# PREDICTING THE NEARBED TURBULENT FLOW IN WAVES AND CURRENTS

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## ABSTRACT

The paper reviews existing models for the prediction of bottom shear stresses under the action of combined waves and currents, and presents the results of an experimental programme investigating the specific case of co-directional waves and currents. Changes in shear stress caused by the waves are deduced from the logarithmic layer and from Reynolds stresses. The turbulence intensities in the flow are also shown to be altered. These results, together with those from three case studies, are compared with theoretical predictions, and conclusions drawn as to the more appropriate models.

## 1. INTRODUCTION

In attempting to predict rates of sediment transport in a coastal regime, it is vital that one can accurately prescribe the hydrodynamics of the prevailing flow field. This is so firstly because the process of sediment entrainment is dependent on instantaneous near-bed fluid velocities, and secondly because the transport rate of suspended sediment is controlled by the mean velocity field. Also, it should be remembered that the distribution of suspended material with height above the seabed is directly related to the turbulent characteristics of the flow. In a typical sea state containing both waves and currents, suspended sediment and mobile bed forms may well interact with the flow and lead to a modified and yet more complex velocity field. However, it remains essential as a first step to be able to describe the initial combined wave and current flow conditions with which the sediment is to interact if the model is to reproduce with accuracy any of the physical processes involved in the transport mechanism.

## 2. WAVE-CURRENT BOUNDARY LAYER MODELS

In the pioneering work of Bijker (1967), the problem of waves propagating at an arbitrary angle to a current was investigated, and, using a mixing length approach, the resulting bed shear stresses were related to the constituent velocities in terms of an empirical constant $\xi$. Application of the theory was limited to the current dominated regime. Swart (1974) refined the determination of the empirical constant $\xi$ by relating it to the wave friction factor $f_w$ introduced earlier by Jonsson (1966). O'Connor and Yoo (1987) have recently developed a more comprehensive model based on the same principle, while Bakker (1974) has extended the mixing length approach using a finite difference solution.

The concept that friction within a turbulent flow is in some way analogous to viscous friction has attracted many theories for the prediction of combined waves and currents. The main difference between the many models of this type so far proposed seems to centre on the eddy viscosity distributions prescribed by the modeller (fig.1) and the consequential complexities of achieving a solution.

Lundgren (1972) was one of the first to use this approach, assuming the mean current eddy viscosity to increase linearly within the wave dominated-bed layer but to vary parabolically above that zone and up to the surface. He also introduced an independent empirical wave eddy viscosity. Fredsoe (1981) later extended this model to cases where wave velocities dominate near the bed.

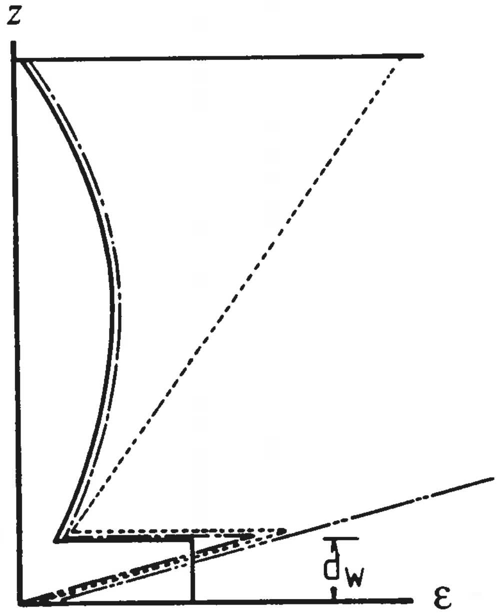


*Fig.1 Assumed Eddy Viscosity distributions*

*—[C], ------ [GM], —·— [CJ], —·-·— [TS]*

Whereas these models considered only the effects of the waves on the mean flow, Smith (1977) developed a model capable of predicting the non-linear interaction between the wave and current components for current dominated flows. Grant and Madsen (1979) proposed a similar model, applicable to the wave-dominated regime, with solutions in terms of Kelvin functions. Tanaka and Shuto (1981) used a simple one-layer model, while Christoffersen (1982) opted for a two layer approach in his model for flows over a very rough bed. In the same paper he also proposed an improved version of the Grant and Madsen model.

