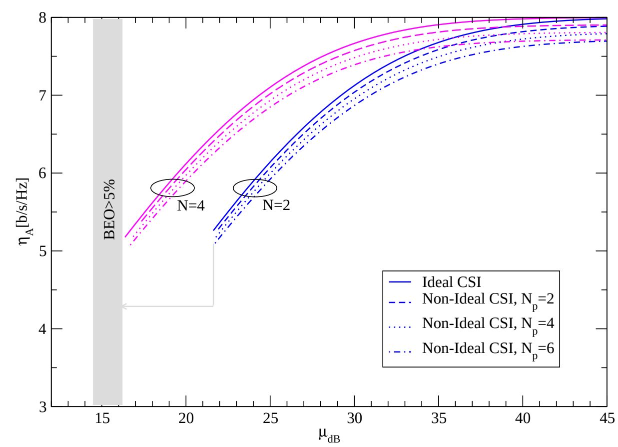
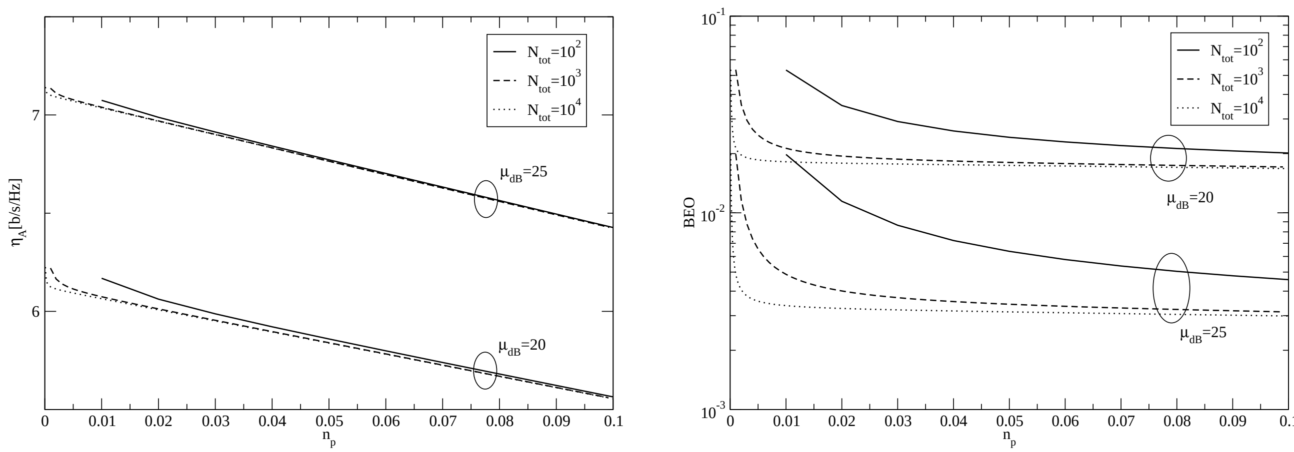


Figure 4. ASE vs. μ_{dB} for SAM systems with MRC ($N = 2$ and 4), $M_{\max} = 256$, maximum BEO 5%, $P_b^\star = 10^{-2}$, $\varepsilon = 1$, $\sigma_{\mathrm{dB}} = 8$, and $N_{\mathrm{tot}}^{\mathrm{SAM}} = 200$. Results are evaluated for both ideal and non-ideal channel estimation.

symbols inserted within the frame of $K \times N_{\mathrm{tot}}^{\mathrm{FAM}}$ symbols is almost constant: in SAM systems, n_{p} varies from 2×10^{-5} to 6×10^{-5}, while in the FAM ones, it varies from 2×10^{-2} to 6×10^{-2}. Moreover, it should be noticed that both FAM and SAM preserve a considerable gain in terms of ASE compared to the fixed modulation schemes. Only for considerably high μ_{dB} values, the fixed modulation system with ideal channel estimation and $M = 256$ can outperform the adaptive modulation systems.

