Two-Way Beamforming Optimization for Full-Duplex SWIPT Systems

(Invited Paper)

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***Abstract*—In this paper, we investigate the problem of two-way relay beamforming optimization to maximize the achievable sum-rate of a simultaneous wireless information and power transfer (SWIPT) system with a full-duplex (FD) multiple-input multiple-output (MIMO) amplify-and-forward (AF) relay. In particular, we address the optimal joint design of the receiver power splitting (PS) ratio and the beamforming matrix at the relay node given the channel state information (CSI). Our contribution is an iterative algorithm based on difference of convex (DC) programming and one-dimensional searching to achieve the joint optimal solution. Simulation results are provided to demonstrate the effectiveness of the proposed algorithm.**

I. Introduction

Conventionally, wireless communication nodes operate in half-duplex (HD) mode under which they transmit and receive signals over orthogonal frequency or time resources. Recent advances, nevertheless, suggest that full-duplex (FD) communications that allows simultaneous transmission and reception of signal over the same radio channel be possible [1], [2]. This brings a new opportunity for simultaneous wireless information and power transfer (SWIPT) [3]-[5].

In addition to the immediate benefit of essentially doubling the bandwidth, full-duplex communications also find applications in SWIPT. Much interest has turned to full-duplex relaying in which information is sent from a source node to a destination node through an intermediate FD relaying node. In the literature, the studies on relay aided SWIPT largely considered HD relaying and adopted a time-switched relaying (TSR) approach [6], [7]. In [8], joint beamforming optimization and power control for full-duplex MIMO two-way relay channel without energy harvesting was considered.

In contrast to the existing results, this paper investigates the joint optimization of the two-way beamforming matrix for SWIPT in a multiple-input multiple-output (MIMO) amplify-and-forward (AF) full-duplex relay system employing a power splitter (PS), where the sum-rate is maximized subject to the energy harvesting and total power constraints.

Notations—We use $\mathbf{X} \in \mathbb{C}^{M\times N}$ to represent a complex matrix with dimension of $M \times N$. Also, we use $(\cdot)^\dagger$ to denote the conjugate transpose, while $\mathrm{trace}(\cdot)$ is the trace operation, and $\|\cdot\|$ denotes the Frobenius norm. In addition, $|\cdot|$ returns the absolute value of a scalar, and $\mathbf{X} \succeq \mathbf{0}$ denotes that the Hermitian matrix $\mathbf{X}$ is positive semidefinite. The expectation operator is denoted by $\mathbb{E}\{\cdot\}$. We define $\Pi_{\mathbf{X}} = \mathbf{X}(\mathbf{X}^\dagger\mathbf{X})^{-1}\mathbf{X}^\dagger$ as the orthogonal projection onto the column space of $\mathbf{X}$; and $\Pi^{\perp}_{\mathbf{X}} = \mathbf{I} - \Pi_{\mathbf{X}}$ as the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{X}$.

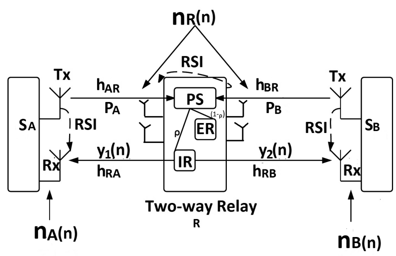

Fig. 1. The model of the two-way full-duplex SWIPT system.

II. System Model

Let us consider SWIPT in a three-node MIMO relay network consisting of two sources $\mathsf{S_A}$ and $\mathsf{S_B}$ wanting to exchange information with the aid of an AF relay R, as shown in Fig. 1. In our model, all the nodes are assumed to operate in FD mode, and we also assume that there is no direct link between $\mathsf{S_A}$ and $\mathsf{S_B}$ so communication between them must be done via R. Both $\mathsf{S_A}$ and $\mathsf{S_B}$ transmit their messages simultaneously to R with transmit power P_A and P_B, respectively.