Christoffersen and Jonsson (1985) later put forward another two layer model, with a range of applicability extended to the small relative roughness regime. Other models of a similar nature include Tanaka, Chian and Shuto (1983), using two layers, Myrhaug (1984), three layer, and Asano and Iwagaki (1984), which is a refined version of Grant and Madsen. The solutions were all analytical, though requiring considerable mathematical skill.

Fredsoe (1984) proposed a model on the premise that both mean and oscillatory velocities would form logarithmic profiles near the bed. He then deduced shear stresses by integrating momentum defect through the depth. The resulting equations required numerical solution, but Fredsoe produced graphs to facilitate the calculations. The model was only applicable for $A_o/k_s>30$.

Davies, Soulsby and King (1988) have recently developed a numerical model for combined wave and current flows which does not require any prior assumptions to be made regarding the general form of the velocity profiles. Instead, they have achieved closure of the turbulent energy equation using expressions for turbulence production, dissipation and diffusion previously established for tidal flow. The model is applicable to any combination of wave and current from unidirectional flow through to pure oscillatory motion, but there is a considerable demand on computer time in achieving a solution.

## 3. EXPERIMENTS

Detailed measurements were made in a recirculating laboratory flume 610 mm wide and 30 m long with a 10mm 3-dimensional bed roughness. In the main test programme, horizontal velocities were measured at over 20 positions through the depth using a

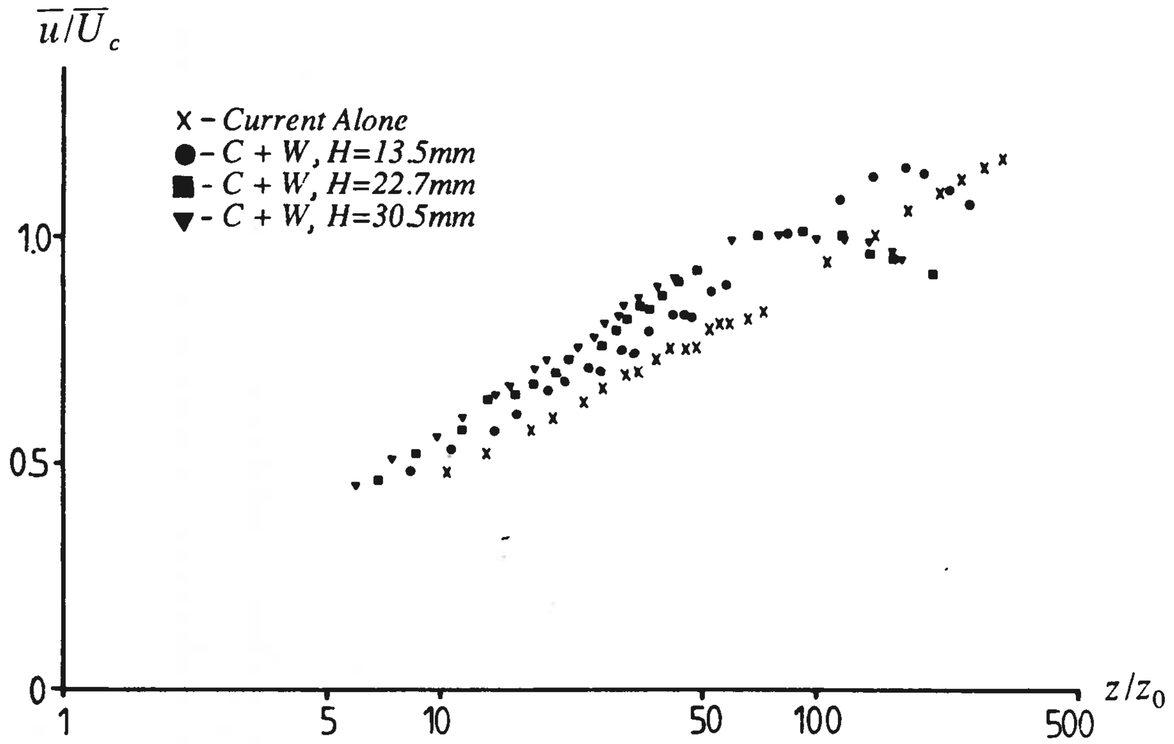


*Fig.2* *Mean Velocity Profiles for Medium Current tests* ($\overline{U} = 195mm/s, T = 1.0s, d = 300mm$)

single channel laser Doppler anemometer (LDA). A two channel LDA was then used at selected points to determine the vertical velocity field and instantaneous Reynolds stresses. The experimental techniques and analysis were similar to those described by Kemp and Simons (1982).