The comparison between FAM and SAM schemes is shown in Fig. 3 in terms of both ASE and BEO for both ideal and non-ideal channel estimation. The systems are compared for several maximum modulation values ($M_{\max} = 4, 16, 64,$ and 256), pilot schemes ($N_{\mathrm{p}} = 2$, and 6, and $\varepsilon = 1$), and for $\sigma_{\mathrm{dB}} = 8$ and $\mu_{\mathrm{dB}} = 20$, and 30. For ideal channel estimation, by increasing the $M_{\max}$ value, the ASE increases while the BEO is constant to the value for $M = 4$, and the FAM system outperforms the SAM one in terms of ASE. Conversely, when non-ideal



(a) ASE

(b) BEO

Figure 6. ASE and BEO vs. $n_{\rm p}$ for SAM systems with $M_{\max} = 256$, four-branches MRC, $\varepsilon = 1$, $\sigma_{\rm dB} = 8$, $\mu_{\rm dB} = 20$ and 25, and $N_{\rm tot} = 10^2, 10^3$, and 10^4.

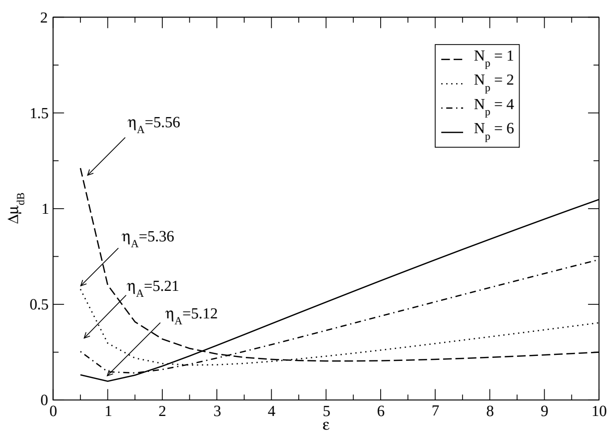


Figure 5. $\Delta\mu_{\rm dB}$ vs. ε for SAM systems with dual-branches MRC, $M_{\max} = 256$, target BEO 5%, $P_{\rm b}^{\star} = 10^{-2}$, $\sigma_{\rm dB} = 8$, $N_{\rm p} = 1, 2$, and 6, and $N_{\rm tot}^{\rm SAM} = 200$.

channel estimation is considered, due to the overestimation of the SNR in the feedback, the BEO increases accordingly with the maximum modulation parameter. In particular, the FAM systems have a non negligible increasing of the BEO value. For example, for $\mu_{\rm dB} = 30$ dB, $M_{\max} = 16$, and $N_{\rm p} = 6$, a FAM system experiences a BEO of 10^{-2}, despite the 3.5×10^{-3} experienced by the SAM system.

In Fig. 4, the ASE as a function of $\mu_{\rm dB}$ is reported for SAM systems with MRC ($N = 2$ and 4 branches), $M_{\max} = 256$, $\varepsilon = 1$, $\sigma_{\rm dB} = 8$, and several values of $N_{\rm p}$. Here, we consider $N_{\rm tot}^{\rm SAM} = 200$ symbols, to emphasize the effect of non-ideal CSI and pilot insertion on the system performance. The tradeoff between channel estimation quality and ASE can be observed in the figure, where for the considered system, $N_{\rm p} = 2$ provides a sufficient quality estimation, with ASE greater than that for $N_{\rm p} = 4$ and 6.

The effect of pilot energy (i.e., the effect of ε) is considered in Fig. 5. In the figure, the median SNR penalty $\Delta\mu_{\rm dB}$ as a function of ε is reported for SAM systems with dual-branches MRC receivers, $M_{\max} = 256$, $N_{\rm tot}^{\rm SAM} = 200$ symbols, target BEO of 5% and various $N_{\rm p}$ values. The numerical values reported represent the ASE evaluated for $\varepsilon = 0.5$ $(i.e., E_{\rm p} = E_{\rm s}/2)$. Low median SNR penalty leads to performance close to the ideal case. In particular, for each $N_{\rm p}$, an ε value that minimizes the penalty can be obtained. By increasing ε the channel estimation accuracy increases, while the symbol energy might decrease.

