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# A unified framework for model-based multi-objective linear process and energy optimisation under uncertainty

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## HIGHLIGHTS

- A framework for multi-objective linear optimisation under uncertainty is proposed.
- The uncertainty and the multiple objectives are modelled as parameters.
- The optimal solution is expressed as explicit functions of the parameters.

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ABSTRACT

Process and energy models provide an invaluable tool for design, analysis and optimisation. These models are usually based upon a number of assumptions, simplifications and approximations, thereby introducing uncertainty in the model predictions. Making model based optimal decisions under uncertainty is therefore a challenging task. This issue is further exacerbated when more than one objective is to be optimised simultaneously, resulting in a Multi-Objective Optimisation ($MO^2$) problem. Even though, some methods have been proposed for $MO^2$ problems under uncertainty, two separate optimisation techniques are employed; one to address the multi-objective aspect and another to take into account uncertainty. In the present work, we propose a unified optimisation framework for linear $MO^2$ problems, in which the uncertainty and the multiple objectives are modelled as varying parameters. The $MO^2$ under uncertainty problem ($MO^2U^2$) is thus reformulated and solved as a multi-parametric programming problem. The solution of the multi-parametric programming problem provides the optimal solution as a set of parametric profiles.



## 1. Introduction

### 1.1. Optimisation under uncertainty

Variations in key parameters and data used to mathematically model a system can often lead to unexpected deviation from the predicted behaviour of the system. For example, parameters like raw material quality, machine availability, safety measures and market requirements can fluctuate with respect to time. In energy and process systems, uncertainty can be either epistemic, such as the value of heat transfer coefficient or the kinetic constant of a reaction, or aleatory such as the demand of energy for the next month or the price of raw material used in a process.

To deal with the uncertainty, a number of formulations and solution techniques, including stochastic programming, fuzzy mathematical programming and multiperiod optimisation, have been proposed in the literature [1–5]. In fuzzy mathematical programming, the random parameters are treated as fuzzy numbers, the constraints as fuzzy sets and some constraint violations are allowed. Fuzzy mathematical programming can be either flexible or possibilistic with regard to where the uncertainty is located in the optimisation problem [6]. In the stochastic programming approach, the decision maker has access to probability distributions which describe the nature of the uncertainty. For the case when the distributions are continuous, a discretisation scheme is employed to compute the discrete probability distributions. The deterministic model is then transformed into a multistage stochastic programming problem and a number of scenarios are considered for different realisation of uncertainty [4]. In the two stage stochastic programming approach the optimisation variables are classified in two groups: the first-stage ones which must be determined before the realisation of the uncertainty and the second-stage ones that enact in a recursive way after the value of uncertain

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**Nomenclature**

*Abbreviations*

| | |
|---|---|
| CR | Critical Region |
| DM | Decision Maker |
| LP | Linear Programming |
| MINLP | Mixed Integer Nonlinear Programming |
| $MO^2$ | Multi-Objective Optimisation |
| $MO^2U^2$ | Multi-Objective Optimisation under Uncertainty |
| mp | Multi-Parametric |
| RES | Renewable Energy Sources |
| RHS | Right Hand Side |

*Greek letters*

| | |
|---|---|
| $\phi$ | uncertain parameter |
| $\psi$ | multi-objective parameter |
| $\theta$ | mp-$MO^2U^2$ parameter |

*Letters*

| | |
|---|---|
| $\boldsymbol{F}$ | vector of objective functions |
| $\boldsymbol{h}$ | vector of equality constraints |
| $\boldsymbol{g}$ | vector of inequality constraints |
| $\boldsymbol{x}$ | vector of decision variables |

*Sets*

| | |
|---|---|
| I | set of objective functions |
| X | set of decision variables |
| $\Theta$ | set of mp-$MO^2U^2$ parameters |
| $\Phi$ | set of uncertain parameters |
| $\Psi$ | set of multi-objective parameters |

*Superscripts*

| | |
|---|---|
| $n_x$ | dimension of decision variables |
| $n_\theta$ | dimension of mp-$MO^2U^2$ parameters |
| $n_\phi$ | dimension of uncertain parameters |
| $n_\psi$ | dimension of multi-objective parameters |
| lo | lower bound |
| up | upper bound |

parameters has been realised. Another technique used to approach uncertainty that was initially introduced from Bellman [7], is stochastic dynamic programming, where multistage decision processes are considered and the uncertainty is part of the dynamic scheme. Grossmann and Morari [8], introduced the concept of flexibility analysis to deal with design and operation of process systems. Multi-parametric programming on the other hand, is an optimisation based methodology that provides a complete map of the optimal solution in the entire range of parametric variability [9].

### 1.2. Multi-objective optimisation

A decision maker has to usually deal with a number of objectives to be optimised, for example, cost, environmental impact, energy efficiency, etc. Multi-objective optimisation, offers a well-founded framework for such problems, with a variety of different approaches such as weighted sum method, goal programming and $\epsilon$-constrained methods [10–12]. In the weighted sum method, the decision maker evaluates the relative importance of each objective function with different weighted coefficients and then performs the optimisation by adding the weighted objective functions together. Although this method can be characterised as computationally efficient, since it generates strong non-inferior solutions, the main disadvantages are the difficulty in the determination of the most adequate weighting coefficients for the problem, as well as the fact that it does not guarantee Pareto optimality [13]. In goal programming, one sets targets for all the objectives that appear in the $MO^2$ problem and then seeks solutions that are closest to the target they have already stated, with the objective to minimise the deviation from the goals set. In the $\epsilon$-constrained method, the optimisation is performed for one objective function, i.e. the most preferred one, with the rest of the objectives bounded between appropriate lower and upper bounds [14,15].

In the $MO^2$ framework, a DM solves a multi-criteria optimisation problem, and chooses between different alternatives acting in pursuit of their own choice and as a result, the concept of optimality in $MO^2$ is replaced with what is known as "Pareto optimality". Energy systems are typical examples of systems in which a performance index can conflict with an environmental or financial restriction as seen in the recent work of Luo et al. [16], where the multi-objective scheme was used for the synthesis of utility systems over the financial cost, the environmental impact and the maximisation of the exergy efficiency. A multi-objective optimisation problem was formulated to account for both the environmental impact and the economic efficiency of the system; the authors solved the resulting $MO^2$ problem with weighted sum and $\epsilon$-constrained method. Zhang et al. [17] examined the optimal design of CHP-based microgrids coupled with life cycle assessment analysis.

