NBER WORKING PAPER SERIES

CONSUMER DURABLES AND INERTIAL BEHAVIOR: ESTIMATION AND AGGREGATION OF (*S,s*) RULES

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Working Paper 5282

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 1995

This paper was financially supported by NSF grant SES-9057511. I also wish to thank the Center for Economic and Policy Research and OTL at Stanford for additional support during the early stages of the project. Cammy Baray, Shawn Blosser, Penny Koujianou and Tom Deleire provided excellent research assistance at various stages of the project. Particular thanks go to Dean Maki. Without his patience in dealing with the data and his dedication, this paper would have been much harder to finish. I would like to thank Tom MaCurdy for some very useful discussions and seminar audiences at Stanford, the Board of Governors, MIT, Boston College, UCL, ECARE, CEMFI, Universitat Pompeu Fabra Barcelona, the University of Bologna, and the Bank of Italy for stimulating comments. The paper was also presented at the workshop 'Empirical macroeconomics: what methods for what problems' held at ECARE, Brussels on June 2nd and 3rd, 1995, where I received useful feedback from the audience and my discussant Giuseppe Bertola. Responsibility for any errors is mine. This paper is part of NBER's research program in Economic Fluctuations. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

This paper presents an (*S*,*s*) model for automobile consumption and estimates it using a data set of US households. The model allows for unobserved heterogeneity in both the target level and the band width, takes into account the possibility of a zero desired level, constrains the band to be non negative and allows asymmetric bands.

The model is estimated on a novel data set which contains information on both stock values and automobile expenditure for a large number of households observed over a period of a year. The (*S*,*s*) rule is specified in terms of the ratio of car stock to non durables. The shortcuts usually employed in the empirical literature on (*S*,*s*) rules can be avoided thanks to the richness of the data set and the rigorous specification of the stochastic model.

Having estimated the model and considered 'goodness of fit' measures, aggregation issues are considered. First, the paper presents a number of negative results. Then, several simulations aimed at evaluating the effects induced by inertial behavior on aggregate dynamics are considered.

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1. Introduction.

In this paper I model the purchase behavior of automobiles by US households as an (S,s) rule. I estimate an extremely flexible specification of such a rule using a large microeconomic data set and study some of the aggregate implications of this type of inertial behavior. The paper is one of the first attempts at estimating directly the parameters of an (S,s) rule using micro data.[1] Because of this, some of the methodological issues concerning the specification and the estimation of (S,s) rules are relevant for many potential applications. Furthermore, the paper is the first to discuss and quantify the aggregate implications of an (S,s) rule estimated on micro data.

The emphasis of the paper is not on brand choice, but rather on the determination of the optimal stock of automobiles at each point in time. [2] The modelling strategy I pursue allows for the presence of transaction costs which introduce important non-convexities into the optimization problem faced by the agent. In such a situation a full and consistent characterization of individual behavior is possible only under very special circumstances and very restrictive assumptions. To describe actual data, however, a more flexible empirical strategy is necessary.

One possibility is to estimate a reduced form equation. While this approach is useful as a tool to summarize and describe the data, it is unable to provide an answer to a large a number of important questions. Alternatively, one can think in terms of an equilibrium model which is relevant in the absence of transaction costs and model individual behavior as an (S,s) rule defined in terms of deviations from such an equilibrium model. In what follows, I define the (S,s) rule in terms of the ratio of the value of the stock of automobiles to non durable consumption. The first order condition for the 'intratemporal' allocation of resources between these two commodities is implicit in this strategy. The use of non durable consumption (a variable which is likely to be adjusted without transaction costs) as the denominator of the ratio implicitely controls for differences in permanent income.

(S,s) rules, originally introduced in the study of inventories (see Arrow et al. (1951), Scarf (1959)), have been proved to be optimal for consumer durables only under special circumstances. However, the stochastic specification I use is flexible enough that could conceivably encompass a large class of models.[3]

The parameters of the model are estimated using a large microeconomic data set, the Consumer Expenditure Survey (CEX), which contains information both on stocks and expenditure

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[1] The only other study trying something similar is that by Lam (1991). Both the specification and the data used here are, however, richer.

[2] See Goldberg (1994) for an empirical model of brand choice.

[3] There are, however, some limitations. In particular, implicit in my approach is the assumption that utility depends on the value of the stock of cars owned by a household *regardless* of the number of cars that compose that stock. In other words, a household is assumed to derive the same utility from two \$ 12,000 cars than from one \$ 24,000 car.

on cars as well as non-durable consumption. The wealth of information in the data allows me to specify and estimate of a very rich and complex model. In particular, I allow for observed and unobserved heterogeneity both in target levels and band width. Specifying the (S, s) rule in terms of the ratio of durable stocks to non-durable consumption, I also implicitely control for differences in permanent income. I consider explicitly the possibility of corner solutions for the desired stock. Finally, the structure I propose can be easily generalized to explore the implications of different parametric assumptions about the distributions of the errors in the system.

Having estimated the parameters of the (S,s) rule, the aim of the second part of the paper is to establish the consequences of inertial behavior for aggregate durable expenditure. Some of the results discussed in this section are negative ones. I show that is extremely difficult to characterize the time series properties of aggregate expenditure from the estimated Ss rules. There are two main reasons for these difficulties. First, to derive aggregate expenditure it is necessary to characterize the processes that move the value of the state variable when no action is undertaken. The model is silent about the nature of these processes. Second, the characterization of the target level implies the presence of traditional aggregation problems *regardless* of the presence of inertial behavior. Furthermore, there is no reason to believe that the non-linearities in the target level do not interact with the determinants of the band width. It is therefore not possible to establish the effect of inertial behavior without considering explicitely the aggregation issues in the target level. The estimation results show that these effects are indeed important.

Not all the results in the Section on aggregation are negative, however. From the estimated parameters I am able to compare aggregate expenditures of systems without transaction costs, with quadratic costs of adjustment and with (S, s) microeconomic rules. While I am not able to characterize completely the aggregate time series properties of any of these systems, I can map the effect that different aggregate shocks have on aggregate expenditure in the three systems. Furthermore, I consider the degree of persistence at the aggregate level induced by individual inertial behavior.

Recently, Caballero and Engel (1992,1994), have proposed a fairly general model of lumpy microeconomic behavior they label as the Generalized (S, s) approach. The basic idea is to have the probability of adjusting a control variable to be a function of the difference between the current level of the variable and the 'desired' level in the absence of transaction or adjustment costs. Caballero and Engel (1992) show that a variety of models, including models with quadratic costs of adjustment and simple (S, s) rules are encompassed by their framework by specifying functional form assumption on the hazard function. My parametrization of (S, s) rules is observationally equivalent, for all practical purposes, to their general hazard function. A specific functional form assumption on their hazard function corresponds to an assumption on the distribution of the residuals in my specification.

The rest of the paper is organized as follows: In section 2, I sketch the theoretical framework used in the paper. In Section 3, I derive the likelihood function and discuss some specification and identification issues. Section 4 describes the main features of the data used in the paper and presents some reduced form evidence. In Section 5, I report the main empirical results and discuss the model's 'goodness of fit'. In section 6, I analyze the aggregation of the estimated (S, s) rules and discuss their implications for aggregate automobile expenditure. Section 7 concludes the paper.

2. The Theoretical Framework

2.1 Review of the literature

The analysis of (S, s) rules for modeling consumption behavior has recently received considerable attention. While the optimality of (S, s) rules can only be proved under fairly restrictive circumstances, they constitute an intuitively appealing way of modelling behavior in the presence of transaction costs of various nature. By choosing a flexible specification for the rule it is should be possible to represent accurately a wide variety of behaviors.

Few theoretical results show the optimality of (S, s) rules. Grossman and Laroque (1991) derive the conditions under which an (S, s) rule is optimal for a consumption and investment problem. Some of their results are extended by Eberly (1991) and Beaulieu (1993). More recently, Eberly (1994) has derived some closed form solutions for durable consumption in the presence of transaction costs while Caballero and Engel (1994) construct a model of investment where an (S, s) rule is optimal.

The major problem with the theoretical results available in the literature is that it is only possible to show optimality of (S, s) rules under extremely stringent conditions. In particular, a necessary condition to derive the results mentioned above, is that it the optimization problem faced by an individual consumer can be reduced to a problem with a single state variable. Unfortunately, as pointed out by Bar-Ilan and Blinder (1991), even simple generalizations of these models, are extremely difficult to analyze, because one looses the possibility of having a unique state variable. For instance, the introduction of labor income in Grossman and Laroque's model has this effect. In what follows, I assume a particular kind of (S, s) behavior without proving its optimality.

To fit the data, simple (S, s) rules have to be modified to allow for individual (unobserved) heterogeneity. In a series of recent papers Caballero and Engel (1992, 1993, 1994) have proposed a model in which the probability an individual adjusts his-her capital stock is increasing in the absolute value of the difference between the actual and desired stock. This model encompasses a simple (S, s) rule (in which the hazard is a step function) and quadratic cost of adjustment model in which the hazard is a constant. The parametrization of the (S, s) rules I propose below is observationally equivalent to the hazard model of Caballero and Engel.

Microeconomic behavior based on (S, s) rules raises a number of interesting aggregation issues. The characterization of aggregate dynamics is not a trivial problem and cannot be summarized by a linear model. Some of the papers that analyzed this problem are those by Caplin (1985), Bertola and Caballero (1990), Caballero and Engel (1991, 1994), Caballero (1993), Beaulieu (1993).

With stringent functional form assumptions it is sometime possible to identify the parameters of the (S, s) rule from the time series behavior of aggregate data. This is the approach taken in various studies including Caballero (1991), Bar-Ilan and Blinder (1991), Bertola and Caballero (1990) and Caballero and Engel (1994).

Very few papers addressed the problem of characterizing (S, s) rules directly from microeconomic data. This is partly due to the lack of data sets containing reliable informations on durable stocks (or more generally, on stocks) and partly to the objective difficulty of the problem. The only attempt at direct estimation of the parameters of an (S, s) rule is by Lam (1991), who uses a panel of observations from the Survey of Consumer Finances from 1966 to 1969. Some other papers (Eberly, 1991, Beaulieau, 1993) exploit the properties of the ergodic cross sectional distribution, derived under the assumption of no aggregate shocks, to infer indirectly the parameters of the (S, s) rule from the observed empirical distribution. The problem, of course, is that this entails assuming that in a given year the observed distributions actually coincides with the ergodic distribution: an extremely unpleasent assumption. More recently, Caballero, Engel and Haltiwanger (1994) have considered data on firm level employment data to characterize the hazard functions of a generalized (S, s) rule.

2.2 The specification of an (S, s) rules

If changing the stock of durables involves a non convex cost it is not easy, in general, to derive a first order condition for the stock (or the expenditure) that could be exploited for empirical analyses. Assuming that (S, s) rules are a good characterization of behavior in the presence of adjustment costs can be a useful first step. However, obtaining a specification suitable for empirical work implies solving two problems. First, one has to specify the variable in terms of which one expresses the (S, s) rule. Second, to be able to fit microeconomic data, one has to specify the (S, s) rule so to allow observed and unobserved individual heterogeneity both in the desired level of the state variable and in the width of the (S, s) band. It is also necessary to take into account the possibility of corner solutions for the desired stock. The choice of a flexible stochastic specification is crucial.