In the broadcast phase, the relay R employs linear processing with an amplification matrix $\mathbf{W}$ to process the received signal and broadcasts the processed signal to the nodes with the harvested power Q. We assume that each source node is equipped with a pair of transmitter-receiver antennas for signal transmission and reception respectively. We use M_T and M_R to denote the number of transmit and receive antennas at R, respectively. We use $\mathbf{h}_{XR} \in \mathbb{C}^{M_R\times 1}$ and $\mathbf{h}_{RX} \in \mathbb{C}^{M_T\times 1}$ to, respectively, denote the directional channel vectors between the source node X's $(\in A, B)$ transmit antenna to R's receive antennas, and that between the relay's transmit antenna(s) to source node X's receive antenna. The concurrent transmission and reception of signals at the nodes produces self-interference (SI) which inhibits the performance of a full-duplex system. We consider using existing SI cancellation mechanisms in the literature to mitigate the SI (e.g., antenna isolation, analog and digital cancellation, and etc.).

Due to imperfect channel estimation, however, the SI cannot be cancelled completely [9]. We therefore denote h_{AA}, h_{BB} and $\mathbf{H}_{RR} \in \mathbb{C}^{M_R \times M_T}$ as the SI channels at the corresponding nodes. For simplicity, we model the residual SI (RSI) channel as a Gaussian distribution random variable with zero mean and variance σ_X^2, for $X \in \{A, B, R\}$ [9]. We further assume that the relay is equipped with a power splitting (PS) device which splits the received signal power at the relay such that a $\rho \in (0, 1)$ portion of the received signal power is fed to the information receiver (IR) and the remaining $(1-\rho)$ portion of the power is fed to the energy receiver (ER) at the relay.

When the source nodes transmit their signals to the relay, the AF relay employs a short delay to perform linear processing. It is assumed that the processing delay at the relay is given by a τ–symbol duration, which denotes the processing time required to implement the full-duplex operation [10]. τ typically takes integer values. We assume that the delay is short enough compared to a time slot which has a large number of data symbols, and thus its effect on the achievable rate is negligible. At time instant n, the received signal $\mathbf{y}_r[n]$ and the transmit signal $\mathbf{x}_R[n]$ at the relay can be written as

$$\mathbf{y}_R[n] = \mathbf{h}_{AR} s_A[n] + \mathbf{h}_{BR} s_B[n] + \mathbf{H}_{RR}\mathbf{x}_R[n] + \mathbf{n}_R[n], \tag{1}$$

$$\mathbf{x}_R[n] = \mathbf{W}\mathbf{y}_R^{IR}(n-\tau), \tag{2}$$

respectively, where $\mathbf{n}_R[n]$ is the AWGN and $\mathbf{y}_r^{\mathrm{IR}}[n]$ is the signal split to the IR at R given by

$$\mathbf{y}_R^{IR}[n] = \sqrt{\rho}\left(\mathbf{h}_{AR} s_A[n] + \mathbf{h}_{BR} s_B[n] + \mathbf{H}_{RR}\mathbf{x}_R[n] + \mathbf{n}_R[n]\right) + \mathbf{n}_p[n]. \tag{3}$$

Here $\mathbf{n}_p$ is the additional processing noise at the IR. Using (2) and (3) recursively, the overall relay output can be written as

$$\mathbf{x}_R[n] = \mathbf{W}\left(\sqrt{\rho}\left(\mathbf{h}_{AR} s_A[n-\tau] + \mathbf{h}_{BR} s_B[n-\tau] + \mathbf{H}_{RR}\mathbf{x}_R[n-\tau] + \mathbf{n}_R[n-\tau]\right) + \mathbf{n}_p[n-\tau]\right). \tag{4}$$

The capacity of a relay network with delay depends only on the relative path delays from the sender to the receiver and not on absolute delays [11]. Thus, the relay output is given as

$$\mathbf{x}_R[n] = \mathbf{W}\sum_{j=0}^{\infty}(\mathbf{H}_{RR}\mathbf{W})^j[\sqrt{\rho}(\mathbf{h}_{AR} s_A[n-j\tau-\tau] + \mathbf{h}_{BR} S_B[n-j\tau-\tau] + \mathbf{n}_R[n-j\tau-\tau]) + \mathbf{n}_p[n-j\tau-\tau]], \tag{5}$$

where j denotes the index of the delayed symbols.