Two particular groups of test conditions are reported here, those with a steady unidirectional current alone, and those with waves propagating co-directionally with a current. Three different current strengths were tested, nominally weak ($\bar{U}$=75mm/s), medium ($\bar{U}$=195mm/s) and strong ($\bar{U}$=250mm/s), and these were combined with waves of 0.7s or 1.0s period. Wave heights ranged between 10mm and 50mm, and the mean water depth was 300 mm in all cases. The test parameters are set out in Table 1.

## 4. RESULTS

Mean velocity profiles for the three unidirectional current conditions all followed the logarithmic curve expected for rough turbulent flow. It should be noted, though, that if the shear velocity, u*, was calculated from the direct Reynolds stress measurements, the Karman constant lay significantly below 0.4. When waves were propagated on the current, there was a general increase in both mean bed shear stress, $\tau_{mean}$, and apparent bed roughness, $z_o$, felt by the current - see fig.2. These increases were more pronounced for the tests with higher $A_o/k_s$, as demonstrated by the results set out in Table 1.

Wave-induced oscillatory velocities were found to follow a potential wave reduction from the surface down to a point some 30 mm above the bed. Below this level they decreased in many of the tests, indicating the existence of a wave boundary layer. However, experimental scatter and local bed effects made it difficult to estimate $d_w$ in this way.