To choose the variable to specify the (S, s) rule, it is useful to think of the optimal allocation of expenditure on durables and non-durables in the absence of transaction costs. In this case, one can derive an intratemporal first order condition which relates the marginal utilities of durables and non durable consumption to their (user) costs. If one expresses the (S, s) rule in terms of the ratio of durable stock and non durable consumption, one can think of the desired level of such a variable as determined by the first order condition of the allocation problem between durables and non durables and interpret the (S, s) rule in terms of deviations from such a first order condition.[4]

The advantage of having information on non durable consumption is that such a variable is likely to adjust quickly to changes in the economic environment and therefore reflect the effect of expectations about future income more directly than other variables affected by adjustment costs or than income itself. Regardless of the presence of adjustment costs in durables, it is possible to derive an intertemporal first order condition for non durable consumption, as long as one conditions on the optimal stock of durables. [5] Expressing the (S, s) rule in terms of the ratio to non durable consumption therefore allows to focus on the behavioral inertia induced by the adjustment costs in durables. The plausibility of the assumption that non durable consumption is bounded away from zero constitutes an additional reason to express the (S, s) rule in terms of the ratio of durables and non durable consumption.

In what follows I consider a particular durable commodity: automobiles. Conditionally on the optimal automobile stock being strictly positive, one can define the state variable governed by the (S, s) rule as $Z_t \equiv \frac{K_t}{c_t}$. I assume that the stock of automobiles will be adjusted to its optimal level only when the deviation of Z_t from it exceeds a certain limit. Both the optimal level and the band around it can be made dependent on a number of observable and unobservable individual characteristics and other variables. In addition the band does not need be symmetric around the optimal level. For instance, the optimal level will presumably depend on the relative prices of durables and non durables, seasonal dummies and various demographic factors, while the size of the band will be related, among other things, to the size of the transaction cost and to factors affecting the opportunity cost of deviating from the optimal level of durables.

These ideas can be formalized in the following way. Let Z_t^d be the target level of Z_t, Z_t^u and Z_t^l the upper and lower bound of the Ss band. The consumer will not adjust the stock of existing cars as long as Z_t is in between Z_t^l and Z_t^u, and will bring it to Z_t^d when Z_t hits one of the two bounds. This means that the consumer will upgrade (downgrade) his/her stock when Z_t hits the lower (upper) bound. The target level Z^d does not necessarily coincides with the optimal level without transaction costs. The consumer might take into account the nature of the transaction costs and the factors that affect future actual and desired stocks when choosing Z^d. If, for instance, the actual stock is expected to depreciate it is likely that Z^d is higher than the optimal level in the absence of transaction costs. Given that I do not characterize with much precision the 'optimal'

[4] In a recent paper Beaulieu (1993) specifies an (S, s) rule for automobiles in terms of the ratio to non durable consumption and, in a simple model, shows the optimality of such a rule.

[5] If the within period utility functions were separable between durables and non durables we would not need to condition on the stock of durables to derive the intertemporal first order condition for non durables.

level of Z without transaction costs, I will not make this distinction explicit in what follows.

The equations that characterize Z_t^d, Z_t^u and Z_t^l are the following:

(1) $$Z_t^d = \beta' w_t^d + u_t^d$$

$$u^d \geq -\beta' w_t^d$$

(2) $$Z_t^u = Z_t^d + exp(\theta^{u'} w_t^b + u_t^b)$$

(3) $$Z_t^l = Z_t^d - exp(\theta^{l'} w_t^b + u_t^b)$$

where W^d and W^b are vectors of observable variables relevant for the target and band equations, and u^d and u^b are gaussian residuals with zero mean and constant variance -covariance matrix (with u^d being truncated). The truncation for u^d is necessary to ensure that the desired stock of automobiles is non negative.

Notice that the band is not necessarily symmetric even though I assume that the random component which affects the lower and upper bound is the same. While this latter assumption is necessary to keep the likelihood function manegeable, I do not believe it is a very strong one. Also notice that the specification in equations (1) to (3) implies that the desired level is between the lower and upper bound of the (S, s) band with probability one.

There are several ways in which one can tackle the issue of corner solutions for the target Z^d. I use a standard Tobit framework, which assumes that equation (1) describes a latent variable Z^* and that the desired stock Z^d equals Z^* if the right hand side of (1) is positive and equals zero otherwise. I have also tried the equivalent of a generalized Tobit model, in which the latent variable Z^* is not necessarily determined by the same variables as Z^d. The results did not change substantially, while the computational burden increases substantially because of the increased number of parameters.

It is also possible to use a log-normal rather than a truncated normal specification for the residuals of the target equation, or, for the band equation, a truncated normal rather than a log-normal. The specifications I tried did not affect the thrust of the results in a substantial way.[6] It is obvious, however, that the assumption about the distribution of the residuals imposes a substantial amount of structure on the problem. The study of more flexible functional forms such as the factor models recently discussed by Mroz and Guilkey (1992) is left for future research.

[6] See Craig (1972) for a discussion of selection models with positive observable dependent variable.

In the derivation of the likelihood function below, I assume that the residuals are i.i.d. across individuals and over time. With panel data, such an assumption would be conflict with the presence household fixed effects. In the present application, however, each household is observed only once.

The households in the sample are naturally divided into two groups: those who engage into a transaction during the observation period and those who do not. The latter are then divided into two groups: those with at least one car and those with no cars. The former can be divided into 4 groups: those who start the observation period with at least one car and up-grade, those who start with no cars and buy at least one, those who down-grade to a positive stock, and those who down-grade to zero.

For households not engaging in a transaction neither the target, nor the band are observable. The difference between the value of Z at the beginning and at the end of the sample period, however, gives a lower bound for the value of the band. [7] For households engaging in a transaction the target level is observed. The observability of the band, on the other hand, is questionable and is discussed in the next section. The value of the upgrading (or downgrading), however, gives at least an upper bound on the width of the (S, s) band. The likelihood function for all these groups is derived in detail in the next section.

The characterization of the (S, s) policy for automobile purchases sketched above is extremely flexible because it allows observable variables to influence both the band and the target level. In addition, I also allow for unobserved heterogeneity by the random terms in (2) and (3). The model can easily encompass the generalized (S, s) rules proposed for firms' fixed investment by Caballero and Engel (1994). It is clear that one can think of equation (2) and (3) as an individual drawing an adjustment cost, corresponding to which there is a band width. From an estimation perspective, without a specific assumption about the distribution that generates u^b, the present model is therefore observationally equivalent to Caballero and Engel's. If the determinants of the band width vary over time for the same individual (which is irrelevant for estimation in the present context because I do not have a panel but a time series of cross sections), even the aggregate dynamic behavior of the two models is observationally equivalent (see Section 6).

While the model is very flexible, it is not vacuous. A number of diagnostic tests and goodness of fit measures can be devised. First of all, having estimated the parameters of the model, it is possible to check that they make sense within the framework outlined above: one would expect, for instance, that the relative price of non durables or family characteristics such as the number of earners, to have a positive effect on the desired level of Z_t, while one would expect variables related to the size of transaction costs to have a positive effect on the size of the band. Furthermore, it is possible to compute the predicted probabilities of adjustment and compare them with the

[7] Z varies over the sample period even when the household does not engage in any transaction because of depreciation.

frequencies observed in the sample.

3. Identification and derivation of the likelihood function.

In this section I derive the likelihood function for the model given by equations (1) to (3). Each household in the sample is observed only for one period. For households who do not engage in any transaction, however, I have observations on the value of the stock of automobiles at the beginning and at the end of the period. For those who buy/and or sell one or more cars I know the value of the stock of cars just before and after the transaction. [8] We can therefore divide the observations in the 6 groups described in the previous section.

To evaluate the likelihood function it is necessary to make an assumption about the nature of the process that changes the variable Z in the absence of an action from the household. If one thinks of time and changes in Z within the observation period as being continuous, one can assume that the level observed before a transaction is equal to either the lower or the upper level bound of the Ss band depending on whether the transaction is an up-grade or a down-grade. Under this interpretation, households adjust their stock of cars as soon as the variable Z crosses the boundaries of the (S, s) band. Because Z^d is observable for households engaging in a transaction, under this assumption the width of the (S, s) band is also observable.

On the other hand, if one assumes that time and/or changes in Z are discrete, the observation of the value of Z before a transaction gives either a lower bound for Z^l(for an up-grade) or an upper bound for Z^u (for a downgrade). In this case, therefore, the width of the (S, s) band is never observable. The amount by which the household upgrades (or downgrades) its stock of automobiles constitutes only an upper bound for the band width.

From this last consideration it follows that the model with unobservable bands is identified only if one observes *changes* in the actual Z_t even for the households that do not engage in a transaction. Because the households engaging in a transaction provide an upper bound, the observations which do not transact must provide a lower bound on the width of the band itself. It is therefore essential to observe changes in the value of Z (maybe because of depreciation) even for the 'inactive' households. This was the reason for dropping from the sample the households which were observed only once.

While the assumption of unobservable bands is probably more plausible, there are at least two reasons to consider the alternative as well. First, assuming the observability of the band gives one a considerable amount of information and therefore the possibility of obtaining much more precise estimates of the model's parameters. More importantly, one can interpret the residual

[8] If a household engages in more than one transaction during the observation period it is assumed that all the transactions occurr simultaneously.

u^b as made of two components, one reflecting unobserved heterogeneity in band widths and the other reflecting the depreciation process. Under this interpretation, the parameters of equations (2) and (3) cannot strictly be interpreted as determining the size of the (S,s) band as they are influenced by the depreciation process. On the other hand, to determine aggregate expenditure we need to know the amount purchased by each individual so that we are indeed interested in the combination of band width and depreciation. In what follows, I present results obtained using both assumptions.

(i) Households who up-grade their stock of cars

For these households we observe the target level. It is therefore possible to derive an expression for the residuals in equation (5). u^b can also be determined if we assume that the stock observed before the purchase equals Z^l. Alternatively, Z consitutes a lower bound for Z^l. The likelihood function in the first case is given by $f(u^d, u^b)$, where f is the joint density of u^d and u^b. In the second case, the likelihood function is given by:

$$f(u^d) \int_{-\infty}^{log(Z_t^d - Z_t) - \theta^{l'} w_t^b} f(u_t^b \mid u_t^d) du_t^b$$

(ii) Households who down-grade their stock of cars

The likelihood function for this group is equivalent to that of group (i), with the only difference that we observe (or determine a lower bound) for Z^u rather than Z^l.

(iii) Households with at least one car that do not execute transactions.