To simplify the signal model and to keep the optimization problem tractable, we add the zero forcing (ZF) solution constraints such that the optimization of $\mathbf{W}$ nulls out the RSI from the relay output to the relay input [8]. To realise this, it is easy to check from (5) that the condition below is sufficient:

$$\mathbf{W}\mathbf{H}_{RR}\mathbf{W} = \mathbf{0}. \tag{6}$$

Consequently, (5) becomes

$$\mathbf{x}_R[n] = \mathbf{W}\left(\sqrt{\rho}\left(\mathbf{h}_{AR} s_A[n-\tau] + \mathbf{h}_{BR} s_B[n-\tau] + \mathbf{n}_R[n-\tau]\right) + \mathbf{n}_p[n-\tau]\right), \tag{7}$$

with the covariance matrix

$$\mathbb{E}[\mathbf{x}_R\mathbf{x}_R^\dagger] = \rho P_A \mathbf{W}\mathbf{h}_{AR}\mathbf{h}_{AR}^\dagger\mathbf{W}^\dagger + \rho P_B \mathbf{W}\mathbf{h}_{BR}\mathbf{h}_{BR}^\dagger\mathbf{W}^\dagger + \rho\mathbf{W}\mathbf{W}^\dagger + \mathbf{W}\mathbf{W}^\dagger. \tag{8}$$

Thus the relay output power can be written as

$$p_R = \mathrm{trace}(\mathbb{E}[\mathbf{x}_R\mathbf{x}_R^\dagger]) = \rho[P_A\|\mathbf{W}\mathbf{h}_{AR}\|^2 + P_B\|\mathbf{W}\mathbf{h}_{BR}\|^2 + \mathrm{trace}(\mathbf{W}\mathbf{W}^\dagger)] + \mathrm{trace}(\mathbf{W}\mathbf{W}^\dagger). \tag{9}$$

In the second time slot, the received signal at $\mathsf{S_A}$ is given by

$$\begin{aligned} y_{S_A}[n] &= \mathbf{h}_{RA}^\dagger\mathbf{x}_R[n] + h_{AA}s_A[n] + n_A[n] \\ &= \sqrt{\rho}(\mathbf{h}_{RA}^\dagger\mathbf{W}\mathbf{h}_{AR}s_A[n-\tau] \\ &+ \mathbf{h}_{RA}^\dagger\mathbf{W}\mathbf{h}_{BR}s_B[n-\tau] + \mathbf{h}_{RA}^\dagger\mathbf{W}\mathbf{n}_R[n]) \\ &+ \mathbf{h}_{RA}^\dagger\mathbf{W}n_p[n] + h_{AA}s_A[n] + n_A[n]. \end{aligned} \tag{10}$$

After cancelling its own signal $s_A[n-\tau]$, it becomes

$$y_{S_A}[n] = \sqrt{\rho}(\mathbf{h}_{RA}^\dagger\mathbf{W}\mathbf{h}_{BR}s_B[n-\tau] + \mathbf{h}_{RA}^\dagger\mathbf{W}\mathbf{n}_R[n]) + \mathbf{h}_{RA}^\dagger\mathbf{W}n_p[n] + h_{AA}s_A[n] + n_A[n]. \tag{11}$$

The received signal-to-interference-plus-noise ratio (SINR) at node $\mathsf{S_A}$, denoted as γ_A, can be expressed as

$$\gamma_A = \frac{\rho P_B|\mathbf{h}_{RA}^\dagger\mathbf{W}\mathbf{h}_{BR}|^2}{\rho\|\mathbf{h}_{RA}^\dagger\mathbf{W}\|^2 + \|\mathbf{h}_{RA}^\dagger\mathbf{W}\|^2 + P_A|h_{AA}|^2 + 1}. \tag{12}$$

Similarly, the received SINR at node $\mathsf{S_B}$ can be written as

$$\gamma_B = \frac{\rho P_A|\mathbf{h}_{RB}^\dagger\mathbf{W}\mathbf{h}_{AR}|^2}{\rho\|\mathbf{h}_{RB}^\dagger\mathbf{W}\|^2 + \|\mathbf{h}_{RB}^\dagger\mathbf{W}\|^2 + P_B|h_{BB}|^2 + 1}. \tag{13}$$

The achievable rates are then given by $R_A = \log_2(1+\gamma_A)$ and $R_B = \log_2(1+\gamma_B)$, at nodes A and B, respectively.

