

Decentralised Stabilisation of Nonlinear Time Delay Interconnected Systems

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Abstract: A decentralised state feedback control scheme is proposed to stabilise a class of nonlinear interconnected systems asymptotically based on the characteristics of the system structure. Under the condition that all the nominal isolated subsystems have uniform relative degree, the considered class of interconnected systems is transferred to a new interconnected system formed of single input systems, which facilitates the decentralised control design. A new term, weak mismatched uncertainty, is introduced for the first time to recognise a class of mismatched uncertainties in the isolated subsystems. The study shows that the effects of both matched and weak mismatched uncertainties in the isolated subsystems can be rejected completely by appropriate choice of control, and the effects of matched interconnections can be largely reduced if the control gain is sufficiently high.

Keywords: Nonlinear interconnected systems, decentralised control, time delay, relative degree.

1. INTRODUCTION

With advances in technology, systems are frequently networked together to form large scale interconnected systems in order to fulfil complex requirements. Such systems appear in power networks, urban traffic networks, ecological systems and energy systems and the corresponding large scale systems are usually distributed geographically in space. This may cause issues with economic cost and reliability of communication links and thus provides an impetus for considering decentralised control which only uses local information within the design (Bakule, 2008).

Decentralised control strategies have been studied for many years and there are numerous results concerning the development of decentralised schemes for interconnected systems including decentralised output feedback schemes (Yan et al., 1998; Bakule, 2008). Much existing work considers interconnected systems with either linear isolated subsystems or linear interconnections, and/or it is required that the nonlinearity or uncertainties satisfy linear growth conditions (Mahmoud, 2009; Ye et al., 2012; Wang et al., 2015). Time delay is another important factor which brings additional complexity to the study of large scale interconnected systems (Yan et al., 2013). A class of time delay interconnected systems is considered in Mahmoud and Bingulac (1998) where delay does not appear in the interconnections. However, the interconnections between

two or more physical systems are often accompanied by phenomena such as material transfer, energy transfer and information transfer, which, from a mathematical point of view, can be represented by delay elements (Michiels and Niculescu, 2007). This has motivated the study of large scale time delay interconnected systems (Bakule, 2008; Hua et al., 2008; Ye et al., 2012).

Due to the richness of nonlinear phenomena, there is no general approach to deal with nonlinear systems as for the linear case. It is necessary to study a class of systems and employ the system structure to complete the design for complex systems. A class of interconnected systems with similar structure is considered in Yan and Zhang (1997) where delay is not involved. A class of nonlinear interconnected systems with triangular structure is considered in Hua et al. (2008), and a large scale system composed of a set of single input single output subsystems with dead zone input is considered in Zhou (2008). In both Hua et al. (2008) and Zhou (2008), the control schemes are based on dynamical feedback which increases the computation greatly. A class of feedforward nonlinear systems are considered in Ye et al. (2012) in which the developed results are only applicable to systems with relatively small delay.

In this paper, a decentralised time delay dependent state feedback controller is synthesised to stabilise a class of large scale time delay interconnected systems with unknown nonlinear interconnections. The interconnections are separated into matched and mismatched parts and

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dealt with separately to reduce the conservatism. Geometric transformations are employed to explore the system structure and transfer the nominal isolated subsystems to a set of single input subsystems to facilitate the design. Discontinuous decentralised controllers are designed using the bounds on the uncertainties relating to each component instead of the bounds on the vector of uncertainties. Using the Lyapunov Razumikhin approach, a set of sufficient conditions are derived so that the corresponding closed-loop systems are uniformly asymptotically stable. The concept of weak mismatched uncertainty is introduced. The study shows that all the matched and weak mismatched uncertainties in the isolated subsystems can be completely rejected by appropriately designed control. Moreover, the effects of the matched interconnections can be largely rejected if the control gain is sufficiently high.

Notation: In this paper, \mathcal{R}^+ denotes the nonnegative set of real numbers $\{t \mid t \geq 0\}$. For a square matrix $A \in \mathcal{R}^{n \times n}$, the expression $A > 0$ represents that the matrix A is symmetric positive definite and the symbol $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) represents its maximum (minimum) eigenvalue. Suppose the function $g : \mathcal{R}^n \mapsto \mathcal{R}$ is differentiable, and $f := (f_1(\cdot), f_2(\cdot), \dots, f_n(\cdot))^T : \mathcal{R}^n \mapsto \mathcal{R}^n$. The notation $L_f g(x)$ denotes the derivative of $g(x)$ along f defined by $L_f g(x) := \sum_{i=1}^n \frac{\partial g}{\partial x} f_i(x)$ and $L_f^k g(x)$ represents a recursion defined by $L_f^k g(x) := \frac{\partial L_f^{k-1} g}{\partial x} f(x)$ with $L_f^0 := g(x)$. Finally, $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

2. PROBLEM FORMULATION

Consider a nonlinear interconnected system described by

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i) (u_i + \phi_i(t, x_i, x_{id_i})) + \xi_i(t, x_i, x_{id_i}) \\ &\quad + \psi_i(t, x, x_d), \quad i = 1, 2, \dots, n \end{aligned} \quad (1)$$