For these households we will have to evaluate the probability of the following event :

$$Z_t^l < Z_t^{min} < Z_t^{max} < Z_t^u$$

where Z^{min} and Z^{max} are the lower and the highest value of Z observed over the sample period. This probability can be written as:

$$Pr\{Z_t^l < Z_t^{min} < Z_t^{max} < Z_t^u\} = Pr\{Z_t^l < Z_t^{min} < Z_t^{max} < Z_t^u \mid Z_t^d > 0\} Pr\{Z_t^d > 0\}$$

$$+Pr\{Z_t^l < Z_t^{min} < Z_t^{max} < Z_t^u \mid Z_t^d = 0\} Pr\{Z_t^d = 0\} \quad =$$

$$Pr\{Z^d - e^{\theta^{l'} w_t + u_t^b} < Z_t^{min} < Z_t^{max} < Z^d + e^{\theta^{u'} w_t + u_t^b} \mid Z_t^d > 0\} Pr\{Z_t^d > 0\} +$$

$$Pr\{-e^{\theta^{l'} w_t + u_t^b} < Z_t^{min} < Z_t^{max} < e^{\theta^{u'} w_t + u_t^b} \mid Z_t^d = 0\} Pr\{Z_t^d = 0\}$$

Notice that for the first probability to be different from zero u_t^b has to be such that:

$$Z_t^{max} - e^{\theta^{u'} w_t^b + u_t^b} < Z_t^{min} + e^{\theta^{l'} w_t^b + u_t^b}$$

This implies that $u_t^b > log(Z_t^{max} - Z_t^{min}) - log(e^{\theta^{u'} w_t^b} + e^{\theta^{l'} w_t^b})$. Therefore the likelihood function is given by:

$$\int_H^{\infty} \int_{\psi_2(u^b)}^{\psi_1(u^b)} f(u^d \mid u^b) du^d \, f(u^b) du^b - \int_{-\infty}^{-\beta' w_t^d} \int_{Z^{max} - \theta^{u'} w_t}^{\infty} f(u^b, u^d) du^d du^b$$

where $f(u^d \mid u^b)$ is the density function of u^d conditional on u^b (which is normal) and $f(u^b)$ is the marginal density of u^b (which is again normal). ψ_1, ψ_2 and H are given by the following expressions.

$$\psi_1(u^b) \quad = \quad Z_t^{min} + e^{\theta^{l'} w_t^b + u_t^b} - \beta' w^d$$

$$\psi_2(u^b) = \begin{cases} Z_t^{max} - e^{\theta^{u'} w_t^b + u_t^b} - \beta' w^d & \text{if } Z_t^{max} - e^{\theta^{u'} w_t^b + u_t^b} > 0; \\ -\beta' w^d & \text{if } Z_t^{max} - e^{\theta^{u'} w_t^b + u_t^b} \leq 0. \end{cases}$$

$$H = \begin{cases} log(Z_t^{max} - Z_t^{min}) - log(e^{\theta^{u'} w_t} + e^{\theta^{l'} w_t}) & \text{if } Z_t^{max} - Z_t^{min} > 0; \\ -\infty & \text{if } Z_t^{max} = Z_t^{min}. \end{cases}$$

(iv) Households with no cars at the beginning of the period and at least one at the end

For these observations we observe the target value at the end of the period, but we do not observe the lower bound. The likelihood function is given by :

$$f(u_t^d) \int_{log(Z_t^d) - \theta^{l'} w_t^b}^{\infty} f(u_t^b \mid u_t^d) du_t^b$$

(v) Households with at least one car at the beginning and no cars at the end of the period.

If we assume that the upper bound is observed, the likelihood function is given by the following expression:

$$f(u_t^b) \int_{-\infty}^{-\beta' w_t^d} f(u_t^d \mid u_t^b) du_t^d$$

Under the alternative assumption we have:

$$\int_{-\infty}^{-\beta' w_t^d} \int_{-\infty}^{log(Z_t) - \theta^{u'} w_t} f(u_t^b, u_t^d) du_t^b du_t^d$$

(vi) Households with no cars throughout the sample period.

For these observations, either the desired stock is zero, or it is positive, but the lower bound of the (S, s) band is below zero. Therefore the likelihood function is given by the following probability:

$$Pr\{Z_t^d - e^{\theta^{l'} w_t + u_t^b} \leq 0 \leq Z_t^d + e^{\theta^{u'} w_t + u_t^b}\} Pr\{Z^d > 0\} +$$

$$Pr\{-e^{\theta^{l'} w_t + u_t^b} \leq 0 \leq +e^{\theta^{u'} w_t + u_t^b}\} Pr\{Z^d = 0\} \quad =$$

$$Pr\{Z_t^d - e^{\theta^{l'} w_t + u_t^b} \leq 0 \leq Z_t^d + e^{\theta^{u'} w_t + u_t^b}\} Pr\{Z^d > 0\} + Pr\{Z^d = 0\} \quad =$$

$$\int_{-\infty}^{\infty} \int_{\psi_2(u^b)}^{\psi_1(u^b)} f(u^d \mid u^b) du^d f(u^b) du^b + \int_{-\infty}^{-\beta' w_t^d} f(u^d) du^d$$

The computation of the likelihood function is numerically intense. A number of issues, concerning in particular the evaluation of the multidimensional integral, have to be tackled. The integrals in the likelihood function were computed using Gaussian quadrature formulae. Details on the numerical methods used are given in Appendix 2.

4. Data

In this section I briefly describe the data sources and illustrate the main features of the data used in the estimation of the (S, s) rules described in Section 2.

4.1 Data sources

The data used in this paper come mainly from the Consumer Expenditure Survey (CEX). Since 1980, the CEX, which is designed to measure expenditure shares to be used in the computation of consumer price indexes, has been run as a rotating panel. The size of the panel is around 7,000 households and each household is interviewed for four consecutive quarters. The CEX survey is the only US micro data set which contains complete and detailed information on consumption. Since 1984, the CEX contains a substantial amount of information on the stock of automobiles owned by each households. Information is available on the model, make and year of each vehicle owned by the household in each of the four interviews. This information enables one to estimate the value of the stock of cars owned by the household at each point in time.

The value of the stock of cars over the period of observation (one year) along with information on the purchases of automobiles and non durable consumption can be used to measure the variable Z described in section 2 at the beginning and at the end of the observation period. Prices of used and new cars are obtained, for the period from 1984 to 1989 and at a monthly frequency, from the *Kelley Blue Books*.

For those households who do not modify actively their stock of cars, the Z variable changes between the beginning and the end of the observation period because of depreciation (appreciation). For those households who engage in a transaction we can measure the value of the stock of cars just before and after the transaction.[9]

[9] The information on the automobiles is far from complete, so that it is necessary to approximate the value of the stock in several ways. Again, the details are provided in Appendix 1. It should be stressed that the relevant concept here is economic as well as physical depreciation. The use of used car prices is consistent with this idea.

The length of the observation period varies between one quarter and one year because some households miss one or more of the four interview.[10] I decided to exclude the households observed for only one quarter. The main reason for this choice is that for the model with unobservable bands to be identified we need to observe two different values for the state variable Z for the households which do not transact. Furthermore, the households observed for only one period present a number of other problems. For instance, many households, when changing the value of their car stock, simultaneously buy a new car and sell or trade in an old one. In the sample, 44% of the households who buy a car sell a car in the same quarter. However, the car sale might be reported or occurr, for various reasons, in a subsequent quarter.

Not surprisingly the households completing only one interview are systematically different, in terms of observables, from the other households. For instance, the former are younger, poorer, more likely to be headed by a black.[11] Furthermore, when I relate the change in the ratio of cars to non durable consumption either at the beginning or at the end of the period (the variable ΔZ), to the number of interviews completed, I find that households completing a single interview have significantly lower ΔZ's, even after controlling for a number of observable variables. These differences are therefore likely to introduce a bias in the estimates of the parameters of the model.

I selected households whose reference person was aged between 21 and 75 and resided in urban areas and for whom the value of their automobile stock could be estimated. Other minor selection criteria are described in Appendix 1.

4.2 Data description.

In table 1, I report mean and standard deviation of a the variables used in the estimation below or otherwise relevant. Income and non durable consumption are deflated by a consumer price index for non durable consumption, while the stock of cars is deflated by a price index for new and used automobiles (base 1982-1984).

The only items in the table worth a mention are those directly related to car ownership. 87% of the households own at least one automobile and the average number of automobiles is 2.02. The average value of the stock of cars conditional on ownership is almos $ 6,600. The ratio of cars value to non durable consumption is about 0.56.

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[10] The non durable consumption figure for the households with missing interviews is adjusted taking into account how many and which months are missed. This adjustment is meant to allow for seasonality in consumption. Details are provided in Appendix 1.

[11] These differences are consistent with those reported by Nelson (1994) who discusses attrition in the CEX. Some households, however, have only one quarter of databecause of the nature of the survey. Households who were at their 5th interview at the beginning of 1984 would not have information on cars in the early interviews that occurred in 1983. Furthermore, because the sample was discountinued in 1986, the same situation occurs for households interviewed at the beginning of that year.

The main concern of the paper is modelling automobile purchases. In Table 2, I report the percentage of households who engage in a transaction during the period over which they are observed. The table also distinguishes the households who actively change the value of their stock of cars depending on the type of transaction. The categories reported match the six groups listed in Section 3 while characterizing the likelihood function.

The figures in Table 2 can be used to have an idea of the frequency with which a car is replaced. For the whole sample, of the households with at least one car, 35% perform a transaction during the observation period. On average, these households are observed for 2.5 quarters and hold 2.24 cars. If one neglects the correlation between length of observation period, number of cars owned and propensity to change cars, these figures imply that the average holding period of a car is 16 quarters.

As a first attempt at describing the purchase of automobiles, I estimate two simple probit equations for the purchase (or sale) of a car. I define a variable d^b to be one if the household upgrades its stock of cars during the observation period and zero otherwise. An analogous variable d^s is defined for those who downgrade. These variables are then related to a number of controls and to the value of the stock of cars at the beginning of the period divided by non durables consumption by means of a simple probit regression. The controls include a quadratic in age, the number of children of age 3 to 15, the number of children of age 0-2, the number of females and males over 16, education, regional and year dummies and the number of earners. The results are reported in table 3.

The interpretation of most coefficient is not straightforward. However, for the present discussion, it is interesting to note that both coefficients on the 'beginning of period' Z are consistent with the model outlined in section 2. A high value of the ratio of the stock of cars to non durable consumption makes household less likely to upgrade and more likely to downgrade its stock of cars.

For the aggregation issues discussed in section 6 the cross sectional distribution of the Z variable and its evolution is important. In Figure 1 I graph the cross sectional density of the Z variable (conditional on a positive initial Z) which presents a pronounced skewness. Similar figures can be obtained if one plots the cross sectional densities in different years.

5. Estimating the (S, s) Model

The estimation of the model described in Section 2 is not easy. The maximization of the likelihood function in Section 3 is numerically intensive and therefore there are limits to the amount of specification search that can be performed. [12] However, the results obtained under the two assumptions about the observability of the band, reported in Table 4 and 5, are reasonably precise.

[12] All results were obtained as the solution of the maximization problems starting from at least two different sets of initial values.

The results obtained using a log-normal specification for the target value and a separate equation for the selection process were not substantially different.

5.1 The specification of target values and band width.

The estimates reported in Tables 4 and 5 for the two models refer to the same specification, so that they are directly comparable. Even though slightly more parsimonious specifications are available, I prefer to report the most general specification I have estimated which contains the same variables for the two models. The advantage of such a choice is to make the two models strictly comparable.