### 1.3. Multi-objective optimisation under uncertainty

Klein et al. [18], proposed an interactive approach for solving $MO^2$ with uncertainty in the RHS of the technology matrix, based on the concept of mutual efficiency. Kheawhom and Kittisupakorn [19], proposed a two stage algorithm, in which the $MO^2$ problem is solved in the first step with a genetic algorithm and via a stochastic modeller in the second step, where problem decomposition techniques and sequential quadratic programming method are employed to solve the subproblems. Kwak et al. [20] proposed a new method for $MO^2$ under uncertainty problems in energy conservation in commercial buildings, which included heuristics and also insights from human subject studies. An improved multi-objective teaching–learning based technique coupled with stochastic optimisation was proposed by Niknam et al. [21], where the authors deal with the operation of microgrids under uncertainty. A stochastic multi-objective optimisation study for the optimal operation of combined cooling, heating and power (CCHP) systems was presented by in Hu and Cho [22]. The authors considered variations in climate conditions and three different objective functions for the minimisation of operational cost, primary energy usage and carbon dioxide emissions. Recently, Sabio [23] proposed a systematic framework, including a multiscenario stochastic MINLP, in order to handle uncertainty explicitly in $MO^2$ problems for LCA of industrial processes. In their approach even though the uncertainty is considered explicitly, it is modelled as multiple scenarios of the same MINLP with equal probability of occurrence. Barteczko-Hibber et al. [24], presented a multi-period MILP model for electricity supply up to 2060 in the UK. They optimised costs and a number of environmental objectives in an LCA scheme separately to evaluate the trade-offs. Uncertainty in future energy demands and carbon reduction targets was treated by considering four different scenarios while sensitivity analysis was conducted for the impact of a certain regulation on the cost objective. Even though the results from this work provide valuable insight about the planning decisions, the authors identified the need for multi-objective optimisation for a more complete and coherent assessment.

The research work reported in literature addresses $MO^2U^2$ in a decoupled way and thus two different solution strategies are employed. For example, $MO^2U^2$ arises in the problem of biomass conversion technologies [25] where one would like to minimise the investment cost while minimising net $CO_2$ footprint and/or minimising operating costs but under uncertain capacity of equipment and variations in the energy demand. Another application area is building energy performance [26] where, e.g. heating/cooling requirements, energy consumption and investment cost form a $MO^2$ problem while uncertain parameters such as weather conditions and characteristics of building material lead to an $MO^2U^2$ problem. In addition, integrated systems of renewable energy resources, such as hydro–photovoltaic power systems [27] can be studied through a $MO^2U^2$ framework as the minimisation of the variance of power output and the maximisation of generated energy form two conflicting objectives and uncertainty in weather conditions and ratio coefficients make the decision making in such system quite complex. In this work, a novel and unified modelling framework for solving multi-objective optimisation problems under uncertainty is presented. The key advantages of this unified framework are that only one optimisation technique is employed and useful insights are obtained from the explicit functions thus obtained.

The remainder of the paper is organised as follows: Section 2.1 presents the mathematical preliminaries for multi-objective optimisation, multi-parametric programming and parametric programming under uncertainty. In Section 2.2 the proposed algorithm for the unified framework is outlined. Then, in Section 3 two case studies for the proposed unified framework are presented, a thermal power generation and distribution system and a turbo–boiler power co-generation system, along with a discussion of the results. Finally, in Section 4, concluding remarks are drawn.

## 2. Methodology

### 2.1. Mathematical preliminaries

#### 2.1.1. Multi-objective optimisation

Multi-objective optimisation aims to simultaneously optimise a number of objective functions, that often conflict with each other. Consider the general case of $MO^2$:

**$MO^2$:**

$$\begin{aligned} &\min_{\mathbf{x}} \boldsymbol{F}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_i(\mathbf{x})]^T \\ &\text{subject to:} \quad \mathbf{h}(\mathbf{x}) = 0 \\ &\qquad\qquad\quad \mathbf{g}(\mathbf{x}) \leqslant 0 \\ &\qquad\qquad\quad \mathbf{x} \in X \subseteq R^{n_x} \end{aligned} \tag{1}$$

where i is the number of objective functions, $\mathbf{h}(\mathbf{x})$ is the vector of equality constraints, $\mathbf{g}(\mathbf{x})$ is the vector of inequality constraints. Typically h corresponds to conservation equations, e.g., mass and energy balances while $\mathbf{g}(\mathbf{x})$ corresponds to specifications, e.g., product purity and maximum safe temperatures, pressures, etc. The decision variables, $\mathbf{x} \in X \subseteq R^{n_x}$, correspond to the optimal design and operating conditions such as generative capacity of a turbine and steam flowrate in combined heat and power generation systems. In such problems, the aim of optimisation is to find the solution that provides the decision maker with acceptable values of the objective functions.

A feasible decision vector that would decrease some objective functions without increasing at least another one is called Pareto optimal, $\mathbf{x}^*$, i.e.:

$$f_i(\mathbf{x}) = f_i(\mathbf{x}^*), \quad \forall i \in I$$

or, at least one $i \in I$ such that:

$$f_i(\mathbf{x}) > f_i(\mathbf{x}^*)$$

In the context of Pareto optimality, the minima are within the boundaries of the feasible region, or in the locus of the tangent points of the objective functions. For example, considering a bi-objective optimisation problem in which the DM wants to reduce cost, namely $f_1$ and simultaneously decrease the environmental impact of the process, namely $f_2$, the set of points defining the bold line in Fig. 1 is called Pareto front. Note that these two objectives are conflicting as a reduction in cost results in increase in the environmental impact and vice versa.