where $x := \text{col}(x_1, \dots, x_n) \in \mathcal{X} := \mathcal{X}_1 \times \dots \times \mathcal{X}_n$, $x_i \in \mathcal{X}_i \subset \mathcal{R}^{n_i}$ is the system state of the i -th subsystem (\mathcal{X}_i is a neighborhood of the origin) and $u_i \in \mathcal{R}^{m_i}$ is the input to the i -th subsystem. $x_d := \text{col}(x_{1d_1}, \dots, x_{nd_n})$ with $x_{id_i} := x_i(t - d_i)$ denote delayed state vectors where $d_i := d_i(t)$ represent a time varying delay which is assumed to be known and bounded by $\bar{d}_i := \sup_{t \in \mathcal{R}^+} \{d_i(t)\} < +\infty$. The initial conditions relating to the time delays are given by $x_i(t) := \varrho_i(t)$ for $t \in [-\bar{d}_i, 0]$ where $\varrho_i(\cdot)$ are continuous. The function matrices $g_i(x_i) := [g_{i1}(x_i), g_{i2}(x_i), \dots, g_{im_i}(x_i)] \in \mathcal{R}^{n_i \times m_i}$ describe the input distributions. The terms $\phi_i(\cdot) \in \mathcal{R}^{m_i}$ represent the uncertainty in the input channel of the i -th subsystems, and $\xi_i(\cdot) \in \mathcal{R}^{n_i}$ denote the mismatched uncertainties in the i -th isolated subsystems. The terms $\psi_i(\cdot) \in \mathcal{R}^{n_i}$ are unknown interconnections of the i -th subsystem. All the vector fields $f_i(x_i) \in \mathcal{R}^{n_i}$ and $g_{il}(\cdot) \in \mathcal{R}^{n_i}$ are assumed to be smooth enough for $i = 1, \dots, n$ and $l = 1, \dots, m_i$.

Definition 1. Consider system (1). The system

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i) (u_i + \phi_i(t, x_i, x_{id_i})) \\ &\quad + \xi_i(t, x_i, x_{id_i}), \quad i = 1, \dots, n \end{aligned} \quad (2)$$

is called the i -th isolated subsystem of (1), and the system

$$\dot{x}_i = f_i(x_i) + g_i(x_i) u_i, \quad i = 1, \dots, n \quad (3)$$

is called the i -th nominal isolated subsystem of system (1).

Definition 2. System (3) is said to have uniform relative degree $(r_{i1}, r_{i2}, \dots, r_{im_i})$ in domain \mathcal{X}_i if there exist functions $h_{ij}(x_i)$ defined in \mathcal{X}_i such that for any $x_i \in \mathcal{X}_i$,

i) $L_{g_{ij}} L_{f_i}^{k_{ij}} h_{il}(x_i) = 0$ for all $j, l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$

ii) the $m_i \times m_i$ function matrices $\Lambda_i(\cdot)$ defined by

$$\Lambda_i := \begin{bmatrix} L_{g_{i1}} L_{f_i}^{r_{i1}-1} h_{i1}(\cdot) & \dots & L_{g_{im_i}} L_{f_i}^{r_{i1}-1} h_{i1}(\cdot) \\ L_{g_{i1}} L_{f_i}^{r_{i2}-1} h_{i2}(\cdot) & \dots & L_{g_{im_i}} L_{f_i}^{r_{i2}-1} h_{i2}(\cdot) \\ \dots & \dots & \dots \\ L_{g_{i1}} L_{f_i}^{r_{im_i}-1} h_{im_i}(\cdot) & \dots & L_{g_{im_i}} L_{f_i}^{r_{im_i}-1} h_{im_i}(\cdot) \end{bmatrix}$$

are nonsingular for $i = 1, \dots, n$

Remark 1. The definition of uniform relative degree above is from Isidori (1995). The uniform relative degree implies that, for any point $x_i \in \mathcal{X}_i$, the system has relative degree, and the relative degree is independent of $x_i \in \mathcal{X}_i$. A full discussion about relative degree appears in Isidori (1995).

Assume that the i -th nominal isolated subsystem of the interconnected system (1) has uniform relative degree $(r_{i1}, r_{i2}, \dots, r_{im_i})$ in the domain \mathcal{X}_i for $i = 1, 2, \dots, n$. The objective of this paper is to design a decentralised control

$$u_i = u_i(t, x_i, x_{id_i}), \quad i = 1, 2, \dots, n \quad (4)$$

such that the corresponding closed-loop systems formed by applying (4) to the interconnected system (1) are uniformly asymptotically stable. This problem is called decentralised state feedback stabilisation. The controllers u_i in (4) depend on the local states x_i and delayed states x_{id_i} but are independent of x_j for $j \neq i$, and are called decentralised time delay dependent controllers.

3. INTERCONNECTED SYSTEM ANALYSIS

Consider the interconnected system (1). The distributions generated by $g_{i1}(\cdot), g_{i2}(\cdot), \dots, g_{im_i}(\cdot)$ in the domain \mathcal{X}_i are denoted by

$$\mathcal{G}_i(x_i) := \text{span} \{g_{i1}(x_i), g_{i2}(x_i), \dots, g_{im_i}(x_i)\} \quad (5)$$

for $i = 1, 2, \dots, n$.

In this section, it is assumed that the nominal isolated subsystem (3) has uniform relative degree

$$(r_{i1}, r_{i2}, \dots, r_{im_i})$$

and the distribution $\mathcal{G}_i(x_i)$ defined in (5) is involutive in the domain \mathcal{X}_i for $i = 1, 2, \dots, n$. The objective is to transfer the system (1) to a new interconnected system to facilitate design.

Let $r_i := \sum_{l=1}^{m_i} r_{il}$ for $i = 1, 2, \dots, n$. From the definition of relative degree, r_{il} are nonnegative constants and $r_i \leq n_i$. Then, the differentials $dh_{il}(x_i), dL_{f_i} h_{il}(x_i), \dots, dL_{f_i}^{r_{il}-1} h_{il}(x_i)$ are linearly independent for $l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$. Let

$$z_{il} := \begin{bmatrix} h_{il}(x_i) \\ L_{f_i} h_{il}(x_i) \\ \vdots \\ L_{f_i}^{r_{il}-1} h_{il}(x_i) \end{bmatrix}, \quad l = 1, 2, \dots, m_i \quad (6)$$

for $i = 1, 2, \dots, n$. Since the distributions $\mathcal{G}_i(x_i)$ are involutive for $i = 1, 2, \dots, n$, there exist $n_i - r_i$ functions