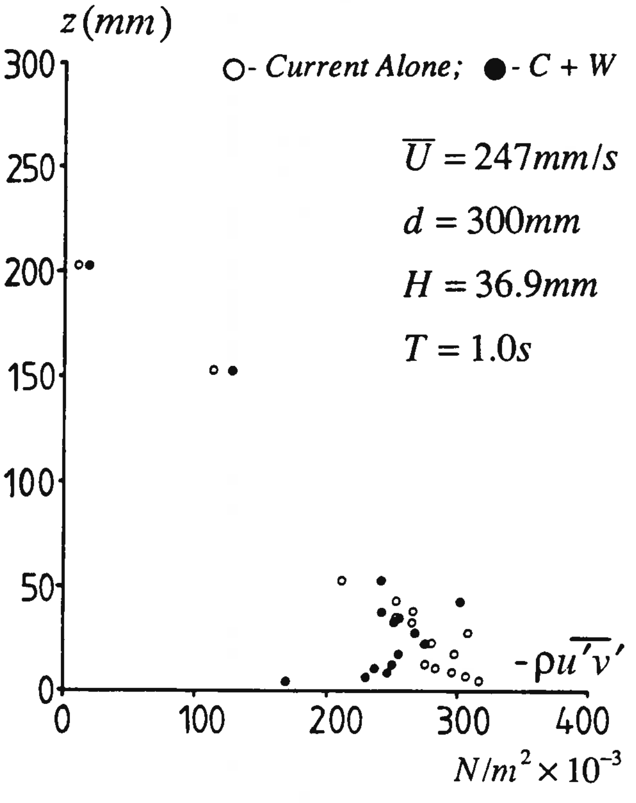
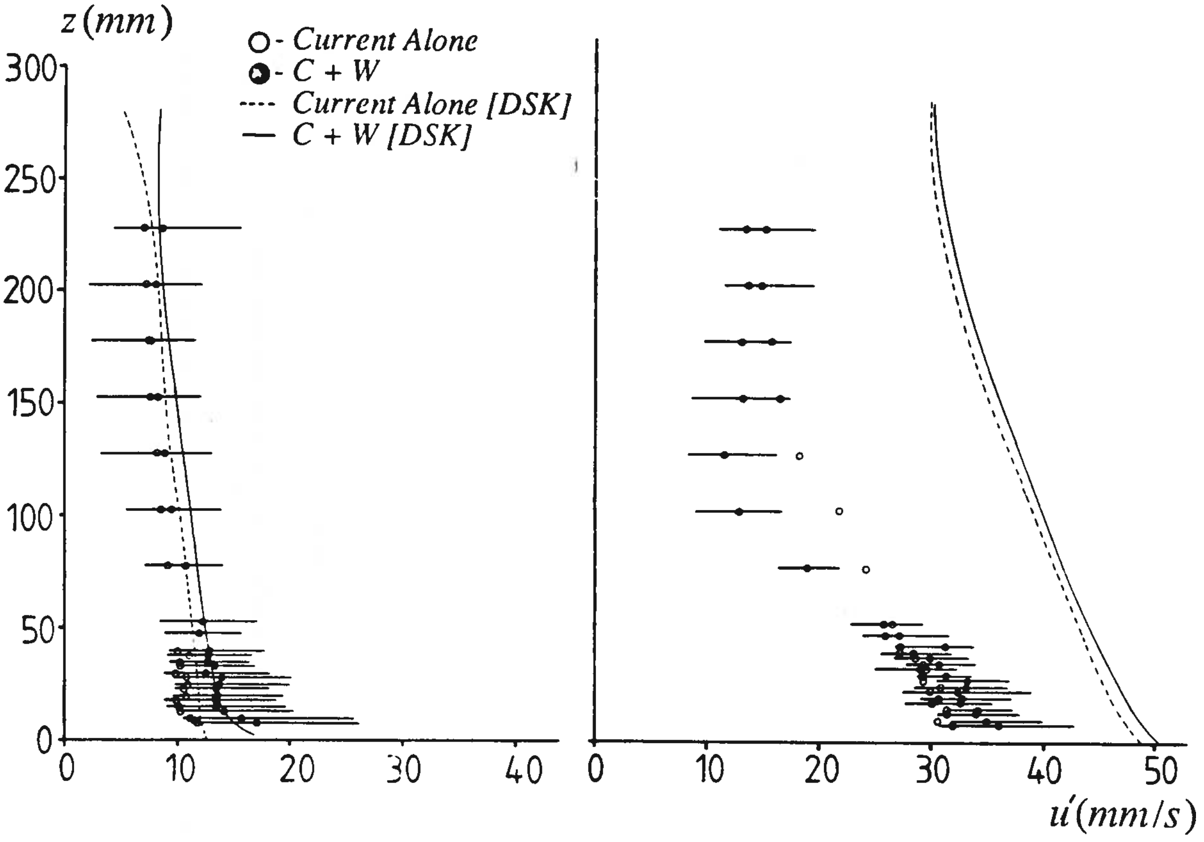


*Fig.3 Distribution of Reynolds Stresses*

Fig.3 shows the distribution of average Reynolds stress for the strong current with and without waves. It can be seen that the waves have induced a reduction in $\overline{u'v'}$ near the bed, so producing a maximum some way from the bed and well into the logarithmic layer. For the larger waves, Reynolds stress was found to vary through the cycle near the bed.

Turbulence intensities (fig.4) were determined from the fluctuations about the ensemble averaged horizontal velocities. Two important observations can be made relating to the superposition of waves, firstly, that $u'$ increased in and below the logarithmic layer by between 15% and 40% (depending on the relative strengths of wave and current), and, secondly, that the extent of the turbulent boundary layer was reduced, in the case of the strong current by almost 50%. The increase in $u'$ was of the same order as the increase in $u^*$ defined from the mean velocity profiles.

The horizontal lines drawn through each value of $u'$ for the combined wave and current results in fig.4 indicate the range of unfiltered values found through the wave cycle. In general this represents random scatter, but within 15 mm of the bed it was noted that $u'$ varied systematically through the wave cycle, with maxima corresponding to the decelerating phase.