#### 2.1.2. Multi-parametric programming

A multi-parametric programming problem is of the following form [28–30]:

**mp-Programming:**

$$\begin{aligned} &z(\boldsymbol{\phi}) = \min_{x} f(\mathbf{x}, \phi) \\ &\text{subject to:} \quad \boldsymbol{h}(\mathbf{x}, \phi) = 0 \\ &\qquad\qquad\quad \mathbf{g}(\mathbf{x}, \phi) \leqslant 0 \\ &\qquad\qquad\quad \mathbf{x} \in X, \quad \phi \in \Phi \subseteq R^{n_\phi} \end{aligned} \tag{2}$$

Solving the system described in (2) results in a solution of the following general structure:

$$\mathbf{x}(\phi) = \begin{cases} x_1(\phi) & \text{if } \phi \in CR_1 \\ x_2(\phi) & \text{if } \phi \in CR_2 \\ \quad\vdots & \\ x_n(\phi) & \text{if } \phi \in CR_n \end{cases} \tag{3}$$

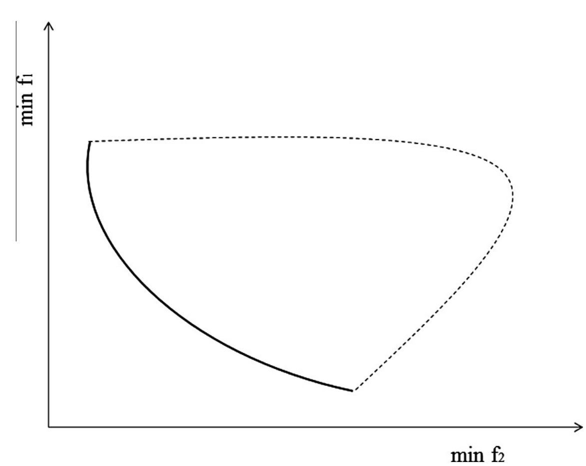


**Fig. 1.** Graphical representation of a bi-objective optimisation problem; the Pareto front is marked with the bold line and the dotted line indicates other feasible solutions of the problem.

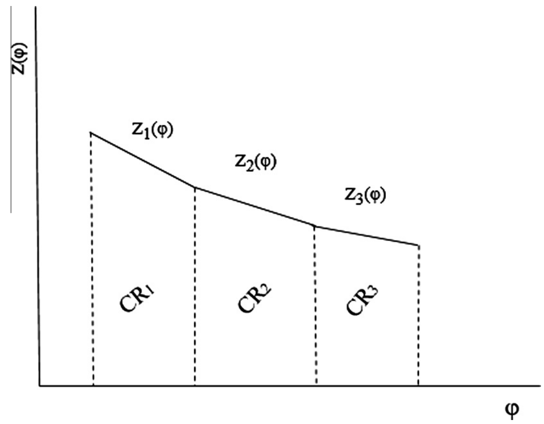


**Fig. 2.** Parametric profile of optimal objective value in different CRs.

where **x** is the vector of optimisation/decision variables and $\phi$ is the vector of (uncertain) parameters. Typically $\phi$ corresponds to uncertain parameters such as raw material quality and product demands [9].

The solution of a multi-parametric program provides the optimal vector of optimisation variables as explicit functions of the problem's parameters as well as a number of Critical Regions (CR) in which each solution is optimal. Graphically this is shown in Fig. 2 where 3 CRs have been computed and solution $z_1(\phi)$ is valid in $CR_1$, $z_2(\phi)$ is valid in $CR_2$ and $z_3(\phi)$ is valid in $CR_3$.

#### 2.1.3. Multi-objective optimisation using multi-parametric programming

$MO^2$ problems have been considered from a parametric programming approach by several authors for the case of linear cost functions [31,32]; while Ghafari-Hadigheh et al. [33] examined the case of quadratic cost functions but only for the case that the quadratic part remains constant. Papalexandri and Dimkou [34] reformulated multi-objective MINLP (Mixed Integer Nonlinear Programming) problems (of the form given in Eq. (1)), as parametric MINLP problem as follows:

***mp-MO$^2$***:

$$
\begin{aligned}
f_1^*(\psi) = \min_x \; & f_1(\mathbf{x}) \\
\text{subject to:} \quad & \boldsymbol{h}(\mathbf{x}) = 0 \\
& f_2(\mathbf{x}) \leqslant \psi_2 \\
& \vdots \\
& f_i(\mathbf{x}) \leqslant \psi_i \\
& \mathbf{g}(\mathbf{x}) \leqslant 0 \\
& \mathbf{x} \in X \subseteq R^{n_x} \\
& \psi_2 \in \left[\psi_2^{lo}, \psi_2^{up}\right], \ldots, \psi_{i-1} \in \left[\psi_i^{lo}, \psi_i^{up}\right]
\end{aligned}
\tag{4}
$$

where the identification of the lower and upper bounds of the parameters, $\psi$, results in a new optimisation problem for each of the scalar objective functions separately. Bemporad and Munoz de la Pena [35], proposed a multi-objective explicit model predictive control framework, where the Pareto optimal solution based on the weighted sum method was computed offline using multi-parametric programming. Recently, [36], presented an approximate algorithm with tunable suboptimality for the explicit calculation of the Pareto front of $MO^2$ problems with convex quadratic objective functions within the framework of $\epsilon$-constraint method. While an extensive research work has been reported for solving $MO^2$ problems using multi-parametric programming, to the best of our knowledge no previous research work has considered for $MO^2$ under uncertainty, within the $\epsilon$-constraint methodology, using multi-parametric programming.