$z_{i(r_i+1)}, z_{i(r_i+2)}, \dots, z_{in_i}$ defined in \mathcal{X}_i such that the Jacobian matrix of the mapping

$$T_i : x_i \mapsto z_i := \text{col}(z_i^a, z_i^b) \quad (7)$$

is nonsingular in \mathcal{X}_i , where $z_i^a := \text{col}(z_{i1}, z_{i2}, \dots, z_{ir_i})$ with $z_{il} \in \mathcal{R}^{r_{il}}$ for $l = 1, 2, \dots, m_i$ and $z_i^b := \text{col}(z_{i(r_i+1)}, z_{i(r_i+2)}, \dots, z_{in_i}) \in \mathcal{R}^{n_i-r_i}$. Thus the transformations $z_i = T_i(x_i)$ are diffeomorphisms in \mathcal{X}_i for $i = 1, 2, \dots, n$, and $z = T(x)$ with

$$T(x) := \text{col}(T_1(x_1) \ T_2(x_2) \ \dots \ T_n(x_n)) \quad (8)$$

defines a new coordinate system $z := \text{col}(z_1, z_2, \dots, z_n)$. Let

$$\mathcal{Z}_i := \{z_i \mid z_i = T_i(x_i), x_i \in \mathcal{X}_i\}$$

It is clear to see that the domain \mathcal{X}_i is transformed to $\mathcal{Z}_i \in \mathcal{R}^{n_i}$ for $i = 1, 2, \dots, n$ in the new coordinate system $z \in \mathcal{R}^{\sum_{i=1}^n n_i}$.

Construct a feedback

$$u_i = -\Lambda^{-1}(x_i)\alpha_i(x_i) + \Lambda^{-1}(x_i)v_i \quad (9)$$

where $v_i \in \mathcal{R}^{m_i}$ are the new inputs, the matrices $\Lambda_i(\cdot) \in \mathcal{R}^{m_i \times m_i}$ defined in Definition 2 are nonsingular for any $x_i \in \mathcal{X}_i$, and

$$\alpha_i(x_i) := \begin{bmatrix} L_{f_i}^{r_{i1}} h_{i1}(x_i) \\ L_{f_i}^{r_{i2}} h_{i2}(x_i) \\ \vdots \\ L_{f_i}^{r_{im_i}} h_{im_i}(x_i) \end{bmatrix} \in \mathcal{R}^{m_i} \quad (10)$$

It follows from Isidori (1995) that in the new coordinates z , the closed-loop systems formed by applying the feedback transformation (9) to the system (1), can be described by

$$\begin{aligned} \dot{z}_i^a &= \text{diag}\{A_{i1}, A_{i2}, \dots, A_{im_i}\} z_i^a + \text{diag}\{B_{i1}, B_{i2}, \\ &\dots, B_{im_i}\} (v_i + \Phi_i(t, z_i, z_{id_i}) + \Xi_i^a(t, z_i, z_{id_i}) \\ &+ \Psi_i^a(t, z, z_d) \end{aligned} \quad (11)$$

$$\dot{z}_i^b = w_i(z_i^a, z_i^b) + \underbrace{\Xi_i^b(t, z_i, z_{id_i}) + \Psi_i^b(t, z, z_d)}_{H_i(t, z, z_d)} \quad (12)$$

where $z_i^a = \text{col}(z_{i1}, z_{i2}, \dots, z_{im_i}) \in \mathcal{R}^{r_i}$ with $z_{il} \in \mathcal{R}^{r_{il}}$ for $l = 1, 2, \dots, m_i$, $z_i^b = \text{col}(z_{i(r_i+1)}, z_{i(r_i+2)}, \dots, z_{in_i}) \in \mathcal{R}^{n_i-r_i}$, $z_i := \text{col}(z_i^a, z_i^b)$, and $z = \text{col}(z_1, z_2, \dots, z_n)$. The symbols $z_{id_i} := z_i(t - d_i(t))$ denote the delayed states where $d_i(t)$ are time varying delays, and $z_d := \text{col}(z_{1d_1}, z_{2d_2}, \dots, z_{nd_n})$. The matrix pairs (A_{il}, B_{il}) have the Brunovsky standard form as follows

$$A_{il} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}_{r_{il} \times r_{il}}, \quad B_{il} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r_{il} \times 1} \quad (13)$$

for $l = 1, 2, \dots, m_i$, and

$$\Phi_i(\cdot) := \phi_i(t, T_i^{-1}(z_i), T_i^{-1}(z_i(d_i))) \quad (14)$$

$$\begin{bmatrix} \Xi_i^a(\cdot) \\ \Xi_i^b(\cdot) \end{bmatrix} := \begin{bmatrix} \frac{\partial T_i}{\partial x_i} \xi_i(t, x_i, x_{id_i}) \end{bmatrix}_{x_i=T_i^{-1}(z_i)} \quad (15)$$

$$\begin{bmatrix} \Psi_i^a(\cdot) \\ \Psi_i^b(\cdot) \end{bmatrix} := \begin{bmatrix} \frac{\partial T_i}{\partial x_i} \end{bmatrix}_{x_i=T_i^{-1}(z_i)} \psi_i(t, T^{-1}(z), T^{-1}(z_d)) \quad (16)$$

where $\Xi_i^a(\cdot) \in \mathcal{R}^{r_i}$. The terms $\Psi_i^a(\cdot) \in \mathcal{R}^{r_i}$ and $\Psi_i^b(\cdot) \in \mathcal{R}^{n_i-r_i}$ denote the interconnections of the i -th subsystems in the new coordinates z .

4. BASIC ASSUMPTIONS

In this section, the focus is on the nonlinear interconnected system (11)–(12). Some basic assumptions are imposed on the system.