In addition to the age of the household head and its square, target equation includes several variables that describe the composition of the family, the labor supply behavior of the household members and the socio-economic status of the household. In particular, the variables describing family composition include the number of male and female adults, the number of male and female children between age 2 and 15 and the number of infants, while those related to labor supply include the number of annual hours worked by the head and the spouse, as well as two dummies for the number of earners (the reference group are households with two or more earners). The last group of variables that complete the description of the household are dummies for the educational attainment, race and gender of the household head and the region of residence.[13]

In addition to the variables just mentioned, I also include the log of two relative prices: the ratios of a price index of non-durable consumption to a measure of the rental cost of cars and to a price index of motor fuels. The price index for motor fuels is taken from the BLS detailed CPI tape. The price for non durables is constructed as a Stone price index from CPI detailed indexes, published at monthly frequencies at the regional level by the BLS. Weights are constructed using average expenditure shares; the averages are taken over households within education and regional groups. Therefore, relative prices exhibit not only time but also cross sectional variability. Most of the actual variability, however, is observed over time. The rental cost of cars is constructed taking into account the change in the price of cars, depreciation and an interest rate. [14]

The variables that determine the width of the (S, s) band are essentialy the same as those that enter the target equation with one exception. The relative price variables are subsituted by year dummies. There are two related reasons for such a subsitution. First, there is no obvious reason why the relative price of durables and non durables should affect transaction costs and therefore the band width. Second, because most of the variability in relative prices is time series variability, it becomes very difficult to obtain precise estimates of these parameters once time dummies are

[13] The reference group is formed of households headed by a college graduate black female residing in the West.

[14] The Appendix 1 gives further details about the construction of the price indexes and, in particular, of the rental cost of cars.

introduced in the band width. The latter are important to capture any time variation in transaction costs and therefore band width. While this procedure is not fully satisfactory, especially if one wants to use the model for predictions, there is no obvious variable that could capture changes in transaction costs.

5.2 Results for the model with observable bands

The estimates of the coefficients of the target equation for the model with observable bands are reported in the top part of Table 4, while those of the two bands are in the bottom part together with the estimated variance covariance matrix and the value of the loglikelihood function.

Starting with the target equation we notice that, as expected, both the coefficients on age and those on the demographic variables are strongly significant, indicating the presence of strong life cycle effects as well as the importance of family composition for the desired ratio of cars to non-durable consumptioin. The estimated polynomial in age peaks at age 37.

The labor supply variables are marginally significant, probably indicating non separability between consumption and leisure in the utility function. The desired ratio of automobiles to non durable consumption increases in the number of hours worked by the head and (more strongly) by the spouse. [15]

It is interesting to notice that the desired ratio of cars to non-durable consumption is lower for households with a lower permanent income, such as those headed by high school dropouts, females and blacks.[16] This evidence is consistent with the hypothesis that cars are a luxury with an elasticity to income greater than unity.

The coefficient on the relative price of fuel is significant and with the expected positive sign. A positive sign indicates that an increase in the price of non durables relative to the price of gasoline increases the desired ratio of automobiles to non durable consumption. The coefficient on the other relative price, on the other hand, is not significantly different from zero (and takes a negative point estimate).

Even though the same variables determine the two bands, they are allowed to have a different effect. The hypothesis that the coefficients of the upper and lower band are equal is overwhelming rejected indicating the presence of strong asymmetries.

There are significant life-cycle and family composition effects on band width. For both bands the age polynomial is convex and has a minimum at age 27 for the lower bound and at age 35 for the upper bound.

The dummies on the number of earners are strongly significant, while the effect of hours of

[15] The coefficients on the number of earners and in particular that on the zero-earners dummy, should be interpreted with care as the number of hours of head and spouse cannot be kept constant when considering the difference between households with zero, one or two and more earners.

[16] The coefficient on black heads is not significantly different from zero.

work is not estimated precisely. Households headed by college graduates have relatively larger bands, maybe indicating higher transaction costs for these groups. If transaction costs are related to search, individuals with higher earnings should face higher search costs. It is also possible that wealthier individuals purchase cars that depreciate more quickly.

Overall, the signs of these coefficients are consistent with the signs of the probit model reported in Table 3. The most striking feature, however, is the width of the band, and in particular of its upper part. The average width of the lower band evaluated at $u^b = 0$ is 1.0 (median = 0.73, st.dev.= 0.85). The average width of the upper band is 1.28 (median = 1.08, st.dev.= 0.74).

The large size of the band can be a consequence of the observability assumption. As mentioned above, if the depreciation process and time are discrete, one can interpret the estimated bands as an upper bound to the actual one. The size of the estimated band depends both on the size of the actual band and on the process for depreciation. A comparison of the band size estimated under the two assumptions can give an idea of the quantitative importance of the depreciation process.

The correlation coefficient between u^b and u^d is estimated quite precisely at 0.8. This implies that households with relatively high target values also face higher transaction costs. This result is of some relevance for the aggregation excercise discussed in Section 6.

5.3 Results for the model with unobservable bands.

The results reported in Table 5 were obtained relaxing the assumption that the lower (upper) bands are observable for those households who up-grade (downgrade) their stock of cars. As discussed above, I assume that the observed upgrade (downgrade) gives us only an upper bound on the size of the band.

The specification reported in the Table is the same as that in Table 4. The sign and significance of coefficients are substantially similar to those estimated for the other model. There are however some differences in magnitude. For instance, the coefficients in Table 5 imply that the minimum of the effect of age on the width of the bands occurs around age 21 for the lower band and 39 for the upper one. The only coefficient that change sign is that on the relative price of non durables and cars (measured as the rental cost) which takes now the expected positive sign but is still insignificant. In addition, the labor supply variables (including the dummy for no earners) are now strongly significant. Both the non-black and the male dummy are insignificant.

The pattern of coefficients of the band equations is substantially similar in the two models, even though those in Table 5 (model with unobservable badns) are more precisely estimated. The main difference between the tables 4 and 5 lies in size of the band: the model estimated under the assumption of unobservable bands implies, for most observations, a much narrower lower band. The median lower band measures now 0.38 (compared to 0.73 of the previous model). [17] The

[17] Given to the presence of a large upper tail the average value for the lower band is actually higher, at 2.3 (st.dev. = 6.9),in the model with unobservable bands.

median value of the upper band is 1.21 (mean = 1.69, st.dev. 1.43). The median level of the sum of the two bands is 1.77 (mean 3.98, st.dev.= 7.52).

The correlation coefficient between the two residuals is now lower at 0.53. Once again the hypothesis of symmetric bands is rejected at any reasonable significance level.

5.4 Goodness of fit.

To establish to what extent the estimated models fit the data, for all the households in the sample, I compute, for the (observed) initial stock, the probabilities of various kinds of adjustment. I then compare the sample averages of these probabilities to sample frequencies.

The probabilities of adjustment can be easily derived from the structure of the model as specified in equations (1) to (3). In particular, given a positive initial stock Z_{t^-}, the probability of an upward adjustment, prior to the realization of u^b and u^d, is given bv the following expression:

$$(4a)\qquad \int_{-\infty}^{\infty}\int_{Z_{t^-}+e^{\beta^{u'}w_t^b+u_t^b}-\beta' w_t^d}^{\infty} f(u^d \mid u^b)du^d f(u^b)du^b \equiv \Lambda^{++}(Z_{t^-})$$

while the probability of a downward adjustment to a positive stock is given by:

$$(4b)\qquad \int_{-\infty}^{\log(Z_{t^-})-\beta^{u'}w_t^b}\int_{-\beta' w^d}^{Z_{t^-}-e^{\beta^{u'}w_t^b+u_t^b}-\beta' w_t^d} f(u^d \mid u^b)du^d f(u^b)du^b \equiv \Lambda^{-+}(Z_{t^-})$$

The probability of upgrading from a zero stock and the probability of downgrading to zero are given by:

$$(4c)\qquad \int_{-\infty}^{\infty}\int_{e^{\beta^{u'}w_t^b+u_t^b}-\beta' w_t^d}^{\infty} f(u^d \mid u^b)du^d f(u^b)du^b \equiv \Lambda^{\prime}(Z_{t^-})$$

$$(4d)\qquad \int_{-\infty}^{\log(Z_{t^-})-\beta^{u'}w_t^b}\int_{-\infty}^{-\beta' w^d} f(u^d, u^b)du^d du^b \equiv \Lambda^{-0}(Z_{t^-})$$

In Table 6, I report the mean of these probabilities computed on the basis of the estimates of the four specifications reported above, as well as the probability of a zero desired stock. In the last column, I report the corresponding frequencies in the sample. Comparing the average estimated probabilities with the actual provides a loose specification test of the estimated model. [18]

The model with observable bands tends to under-predict substantially the probability of an upward adjustment. The model with unobservable band, on the other hand, slightly overpredicts

[18] The mean could also be computed for various groups, selected for instance, on the basis of the initial z. These computations did not yield large differences in goodness of fit among subgroups.

the probability of upgrading (both from zero and from a positive stock). Both models slightly underpredict the probability of a downward adjustment, while both considerably overpredict the probability of a downward adjustment to zero.

Overall the model with unobservable bands seems to preform considerably better than the one with observable ones. The systematic underprediction of upward adjustment in the latter is due to the large size of the lower band.

6. Aggregation of the estimated (S, s) rules.

6.1 Generalities

If microeconomic consumption behavior is based on (S, s) rules, the implied aggregate dynamics can be extremely rich and complex.[19] The specific properties of the aggregate system obviously depend on the details of the (S, s) system. Caballero and Engel (1994) analyze the properties induced by their porportional hazard model, which they label 'generalized (S, s) system', on the time series behavior of aggregate investment. The (S, s) system proposed in section 2 can encompass, with a flexible enough parametrization of its stochastic components, the generalized (S, s) system of Caballero and Engel, even in terms of the implied aggregate dynamics.

In any model with inertial behavior caused by adjustment cost, aggregate dynamics depends on the distribution of individual differences between actual and desired controls. The nature of the microeconomic behavior in turns affects the evolution of this density distribution. In the system of Caballero and Engel (1994) each firm draws a cost of adjustment (with the draws being independent over time and across agents) and on the basis of this cost decides whether to adjust or not its capital stock. Indeed, *given the adjustment cost*, each firm behaves according to a simple (S, s) rule. Caballero and Engel characterize this sort of behavior in terms of an hazard function: conditional on the difference between actual and desired stock each firm has a certain probability of adjusting, which depends on the density function from which adjustment costs are drawn.

From these considerations should be immediately clear how the model sketched in section 2 is, with appropriate assumptions about the nature of the residuals u^b and u^d, observationally equivalent even in terms of the implied aggregate dynamics, to the 'generalized (S, s) model' of Caballero and Engel. If the u^b's are i.i.d. over time and across individuals, given the actual and desired stock (and the observable components of the band), one can think of each household as having an ex-ante probability of adjusting which depends on the distribution function generating the shocks. For any hazard function, one can derive the properties of the density that generates it.

[19] Caballero and Engel (1991) have studied the convergence properties of (S, s) economies, while Caballero (1992) uses the same model to explain the slow adjustment of aggregate durable expenditure.

The model presented in Section 2 is indeed able to generate aggregate behavior richer than a simple hazard model because it incorporates a variety of factors ignored in the latter. [20] The availability of micro data makes possible the estimation and study of a much more complex model which embeds a substantial amount of individual heterogeneity. The aggregation issues that arise within this approach are not necessarily linked to inertial behavior, but are nonetheless important, as it is well known from demand analysis.[21]

First, the target level for Z is systematically different across individuals. This aspect can be important if the sample is composed of individuals with different levels of income: if cars are 'luxuries' and non-durable consumption a 'necessity', the desired level of their ratio will depend, for any given level of relative prices, on the household (permanent) income. In the presence of binding liquidity constraints, even the level of current income is likely to be important. Furthermore, the desired level of Z is likely to depend on a number of factors such as family composition, labor force participation, occupation and so on which therefore constitute an additional source of heterogeneity. All these effects are likely to be important in that aggregate demand will depend in a crucial way on composition effects. Some of these factors are fixed or move at very low frequencies (family composition, to a certain extent permanent income), others, such as current income or employment status, are likely to exhibit important movements even at high frequencies with a varying degree of synchronization.