(a) U=74mm/s, H=38.5mm, T=1.0s

(b) U=247mm/s, H=36.9mm, T=1.0s

*Fig.4 Turbulence Intensities through the depth for Combined Waves and Current.*

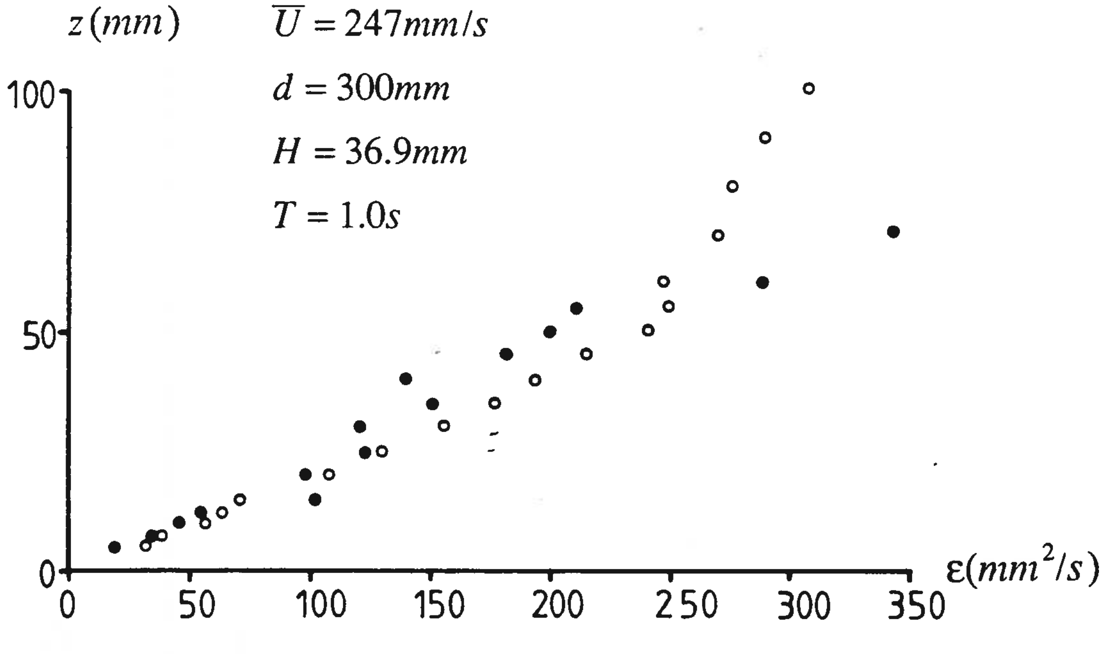


*Fig.5 Measured Eddy Viscosity Distribution;* ○ *- Current Alone;* ● *- C + W*

## 5. COMPARISON WITH MODELS

Comparing the experimental mean shear stresses, corrected for $\kappa = 0.4$, with those predicted by the various models (Table 2), the average deviation for Christoffersen [C], Christoffersen and Jonsson [CJ], and Davies et al. [DSK] lies between 15% and 20%, with [CJ] and [DSK] generally under-estimating, while [C] predicts values greater than those found experimentally. The Grant and Madsen model, as modified by Christoffersen, [MGM], leads to values consistently 40% high. If the wave dominated, weak current results are ignored, then O'Connor and Yoo [OY] and Tanaka and Shuto [TS] also lie within 15% and Bijker [B] within 20%. However, Swart [S] produces a gross over-prediction in all cases involving significant wave action at the bed.

Turning now to the maximum stresses felt at the bed, the present experimental values were deduced assuming that a logarithmic profile exists between the top of the wave boundary layer and the bed origin, $z_o$, determined from the current only tests. Comparing these experimental values with the models, it was found that [DSK] gave good agreement in most cases, as did [TS]. However, [C], [CJ] and [MGM] all produced figures between two and five times too large, results which emphasize that these models are intended for use at higher $A_o/k_s$ than found in the present tests. Of the three, [C] was consistently closer to the experimental values.

To widen the parametric range of the study, it was decided to adopt the three case studies published by [DSK]. Calculated values of mean and maximum shear stress are shown in Table 3. In many respects, these confirm the previous comments, in particular, that [OY], and all the eddy viscosity models considered, [C], [CJ], [MGM], [TS] and Grant and Madsen, lay well within 50% (the [DSK] results were assumed "correct" for the purposes of this comparison), while the earlier mixing length models, [B] and [S], produced progressively greater over-estimates with increasing wave strength. However, the most impressive results were those from Fredsoe's (1984) momentum integral model; this had not figured in the earlier comparisons as conditions lay well outside its range of applicability and no solutions were available.