In Fig. 6 we provide performance of SAM systems for various $N_{\rm tot}$ values (i.e., several frame lengths). In particular, we compare the systems performance in terms of ASE (Fig. 6(a)) and in terms of BEO (Fig. 6(b)) as function of $n_{\rm p}$ for SAM systems with $M_{\max} = 256$, four-branches MRC, $\varepsilon = 1$, $\sigma_{\rm dB} = 8$, $\mu_{\rm dB} = 20$ and 25. Fig. 6(a) shows the ASE vs. $n_{\rm p}$ when $N_{\rm tot}$ equals $10^2, 10^3$, and 10^4. For low $n_{\rm p}$ the system with the lowest number of pilot symbols (i.e., the system with $N_{\rm tot} = 10^2$) outperforms the others. From (6), it can be noticed that, the lower the $N_{\rm p}$, the higher the $\Delta\overline{\gamma}$. It is worth noting that, the overestimation of the mean SNR achieves an ASE higher than the ideal systems, but it leads to an increasing of the BEO, as it can be evaluated from (15). This drawback is depicted in Fig. 6(b), where the BEO as a function of $n_{\rm p}$ is provided. The greater the $N_{\rm p}$, the greater the $\Delta\overline{\gamma}$ and thus the greater is the BEO. For example, when $\mu_{\rm dB} = 25$ and $n_{\rm p} = 0.01$, the system with $N_{\rm tot} = 10^2$ (and thus $N_{\rm p} = 1$) achieves a BEO of 2×10^{-2} despite the 5×10^{-3} and 3.5×10^{-3} experienced by the systems with $N_{\rm tot} = 10^3$ and 10^4, respectively.

Finally, in Fig. 7, we provide a comparison between FAM and SAM performance in terms of ASE penalty in (19), for systems with dual-branches MRC, $M_{\max} = 256$, target BEO 5%, $P_{\rm b}^{\star} = 10^{-2}$, $\sigma_{\rm dB} = 8$, $N_{\rm tot}^{\rm FAM} = 100$ and $K = 1000$. The ASE penalty needs to be minimized, and in particular, $\Delta\eta$ approaches 1 for systems with $\eta_{\rm A}$ that tends to $\eta^{(\rm ideal)}$. For both FAM and SAM, the penalty increases accordingly with

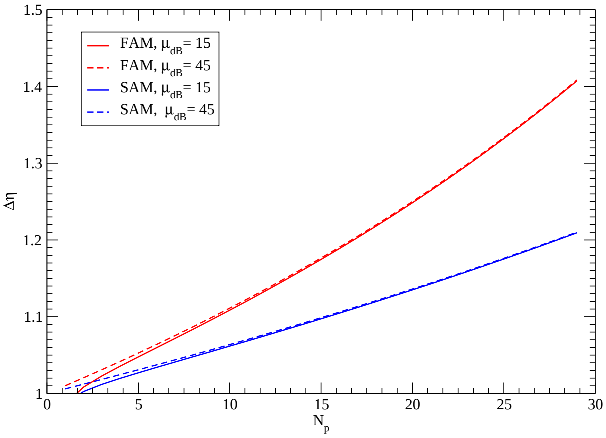


Figure 7. $\Delta\eta$ vs. N_p for FAM and SAM systems with dual-branches MRC, $M_{\max} = 256$, $P_\mathrm{b}^\star = 10^{-2}$, $\sigma_{\mathrm{dB}} = 8$, $N_{\mathrm{tot}}^{\mathrm{FAM}} = 100$, $K = 1000$, and non-ideal CSI.

the number of pilot symbols within each frame. In addition, it is noticeable that the penalty on the ASE for SAM is almost always lower than that of FAM.

From the above results, the system designer can obtain the minimum value of the median SNR for specified target BEP, BEO, and ASE. Since the median SNR is tied to the propagation law and location of the user, one can design the wireless system (e.g., cell size, and power levels for cellular systems) that fulfills the requirements.