Second, as discussed above, the size of the band can be systematically different across individuals and over time. This source of heterogeneity is likely to be important for the properties of aggregate dynamics.

Finally, it is likely that the determinants of the target level and band width heterogeneity are correlated, both through common observable determinants and through the stochastic (unbobservable) components u^b and u^d. Indeed, the results in Section 5 indicate the presence of a substantial amount of correlation between the two residuals. This correlation is important because it implies that households with high target stock to non durable ratios are less likely to adjust, ***for any given level of*** Z, because they are likely to have a wider (S, s) band.[22]

6.2 Target and actual values

To establish the dynamic properties of aggregate automobile expenditure it is necessary to characterize the evolution of the desired stock of cars to non-durable ratio in the population and

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[20] Caballero and Engel are forced to keep the model extremely simple because of the necessity of estimating it with aggregate time series data.

[21] Attanasio and Weber (1993) discuss the importance of various aggregation issues, and non linearities in particular, for estimating and testing Euler equations for non durable consumption. See Stoker (1993) for a survey on aggregation.

[22] This correlation prevents a number of simplifications typically used in the aggregation of (S, s) rules.

that of the cross sectional distribution of the difference between actual and desired stock. Given equation (1) to (3), the former is conceptually straightforward: the evolution of Z^d depends on the evolution of the variables on the right-hand-side of the equation. These may include macroeconomic variables, such as the relative price of non-durable consumption and automobiles, and individual specific variable, such as various determinants of the marginal rate of substitution between cars and non-durable consumption.

To characterize the evolution of the cross sectional distribution of the ratio Z, however, some additional elements not included in the model are necessary. In particular, it is necessary to specify the process which governs the evolution of Z when the stock of cars is not actively changed.

By definition, if a household does not change its stock of cars, Z changes because of changes in non durable consumption and because of depreciation. In theory, the evolution of non durable consumption can be described by an Euler equation which holds, in the absence of binding liquidity constraints, regardless of the presence of transaction costs for durable consumption.

In the literature, it is typically assumed that the stock of durables depreciates at a constant rate δ. In reality this assumption is easily falsified. As indirect evidence of this, one can point at the relative price of new and used cars which exhibits wide fluctuations.[23] More direct evidence can be obtained by a cursory look at the rate of depreciation of cars of different vintages and different makes over time. It is not unusual for used cars to appreciate in some years. These movements in the rate of depreciation can introduce an additional source of heterogeneity and therefore aggregation biases insofar as different groups of the population hold cars whose depreciation rates is systematically different and these differences vary over time. [24]

In the sample I use, households who do not modify the composition of their stock of cars experience changes in their value due to economic depreciation. If a household is observed over a period of one year, the available estimate of the value of its cars will pick up these changes (see Appendix 1 for details).

In theory, therefore, it would be possible to use our sample to estimate the time series process which characterizes depreciation. With a large enough sample it would also be possible to control for the implications that different compositions of the car stock have for depreciation. Unfortunately, the time dimension of the sample is too small for a reliable characterization of the time properties of depreciation and the cross sectional dimension is not sufficiently large to control for the many dimensions of stock composition. Furthermore, it is beyond the scope of this paper to

[23] Obviously these fluctuations depend not only on differences over time in the rate of depreciation, but also on the composition of the stock of old cars which in turns depends on the dynamics of car replacement.

[24] It is well known, for instance, that during the 1980s, japanese cars depreciated much more slowly than domestic cars. It is also well known that japanese cars are much more widely held in the West than, say, in the Midwest.

characterize fully the changes in non durable consumption by estimating an Euler equation.

In what follows, I circumvent the necessity of specifying or estimating the process for depreciation or for non-durable consumption by focussing on the difference between systems with and without transaction costs.

6.3 The evolution of the cross sectional density and aggregation

Given the cross sectional distribution of the variable Z at the end of period $t-1$, $f(Z,t-1)$, it is possible to derive its cross sectional density at time t. To do so, it is necessary to specify the timing of the shocks that affect the system. I assume that any given household at the end of period $t-1$, decides non durable consumption for period t. At the same time the depreciation process takes place giving rise to Z_{t^-}. After that the shocks determining the target value and the position of the band occurr. The household then decides whether to adjust or not. If it adjusts, the ratio Z is set at Z_t^d, otherwise $Z_t = Z_{t^-}$.[25]

Given the expressions for the adjustment probabilities in equations (4), an initial cross sectional distribution and a process for depreciation, it is possible to derive the evolution of the cross sectional distribution. Suppose that the inital distribution has mass $f(0,t-1)$ at zero and is given by a known density $f^+(Z,t-1)$ for $Z>0$. If the 'depreciation' process is given by:

$$log(Z_{t^-}) = log(Z_{t-1}) - \delta_t \tag{5}$$

and the density of the target Z_t^d is indicated with f_d, the two components of the cross sectional density of Z at t are given by:

$$f(0,t) = f(0,t-1)(1-\Lambda^{+0}) + \int_0^\infty f(ze^{\delta_t},t-1)\Lambda^{-0}(z)dz$$

$$f^+(Z,t) = \left(\int_0^\infty (\Lambda^{++}(z)+\Lambda^{-+}(z))f^+(z\,e^{\delta_t},t-1)dz + f(0,t-1)\Lambda^{+0}(0)\right)f_d(Z \mid Z^d>0)+$$

$$+(1-\Lambda^{-+}(Z)-\Lambda^{++}(Z)-\Lambda^{-0}(Z))f^+(Z\,e^{\delta_t},t-1) \tag{6}$$

Equation (6) and a process for the 'depreciation' δ_t will characterize the evolution of the cross sectional distribution of Z. Average Z can be obtained, conditional on a depreciation process and the distribution of Z at time $t-1$ integrating $Zf(Z,t)$. The unconditional average can then be obtained integrating out δ_t and Z_{t-1}. It is not hard to allow for heterogeneity in δ_t.

[25] The mechanism outlined here is consistent with the model with unobservable bands or with the interpretation of the residual u^b as a combination of heterogeneity in band width and stochastic depreciation.

From equation (6), it is clear that the presence of transaction costs has two effects. On the one hand, it introduces an element of sluggishness, given by the households who do not change their stocks and is represented by the second element in the equation. On the other hand, an important element of non linearity is introduced by the first element of equation (6). The dynamics of Z^d is filtered into the dynamics of the actual Z through a time varying function which depends on the lagged cross sectional distribution of the actual Z.

In a model without transaction costs and inertial behavior, the variance of u^b is equal to zero and both θ^l and θ^u are equal to zero. It is easily seen from equations (4) and (5) that, under these circumstances, $\Lambda^{++}(Z) + \Lambda^{-+}(Z) + \Lambda^{-0}(Z) = 1$, i.e. households adjust their stock of cars to the desired level (downward or upward) in each period. The evolution of the aggregate stock mirrors the evolution of the desired stock, while aggregate expenditure reflects its changes. Unfortunately, in our model individual desired stocks are heterogeneous and expressed in terms of ratios to non durable consumption. Therefore, even in the absence of transaction costs and inertial behavior, one would be faced by standard aggregation problems which arise because of the non-linearities involved in the model.

The net change in the aggregate stock of cars is given by the change induced by depreciation plus the net adjustment of the households who do adjust. The average expenditure on cars is obtained aggregating the changes of those households that adjust upwards. Consider a generic household which enters time t with $Z_{t^-} = Z_{t-1}e^{-\delta_t}$.[26] After the realization of u^d and u^b the household decides whether to adjust or not. If it does adjust, the change in Z will be given by $Z_t^d - Z_{t^-}$. Therefore, the size of adjustment depends (given Z_{t^-})) on u^d, while the decision to adjust depends on both u^d and u^b. The expected expenditure for a generic household will be given by the size of the adjustment times the *ex-ante* probability of adjusting. To obtain average expected expenditure (relative to non durable consumption), one can sum the expected expenditure of all the households. The expression for aggregate expenditure is therefore given by:

$$(7) \qquad \sum_i h(Z_{t-1}^i, \beta' w_t^{d,i}, \theta^{l'} w_t^{b,i})$$

where

$$h(Z_{t-1}, \beta' w_t^d, \theta^{l'} w_t^b) = \int_{Z_{t-1}e^{-\delta_t} - \beta' w_t^d}^{\infty} \Big((\beta' w_t^d + u_t^d - Z_{t-1}e^{-\delta_t}) g(u^d, Z_{t^-}) \Big) f(u_t^d \mid Z_{t-1}) du_t^d \ ,$$

$Z_{t^-} = Z_{t-1}e^{-\delta_t}$, and

$$g(u^d, Z_{t^-}) = \int_{-\infty}^{log(\beta' w_t^d + u_t^d - Z_{t^-}) - \theta^{l'} w_t^b} f(u^b \mid u_t^d, Z_{t-1}) du^b$$

[26] For notational simplicity I am assuming that the depreciation is common across households. One can assume that depreciation is made of two components one that is realized immediately and is common across periods and one that is idyosincratic and is included in u^b.

In the absence of transaction costs, the expenditure on cars (relative to consumption) is given by:

(8) $$\sum_i \left[\int_{Z^i_{t^-} - \beta' w^{d,i}_t}^{\infty} (\beta' w^{d,i}_t + u^d - Z^i_{t-1} e^{-\delta_t}) f(u^d \mid Z_{t-1}) du^d \right]$$

Notice that in equations (7) and (8), $\beta' w^d$ and Z_{t^-} always appear as $\tilde{Z}_t \equiv Z_{t^-} - \beta' w^d$. If one is knows or can estimate the joint cross sectional distribution of $\tilde{Z}_t$ and $\theta^{l'} w^h_t \equiv x_t$, one can evaluate average aggregate expenditure for the two models using the following expressions. [27]

(7') $$\int_0^{\infty} \int_{-\infty}^{\infty} h(Z_{t-1}) f(\tilde{Z}_t, x_t) dx_t d\tilde{Z}_t$$

(8') $$\int_0^{\infty} \int_{-\infty}^{\infty} \left[\int_{Z_{t^-} - \beta' w^d_t}^{\infty} (\beta' w^d_t + u^d_t - Z_{t-1} e^{-\delta_t}) f(u^d_t \mid Z_{t-1}) du^d_t \right] \tilde{f}(\tilde{Z}_t, x_t) dx_t d\tilde{Z}_t$$

From equation (7) and (8) one can see that, as far as the changes in Z are concerned, the main differences between a system with and one without lumpy adjustment are two: first, the presence of the function g which represents the probability of upward adjustment and, second, the evolution of the cross sectional distribution of Z. Without adjustment costs, g is equal to unity. The cross sectional density function $f(\tilde{Z}_{t-1})$ obvioulsy evolves differently from $\tilde{f}(\tilde{Z}_{t-1})$ which, for every t, coincides with the cross sectional distribution of u^d.

Equation (7) is also useful to chatacterize the difference between the (S, s) model and a model with partial adjustment derived from quadratic costs. Aggregate adjustment in the latter is characterized by equation (7) with g independent of the initial Z_{t^-} and constant across households and over time. The difference in the impact effect of a change in the desired stock depends on the variability of the g's. The dynamics induced by the (S, s) model is then richer because of the dependence of g on Z_{t^-}.