### 2.2. Multi-objective optimisation under uncertainty

In this work, we consider linear multi-objective optimisation problems (of the form given in Eq. (4)) when they also involve uncertain parameters. This is achieved by augmenting the uncertain parameters ($\boldsymbol{\phi}$) with the parameters corresponding to the multiple objectives ($\boldsymbol{\psi}$), resulting in the following problem:

**mp-MO$^2$ under uncertainty (mp-**MO$^2$U$^2$**):**

$$
\begin{aligned}
z(\theta) = \min_x \; & f_1(\mathbf{x}, \theta) \\
& \boldsymbol{h}(\mathbf{x}, \theta) = 0 \\
& \mathbf{g}(\mathbf{x}, \theta) \leqslant 0 \\
& \mathbf{x} \in X \subseteq R^{n_x} \\
& \theta \in \Theta \subseteq R^{n_\theta}, \quad \Theta = \left[\phi_j^{lo}, \phi_j^{up}\right] \times \left[\psi_i^{lo}, \psi_i^{up}\right]
\end{aligned}
\tag{5}
$$

where I denotes the number of the multiple objective functions apart from the main one, i.e. $f_1$, that are involved in the $MO^2$ problem, **x** is the vector of the decision variables and $\boldsymbol{\theta}$ is the vector of the augmented parameters that refer to both the uncertainty ($\boldsymbol{\phi}$) and the multi-objective ($\boldsymbol{\psi}$) parameters for the scalar functions. Adopting this framework for $MO^2$ under uncertainty problems the solution is computed once, as a multi-parametric program. An outline of the proposed algorithm is given in Algorithm 1. The solution is given by $\mathbf{x}(\boldsymbol{\theta})$ i.e., the optimal decision variables as a function of uncertain parameters as well as multiple-objectives. Two case studies are presented in the next section to illustrate the key concepts and ideas.

**Algorithm 1.** mp-$MO^2$ under uncertainty

---

Step 1: Choose the main objective function of the $MO^2$, i.e. $f_1$ as it is shown in problem (4). Reformulate the $MO^2$ as an mp-$MO^2U^2$, i.e. Problem (5), by treating the scalar objective functions as inequality constraints with respect to the parameters, $\boldsymbol{\psi}$.
Step 2: Solve two optimisation problems for each of the rest of the scalar objective functions in order to compute the lower ($\psi^{lo}$) and upper ($\psi^{up}$) bounds of the parameters $\boldsymbol{\psi}$.
Step 3: Solve the resulting mp-$MO^2U^2$, compute the optimal values, $z_i$, as explicit functions of the mp-$MO^2U^2$ parameters, i.e $\boldsymbol{\theta}$ as shown in problem (5), along with the corresponding critical regions, $CR_i$.

---

## 3. Case studies

### 3.1. Thermal power generation and distribution under uncertainty

Consider the following power generation problem where four different types of power generation exist, namely, lignite fired,

**Table 1**
Raw material and power demand data.

| | Lignite | Oil | Natural gas | RES |
|---|---|---|---|---|
| Maximum production per year (GW h) | 31,000 | 15,000 | 22,000 | 10,000 |
| Cost of production (€/MW h) | 30 | 75 | 60 | 90 |
| $CO_2$ emission coefficient (t/MW h) | 1.44 | 0.72 | 0.45 | 0 |

oil fired, natural gas fired and units exploiting Renewable Energy Sources (RES). A similar version of this example has been examined in the literature by Mavrotas [37] and the data are shown in Table 1.

The yearly demand is 64,000 GW h and is characterised by a load duration curve which can be divided into three types of load: base load (60%), medium load (30%) and peak load (10%). The lignite fired units can be used only to cover base and medium load, the oil fired units for medium and peak load, the RES units for base and peak load and the natural gas fired units for all types of load. The endogenous sources are lignite and RES. We consider two objective functions: $f_1$, for the minimisation of production cost and $f_2$, for the minimisation of $CO_2$ emissions.

The mathematical formulation of this $MO^2$ problem is given as follows:

$$\min_{\mathbf{x}} f_1(\mathbf{x}) = 30x_1 + 75x_2 + 60x_3 + 90x_4 \quad (6)$$

$$\min_{\mathbf{x}} f_2(\mathbf{x}) = 1.44x_1 + 0.72x_2 + 0.45x_3 \quad (7)$$

$$\text{subject to}: \quad x_1 = x_{11} + x_{12} \quad (8)$$

$$x_2 = x_{22} + x_{23} \quad (9)$$

$$x_3 = x_{31} + x_{32} + x_{33} \quad (10)$$

$$x_4 = x_{41} + x_{43} \quad (11)$$

$$x_1 \leqslant 31{,}000 \quad (12)$$

$$x_2 \leqslant 15{,}000 \quad (13)$$

$$x_3 \leqslant 22{,}000 \quad (14)$$

$$x_4 \leqslant 10{,}000 \quad (15)$$

$$x_{11} + x_{31} + x_{41} \geqslant 38{,}400 \quad (16)$$

$$x_{12} + x_{22} + x_{32} \geqslant 19{,}200 \quad (17)$$

$$x_{23} + x_{33} + x_{43} \geqslant 6400 \quad (18)$$

The $MO^2$ problem is then reformulated as a multi-parametric problem:

$$\min_{\mathbf{x}} f_1(\mathbf{x}) = 30x_1 + 75x_2 + 60x_3 + 90x_4 \quad (19)$$

$$\text{subject to}: \quad x_1 = x_{11} + x_{12} \quad (20)$$

$$x_2 = x_{22} + x_{23} \quad (21)$$

$$x_3 = x_{31} + x_{32} + x_{33} \quad (22)$$

$$x_4 = x_{41} + x_{43} \quad (23)$$

$$x_1 \leqslant 31{,}000 \quad (24)$$

$$x_2 \leqslant 15{,}000 \quad (25)$$

$$x_3 \leqslant 22{,}000 \quad (26)$$

$$x_4 \leqslant 10{,}000 \quad (27)$$

$$x_{11} + x_{31} + x_{41} \geqslant 38{,}400 \quad (28)$$

$$x_{12} + x_{22} + x_{32} \geqslant 19{,}200 \quad (29)$$

$$x_{23} + x_{33} + x_{43} \geqslant 6400 \quad (30)$$

$$f_2(x) = 1.44x_1 + 0.72x_2 + 0.45x_3 \leqslant \theta_2 \quad (31)$$

$$\theta_2 \in [45{,}180, 82{,}620] \quad (32)$$

In addition to the original problem the existence of uncertainty in the capacity of lignite is considered, i.e. $\theta_1$ which can be expressed as $x_1 \leqslant 31{,}000 - \theta_1$. The mp-$MO^2U^2$ problem is therefore formulated as follows:

$$\min_{\mathbf{x}} f_1(\mathbf{x}) = 30x_1 + 75x_2 + 60x_3 + 90x_4 \quad (33)$$

$$\text{subject to}: \quad x_1 = x_{11} + x_{12} \quad (34)$$

$$x_2 = x_{22} + x_{23} \quad (35)$$

$$x_3 = x_{31} + x_{32} + x_{33} \quad (36)$$

$$x_4 = x_{41} + x_{43} \quad (37)$$

$$x_1 \leqslant 31{,}000 - \theta_1 \quad (38)$$

$$x_2 \leqslant 15{,}000 \quad (39)$$

$$x_3 \leqslant 22{,}000 \quad (40)$$

$$x_4 \leqslant 10{,}000 \quad (41)$$

$$x_{11} + x_{31} + x_{41} \geqslant 38{,}400 \quad (42)$$

$$x_{12} + x_{22} + x_{32} \geqslant 19{,}200 \quad (43)$$

$$x_{23} + x_{33} + x_{43} \geqslant 6400 \quad (44)$$

$$f_2(x) = 1.44x_1 + 0.72x_2 + 0.45x_3 \leqslant \theta_2 \quad (45)$$

$$\theta_1 \in [9000, 12{,}000], \quad \theta_2 \in [45{,}180, 82{,}620] \quad (46)$$

In the present problem, two parameters were considered: $\theta_1$ for the variations in the capacity of lignite and $\theta_2$ which is the parameter for the objective function for minimum $CO_2$ emissions. In order to compute the boundaries for $\theta_2$, two additional optimisation problems were solved for the second objective function of the problem with the same constraints. The problem was solved using multi-parametric programming on the space of $\theta_1$ and $\theta_2$, resulting in the optimal values as explicit functions of the parameters as well as the critical regions in which those values are valid. A discussion of the numerical results follows.

In Fig. 3, the evolution of minimum production cost is depicted with respect to $\theta_1$ and $\theta_2$ space. The minimum production cost as shown, is affected by the uncertainty in lignite capacity and by the environmental restrictions concerning the $CO_2$ emissions. Less use of lignite as a source of energy leads to less production cost and stricter environmental policies tend to decrease the production cost. The optimal values of the minimum production cost, $z_i(\theta)$, with the corresponding critical regions, $CR_i$, are as follow:

$$z_1(\theta) = -62.5\theta_2 + 6{,}678{,}750,$$

$$CR_1 = \begin{cases} -30\theta_1 - 41.667\theta_2 + 2{,}302{,}500 \geqslant 0 \\ \theta_2 \geqslant 45{,}180, \quad 9000 \leqslant \theta_1 \leqslant 12{,}000 \end{cases}$$

$$z_2(\theta) = 30\theta_1 - 20.833\theta_2 + 4{,}376{,}250,$$

$$CR_2 = \begin{cases} 30\theta_1 + 20.833\theta_2 - 1{,}361{,}250 \leqslant 0 \\ -30\theta_1 - 41.667\theta_2 + 2{,}302{,}500 \leqslant 0 \\ 9000 \leqslant \theta_1 \leqslant 12{,}000 \end{cases}$$

$$z_3(\theta) = 60\theta_1 + 3{,}015{,}000,$$

$$CR_3 = \begin{cases} 30\theta_1 + 20.833\theta_2 - 1{,}361{,}250 \geqslant 0 \\ \theta_2 \leqslant 82{,}620, \; 9000 \leqslant \theta_1 \leqslant 12{,}000 \end{cases}$$

Graphically the critical regions, in the parametric space of $\theta_1$ and $\theta_2$, are shown in Fig. 4:

As shown Figs. 3 and 4 and also from the parametric solutions, the optimal solutions, $z_2$, $z_3$, are more sensitive to $\theta_1$ (uncertainty in lignite capacity) in $CR_2$ and $CR_3$, respectively. In $CR_1, \theta_1$ has no impact as the minimum production cost is an explicit function only

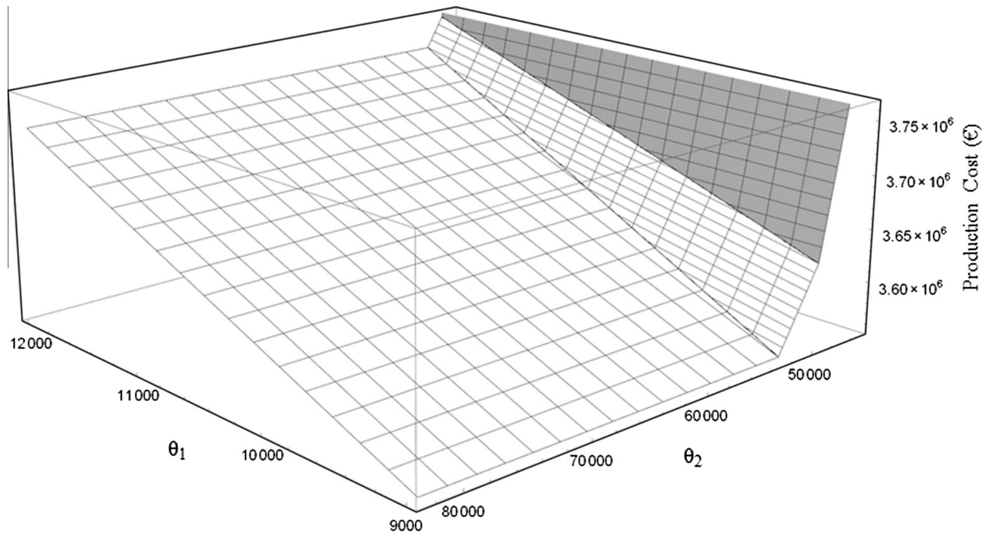


**Fig. 3.** Evolution of minimum production cost with respect to uncertainty in lignite capacity ($\theta_1$) and minimum $CO_2$ emissions ($\theta_2$).