Assumption 1. The interconnection terms $\Psi_i^a(t, z, z_d)$ have the decomposition

$$\Psi_i^a(t, z, z_d) = \text{diag}\{B_{i1}, B_{i2}, \dots, B_{im_i}\} \begin{bmatrix} \Theta_{i1}(\cdot) \\ \Theta_{i2}(\cdot) \\ \vdots \\ \Theta_{im_i}(\cdot) \end{bmatrix} \quad (17)$$

where the scalars $\Theta_{il}(t, z, z_d)$ are uncertainties satisfying

$$|\Theta_{il}(t, z, z_d)| \leq \vartheta_{il}^a(t, z_i, z_{id_i}) \vartheta_{il}^b(t, z) \|z_d\| \quad (18)$$

and the terms $H_i(t, z, z_d)$ in (12) satisfy

$$\|H_i(t, z, z_d)\| \leq \varpi_i(t, z) \|z_d\| \quad (19)$$

where the functions $\vartheta_{il}^a(\cdot)$, $\vartheta_{il}^b(\cdot)$ and $\varpi_i(\cdot)$ are known nonnegative continuous in the considered domain for $l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$.

Remark 2. Assumption 1 is the limitation on the interconnections. The equation (17) is equivalent to that the interconnection terms $\Psi_i^a(\cdot)$ are matched. The terms $H_i(t, z, z_d)$ in (12) include all the mismatched interconnections in the i -th subsystems. The conditions (18) and (19) imply that the interconnections of the systems considered are required to be bounded by known functions.

Under the condition (17), system (11)–(12) can be rewritten by

$$\begin{aligned} \dot{z}_{i1} &= A_{i1}z_{i1} + B_{i1}(v_{i1} + \Phi_{i1}(t, z_i, z_{id_i}) + \Theta_{i1}(t, z, z_d) \\ &+ \Xi_{i1}(t, z_i, z_{id_i})) \end{aligned} \quad (20)$$

$$\begin{aligned} \dot{z}_{i2} &= A_{i2}z_{i2} + B_{i2}(v_{i2} + \Phi_{i2}(t, z_i, z_{id_i}) + \Theta_{i2}(t, z, z_d) \\ &+ \Xi_{i2}(t, z_i, z_{id_i})) \end{aligned} \quad (21)$$

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$$\begin{aligned} \dot{z}_{im_i} &= A_{im_i}z_{im_i} + B_{im_i}(v_{im_i} + \Phi_{im_i}(t, z_i, z_{id_i}) \\ &+ \Theta_{im_i}(t, z, z_d) + \Xi_{im_i}(t, z_i, z_{id_i})) \end{aligned} \quad (22)$$

$$\dot{z}_i^b = w_i(z_i^a, z_i^b) + H_i(t, z, z_d) \quad (23)$$

where $z_{il} \in \mathcal{R}^{r_{il}}$, $z_i^b \in \mathcal{R}^{n_i-r_i}$, $z_i := \text{col}(z_i^a, z_i^b)$, $z = \text{col}(z_1, z_2, \dots, z_n)$, $z_{id_i} := z_i(t - d_i(t))$, $z_d := \text{col}(z_{1d_1}, z_{2d_2}, \dots, z_{nd_n})$, $\Theta_{il}(\cdot)$ satisfy (17) and (18) for $l = 1, 2, \dots, m_i$, the interconnection $H_i(t, z, z_d)$ is defined in (12) and

$$\begin{bmatrix} v_{i1} \\ v_{i2} \\ \vdots \\ v_{im_i} \end{bmatrix} := v_i, \quad \begin{bmatrix} \Phi_{i1}(\cdot) \\ \Phi_{i2}(\cdot) \\ \vdots \\ \Phi_{im_i}(\cdot) \end{bmatrix} := \Phi_i, \quad \begin{bmatrix} \Xi_{i1}(\cdot) \\ \Xi_{i2}(\cdot) \\ \vdots \\ \Xi_{im_i}(\cdot) \end{bmatrix} := \Xi_i^a$$

where $\Phi_i(\cdot)$ and $\Xi_i^a(\cdot)$ are defined in (14)–(15) for $i = 1, 2, \dots, n$.

From the fact that (A_{il}, B_{il}) have Brunovsky standard form, it is follows that there exist $K_{il} \in \mathcal{R}^{1 \times r_{il}}$ such that

the matrices $A_{il} - B_{il}K_{il}$ are Hurwitz stable. Therefore, for any matrix $Q_{il} > 0$, the Lyapunov equations

$$(A_{il} - B_{il}K_{il})^T P_{il} + P_{il}(A_{il}P_{il} - B_{il}K_{il}) = -Q_{il} \quad (24)$$

have unique solutions $P_{il} > 0$ for $l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$.

Now, consider the mismatched uncertainties $\Xi_i^a(\cdot)$ existing in the isolated subsystems (20)–(22). For convenience, the following definition is introduced.

Definition 3. Consider system (20)–(22). The mismatched uncertainty $\Xi_{il}(t, z_i, z_{id_i})$ is called weakly mismatched if

$$\|z_{il}^T P_{il} \Xi_{il}(t, z_i, z_{id_i})\| \leq \zeta_{il}(t, z_i, z_{id_i}) |B_{il}^T P_{il} z_{il}| \quad (25)$$

for some nonnegative known continuous function $\zeta_{il}(\cdot)$, where P_{il} satisfy (24) for $l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$.

Remark 3. It should be noted that the weakly mismatched uncertainty identified in Definition 3 describes a class of mismatched uncertainties. It only has limitations on the bounds of the uncertainties and has no structural limitation on the uncertainties. It implies that the weakly mismatched uncertainty vanishes in the kernel space of the image space of the matrix $B_{il}P_{il}$ where B_{il} is the input matrix.

Assumption 2. All the uncertainties $\Xi_{il}(t, z_i, z_{id_i})$ are weakly mismatched and satisfy (25) with $\Phi_{il}(\cdot)$ satisfying

$$\|\Phi_{il}(t, z_i, z_{id_i})\| \leq \eta_{il}(t, z_i, z_{id_i}) \quad (26)$$

where $\eta_{il}(\cdot)$ are nonnegative known continuous functions for $l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$.