6.4 Evaluating the importance of inertial behavior.

The considerations above and the expressions in equations (7) and (8) make it clear that the implications of the estimated model for the time series properties of aggregate expenditure on automobiles depend on several elements about which the evidence presented in Section 5 is silent. First, one needs to characterize the evolution of the desired stock Z^d as well as of the cross sectional

[27] In equations (7') and (8'),I allow for the possibility of Z_{t-1} to be correlated with u_d as the latter represents idiosyncratic taste shocks. This does not affect the estimation in Section (5) because each household is observed only once.

distribution of the variables that determine the width of the (S, s) band, w^b. Second, one has to be specific about the nature of the 'depreciation' process and its relationship to Z^d. Finally, one has to specify the dynamic properties of the residuals u_t^d and u_t^b.[28] All this cannot be done without a number of very strong simplifying assumptions whose effect is difficult to evaluate.

The scope of this section is to quantify the effect that inertial behavior, as characterized by the system estimated in Section 5, has on aggregate automobile expenditure. Given the difficulties just outlined, instead of presenting a full characterization of the dynamic properties of the model estimated in Section 5, I analyze the main determinants of the dynamic behavior of the (S, s) system and then focus on the differences between systems with and without transaction costs.

As discussed above, an important determinant of the evolution of $f(Z)$ are the probabilities of adjusting upward and downward whose means are reported in Table 6 and discussed in Section 5.4. To show the relationship between the probability of adjusting and the level of the ratio Z at the beginning of the period, I plot in Figure 2 the smoothed medians of the the individual probabilities of upgrading and downgrading against the beginning of period value of Z. Not surprisingly, the figure documents the existence of a negative ralationship between the probability of upgrading and Z and a positive one between the probability of downgrading and the initial stock to non durables ratio.[29]

From equation (7), it is clear that the quantity $g(u^d, Z_{t^-})$ plays an important role in determining the difference between the (S, s) model estimated in Section 5 and systems both without transaction costs and with partial adjustment. Using the estimates of the parameters of both models, I evaluate g at $u^d = 0$ and at the observed Z_{t^-} for each observation in the sample. The computations are performed under the assumption that the distribution of u^b conditional on Z_{t-1} and u^d depends only on the latter.[30] The mean and median of g are below 0.25 for the model with observable bands and just below 0.45 for the model with unobservable bands. Furthermore, both mean and median decline considerably, as expected, as a function of Z_{t^-}. Finally there is a substantial amount of variability in the $g's$, even after controlling for the desired stock.[31]

To evaluate some of the dynamic properties of the system I simulate a special version of the model estimated in Section 5. This is done to study the autocorrelation properties of aggregate changes in Z and the impulse response function to an unexpected change in Z. Along with the

[28] The availability of a true panel would solve some of these problems, but would also make the estimation problem much more complicated.

[29] Both curves in Figure 2 are computed using the model with unobservable bands. Analogous pictures are obtained using the alternative model. Figure 2 makes clear the relationship between the model estimated here and the model based on the hazard function proposed by Caballero and Engel (1994).

[30] Notice that g is not defined for those observations for which Z_{t^-} is greater than $\beta' w_t^d + u_t^d$.

[31] Notice that averaging the function g for different u^d, one obtains the probability of upgrading for an initial value of Z_{t^-}, which is plotted in Figure 2.

(S, s) model I simulate a model without transaction costs and one with partial adjustment. The main focus is on the relative performances of the three models.

I consider an economy populated by agents with characteristics identical to the households present in my 1987 sample. I assume that the systematic part of the target and of the bands equations are fixed for these households through time. The shocks u^d and u^b are assumed to be i.i.d. over time and across consumers and to have the variance covariance matrix estimated in Section 5. The system is started with the observed cross sectional distribution in 1987 and is simulated under the assumption that when a household does not engage in a transaction Z changes according to equation (5) with δ_t distributed as a normal random variable with a mean of 0.15 and a standard deviation of 0.05. The (S, s) model evolves according to equations (1) to (3), with the parameters estimates reported in Section 5. For the models without adjustment costs and with partial adjustment I assume that the desired level is identical to that of the (S, s) model. In the model with quadratic costs the coefficient of partial adjustment is set to 0.5.

Each model is run for 50 periods and each of these paths is replicated 300 times. In the first experiment I use each of the simulated paths of aggregate changes in Z to evaluate the autocorrelation function of the system. The only thing that is moving the system in these simulations is the depreciation process and the stochastic shocks u^d and u^b. To evaluate the autocorrelation, I disregard the first 7 observations to minimize the effect of the initial (observed) distribution.

The experiment is then repeated by introducing an unexpected change in the desired level of Z which is the same for all households in the economy. The simulations are used to compute the impulse response function of aggregate changes in Z. I consider both permanent shocks and shocks that are undone after one period.

It should be stressed that these exercises have four major limitations. First, I neglect the cross sectional correlation between the initial stock Z_{t-1} and the desired level Z^d. Second, I assume that the distribution of the desired ratio Z^d is the same with and without transaction costs. Third, in the study of the dynamic properties of the system I do not consider any dynamic effects on the stochastic components of the system. Fourth, I do not consider the effects of systematic changes in the size of the band which would have no effects on the model without transaction costs and with partial adjustment but potentially large effects in the model with (S, s) rules. The only justifications for these unpleasent assumptions is the inability of performing the exercises below without them and/or our ignorance of some crucial parameters of the problem, such as the autocorrelation structure of the error terms. While this procedure is not fully satisfactory, I believe it is useful to quantify the importance of the inertial behavior induced by the estimated (S, s) rule.

The autocorrelation function of the 3 models is reported in Table 7. The figures are the mean of the autocorrelations computed for each simulation. The autocorrelogram of the changes in Z for the model without transaction costs resembles, as expected, that of an MA(1) with a negative unit root: the first order autocorrelation is close to -0.5 and the higher order ones are close to zero. Both the (S, s) and the partial adjustment models present substantial deviations from the frictionless case. The first order autocorrelation is substantially higher than -0.5 and higher order correlations are slightly different from zero. The (S, s) model also exhibit some differences relative to the model with partial adjustment.

In Table 8a and 8b, I report the first 5 values of the impulse response function to a shock of 0.1 to the desired Z in the three models. The shock is permanent in Table 8a and transitory in Table 8b. The table reports the difference between changes in Z after and in the period preceding the shock. As expected, in the model without transaction costs the aggregate change is approximately equal to the shock on impact. The aggregate change in Z in the periods after the first is higher than in the previous equilibrium because of the larger 'replacement' expenditure implied by a higher equilibrium stock.

The impact effect in the two models with transaction costs is obviously lower than in the frictionless model.[32] The shock, however, has an effect for 3 periods. The dynamics of the effect is slightly different in the two models in that it is higher on impact in the (S, s) model and lower afterwards. The difference between the impact effect in the (S, s) and the frictionless models declines when I increase the size of the shock. The reason for this is that by increasing the size of the shock in the (S, s) model, one increases both the amount of the change of the households that do adjust and the proportion of the households who adjust.

Figure 3 summarizes the dynamics of a change in Z^d in the 3 models. The Figure plots the aggregate change in Z in the 3 models over time. To stress the dynamics the three aggregate series are normalized. The slower adjustment of the two models with transaction costs as well as the differences between these two are evident in the picture.

Table 8b illustrates that the differences between the two (S, s) and partial adjustment models are even more pronounced when one considers temporary changes in Z^d. In the frictionless case, in period 1 the initial change is almost completely undone. The asymmetry built in the (S, s) model makes the difference with respect to the frictionless case stronger than in the quadratic cost of adjustment one.

Before concluding this section, it is worth stressing that the differences in dynamic behavior between models with partial adjustment and the (S, s) model are not fully captured by the exercises proposed here in which no much happens to band width and to the cross sectional distribution of the Z. The best evidence in this respect is the amount of variability in the estimated $g(.)$'s and their strong dependence on the initial Z.

[32] The impact effect in the model with partial adjustment is (obviously) half that of the model with full adjustment.

7. Conclusions

The main goals of this paper were two. First, I wanted to characterize empirically an (S, s) rule for automobile expenditure using microeconomic data. Second, I studied how aggregate expenditure is affected by the presence of inertial behavior.

Rather than deriving the parameters of the (S, s) rule from aggregate time series data (using strong identifying assumptions) or from the features of the cross- sectional density (assuming that coincides with the ergodic distribution which would prevail in the absence of aggregate shocks), I specify a rich stochastic model which allows for observed and unobserved heterogeneity both in the target level and in band width. The model is then estimated using a micro data set and maximum likelihood techniques. While very specific functional form assumptions are made in the estimation process, the procedure used here is easily extended to allow for more flexible semi-parametric techniques.

Even with a data set as rich as the one I use, the estimation of the (S, s) model is not trivial and implies the solution of a number of conceptual problems. The main issue that has to be solved is whether the value of the stock observed before a transaction is on the boundary of the (S, s) band or is outside the band. While the latter assumption is probably more plausible, the former allows a more precise estimation and, as discussed in Section 5, might be more useful for the characterization of aggregate expenditure. I present results obtained under both assumptions.

In general, both models yield sensible results, in that the coefficients on most variables have the expected sign and the predicted probabilities of adjustment match reasonably well the observed frequencies. The most striking feature, however, is the large size of the (S, s) band. A large band implies that inertial behavior is pretty pervasive and therefore is likely to be important in the determination of aggregate expenditure.

The part of the paper that tackles aggregation issues is partly negative, in that, even after having estimated the parameters of the (S, s) model, I show that it is very difficult to characterize the dynamic properties of aggregate expenditure. This negative result is explained mainly by two facts. First, even in the absence of inertial behavior, the study of durable consumption is affected by traditional aggregation problems which prevent the use of a representative agent framework. Second, to characterize the evolution of aggregate expenditure is necessary to characterize the process which moves the state variable in the absence of active actions on the part of the household. While some of the elements that determine this behavior can be studied within an equilibrium framework, the model presented in this paper is silent about them.

In the last part of the paper, I circumvent these problems by focussing on the differences, implied by the estimated parameters, between models with and without inertial behavior.

Appendix 1. Data

Most of the data used in this paper come from the Consumer Expenditure Survey. The measure of non durable consumption is constructed according to the following procedure. First, non durable consumption is defined as total consumption minus expenditure on durable commodities, health, education and housing. For those observations that complete four interviews, annual consumption is defined as the sum of the twelve monthly figures they report. The observations with complete interviews are then used to estimate a seasonality model for non durable consumption. The coefficient on the monthly dummies estimated from this model are then used to correct the total consumption figure for those households who do not complete four interviews. Basically, rather than multiplying the total observed consumption by 12/n (where n is the number of months for which we have data), the procedure adopted keeps into account which months each household misses.

The price indexes for non durable consumption and automobiles are computed as a Stone price indexes of the various components (new and used cars for the latter) using group expenditure means as weights. The original price indexes are taken from the BLS detailed CPI tape and are monthly, regional prices. The cells are defined by the interactions of cohort, education, regional and time dummies. Therefore, the price indexes present both time series and cross sectional variation. The latter is partly across education groups (and it is caused by difference in expenditure shares) and partly regional.

The family variables used in the analysis come from the family income and charateristic file and refer to the last interview completed by each household.