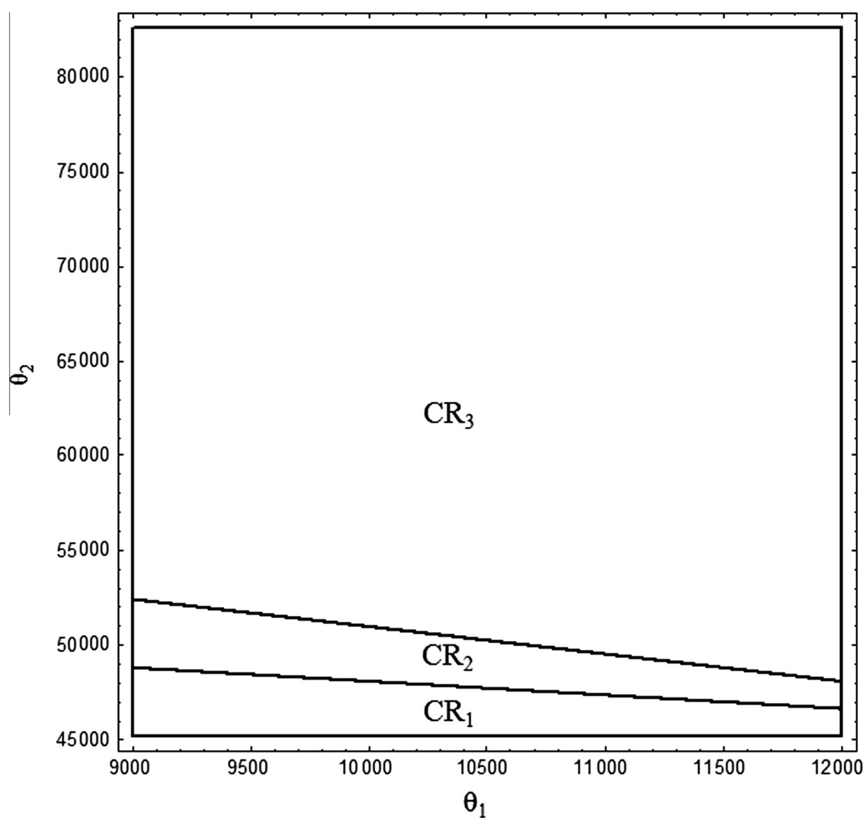


**Fig. 4.** Critical regions in the space of $\theta_1$ (uncertainty in lignite capacity) and $\theta_2$ (minimum $CO_2$ emissions).

of $\theta_2$ (minimum $CO_2$ emissions). Such results are very useful for the decision making for the optimal operation of power plants under uncertainty in the presence of more than one objectives to be optimised. Furthermore, by calculating the optimal explicit function of the main objective in the $MO^2U^2$ problem, the decision maker can systematically analyse cases as the one demonstrated in the present example where if the uncertainty is located in $CR_1$, then any variation in the demand has no impact on the profitability of the process. Explicit solution of the optimisation problem and the additional insight obtained through the explicit solution, are useful for fast and efficient decision making especially for the complex problem of $MO^2U^2$ that would otherwise require the employment of two different solution strategies.

### 3.2. Turbo–boiler power co-generation under uncertainty

In the second study a slightly modified version of the boiler/turbo-generator system from Edgar et al. [38] was examined, as shown in Fig. 5.

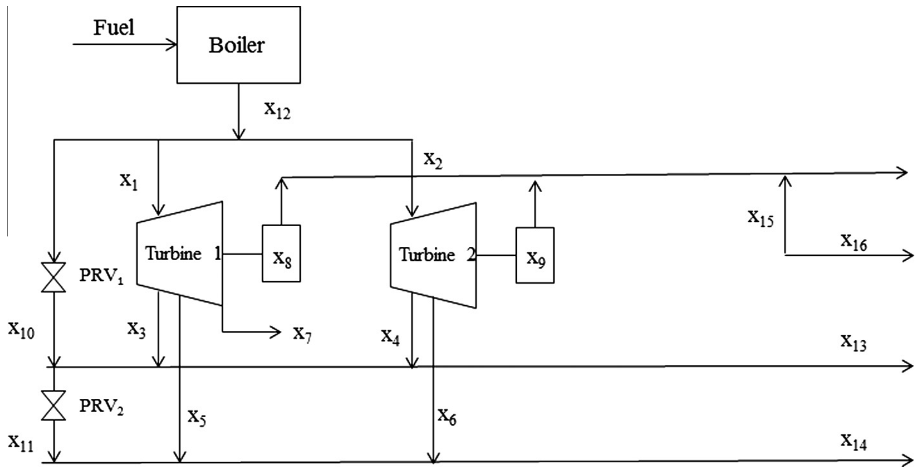


**Fig. 5.** Turbo–boiler power co-generation system.

**Table 2**
Turbo–boiler power system.

| Process variables | |
|---|---|
| $x_1$ | Inlet flowrate for turbine 1 [lbm/h] |
| $x_2$ | Inlet flowrate for turbine 2 [lbm/h] |
| $x_3$ | Exit flowrate from turbine 1 to 195 psi header [lbm/h] |
| $x_4$ | Exit flowrate from turbine 2 to 195 psi header [lbm/h] |
| $x_5$ | Exit flowrate from turbine 1 to 62 psi header [lbm/h] |
| $x_6$ | Exit flowrate from turbine 2 to 62 psi header [lbm/h] |
| $x_7$ | Condensate flow rate from turbine 1 [lbm/h] |
| $x_8$ | Power generated by turbine 1 [kW] |
| $x_9$ | Power generated by turbine 2 [kW] |
| $x_{10}$ | Bypass flow rate from 635 psi to 195 psi header [lbm/h] |
| $x_{11}$ | Bypass flow rate from 195 psi to 62 psi header [lbm/h] |
| $x_{12}$ | High pressure steam (635 psi) [lbm/h] |
| $x_{13}$ | Medium pressure steam (195 psi) [lbm/h] |
| $x_{14}$ | Low pressure steam (62 psi) [lbm/h] |
| $x_{15}$ | Purchased power [kW] |
| $x_{16}$ | Excess power [kW] |

The notation for the turbo–boiler co-generation system is shown in Table 2.