Now the signal split to ER at the relay node is given as

$$\mathbf{y}_R^{ER} = \sqrt{(1-\rho)}(\mathbf{h}_{AR}s_A[n] + \mathbf{h}_{BR}s_B[n] + \mathbf{H}_{RR}\mathbf{x}_R[n] + \mathbf{n}_R[n]).$$

Thus, the harvested energy at the relay is given by

$$Q = \beta(1-\rho)\left(|\mathbf{h}_{AR}|^2 P_A + |\mathbf{h}_{BR}|^2 P_B + \bar{\mathrm{E}} + M_T\right), \tag{14}$$

where $\bar{\mathrm{E}} = \mathbb{E}[\mathbf{x}_R\mathbf{x}_R^\dagger]$ and β denotes the energy conversion efficiency of the ER at the relay which accounts for the loss in energy transducer for converting the RF energy to electrical energy to be stored. For simplicity, we assume $\beta = 1$.

Note that the conventional HD relay communication system requires two phases for $\mathsf{S_A}$ and $\mathsf{S_B}$ to exchange information. FD relay systems on the other hand reduce the whole operation to only one phase, hence increasing the spectrum efficiency. For simplicity, we assume that the transmit power at the source nodes are intelligently selected by the sources. Therefore, in this work, we do not consider optimization at the source nodes. To ensure a continuous information transfer between the two sources, the harvested energy at the relay should be above a given threshold so that a useful level of harvested energy is reached. As a result, we formulate the joint relay beamforming and receive PS ratio (ρ) optimization problem as a maximization problem of the sum-rate. Mathematically, this problem is formulated as

$$\max_{\mathbf{W},\rho\in(0,1)} \quad R_A + R_B$$
$$\text{s.t.} \quad Q \geq \bar{Q}, \quad p_R \leq P_R, \tag{15}$$

where P_R is the maximum transmit power at the relay and $\bar{Q}$ is the minimum amount of harvested energy required to maintain the relay's operation.

III. Proposed solution

Considering the fact that each source only transmits a single data stream and the network coding principle encourages mixing rather than separating the data streams from the two sources, we decompose $\mathbf{W}$ as $\mathbf{W} = \mathbf{w}_t\mathbf{w}_r^\dagger$, where $\mathbf{w}_t$ is the transmit beam forming vector and $\mathbf{w}_r$ is the receive beam forming vector at the relay. Then the ZF condition is simplified to $(\mathbf{w}_r^\dagger\mathbf{H}_{RR}\mathbf{w}_t)\mathbf{W} = 0$ or equivalently $(\mathbf{w}_r^\dagger\mathbf{H}_{RR}\mathbf{w}_t) = 0$ since in general $\mathbf{W} \neq 0$ [8]. We further assume without loss of optimality that $\|\mathbf{w}_r\| = 1$. Therefore, the optimization problem in (15) can be rewritten as (16) (at the top of the next page) where $C_{rA} \triangleq |\mathbf{w}_r^\dagger\mathbf{h}_{AR}|^2$ and $C_{rB} \triangleq |\mathbf{w}_r^\dagger\mathbf{h}_{BR}|^2$.

A. Parametrization of the receive beamforming vector $\mathbf{w}_r$

Observe in (16) that $\mathbf{w}_r$ is mainly involved in $|\mathbf{w}_r^\dagger\mathbf{h}_{AR}|^2$ and $|\mathbf{w}_r^\dagger\mathbf{h}_{BR}|^2$, so it has to balance the signals received from the sources. According to the result obtained in [12], $\mathbf{w}_r$ can be parameterized by $0 \leq \alpha \leq 1$ as

$$\mathbf{w}_r = \alpha\frac{\Pi_{\mathbf{h}_{BR}}\mathbf{h}_{AR}}{\|\Pi_{\mathbf{h}_{BR}}\mathbf{h}_{AR}\|} + \sqrt{1-\alpha}\frac{\Pi^\perp_{\mathbf{h}_{BR}}\mathbf{h}_{AR}}{\|\Pi^\perp_{\mathbf{h}_{BR}}\mathbf{h}_{AR}\|}. \tag{17}$$

It should be made clear that (17) is not the complete characterization of $\mathbf{w}_r$ because it is also involved in the ZF constraint $\mathbf{w}_r^\dagger\mathbf{H}_{RR}\mathbf{w}_t = 0$, but this parametrization makes the problem more tractable. Thus, given α, we can optimize $\mathbf{w}_t$ for fixed PS ratio ρ. Then perform a 1-D search to find the optimal α^*.