Assumption 3. There exist C^1 functions $V_i^b(t, z_i^b) : \mathcal{R} \times \mathcal{R}^{n_i - r_i} \mapsto \mathcal{R}^+$ such that

$$c_{i1} \|z_i^b\|^2 \leq V_i^b(t, z_i^b) \leq c_{i2} \|z_i^b\|^2 \quad (27)$$

$$\frac{\partial V_i^b}{\partial t} + \frac{\partial V_i^b}{\partial z_i^b} w_i(0, z_i^b) \leq -c_{i3} \|z_i^b\|^2 \quad (28)$$

$$\left\| \frac{\partial V_i^b}{\partial z_i^b} \right\| \leq c_{i4} \|z_i^b\| \quad (29)$$

where c_{i1} , c_{i2} , c_{i3} and c_{i4} are positive constants, $\frac{\partial V_i^b}{\partial z_i^b} =: \left(\frac{\partial V_i^b}{\partial z_{i(r_i+1)}}, \frac{\partial V_i^b}{\partial z_{i(r_i+2)}}, \dots, \frac{\partial V_i^b}{\partial z_{in_i}} \right)$, and the nonlinear functions $w_i(z_i^a, z_i^b)$ in (23) satisfy Lipschitz conditions with respect to z_i^a and uniformly for z_i^b in the considered domain \mathcal{Z}_i for $i = 1, 2, \dots, n$.

If $w_i(z_i^a, z_i^b)$ are Lipschitz with respect to z_i^a and uniformly for z_i^b in the considered domain \mathcal{Z}_i , then, for any $\text{col}(z_i^a, z_i^b) \in \mathcal{Z}_i$ and $\text{col}(\tilde{z}_i^a, \tilde{z}_i^b) \in \mathcal{Z}_i$, there exist nonnegative functions $\mathcal{L}_{w_i}(\cdot)$ such that

$$\|w_i(z_i^a, z_i^b) - w_i(\tilde{z}_i^a, \tilde{z}_i^b)\| \leq \mathcal{L}_{w_i}(z_i^b) \|z_i^a - \tilde{z}_i^a\| \quad (30)$$

5. DECENTRALISED CONTROL SYNTHESIS

In this section, controllers which only use local state information are designed for the nonlinear interconnected systems (1) such that the corresponding closed-loop systems are uniformly asymptotically stable.

Consider the control law defined by

$$v_i = -K_i z_i^a + v_i^a + v_i^b + v_i^c \quad i = 1, 2, \dots, n \quad (31)$$

where $K_i := \text{diag}\{K_{i1}, K_{i2}, \dots, K_{im_i}\}$ with $K_{il} \in \mathcal{R}^{1 \times r_{il}}$ satisfying (24), $z_i^a = \text{col}(z_{i1}, z_{i2}, \dots, z_{im_i})$ with $z_{il} \in \mathcal{R}^{r_{il}}$, and

$$v_i^a := \begin{bmatrix} v_{i1}^a(\cdot) \\ v_{i2}^a(\cdot) \\ \vdots \\ v_{im_i}^a(\cdot) \end{bmatrix}, \quad v_i^b := \begin{bmatrix} v_{i1}^b(\cdot) \\ v_{i2}^b(\cdot) \\ \vdots \\ v_{im_i}^b(\cdot) \end{bmatrix}, \quad v_i^c := \begin{bmatrix} v_{i1}^c(\cdot) \\ v_{i2}^c(\cdot) \\ \vdots \\ v_{im_i}^c(\cdot) \end{bmatrix} \quad (32)$$

where

$$v_{il}^a(\cdot) := -\eta_{il}(t, z_i, z_{id_i}) \text{sgn}(B_{il}^T P_{il} z_{il}) \quad (33)$$

$$v_{il}^b(\cdot) := -\frac{(\vartheta_{il}^a(t, z_i, z_{id_i}))^2}{2\varepsilon_{il}} |B_{il}^T P_{il} z_{il}| \text{sgn}(B_{il}^T P_{il} z_{il}) \quad (34)$$

$$v_{il}^c(\cdot) := -\zeta_{il}(t, z_i, z_{id_i}) \text{sgn}(B_{il}^T P_{il} z_{il}) \quad (35)$$

where the symbol ‘sgn’ is the usual signum function, the functions $\eta_{il}(\cdot)$, $\vartheta_{il}^a(\cdot)$ and $\zeta_{il}(\cdot)$ are defined in (26), (18) and (25) respectively, and ε_{il} are any positive constants.

Now, applying control (31) into the interconnected system (20)–(23), the corresponding closed-loop system can be described by

$$\begin{aligned} \dot{z}_{i1} = & (A_{i1} - B_{i1}K_{i1}) z_{i1} + B_{i1} \left(v_{i1}^a(\cdot) + v_{i1}^b(\cdot) + v_{i1}^c(\cdot) \right. \\ & \left. + \Phi_{i1}(t, z_i, z_{id_i}) + \Theta_{i1}(t, z, z_d) \right) + \Xi_{i1}(\cdot) \end{aligned} \quad (36)$$

$$\begin{aligned} \dot{z}_{i2} = & (A_{i2} - B_{i2}K_{i2}) z_{i2} + B_{i2} \left(v_{i2}^a(\cdot) + v_{i2}^b(\cdot) + v_{i2}^c(\cdot) \right. \\ & \left. + \Phi_{i2}(t, z_i, z_{id_i}) + \Theta_{i2}(t, z, z_d) \right) + \Xi_{i2}(\cdot) \end{aligned} \quad (37)$$

.....

$$\begin{aligned} \dot{z}_{im_i} = & (A_{im_i} - B_{im_i}K_{im_i}) z_{im_i} + B_{im_i} \left(v_{im_i}^a(\cdot) + v_{im_i}^b(\cdot) \right. \\ & \left. v_{im_i}^c(\cdot) + \Phi_{im_i}(\cdot) + \Theta_{im_i}(t, z, z_d) \right) + \Xi_{im_i}(\cdot) \end{aligned} \quad (38)$$

$$\dot{z}_i^b = w_i(z_i^a, z_i^b) + H_i(t, z, z_d) \quad (39)$$

where $v_{il}^a(\cdot)$, $v_{il}^b(\cdot)$ and $v_{il}^c(\cdot)$ are given by (33)–(35) respectively for $l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$.