## 6. DISCUSSION

From the foregoing calculations, it would appear that acceptable predictions of mean bed shear stress come from at least five of the models, but that if the maximum stress is also required, then Fredsoe (1984) or Davies et al (1988) produce the best results. However, both of these require considerable computation, if graphical solutions are to be avoided, and the engineer may be attracted by the eddy viscosity models considered here, which offer a more direct solution. The simple mixing length models, although easy to apply, are only safely used in current-dominated conditions.

It is interesting to note that, irrespective of the initial assumptions made, the shear stresses showed very little sensitivity to the distribution of eddy viscosity. It remains to be seen whether the additional complexity of incorporating a time-varying eddy viscosity is rewarded with any improvement in accuracy. Nevertheless, Fig.5 shows the eddy viscosity distribution determined from the present tests, for a current with and without waves. The results are clearly linear in both cases within 50 mm of the bed, although above this level the combined flow values continue to increase while the current only values curve back to zero.

Of the models considered, only Davies et al. have attempted to predict changes in turbulence energy distribution when waves and currents are combined; their results are included in fig.4. For the weak current, the model produced quite impressive agreement with experiment, predicting a significant increase in u' near the bed. However, the strong current values showed a large discrepancy, even for the current alone, due possibly to low experimental results, or alternatively an incorrect constants in the model. The present tests suggest that if a prediction of nearbed turbulence is required from one of the other models, then a reasonable estimate may be obtained by using the modified u* as a turbulent velocity scale.

None of the models tested above makes any allowance for the possibility of a change in mean current boundary layer thickness, although such a change is suggested by some of the present results. Nor do they take any account of the effects of vertical velocities on the combined flow field. Instead, most require the user to prescribe the wave motion in terms of a plane oscillatory velocity at the bed.

## 7. ACKNOWLEDGEMENTS

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| | | EXPERIMENTAL PARAMETERS | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $H$ (mm) | $U_0^{wc}$ (mm/s) | $\overline{U}_c/\tilde{U}_0^{wc}$ | $A_0/k_s$ | $\tau_{mean}^{c}$ N/m²x10$^{-3}$ | $\tau_{mean}^{wc}$ N/m²x10$^{-3}$ | $k_s$ (mm) | $k_A/k_S$ |
| Weak Current | *T=0.7s* | | | | | | | | |
| | RDWCW1 | 14.1 | 11 | 7.4 | 0.06 | 43.2 | 43.6 | 20.4 | 1.38 |
| | RDWCW2 | 18.4 | 13 | 6.2 | 0.07 | 43.2 | 45.8 | 20.4 | 1.47 |
| | RDWCW3 | 21.1 | 14 | 5.8 | 0.09 | 38.2 | 52.9 | 18.0 | 1.83 |
| | RDWCW4 | 22.2 | 17 | 4.8 | 0.11 | 38.2 | 51.8 | 18.0 | 2.03 |
| | *T=1.0s* | | | | | | | | |
| | RIWCW1 | 19.6 | 25 | 2.8 | 0.20 | 31.4 | 40.8 | 20.4 | 1.62 |
| | RIWCW2 | 29.5 | 39 | 2.0 | 0.31 | 37.2 | 37.2 | 20.1 | 1.73 |
| | RIWCW3 | 38.5 | 58 | 1.3 | 0.59 | 31.7 | 38.9 | 15.6 | 2.48 |
| | RIWCW4 | 50.5 | 71 | 1.0 | 0.72 | 31.7 | 67.1 | 15.6 | 4.94 |
| Medium Current | *T=0.7s* | | | | | | | | |
| | RDWCM1 | 11.8 | 12 | 15.9 | 0.06 | 234.7 | 276.9 | 22.2 | 1.05 |
| | RDWCM2 | 15.0 | 15 | 13.0 | 0.08 | 239.9 | 316.5 | 21.6 | 1.22 |
| | RDWCM3 | 17.2 | 16 | 12.0 | 0.08 | 232.6 | 350.4 | 21.9 | 1.58 |
| | RDWCM4 | 18.2 | 15 | 13.1 | 0.08 | 250.0 | 353.8 | 21.6 | 1.67 |
| | *T=1.0s* | | | | | | | | |
| | RIWCM1 | 13.5 | 23 | 8.4 | 0.17 | 234.1 | 294.1 | 21.9 | 1.23 |
| | RIWCM2 | 22.7 | 35 | 5.6 | 0.26 | 246.5 | 347.1 | 21.6 | 1.53 |
| | RIWCM3 | 30.5 | 47 | 4.2 | 0.35 | 246.5 | 371.7 | 21.6 | 1.75 |
| | RIWCM4 | 40.7 | 61 | 3.2 | 0.45 | 237.2 | 347.8 | 21.6 | 2.17 |
| Strong Current | *T=0.7s* | | | | | | | | |
| | RDWCS1 | 9.9 | 13 | 19.5 | 0.05 | 474.8 | 525.8 | 29.4 | 1.02 |
| | RDWCS2 | 12.9 | 15 | 16.8 | 0.06 | 482.6 | 540.1 | 29.4 | 1.12 |
| | RDWCS3 | 15.1 | 16 | 15.8 | 0.06 | 482.6 | 599.3 | 29.4 | 1.12 |
| | RDWCS4 | 16.3 | 15 | 16.7 | 0.06 | 471.3 | 578.4 | 30.0 | 1.05 |
| | *T=1.0s* | | | | | | | | |
| | RIWCS1 | 12.0 | 21 | 12.0 | 0.11 | 466.6 | 540.1 | 29.4 | 1.12 |
| | RIWCS2 | 20.1 | 34 | 7.4 | 0.18 | 466.6 | 625.5 | 29.4 | 1.12 |
| | RIWCS3 | 27.7 | 44 | 5.7 | 0.24 | 462.3 | 542.4 | 29.7 | 1.05 |
| | RIWCS4 | 36.9 | 56 | 4.4 | 0.30 | 458.0 | 542.9 | 30.0 | 1.22 |