B. Optimization of the receive power splitter (ρ)

For given $\mathbf{w}_r$ and $\mathbf{w}_t$, the optimal receive PS ratio ρ can be determined. Firstly, using the monotonicity between SINR and the rate, (16) can be rewritten as

$$\max_{\rho\in(0,1)} \quad \frac{\rho P_B C_{rB}|\mathbf{h}_{RA}^\dagger\mathbf{w}_t|^2}{\rho\|\mathbf{h}_{RA}^\dagger\mathbf{w}_t\|^2 + \|\mathbf{h}_{RA}^\dagger\mathbf{w}_t\|^2 + P_A|h_{AA}|^2+1}$$
$$+ \frac{\rho P_A C_{rA}|\mathbf{h}_{RB}^\dagger\mathbf{w}_\mathbf{t}|^2}{\rho\|\mathbf{h}_{RB}^\dagger\mathbf{w}_t\|^2 + \|\mathbf{h}_{RB}^\dagger\mathbf{w}_t\|^2 + P_B|h_{BB}|^2+1} \tag{18a}$$
$$\text{s.t.} \; (1-\rho)(|\mathbf{h}_{AR}|^2 P_A + |\mathbf{h}_{BR}|^2 P_B + \bar{\mathrm{E}} + M_T) \geq \bar{Q} \tag{18b}$$
$$\rho(P_A\|\mathbf{w}_t\|^2 C_{rA} + P_B\|\mathbf{w}_t\|^2 C_{rB} + \|\mathbf{w}_t\|^2) + \|\mathbf{w}_t\|^2 \leq P_R. \tag{18c}$$

It is easy to verify that the objective of the problem (18) is an increasing function of ρ. Hence the optimal receive power splitter ρ^* can be determined based on constraints (18b) and (18c) only. The optimal point will be the largest ρ satisfying both constraints. Note that the left-hand side of constraint (18b) is a decreasing function of ρ whereas that of constraint (18c) is an increasing function of ρ. Now the largest ρ satisfying constraint (18b) to equality is given by

$$\rho_l = 1 - \frac{\bar{Q}}{|\mathbf{h}_{AR}|^2 P_A + |\mathbf{h}_{BR}|^2 P_B + \bar{\mathrm{E}} + M_T}. \tag{19}$$

On the other hand, the minimal ρ satisfying constraint (18c) to equality is given by

$$\rho_m = \frac{P_R - \|\mathbf{w}_t\|^2}{P_A\|\mathbf{w}_t\|^2 C_{rA} + P_B\|\mathbf{w}_t\|^2 C_{rB} + \|\mathbf{w}_t\|^2}. \tag{20}$$

We check whether ρ_l satisfies the constraint (18c). If it does, then it is the optimal solution ρ^*. Otherwise, we perform a one-dimensional search over ρ until ρ_m is reached. Obviously, if $\rho_m > \rho_l$, then the problem (18) turns to be infeasible.

C. Optimization of the Transmit Beamforming Vector ($\mathbf{w}_t$)

In this subsection, we first study how to optimize $\mathbf{w}_t$ for given α and ρ. Then we perform a 1-D search on α to find optimal α^* which guarantees an optimal $\mathbf{w}_r^*$ as defined in (17) for the given ρ. For convenience, we define a semidefinite matrix $\mathbf{W}_t \triangleq \mathbf{w}_t\mathbf{w}_t^\dagger$. Then the problem (16) becomes

$$\max_{\mathbf{W}_t \succeq 0} \quad F(\mathbf{W}_t)$$
$$\text{s.t.} \quad \mathrm{trace}(\mathbf{W}_t) \leq \frac{P_R}{\rho(P_A C_{rA} + P_B C_{rB} + 1) + 1}$$
$$(1-\rho)(|\mathbf{h}_{AR}|^2 P_A + |\mathbf{h}_{BR}|^2 P_B + \bar{\mathrm{E}} + 1) \geq \bar{Q}$$
$$\mathrm{trace}(\mathbf{W}_t\mathbf{H}_{RR}^\dagger\mathbf{w}_r\mathbf{w}_r^\dagger\mathbf{H}_{RR}) = 0$$
$$\mathrm{rank}(\mathbf{W}_t) = 1, \tag{21}$$