In the present version, except from the original objective function which stands for the minimisation of the hourly operational cost, a second objective function for minimisation is considered, namely $f_2$, which represents the environmental impact of the power generated by the system. Uncertainty in this example is considered in the power demand, i.e. $\theta_1$. Data about the process is given in Table 3 for the turbines, in Table 4 for the steam headers, in Table 5 for the energy and in Table 6 the demands on the system are listed.

The mathematical model of the system is given by (47)–(72) and the resulting problem is a Linear Programming (LP) problem. Uncertainty in the power demand, $\theta_1$, is considered to vary as: $-1000 \leqslant \theta_1 \leqslant 8000$ and for the second objective function, $\theta_2$, of the $MO^2$ LP, by solving a minimisation and a maximisation problem. The bounds of $\theta_2$ , are obtained as: $18607.95 \leqslant \theta_2 \leqslant 25913.309$. Treating $f_2$, instead of an equation as an inequality constraint set to be less than $\theta_2$ the corresponding $MO^2U^2$ problem is a mp-LP with two parameters namely, $\theta_1$ and $\theta_2$.

**Table 3**
Turbines data.

| Turbine 1 | | Turbine 2 | |
|---|---|---|---|
| Maximum generative capacity | 6250 kW | Maximum generative capacity | 9000 kW |
| Minimum load | 2500 kW | Minimum load | 3000 kW |
| Maximum inlet flow | 92,000 lbm/h | Maximum inlet flow | 244,000 lbm/h |
| Maximum condensate flow | 62,000 lbm/h | Maximum 62 psi exhaust | 142,000 lbm/h |
| Maximum internal flow | 132,000 lbm/h | High-pressure extraction at | 195 psig |
| High-pressure extraction at | 195 psig | Low-pressure extraction at | 62 psig |
| Low-pressure extraction at | 62 psig | | |

**Table 4**
Steam header data.

| Header | Pressure (psig) | Temperature (F) | Enthalpy (Btu/lbm) |
|---|---|---|---|
| High-pressure steam | 635 | 720 | 1359.8 |
| Medium-pressure steam | 195 | 130 superheat | 1267.8 |
| Low-pressure steam | 62 | 130 superheat | 1251.4 |
| Feedwater (condensate) | | | 193.0 |

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**Table 5**
Energy data for the turbo–boiler cogeneration energy system.

| | |
|---|---|
| Fuel cost | $1.68/10^6 Btu |
| Boiler efficiency | 0.75 |
| Steam cost (635 psi) | $0.002614/lbm |
| Purchased electric power | $0.0239kW h average |
| Demand penalty | $0.009825/kW h |
| Environmental impact penalty | $0.05/kW h |

**Table 6**
Demands of the turbo–boiler cogeneration system.

| Resource | Demand |
|---|---|
| Medium-pressure steam (195 psig) | 271,536 lbm/h |
| Low-pressure steam (62 psig) | 100,623 lbm/h |
| Electric power | 24,550 kW |

The problem includes 16 optimisation variables, 8 equality constraints, 35 inequality constraints and 2 objective functions.

The solution procedure is initialised by treating the problem's parameters as continuous variables varying between their upper and lower bounds in order to compute a feasible solution for a certain point in the parametric space. Then according to the basic sensitivity theorem and a redundancy test [39] the CRs and the explicit solutions of the mp-LP were computed. Because of the linear nature of the problem, the corresponding CRs are polyhedral. Despite the fact that the problem is linear, the methodology presented is generic.

$$z(\boldsymbol{\theta}) = \min_{\mathbf{x}} f_1(\mathbf{x}) = 0.00261x_{12} + 0.0239x_{15} + 0.00983x_{16} \quad (47)$$

subject to:

$$h_1(\mathbf{x}) = x_{12} - x_1 - x_2 - x_{10} \quad (48)$$
$$h_2(\mathbf{x}) = x_1 + x_2 - x_7 + x_{10} - x_{13} - x_{14} \quad (49)$$
$$h_3(\mathbf{x}) = x_1 - x_3 - x_5 - x_7 \quad (50)$$
$$h_4(\mathbf{x}) = x_2 - x_4 - x_6 \quad (51)$$
$$h_5(\mathbf{x}) = x_3 + x_4 + x_{10} - x_{11} - x_{13} \quad (52)$$
$$h_6(\mathbf{x}) = x_5 + x_6 + x_{11} - x_{14} \quad (53)$$
$$h_7(\mathbf{x}) = 1359.8x_1 - 1267.8x_3 - 1251.4x_5 - 192x_7 - 3413x_8 \quad (54)$$
$$h_8(\mathbf{x}) = 1359.8x_2 - 1267.8x_4 - 1251.4x_6 - 3413x_9 \quad (55)$$
$$x_8 \geqslant 2500 \quad (56)$$
$$x_8 \leqslant 6250 \quad (57)$$
$$x_3 \leqslant 192,000 \quad (58)$$
$$x_7 \leqslant 62,000 \quad (59)$$
$$x_1 \leqslant x_3 + 132,000 \quad (60)$$
$$x_9 \leqslant 9000 \quad (61)$$
$$x_9 \geqslant 3000 \quad (62)$$
$$x_2 \leqslant 244,000 \quad (63)$$
$$x_6 \leqslant 142,000 \quad (64)$$
$$x_{15} + x_{16} \geqslant 12,000 \quad (65)$$
$$x_{13} \geqslant 271,536 \quad (66)$$
$$x_{14} \geqslant 100,623 \quad (67)$$
$$x_8 + x_9 + x_{15} \geqslant 24,550 + \theta_1 \quad (68)$$
$$x_{13} \leqslant 350,000 \quad (69)$$
$$x_{14} \leqslant 150,000 \quad (70)$$
$$0.05x_{12} \leqslant \theta_2 \quad (71)$$
$$\theta_1 \in [-1000, 8000], \quad \theta_2 \in [18607.95, 25913.309] \quad (72)$$