The estimation of the value of the stock of cars was by far the most difficult task of the data construction process. First of all, I collected data on used cars on a large number of models and makes from the Kelley Blue Books for used cars and the Kelley blue Books for Older Cars. The price of each model was taken from the July issue of each year from 1984 to 1988. I used the retail price and averaged among the prices for the various options.[1]

These prices were then matched with the information on the cars to estimate a value of the stock available to each household at the beginning and at the end of the observation period. In some cases I had to average over prices of different 'vintages' because the CEX, for older automobiles, gives only an interval for the year in which the car was built. Because the price information was sampled only once a year the value of the stock of cars varies without changing the composition of the stock only for those households who complete the four interview. In this sense the use of the variable measuring the length of the observation period in the band equation is important.

For some households the information on its cars was too vague to allow a reliable estimate of the value of one or more cars. All households with incomplete information on the value of the car stock were dropped from the sample. I also dropped from the sample all households headed by somebody younger than 21 or older than 75, those residing in rural areas, those living in student accomodation and those with incomplete or missing information on consumption.

The rental cost of cars rc is construced using the following expression:

$$rc = p^k \frac{(1+r) - (1-\delta)\pi^k}{1+r}$$

where p^k is the BLS price index on new cars, π^k its rate of change, r a measure of the real interest rate (computed as the municipal bond rate minus the actual rate of inflation in the CPI), and δ

[1] In theory one could collect monthly price information and prices for several options on which information exists in the CEX vehicle file exist. Limitation of resources prevented this level of detail.

is a household specific depreciation rate. This last quantity is obtained by first estimating an age specific depreciation profile using the Blue Book price data from 1984 to 1988 (without allowing for vintage or model effects) and then assigning to each household a rate of depreciation based on the age of the cars owned.

Appendix 2. Numerical methods used in the maximization of the likelihood function

The likelihood function derived in section (3) was maximized using a sequential quadratic programming algorithm in which the search direction is the solution to a quadratic programming problem. The gradient of the likelihood function was computed numerically. The algorithm is implemented by NAG subroutine E04UCF and allows the consideration of linear, non-linear and inequality constraints on the parameters.

The likelihood function involves the evaluation of a large number of multiple integrals that have to be computed numerically. Notice that all the integrals in the likelihood function can be written so that the interior integrals are the distribution function of a univariate or bivariate gaussian function for which standard Fortran functions exist in most numerical libraries. The last integral is then obtained using either Gauss-Legendre or Gauss-Hermite quadrature formulae.

Some attention was devoted to the number of points used in the integration formulae: I increased the number of points until the likelihood function did not change to the seventh significant figure. I also checked, for different parameter values, the value obtained by the quadrature formulae with those obtained computing the integrals with an adaptive method with an absolute error of 10^{-5}. In the end, I use 20 points of integration. One evaluation of the likelihood function with 20 points of integration takes about 10 CPU seconds for the reduced sample on a Sun Sparc-10 Mod. 51. As a comparison, using the adaptive method of integration, it would take more than 10 times as much.

Given that I typically estimate 50 to 70 parameters, and that derivatives are computed numerically, each iteration takes from 500 to 700 seconds. It typically takes about 100 iterations to achieve convergence.

Standard errors are computed by using the outer product of the score evaluated at the convergence point.

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Table 1

Descriptive Statistics

	Mean	St. dev.
Age	44.4	15
Beg. of period K	5719	5760
Beg. of period K ($K > 0$)	6832	5124
Beg. of period Z	0.561	0.520
Beg. of period Z ($Z > 0$)	0.656	0.505
Non dur. consum.	11560	7237
Income	22658	21227
No. of cars	2.10	1.72
No. of children 0-2	0.09	0.30
No. of children 3-15	0.61	1.00
No. of males older than 16	0.97	0.68
No. of females older than 16	1.05	0.63
No of earners	2.44	0.87
% with at least one car	87.8	
% of high sch. dropouts	20.90	
% of high sch. graduates	54.16	
% of college graduates	24.94	
% blacks	12.6	
% female heads	31.6	
Number of obs.	21628	

Table 2

Composition of the sample by transaction types

	Frequency	Percentage
Households who downgrade to zero	466	2.15
Households who downgrade	2002	9.26
Households who do not transact	11789	54.51
Households who up-grade	4020	18.59
Households who up-grade from zero	718	3.32
Households with no cars	2633	12.17
Total	21628	100.00

Table 3

Probit models for upgrading and downgrading

	Upgrade	Downgrade
age	-0.002 (0.006)	0.010 (0.006)
age^2	-0.000 (0.000)	0.000 (0.000)
males over 16	0.153 (0.021)	0.185 (0.022)
females over 16	0.101 (0.020)	0.096 (0.022)
children 3-15	-0.005 (0.011)	0.038 (0.012)
children 0-2	0.023 (0.034)	0.048 (0.038)
high sch. dropouts	0.111 (0.034)	0.125 (0.038)
high sch. graduates	0.128 (0.026)	0.129 (0.029)
female head	-0.168 (0.028)	0.030 (0.031)
black head	-0.251 (0.035)	0.090 (0.038)
car bought in the last year	-1.808 (0.037)	0.319 (0.047)

Table 3 (ctd.)

	Upgrade	Downgrade
no earners	-0.588	-0.350
	(0.060)	(0.066)
1 earner	-0.250	-0.217
	(0.045)	(0.049)
2 earners	-0.073	-0.097
	(0.039)	(0.042)
northeast	-0.070	-0.142
	(0.031)	(0.036)
midwest	0.108	0.016
	(0.029)	(0.032)
south	0.052	-0.042
	(0.029)	(0.032)
beginning	-0.747	0.485
of period Z	(0.028)	(0.022)
log likelihhod	-9218.7	-7167.5

Note: Asymptotic standard errors in parentheses. The explanatory variables include also year dummies and a constant.

Table 4

Estimates of Ss Rule
Observable bands

Target equation

	Coeff.	St. Err.		Coeff.	St. Err.
const.	1.274	0.285	spouse hours	0.130	0.066
age	-0.019	0.008	No earner	0.078	0.078
age squared	0.00025	0.0001	1 earner	-0.043	0.051
Males > 16	-0.071	0.029	high sc. drop.	-0.086	0.051
Females > 16	-0.023	0.029	high sc. grad	0.004	0.039
children 3-15	-0.066	0.018	male head	0.108	0.045
children 0-2	-0.063	0.054	non-black head	0.053	0.051
rental pr. cars	-0.231	0.142	north east	-0.032	0.018
rel.price fuel	0.350	0.087	mid west	0.086	0.017
head hours	0.085	0.058	south	0.072	0.017

	Lower band equation		Upper Band equation		
	Coeff.	St. Err.		Coeff.	St. Err.
age	-0.028	0.023	age	-0.070	0.021
age squared	0.0004	0.0003	age squared	0.001	0.000
Males > 16	-0.383	0.077	Males > 16	-0.375	0.074
Females > 16	-0.203	0.076	Females > 16	-0.197	0.074
children 3-15	-0.084	0.046	children 3-15	-0.203	0.045
children 0-2	-0.091	0.144	children 0-2	-0.067	0.132
head hours	0.078	0.159	head hour	0.239	0.147
spouse hours	0.253	0.176	spouse hours	0.128	0.160
no earners	1.020	0.203	no earners	0.641	0.197
one earner	0.143	0.137	one earner	0.439	0.125
high sc. drop.	-0.152	0.137	high sc. drop.	-0.144	0.125
high sc. grad.	-0.069	0.106	high sc. grad.	-0.189	0.096
male head	0.017	0.121	male head	-0.026	0.112
non black head	-0.306	0.142	non black head	-0.123	0.129
constant	0.810	0.522	constant	1.953	0.481
σ_d	0.908	0.008	σ_b	2.351	0.011
$\rho_{d,b}$	0.802	0.004	log likelihood	-24317.57	

Note: the equations for the band also include time dummies, which are not reported to save space.

Table 5

Estimates of Ss Rule
Unobservable bands

Target equation

	Coeff.	St. Err.		Coeff.	St. Err.
const.	0.702	0.261	spouse hours	0.112	0.042
age	-0.021	0.005	No earner	0.143	0.051
age squared	0.0003	0.0001	1 earner	0.004	0.032
Males > 16	-0.062	0.018	high sc. drop.	-0.065	0.032
Females > 16	-0.045	0.019	high sc. grad	0.024	0.025
children 3-15	-0.084	0.011	male head	0.017	0.028
children 0-2	-0.075	0.036	non black head	-0.024	0.031
rental pr. cars	0.154	0.156	north east	-0.041	0.020
rel.price fuel	0.329	0.064	mid west	0.112	0.019
head hours	0.082	0.037	south	0.115	0.019

	Lower band equation		Upper Band equation		
	Coeff.	St. Err.		Coeff.	St. Err.
age	-0.031	0.020	age	-0.056	0.018
age squared	0.001	0.000	age squared	0.001	0.000
Males > 16	-0.890	0.077	Males > 16	-0.787	0.062
Females > 16	-0.698	0.075	Females > 16	-0.390	0.060
children 3-15	-0.236	0.047	children 3-15	-0.191	0.035
children 0-2	-0.258	0.152	children 0-2	-0.116	0.105
head hours	-0.132	0.147	head hour	0.343	0.118
spouse hours	0.288	0.176	spouse hours	0.586	0.129
no earners	1.913	0.180	no earners	0.346	0.186
one earner	0.608	0.131	one earner	0.929	0.100
high sc. drop.	-0.091	0.124	high sc. drop.	-0.487	0.103
high sc. grad.	-0.092	0.099	high sc. grad.	-0.454	0.076
male head	-0.464	0.109	male head	0.116	0.091
non black head	-1.174	0.117	non black head	-0.128	0.109
constant	1.731		constant	2.001	0.404
σ_d	0.701	0.007	σ_b	2.574	0.013
$\rho_{d,b}$	0.530	0.004	log likelihood	-20021.03	

Note: the equations for the band also include time dummies, which are not reported to save space.

Table 6

Mean estimated adjustment probabilities

	Observ. bands from 4	Unobservable bands from 5	Sample Frequencies
(1)	0.133	0.234	0.220
(2)	0.174	0.239	0.213
(3)	0.082	0.100	0.110
(4)	0.155	0.109	0.025
Prob. of zero	0.188	0.159	0.156

Note: (1) upgrading (conditional on $Z_{t^-} > 0$).

(2) upgrading (conditional on $Z_{t^-} = 0$).

(3) downgrading to positive (conditional on $Z_{t^-} > 0$).

(4) downgrading to zero (conditional on $Z_{t^-} > 0$).