where $F(\mathbf{W}_t)$ is given in (22) (at the top of the next page). Clearly, $F(\mathbf{W}_t)$ is not a concave function, making the problem challenging. To solve (22), we propose to use the difference of convex programming (DC) to find a local optimum point. To this end, we express $F(\mathbf{W}_t)$ as a difference of two concave functions $f(\mathbf{W}_t)$ and $g(\mathbf{W}_t)$ i.e.,

$$\begin{aligned} F(\mathbf{W}_t) \; = \; & \log_2((\rho P_B C_{rB} + \rho + 1)\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^\dagger) \\ + \; & P_A|h_{AA}|^2 + 1) - \log_2(\rho\,\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^\dagger) \\ + \; & \mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^\dagger) + P_A|h_{AA}|^2 + 1) \\ & + \log_2((\rho P_A C_{rA} + \rho + 1)\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^\dagger) \\ + \; & P_B|h_{BB}|^2 + 1) - \log_2(\rho\,\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^\dagger) \\ + \; & \mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^\dagger) + P_B|h_{BB}|^2 + 1) \\ = \; & f(\mathbf{W}_t) - g(\mathbf{W}_t), \end{aligned} \tag{23}$$

where

$$\begin{aligned} f(\mathbf{W}_t) \; \triangleq \; & \log_2((\rho P_B C_{rB} + \rho + 1)\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^\dagger) \\ + \; & P_A|h_{AA}|^2 + 1) + \log_2((\rho P_A C_{rA} + \rho + 1) \\ \times \; & \mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^\dagger) + P_B|h_{BB}|^2 + 1), \end{aligned} \tag{24}$$

$$
\begin{aligned}
\max_{\mathbf{w}_r, \mathbf{w}_t \rho\in(0,1)} \quad & \log_2\left(1+\frac{\rho P_B C_{rB}|\mathbf{h}_{RA}^{\dagger}\mathbf{w}_t|^2}{\rho\|\mathbf{h}_{RA}^{\dagger}\mathbf{w}_t\|^2+\|\mathbf{h}_{RA}^{\dagger}\mathbf{w}_t\|^2+P_A|h_{AA}|^2+1}\right)\\
& +\log_2\left(1+\frac{\rho P_A C_{rA}|\mathbf{h}_{RB}^{\dagger}\mathbf{w}_t|^2}{\rho\|\mathbf{h}_{RB}^{\dagger}\mathbf{w}_t\|^2+\|\mathbf{h}_{RB}^{\dagger}\mathbf{w}_t\|^2+P_B|h_{BB}|^2+1}\right)\\
\text{s.t.} \quad & (1-\rho)(|\mathbf{h}_{AR}|^2 P_A+|\mathbf{h}_{BR}|^2 P_B+\bar{\mathrm{E}}+M_T)\geq \bar{Q}\\
& \rho(P_A\|\mathbf{w}_t\|^2 C_{rA}+P_B\|\mathbf{w}_t\|^2 C_{rB}+\|\mathbf{w}_t\|^2)+\|\mathbf{w}_t\|^2\leq P_R\\
& \mathbf{w}_r^{\dagger}\mathbf{H}_{RR}\mathbf{w}_t=0. \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
F(\mathbf{W}_t)\triangleq &\log_2\left(1+\frac{\rho P_B C_{rB}\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})}{\rho\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})+\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})+P_A|h_{AA}|^2+1}\right)\\
&+\log_2\left(1+\frac{\rho P_A C_{rA}\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})}{\rho\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+\mathrm{trace}\mathbf{W}_t(\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+P_B|h_{BB}|^2+1}\right). \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
g(\mathbf{W}_t)\triangleq &\log_2(\rho\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})+\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})\\
&+P_A|h_{AA}|^2+1)+\log_2(\rho\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})\\
&+\mathrm{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+P_B|h_{BB}|^2+1). \qquad (25)
\end{aligned}
$$