VI. Conclusions

We analyzed fast adaptive modulation (FAM) and slow adaptive modulation (SAM) systems with multichannel reception and non-ideal channel estimation under energy constraints. An appropriate figure of merit for the evaluation of the achieved SE (ASE) is defined. It takes into account the tradeoff between channel estimation and data reconstruction for a given total amount of energy per frame. The mathematical framework enables a system designer to evaluate the amount of energy and resources to be devoted to channel estimation for given target bit error probability and outage. Numerical results show that for some system configurations, SAM systems, despite the lower feedback rate, can outperform the FAM systems. This gives a different prospective for the design of adaptive communication systems.

Appendix
Bit Error Outage in FAM Systems

For FAM systems the $\Delta\chi = \Delta\gamma_\mathrm{T}$ is the RV resulting from the difference of two correlated chi-square distributed RVs

$$\Delta\gamma_\mathrm{T} = \widehat{\gamma}_\mathrm{T} - \gamma_\mathrm{T} = \Delta h \frac{E_\mathrm{s}}{N_0}$$

where

$$\Delta h = \sum_{n=1}^{N} \left(|\widehat{h}_n|^2 - |h_n|^2 \right) .$$

and $\rho^2 = \sigma_\mathrm{h}^2/(\sigma_\mathrm{h}^2 + \sigma_\mathrm{e}^2) = \overline{\gamma}/(\overline{\gamma} + (N_\mathrm{p}\varepsilon)^{-1})$ is the normalized correlation. As reported in [31], the PDF of Δh can be expressed as (30) at the bottom of the page, where $\sigma_1^2 = \sigma_\mathrm{h}^2$, $\sigma_2^2 = \sigma_\mathrm{h}^2 + \sigma_\mathrm{e}^2$, and

$$\gamma^- = \frac{\left[\left(\sigma_2^2 - \sigma_1^2 + 4\sigma_1^2\sigma_2^2 \left(1-\rho^2\right) \right)^2 \right]^{1/2}}{\sigma_1^2\sigma_2^2\left(1-\rho^2\right)}$$

$$\alpha^{\pm} = \gamma^- \pm \frac{\sigma_2^2 - \sigma_1^2}{\sigma_1^2\sigma_2^2\left(1-\rho^2\right)} .$$

Thus, the PDF of $\Delta\gamma_\mathrm{T}$, conditioned to $\overline{\gamma}$ results

$$f_{\Delta\gamma_\mathrm{T}|\overline{\gamma}}(\zeta) = f_{\Delta h}(\xi)\big|_{\xi = \Delta\gamma_\mathrm{T} 2\sigma_\mathrm{h}^2/\overline{\gamma}} \frac{2\sigma_\mathrm{h}^2}{\overline{\gamma}} . \qquad (31)$$

The unconditioned PDF of the mean SNR can be written as

$$f_{\Delta\gamma_\mathrm{T}}(\zeta) = \int_{-\infty}^{\infty} f_{\Delta\gamma_\mathrm{T}|\overline{\gamma}}(\zeta) f_{\overline{\gamma}}(\nu) d\nu . \qquad (32)$$

Knowing the distribution of $\Delta\gamma_\mathrm{T}$, the unconditioned BEO for FAM systems can be evaluated.

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$$f_{\Delta h}(\xi) = \begin{cases} \frac{|\xi|^{N-1}}{(N-1)![2\sigma_1^2\sigma_2^2(1-\rho^2)\gamma^-]^N} \exp\left[\frac{1}{4}\alpha^- \xi\right] \sum_{i=0}^{N-1} \frac{(m+i-1)!}{i!(m-i-1)!} \left(\frac{2}{\gamma^-|\xi|}\right)^i & \xi < 0 \\ \frac{|\xi|^{N-1}}{(N-1)![2\sigma_1^2\sigma_2^2(1-\rho^2)\gamma^-]^N} \exp\left[-\frac{1}{4}\alpha^+ \xi\right] \sum_{i=0}^{N-1} \frac{(m+i-1)!}{i!(m-i-1)!} \left(\frac{2}{\gamma^-|\xi|}\right)^i & \xi \geq 0 \end{cases} \qquad (30)$$

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