ORTHOGONAL REPRESENTATIONS, PROJECTIVE RANK, AND FRACTIONAL MINIMUM POSITIVE SEMIDEFINITE RANK: CONNECTIONS AND NEW DIRECTIONS[*]

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Abstract. Fractional minimum positive semidefinite rank is defined from r-fold faithful orthogonal representations and it is shown that the projective rank of any graph equals the fractional minimum positive semidefinite rank of its complement. An r-fold version of the traditional definition of minimum positive semidefinite rank of a graph using Hermitian matrices that fit the graph is also presented. This paper also introduces r-fold orthogonal representations of graphs and formalizes the understanding of projective rank as fractional orthogonal rank. Connections of these concepts to quantum theory, including Tsirelson's problem, are discussed.

Key words. Projective rank, Orthogonal representation, Minimum positive semidefinite rank, Fractional, Tsirelson's problem, Graph, Matrix.

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