Note that $f(\mathbf{W}_t)$ is a concave function while $g(\mathbf{W}_t)$ is a convex function. The main idea is to approximate $g(\mathbf{W}_t)$ by a linear function. The linearization (first-order approximation) of $g(\mathbf{W}_t)$ around the point $f(\mathbf{W}_{t,k})$ is given in (26), (at the top of the next page). Then, the DC programming is applied to sequentially solve the following convex problem

$$
\begin{aligned}
\mathbf{W}_{t,k+1} \;=\; & \arg\max_{\mathbf{W_t}} f(\mathbf{W}_t)-g_L(\mathbf{W}_t;\mathbf{W}_{t,k})\\
\text{s.t.} \quad & \mathrm{trace}(\mathbf{W}_t)=\frac{P_R}{\rho(P_A C_{rA}+P_B C_{rB}+1)+1}\\
& (1-\rho)(|\mathbf{h}_{AR}|^2 P_A+|\mathbf{h}_{BR}|^2 P_B+\bar{\mathrm{E}}+1)\geq\bar{Q}\\
& \mathrm{trace}(\mathbf{W}_t\mathbf{H}_{RR}^{\dagger}\mathbf{w}_r\mathbf{w}_r^{\dagger}\mathbf{H}_{RR})=0. \qquad (27)
\end{aligned}
$$

Now the problem (21) can be solved by (i) Choosing an initial point $\mathbf{W}_t$ and ii) For $k=0,1,\cdots$, solving (27) until convergence. Notice that in (27), we have ignored the rank–1 constraint on $\mathbf{W}_t$. This constraint is guaranteed to be satisfied by the results in [13, Theorem 2] when $\mathrm{M}_T>2$, therefore, the decomposition of $\mathbf{W}_t$ leads to the optimal solution $\mathbf{w}_t^{\dagger}$.

D. Optimization of the Receive Beamforming Vector $(\mathbf{w}_r)$

Given $\mathbf{w}_t$, the value of the optimal receive beamforming vector $\mathbf{w}_r$ can be obtained by performing a 1-D search on α to find the maximum α^* which maximises $R_{sum}(\mathbf{w}_r)$ for a fixed value of $\rho\in(0,1)$. Algorithm 1 summarises this procedure. The bounds of the rate search interval are obtained as follows. The lower bound $(R_A+R_B)_{low}$ is obviously zero while the upper bound $(R_A+R_B)_{max}$ is defined as the achievable sum-rate at zero RSI. With optimal α^*, optimal $\mathbf{w}_r^*$ can be obtained from (17).

Algorithm 1 Procedure for solving problem (21)

1: Set $0\leq\alpha\leq1$ and $0\leq\rho\leq1$ as non-negative real-valued scaler and obtain $\mathbf{w}_r$ as given in (17).
2: At step k, set $\alpha(k)=\alpha(k-1)+\triangle\alpha$ until $\alpha(k)=1$, where $\triangle\alpha$ is the searching step size.
3: Initialise $(R_A+R_B)_{low}=0$ and $(R_A+R_B)_{up}=(R_A+R_B)_{max}$.
4: **Repeat**
  a) Set $R\leftarrow\frac{1}{2}((R_A+R_B)_{low}+(R_A+R_B)_{up})$
  b) Obtain the optimal relay transmit beamforming vector $\mathbf{w}_t$ by solving problem (27).
  iii) Update the value of R with the bisection search method: if (ii) is feasible, set $(R_A+R_B)_{low}=R$; otherwise, $(R_A+R_B)_{up}=R$.
5: **Until** $(R_A+R_B)_{up}-(R_A+R_B)_{low}<\epsilon$, where ϵ is a small positive number. Thus we get $R(\alpha(k))$.
6: k = k+1
7: Find optimal α^* by comparing all $R(\alpha(k))$ that yields maximal R. Corresponding $\mathbf{w}_t$ is the optimal one.
8: Obtain the optimal $\mathbf{w}_r^*$ from (17) using α^*.

E. Iterative update

Now, the original beamforming and receive power splitter optimization problem (16) can be solved by an iterative technique shown in Algorithm 2. Algorithm 2 continually updates the objective function in (16) until convergence.