Theorem 1. Under Assumptions 1–3, the nonlinear interconnected systems (20)–(23) are stabilised uniformly asymptotically by the control (31)–(35) if the $4n \times 4n$ function matrix $W(\cdot) := (W_{ij})$ is positive definite in the considered domain where the $n \times n$ matrices W_{ij} for $i, j = 1, 2, 3, 4$ are defined by

$$W_{11} = \text{diag}\{\lambda_{\min}(Q_1) - q\lambda_{\max}(P_1), \dots, \lambda_{\min}(Q_n) - q\lambda_{\max}(P_n)\}$$

$$W_{22} = \text{diag}\{c_{13} - qc_{12}, \dots, c_{n3} - qc_{n2}\}$$

$$W_{33} = \text{diag}\left\{ \lambda_{P_1} - \sum_{j=1}^n \sum_{l=1}^{m_j} (\varepsilon_{jl} (\vartheta_{jl}^b(t, z))^2), \dots, \right.$$

$$\left. \lambda_{P_n} - \sum_{j=1}^n \sum_{l=1}^{m_j} (\varepsilon_{jl} (\vartheta_{jl}^b(t, z))^2) \right\}$$

$$W_{12} = W_{21} = \text{diag}\{c_{14}\mathcal{L}_{w_1}(z_1^b), \dots, c_{n4}\mathcal{L}_{w_n}(z_n^b)\}$$

$$W_{23} = W_{32} = W_{24} = W_{42} \\ = \text{diag}\{c_{14}\varpi_1(t, z), \dots, c_{n4}\varpi_n(t, z)\}$$

$$W_{13} = W_{14} = W_{23} = W_{24} = W_{34} = W_{43} = 0$$

where $P_i := \text{diag}\{P_{i1}, \dots, P_{im_i}\}$ and $Q_i := \text{diag}\{Q_{i1}, \dots, Q_{im_i}\}$ satisfy (24), $q > 1$, $\varepsilon_{jl} > 0$, $\vartheta_{jl}^b(\cdot)$ and $\varpi_1(\cdot)$ are defined in Assumption 1, and c_{ij} with $j = 1, 2, 3, 4$ are defined in Assumption 3 for $i = 1, 2, \dots, n$.

Proof: From the analysis above, it is only necessary to prove that the closed-loop systems (36)–(39) are uniformly asymptotically stable.

For system (36)–(39), consider the Lyapunov function candidate

$$V(t, z(t)) := V(t, z_1(t), z_2(t), \dots, z_n(t)) \\ = \sum_{i=1}^n \sum_{l=1}^{m_i} z_{il}^T(t) P_{il} z_{il}(t) + \sum_{i=1}^n V_i^b(t, z_i^b) \quad (40)$$

where $P_{il} > 0$ satisfy equation (24) for $l = 1, 2, \dots, m_i$ and $i = 1, 2, \dots, n$, and $V_i^b(\cdot)$ satisfy Assumption 3 for $i = 1, 2, \dots, n$.

Then, the time derivative of $V(\cdot)$ along the trajectories of system (36)–(39) is given by

$$\dot{V} = \sum_{i=1}^n \left\{ - \sum_{l=1}^{m_i} z_{il}^T(t) Q_{il} z_{il}(t) + \sum_{l=1}^{m_i} 2z_{il}^T P_{il} B_{il} (v_{il}^a(\cdot) \right. \\ \left. + \Phi_{il}(t, z_i, z_{id_i})) + \sum_{l=1}^{m_i} 2z_{il}^T P_{il} B_{il} (v_{il}^b(\cdot) + \Theta_{il}(t, z, z_d)) \right. \\ \left. + \sum_{l=1}^{m_i} 2z_{il}^T P_{il} (B_{il} v_{il}^c(\cdot) + \Xi_{il}(t, z_i, z_{id_i})) \right\} \\ + \sum_{i=1}^n \left\{ \frac{\partial V_i^b}{\partial t} + \frac{\partial V_i^b}{\partial z_i^b} (w_i(z_i^a, z_i^b) + H_i(t, z, z_d)) \right\} \quad (41)$$

From (26) and the design of v_{il}^a in (33),

$$\sum_{l=1}^{m_i} 2z_{il}^T P_{il} B_{il} (v_{il}^a(\cdot) + \Phi_{il}(t, z_i, z_{id_i})) \\ = \sum_{l=1}^{m_i} 2z_{il}^T P_{il} B_{il} (-\eta_{il}(t, z_i, z_{id_i}) \text{sgn}(B_{il}^T P_{il} z_{il}) \\ + \Phi_{il}(t, z_i, z_{id_i})) \\ = \sum_{l=1}^{m_i} 2\{-\eta_{il}(t, z_i, z_{id_i}) |B_{il}^T P_{il} z_{il}| \\ + \Phi_{il}(t, z_i, z_{id_i}) B_{il}^T P_{il} z_{il}\} \\ \leq 0 \quad (42)$$