Table 7

Autocorrelations of aggregate changes in Z

model with observable bands
(coefficients from table 4)

lag	Ss rule	without frictions	quadratic costs of adj.
1	-0.3299	-0.4937	-0.2721
2	-0.1063	0.0019	-0.1135
3	-0.0245	-0.0059	-0.0553
4	-0.0134	0.0039	-0.0192

model with unobservable bands
(coefficients from table 5)

lag	Ss rule	without frictions	quadratic costs of adj.
1	-0.3415	-0.4947	-0.2735
2	-0.0999	0.0027	-0.1129
3	-0.0193	-0.0054	-0.0545
4	-0.0167	0.0032	-0.0196

Table 8a

Aggregate impulse response functions in different models

Permanent shock of 0.1 to desidered stock

model with observable bands
(coefficients from table 4)

lag	Ss rule	without frictions	quadratic costs of adj.
0	0.0611	0.0994	0.0499
1	0.0274	0.0125	0.0280
2	0.0171	0.0129	0.0188
3	0.0148	0.0145	0.0156
4	0.0124	0.0118	0.0129

model with unobservable bands
(coefficients from table 5)

lag	Ss rule	without frictions	quadratic costs of adj.
0	0.0600	0.0995	0.0499
1	0.0254	0.0128	0.0281
2	0.0164	0.0131	0.0188
3	0.0149	0.0142	0.0154
4	0.0118	0.0123	0.0130

Table 8b

Aggregate impulse response functions in different models

Transitory shock of 0.1 to desidered stock

model with observable bands
(coefficients from table 4)

lag	Ss rule	without frictions	quadratic costs of adj.
0	0.0611	0.1013	0.0502
1	-0.0319	-0.0853	-0.0214
2	-0.0093	0.0016	-0.0088
3	-0.0021	0.0007	-0.0038
4	-0.0012	0.0004	-0.0018

model with unobservable bands
(coefficients from table 5)

lag	Ss rule	without frictions	quadratic costs of adj.
0	0.0601	0.1008	0.0501
1	-0.0325	-0.0854	-0.0214
2	-0.0075	0.0010	-0.0090
3	-0.0023	0.0007	-0.0038
4	-0.0013	0.0001	-0.0019

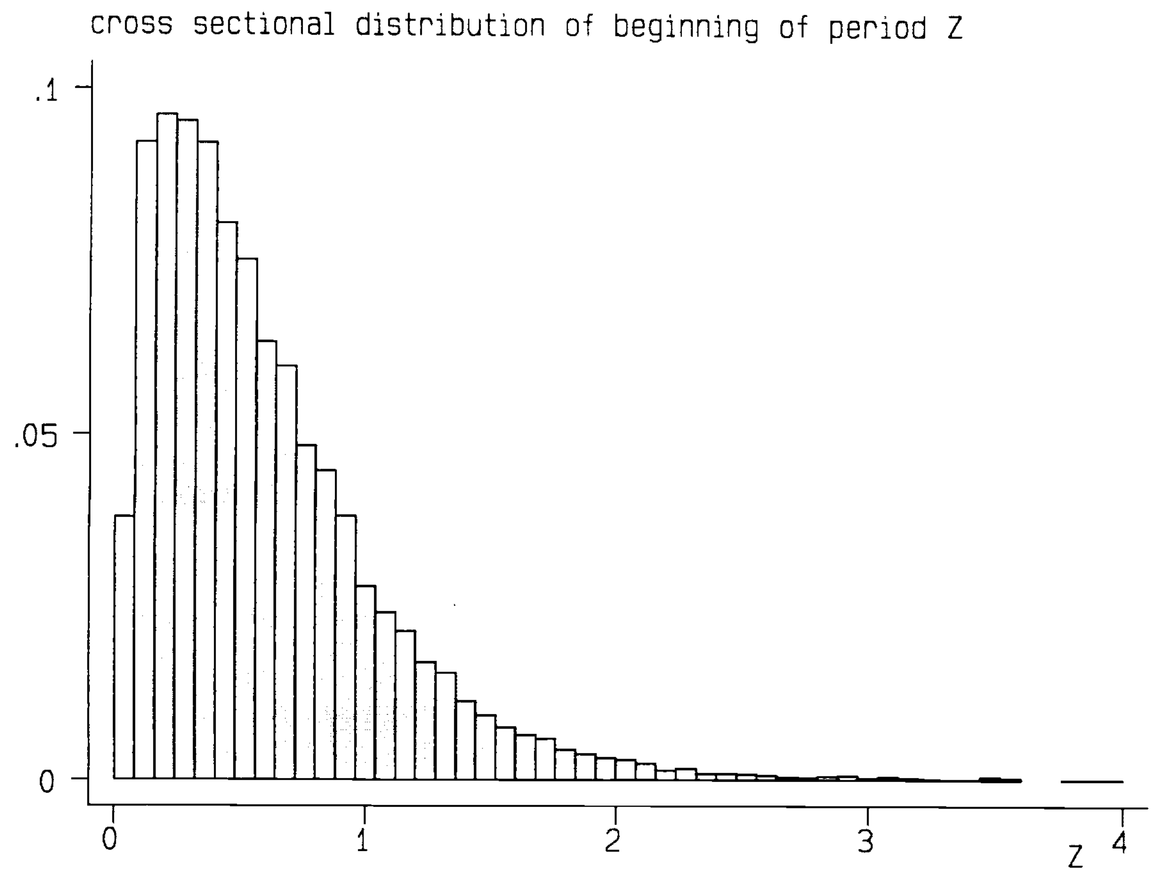
cross sectional distribution of beginning of period Z
.1
.05
0
0
1
2
3
4
Z

Figure 1

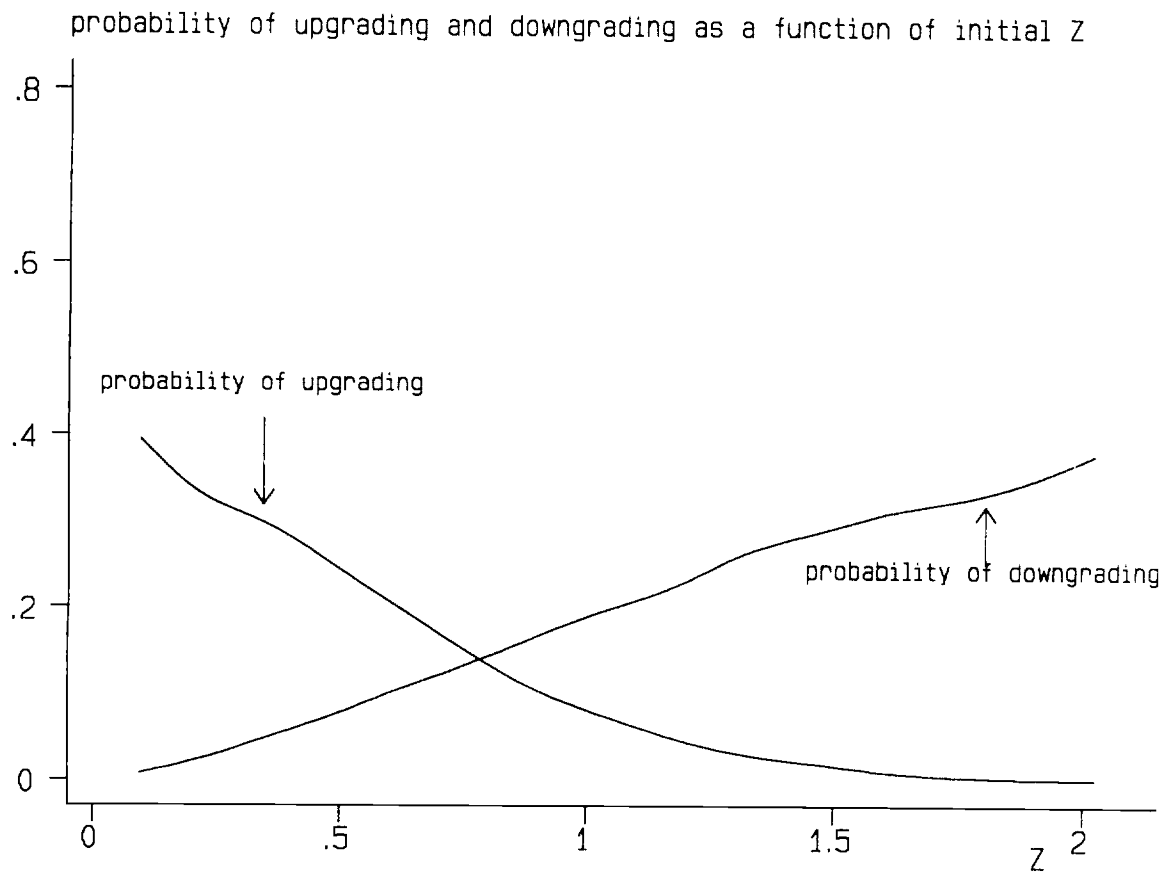
probability of upgrading and downgrading as a function of initial Z
.8
.6
.4
.2
0
probability of upgrading
probability of downgrading
0
.5
1
1.5
2
Z

Figure 2

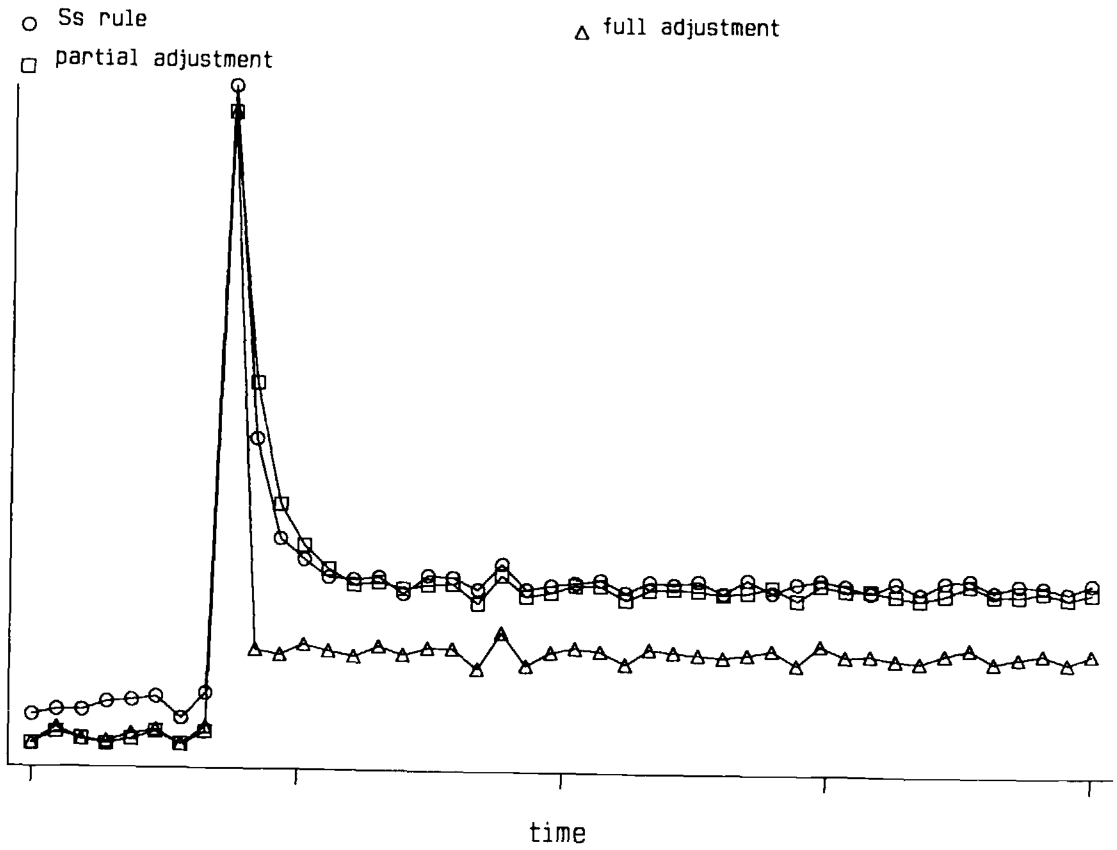

Figure 3