IV. Numerical example

In this section, we evaluate the performance of the proposed algorithm through computer simulations assuming flat Rayleigh fading environments. In order to ensure that the relay harvests the maximum possible energy, we assume that the two source nodes transmit at their maximum power budget, i.e., $P_A=P_B=P_{\max}$ and $P_R=4$ (dB). All simulations are

$$
\begin{aligned}
g_L(\mathbf{W}_t;\mathbf{W}_{t,k}) = {} & \frac{1}{\ln(2)}\frac{\rho\text{trace}((\mathbf{W}_t-\mathbf{W}_{t,k})\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})+\text{trace}((\mathbf{W}_t-\mathbf{W}_{t,k})\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})}{\rho\text{trace}(\mathbf{W}_{t,k}\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})+\text{trace}(\mathbf{W}_t\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})+P_A|h_{AA}|^2+1} \\
& +\frac{1}{\ln(2)}\frac{\rho\text{trace}((\mathbf{W}_t-\mathbf{W}_{t,k})\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+\text{trace}((\mathbf{W}_t-\mathbf{W}_{t,k})\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})}{\rho\text{trace}(\mathbf{W}_{t,k}\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+\text{trace}(\mathbf{W}_t\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+P_B|h_{BB}|^2+1}+\log_2(\rho\text{trace}(\mathbf{W}_{t,k}\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger}) \\
& +\text{trace}(\mathbf{W}_{t,k}\mathbf{h}_{RA}\mathbf{h}_{RA}^{\dagger})+P_A|h_{AA}|^2+1)+\log_2(\rho\text{trace}(\mathbf{W}_{t,k}\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+\text{trace}(\mathbf{W}_{t,k}\mathbf{h}_{RB}\mathbf{h}_{RB}^{\dagger})+P_B|h_{BB}|^2+1). \quad (26)
\end{aligned}
$$

Algorithm 2 Procedure for solving problem (16)

1: Initialise $0 \leq \rho \leq 1$.
2: **Repeat**
   a) Obtain $\mathbf{w}_t^*$ and $\mathbf{w}_r^*$ using Algorithm 1.
   b) Obtain optimal ρ^* following the procedure in subsection III-B
3: **Until** convergence.

averaged over 500 independent channel realizations.

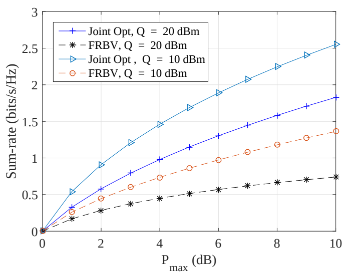

Fig. 2. Sum-rate versus P_{max}.

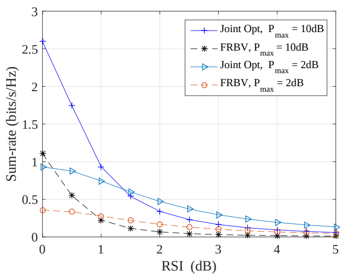

Fig. 3. Sum-rate versus residual self-interference.

In Fig. 2, we show the sum-rate results against the transmit power budget P_{max} (dB) for various harvested energy constraint. The proposed scheme ('Joint Opt' in the figure) is compared with those of the fixed receive beamforming vector ($\mathbf{w}_r$) ('FRBV'= 0.583) at optimal PS coefficient (ρ^*). Remarkably, the proposed scheme yields higher sum-rate compared to the sum-rate of the FRBV schemes which essentially necessitates joint optimization. Also, as the harvested energy constraint decreases from 20 dBm to 10 dBm, the achievable sum-rate for both schemes increases.

In the last figure, we analyze the impact of the residual self-interference on the sum-rate. We can observe from Fig. 3 that an increase in the residual self-interference results in a corresponding decrease in the achievable sum- rate. Also, we see that the sum-rate decreases faster at higher transmit power in the low RSI region.

V. Conclusion

In this paper, we investigated the joint beamforming optimization for SWIPT in FD MIMO two-way relay channel and proposed an algorithm which maximizes the sum-rate subject to the relay transmit power and harvested energy constraints. Using DC and a 1-D search, we jointly optimized the receive beamforming vector, the transmit beamforming vector, and receive PS ratio to maximize the sum-rate. Simulation results confirm the importance of joint optimization.

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