*Table 1. Experimental Parameters. In all cases Water Depth d=300mm.*

|  |  | MODEL PREDICTIONS |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|
|  |  | B | C | CJ | DSK | MGM | OY | S | TS |
| Weak Current | *T=0.7s* |  |  |  |  |  |  |  |  |
|  | RDWCW1 | -2.76 | 22.53 | -2.76 | - | 77.93 | 12.64 | 19.08 | 6.90 |
|  | RDWCW2 | -4.80 | 22.93 | -7.64 | - | 65.28 | 13.10 | 36.24 | 40.39 |
|  | RDWCW3 | -17.58 | 3.59 | -22.50 | - | 53.12 | -2.08 | 27.03 | 20.98 |
|  | RDWCW4 | -13.32 | 5.79 | -20.85 | - | 93.05 | 2.90 | 36.29 | 26.06 |
|  | *T=1.0s* |  |  |  |  |  |  |  |  |
|  | RIWCW1 | 20.10 | 20.10 | -17.65 | - | 94.12 | 41.67 | 201.96 | 72.55 |
|  | RIWCW2 | 117.74 | 67.74 | 17.20 | - | 127.42 | 137.63 | 570.97 | 161.29 |
|  | RIWCW3 | 178.15 | 44.73 | -1.29 | -3.08 | 90.23 | 162.21 | 917.99 | 145.50 |
|  | RIWCW4 | 144.11 | -13.86 | -40.83 | -42.77 | -2.24 | 100.60 | 876.75 | 65.42 |
| Medium Current | *T=0.7s* |  |  |  |  |  |  |  |  |
|  | RDWCM1 | -17.66 | 11.88 | -9.86 | - | 40.16 | -8.70 | -13.22 | -2.89 |
|  | RDWCM2 | -25.06 | -3.51 | -21.14 | - | 26.38 | -16.05 | -19.12 | -9.76 |
|  | RDWCM3 | -33.19 | -7.53 | -29.65 | - | 14.16 | -25.11 | -26.94 | -18.52 |
|  | RDWCM4 | -30.33 | -3.25 | -26.74 | - | 17.64 | -22.10 | -23.04 | -15.40 |
|  | *T=1.0s* |  |  |  |  |  |  |  |  |
|  | RIWCM1 | -17.24 | 18.91 | -12.95 | - | 31.96 | -6.29 | -2.89 | 4.08 |
|  | RIWCM2 | -22.50 | -44.68 | -20.60 | - | 23.45 | -8.73 | 10.69 | 2.91 |
|  | RIWCM3 | -20.42 | 7.61 | -23.97 | - | 22.06 | -4.92 | 36.21 | 5.38 |
|  | RIWCM4 | -1.58 | 20.85 | -15.93 | -19.47 | 39.16 | 16.16 | 104.97 | 25.68 |
| Strong Current | *T=0.7s* |  |  |  |  |  |  |  |  |
|  | RDWCS1 | -15.33 | 15.10 | -10.44 | - | 43.84 | -8.79 | -12.89 | -4.56 |
|  | RDWCS2 | -17.57 | 13.87 | -12.81 | - | 41.05 | -11.20 | -14.40 | -5.42 |
|  | RDWCS3 | -25.71 | 3.45 | -21.42 | - | 28.03 | -19.24 | -22.14 | -13.27 |
|  | RDWCS4 | -23.03 | 7.19 | -18.59 | - | 33.61 | -17.08 | -18.59 | -9.34 |
|  | *T=1.0s* |  |  |  |  |  |  |  |  |
|  | RIWCS1 | -16.00 | 22.29 | -10.39 | - | 33.98 | -7.92 | -9.57 | 0.43 |
|  | RIWCS2 | -24.72 | 12.28 | -19.78 | - | 23.55 | -15.43 | -9.45 | -4.03 |
|  | RIWCS3 | -9.96 | 33.41 | -6.66 | -10.93 | 49.76 | 3.56 | 23.67 | 19.08 |
|  | RIWCS4 | -5.08 | 34.28 | -5.91 | -9.80 | 54.91 | 10.57 | 54.91 | 27.39 |
|  | **Average** | **4.84** | **13.45** | **-15.13** | **-17.21** | **46.78** | **13.64** | **114.36** | **22.54** |

