



ORTHOGONAL REPRESENTATIONS, PROJECTIVE RANK, AND FRACTIONAL MINIMUM POSITIVE SEMIDEFINITE RANK: CONNECTIONS AND NEW DIRECTIONS*

LESLIE HOGBEN[†], KEVIN F. PALMOWSKI[‡], DAVID E. ROBERSON[§], AND SIMONE SEVERINI[¶]

Abstract. Fractional minimum positive semidefinite rank is defined from r -fold faithful orthogonal representations and it is shown that the projective rank of any graph equals the fractional minimum positive semidefinite rank of its complement. An r -fold version of the traditional definition of minimum positive semidefinite rank of a graph using Hermitian matrices that fit the graph is also presented. This paper also introduces r -fold orthogonal representations of graphs and formalizes the understanding of projective rank as fractional orthogonal rank. Connections of these concepts to quantum theory, including Tsirelson's problem, are discussed.

Key words. Projective rank, Orthogonal representation, Minimum positive semidefinite rank, Fractional, Tsirelson's problem, Graph, Matrix.

AMS subject classifications. 15B10, 05C72, 05C90, 15A03, 15B57, 81P45.

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[†]Department of Mathematics, Iowa State University, Ames, IA 50011, USA (hogben@iastate.edu), and American Institute of Mathematics, 600 E. Brokaw Rd., San Jose, CA 95112, USA (hogben@aimath.org).

[‡]Department of Mathematics, Iowa State University, Ames, IA 50011, USA (kevin.palmowski@gmail.com).

[§]Division of Mathematical Sciences, Nanyang Technological University, SPMS-MAS-03-01, 21 Nanyang Link, Singapore 637371 (davideroberson@gmail.com). Research supported in part by the Singapore National Research Foundation under NRF RF Award no. NRF-NRFF2013-13.

[¶]Department of Computer Science, University College London, Gower Street, London WC1E 6BT, United Kingdom (simoseve@gmail.com). Research supported by the Royal Society and EPSRC.