From (18), the designed $v_{il}^b(\cdot)$ in (34) and Young's inequality, it follows that for any constant $\varepsilon_{il} > 0$,

$$\sum_{l=1}^{m_i} 2z_{il}^T P_{il} B_{il} (v_{il}^b(\cdot) + \Theta_{il}(t, z, z_d)) \\ \leq \sum_{l=1}^{m_i} \left\{ 2z_{il}^T P_{il} B_{il} v_{il}^b(\cdot) + 2|z_{il}^T P_{il} B_{il}| |\Theta_{il}(t, z, z_d)| \right\}$$

$$\leq \sum_{l=1}^{m_i} \left\{ -\frac{1}{\varepsilon_{il}} z_{il}^T P_{il} B_{il} (\vartheta_{il}^a(t, z_i, z_{id_i}))^2 |B_{il}^T P_{il} z_{il}| \right. \\ \left. \cdot \text{sgn}(B_{il}^T P_{il} z_{il}) + 2|B_{il}^T P_{il} z_{il}| \vartheta_{il}^a(\cdot) \vartheta_{il}^b(t, z) \|z_d\| \right\} \\ \leq \sum_{l=1}^{m_i} \left\{ -\frac{1}{\varepsilon_{il}} (\vartheta_{il}^a(t, z_i, z_{id_i}))^2 |B_{il}^T P_{il} z_{il}|^2 + \right. \\ \left. \frac{1}{\varepsilon_{il}} |B_{il}^T P_{il} z_{il}|^2 (\vartheta_{il}^a(t, z_i, z_{id_i}))^2 + \varepsilon_{il} (\vartheta_{il}^b(t, z))^2 \|z_d\|^2 \right\} \\ = \sum_{l=1}^{m_i} \varepsilon_{il} (\vartheta_{il}^b(t, z))^2 \|z_d\|^2 \quad (43)$$

From the definition of $v_{il}^c(\cdot)$ in (35),

$$\sum_{l=1}^{m_i} 2z_{il}^T P_{il} (B_{il} v_{il}^c(\cdot) + \Xi_{il}(t, z_i, z_{id_i})) \\ = 2 \sum_{l=1}^{m_i} \left\{ -z_{il}^T P_{il} B_{il} \zeta_{il}(t, z_i, z_{id_i}) \text{sgn}(B_{il}^T P_{il} z_{il}) \right. \\ \left. + z_{il}^T P_{il} \Xi_{il}(t, z_i, z_{id_i}) \right\} \\ \leq 2 \sum_{l=1}^{m_i} \left\{ -|z_{il}^T P_{il} B_{il}| \zeta_{il}(t, z_i, z_{id_i}) + z_{il}^T P_{il} \Xi_{il}(t, z_i, z_{id_i}) \right\} \\ \leq 0 \quad (44)$$

where Assumption 2 and (25) are used to obtain the inequality above. From Assumption 3, (30) and (19),

$$\frac{\partial V_i^b}{\partial t} + \frac{\partial V_i^b}{\partial z_i^b} (w_i(z_i^a, z_i^b) + H_i(t, z, z_d)) \\ = \frac{\partial V_i^b}{\partial t} + \frac{\partial V_i^b}{\partial z_i^b} w_i(0, z_i^b) + \frac{\partial V_i^b}{\partial z_i^b} (w_i(z_i^a, z_i^b) - w_i(0, z_i^b)) \\ + \frac{\partial V_i^b}{\partial z_i^b} H_i(t, z, z_d) \\ \leq -c_{i3} \|z_i^b\|^2 + \left\| \frac{\partial V_i^b}{\partial z_i^b} \right\| \|w_i(z_i^a, z_i^b) - w_i(0, z_i^b)\| \\ + \left\| \frac{\partial V_i^b}{\partial z_i^b} \right\| \|H_i(t, z, z_d)\| \\ \leq -c_{i3} \|z_i^b\|^2 + c_{i4} \mathcal{L}_{w_i}(z_i^b) \|z_i^b\| \|z_i^a\| \\ + c_{i4} \varpi_i(t, z) \|z_i^b\| \|z_d\| \quad (45)$$

Since

$$\|z_d\| \leq \sum_{i=1}^n (\|z_{id_i}^a\| + \|z_{id_i}^b\|) \\ \|z_d\|^2 = \sum_{i=1}^n (\|z_{id_i}^a\|^2 + \|z_{id_i}^b\|^2),$$

submitting (42)–(45) into (41) yields that

$$\dot{V} = - \sum_{i=1}^n \sum_{l=1}^{m_i} \lambda_{\min}(Q_{il}) \|z_{il}\|^2 + \sum_{i=1}^n \left\{ -c_{i3} \|z_i^b\|^2 \right. \\ \left. + \sum_{l=1}^{m_i} \varepsilon_{il} (\vartheta_{il}^b(t, z))^2 \|z_d\|^2 + c_{i4} \mathcal{L}_{w_i}(z_i^b) \|z_i^b\| \|z_i^a\| \right. \\ \left. + c_{i4} \varpi_i(t, z) \|z_i^b\| \|z_d\| \right\}$$

$$\begin{aligned}
&\leq -\sum_{i=1}^n \lambda_{\min}(Q_{il}) \|z_{il}\|^2 - \sum_{i=1}^n c_{i3} \|z_i^b\|^2 \\
&+ \sum_{i=1}^n \sum_{j=1}^n \left(\sum_{l=1}^{m_i} \varepsilon_{il} (\vartheta_{il}^b(t, z))^2 \right) \left(\|z_{jd_j}^a\|^2 + \|z_{jd_j}^b\|^2 \right) \\
&+ \sum_{i=1}^n c_{i4} \mathcal{L}_{w_i}(z_i^b) \|z_i^a\| \|z_i^b\| \\
&+ \sum_{i=1}^n \sum_{j=1}^n c_{i4} \varpi_i(t, z) \|z_i^b\| \left(\|z_{jd_j}^a\| + \|z_{jd_j}^b\| \right) \quad (46)
\end{aligned}$$

From the design of $V(\cdot)$ in (40), the fact that

$$V(z_{1d_1}, z_{2d_2}, \dots, z_{nd_n}) \leq qV(z_1, z_2, \dots, z_n), \quad (q > 1)$$

implies that

$$\begin{aligned}
&\sum_{i=1}^n (\lambda_{\min}(P_i) \|z_{id_i}^a\|^2 + c_{i1} \|z_{id_i}^b\|^2) \\
&\leq q \sum_{i=1}^n (\lambda_{\max}(P_i) \|z_i^a\|^2 + c_{i2} \|z_i^b\|^2) \quad (47)
\end{aligned}$$