Percentage Deviation from Experimental Mean Bed Shear Stress

| MODEL | CASE 1<br>$\overline{U} = 0.795 m/s$ ; $\tilde{U}_0 = 0.5 m/s$ | | CASE 2<br>$\overline{U} = 0.672 m/s$ ; $\tilde{U}_0 = 1 m/s$ | | CASE 3<br>$\overline{U} = 0.612 m/s$ ; $\tilde{U}_0 = 1.5 m/s$ | |
|---|---|---|---|---|---|---|
| | $\tau_{mean} N/m^2$ | $\tau_{\max} N/m^2$ | $\tau_{mean} N/m^2$ | $\tau_{\max} N/m^2$ | $\tau_{mean} N/m^2$ | $\tau_{\max} N/m^2$ |
| Bijker (1966) [B] | 6.3 | - | 17.8 | - | 37.8 | - |
| Christoffersen (1982) [C] | 3.92 | 21.99 | 3.25 | 46.9 | 2.95 | 77.1 |
| Christoffersen & Jonsson (1985) [CJ] | 3.23 | 20.26 | 2.81 | 43.4 | 2.67 | 72.7 |
| **Davies, Soulsby & King (1988) [DSK]** | **3.38** | **13.03** | **3.38** | **27.91** | **3.38** | **48.85** |
| Fredsoe (1984) [F] | 3.25 | 10.53 | 3.36 | 22.74 | 3.60 | 36.86 |
| Grant & Madsen (1979) [GM] | 4.67 | 17.69 | 4.73 | 37.75 | 4.60 | 63.26 |
| Jonnson (1966) [J] | 20.3 | 44.9 | 22.8 | 69.1 | 27.0 | 100.4 |
| Mod GM (1982) [MGM] | 4.49 | 21.78 | 4.49 | 45.63 | 4.68 | 75.46 |
| O'Connor & Yoo (1986) [OY] | 5.2 | - | 5.7 | - | 6.4 | - |
| Swart (1974) [S] | 10.5 | - | 21.8 | - | 36.4 | - |
| Tanaka & Shuto (1981) [TS] | 6.8 | 19.67 | 8.3 | 41.72 | 9.8 | 69.57 |

*Table 3. Case Studies. In all cases Wave Period T=8s, Water Depth d=10m and Nikuradse Roughness* $k_S = 150mm$.