Therefore, from (47) and (46), it follows that when $V(z_{1d_1}, \dots, z_{nd_n}) \leq qV(z_1, \dots, z_n)$,

$$\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^n (z_i^a)^T Q_i z_i^a + \sum_{i=1}^n \sum_{j=1}^n \left(\sum_{l=1}^{m_i} \varepsilon_{il} (\vartheta_{il}^b(t, z))^2 \right) \\
&\cdot \left(\|z_{jd_j}^a\|^2 + \|z_{jd_j}^b\|^2 \right) - \sum_{i=1}^n c_{i3} \|z_i^b\|^2 \\
&+ \sum_{i=1}^n c_{i4} \mathcal{L}_{w_i}(z_i^b) \|z_i^a\| \|z_i^b\| \\
&+ \sum_{i=1}^n \sum_{j=1}^n c_{i4} \varpi_i(t, z) \|z_i^b\| \left(\|z_{jd_j}^a\| + \|z_{jd_j}^b\| \right) \\
&+ q \sum_{i=1}^n (\lambda_{\max}(P_i) \|z_i^a\|^2 + c_{i2} \|z_i^b\|^2) \\
&- \sum_{i=1}^n (\lambda_{\min}(P_i) \|z_{id_i}^a\|^2 + c_{i1} \|z_{id_i}^b\|^2) \\
&\leq -\sum_{i=1}^n (\lambda_{\min}(Q_i) - q\lambda_{\max}(P_i)) \|z_i^a\|^2 - \sum_{i=1}^n (c_{i3} \\
&- qc_{i2}) \|z_i^b\|^2 + \sum_{i=1}^n \sum_{j=1}^n \left(\sum_{l=1}^{m_i} \varepsilon_{il} (\vartheta_{il}^b(t, z))^2 \right) \\
&\cdot \left(\|z_{jd_j}^a\|^2 + \|z_{jd_j}^b\|^2 \right) + \sum_{i=1}^n c_{i4} \mathcal{L}_{w_i}(z_i^b) \|z_i^a\| \|z_i^b\| \\
&+ \sum_{i=1}^n \sum_{j=1}^n c_{i4} \varpi_i(t, z) \left(\|z_{jd_j}^a\| + \|z_{jd_j}^b\| \right) \|z_i^b\| \\
&- \sum_{i=1}^n (\lambda_{\min}(P_i) \|z_{id_i}^a\|^2 + c_{i1} \|z_{id_i}^b\|^2) \quad (48)
\end{aligned}$$

$$= -Z^T W(z) Z \quad (49)$$

where $Z := (\|z_1^a\|, \|z_1^b\|, \dots, \|z_n^a\|, \|z_n^b\|, \|z_{1d_1}^a\|, \|z_{1d_1}^b\|, \dots, \|z_{nd_n}^a\|, \|z_{nd_n}^b\|)^T$.

Hence the result follows from $W(\cdot) > 0$ by using the Razumikhin Theorem. \square

6. CONCLUSIONS

A discontinuous decentralised control scheme has been proposed to stabilise nonlinear interconnected systems under the assumption that all the system states and time delays are available for design. The designed controllers can reject the effects of the matched and weakly mismatched uncertainties in the isolated subsystems completely if bounds on the uncertainties are known. It is not required that the nominal isolated subsystems are linearisable and there is no limitation on the rate of change of the time delays. Therefore, the obtained results can be applied to a wide class of nonlinear interconnected systems. Further study of interconnected systems using output feedback control scheme may be carried out under the framework of this paper.

REFERENCES

- Bakule, L. (2008). Decentralized control: an overview. *Annual Reviews in Control*, 32(1), 87–98.
- Hua, C., Wang, Q., and Guan, X. (2008). Memoryless state feedback controller design for time delay systems with matched uncertain nonlinearities. *IEEE Trans. on Automat. Control*, 53(3), 801–807.
- Isidori, A. (1995). *Nonlinear Control Systems (Third Edition)*. Great Britain: Springer-Verlag London Limited.
- Mahmoud, M.S. (2009). Decentralized stabilization of interconnected systems with time-varying delays. *IEEE Trans. on Automat. Control*, 54(11), 2663–68.
- Mahmoud, M.S. and Bingulac, S. (1998). Robust design of stabilizing controllers for interconnected time-delay systems. *Automatica*, 34(5), 795–800.
- Michiels, W. and Niculescu, S.I. (2007). *Stability and stabilization of time-delay systems: an eigenvalue-based approach*. Philadelphia: the Society for Industrial and Applied Mathematics.
- Wang, Z., Wang, X., Liu, L., and Huang, M. (2015). Optimal state feedback control for wireless networked control systems with decentralised controllers. *IET Control Theory and Applications*, 9(6), 852–862.
- Yan, X.G., Spurgeon, S.K., and Edwards, C. (2013). Decentralised stabilisation for nonlinear time delay interconnected systems using static output feedback. *Automatica*, 49(2), 633–641.
- Yan, X.G., Wang, J., Lü, X., and Zhang, S. (1998). Decentralized output feedback robust stabilization for a class of nonlinear interconnected systems with similarity. *IEEE Trans. on Automat. Control*, 43(2), 294–299.
- Yan, X.G. and Zhang, S.Y. (1997). Design of robust controllers with similar structure for nonlinear uncertain composite large-scale systems possessing similarity. *Control Theory & Applications*, 14(4), 513–519.
- Ye, H., Jiang, Z., Gui, W., and Yang, C. (2012). Decentralized stabilization of large-scale feedforward systems using saturated delayed controls. *Automatica*, 48(1), 89–94.
- Zhou, J. (2008). Decentralized adaptive control for large-scale time-delay systems with dead-zone input. *Automatica*, 44(7), 1790–99.