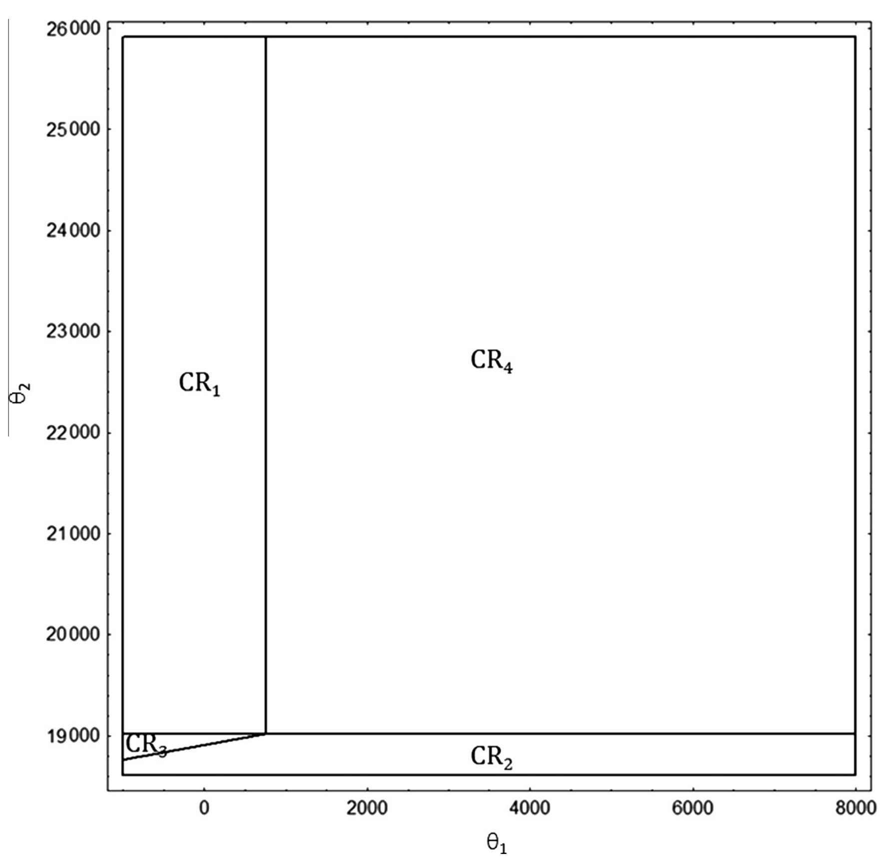


**Fig. 6.** Critical regions for the turbo–boiler cogeneration system.

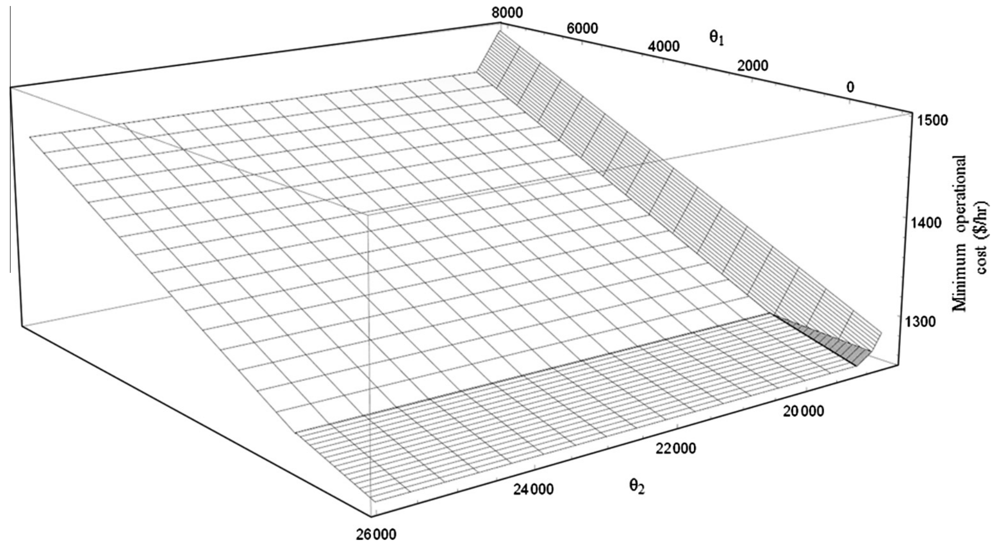


**Fig. 7.** 3D graph of the minimum hourly operational cost in the parametric space of $\theta_1$ (uncertainty in power demand) and $\theta_2$ (minimum environmental penalty).

The CRs of the case study are presented in Fig. 6 while the explicit optimal solutions for the mp-MO$^2$U$^2$ are given by Eqs. (77)–(80).

The mathematical expressions of the CRs are given by the conditional expressions (73)–(76).

$$\mathrm{CR}_1 = \begin{cases} -1000 \leqslant \theta_1 \leqslant 760 \\ 19016.45 \leqslant \theta_2 \leqslant 25913.309 \end{cases} \quad (73)$$

$$\mathrm{CR}_2 = \begin{cases} 18607.95 \leqslant \theta_2 \leqslant 19016.45 \\ 6.843436\theta_2 - \theta_1 - 129,377 \leqslant 0 \end{cases} \quad (74)$$

$$\mathrm{CR}_3 = \begin{cases} 18607.95 \leqslant \theta_2 \leqslant 19016.45 \\ 6.843436\theta_2 - \theta_1 - 129,377 \geqslant 0 \end{cases} \quad (75)$$

$$\mathrm{CR}_4 = \begin{cases} 760 \leqslant \theta_1 \leqslant 8000 \\ 19016.455 \leqslant \theta_2 \leqslant 25913.309 \end{cases} \quad (76)$$

$$z_1(\boldsymbol{\theta}) = 1269.7655 + 0.01407\theta_1 \quad \text{if} \quad [\theta_1, \theta_2] \in \mathrm{CR}_1 \quad (77)$$

$$z_2(\boldsymbol{\theta}) = 3378.9103 - 0.11135\theta_2 + 0.0239\theta_1 \quad \text{if} \quad [\theta_1, \theta_2] \in \mathrm{CR}_2 \quad (78)$$

$$z_3(\boldsymbol{\theta}) = 2107.135 + 0.014\theta_1 - 0.04408\theta_2 \quad \text{if} \quad [\theta_1, \theta_2] \in \mathrm{CR}_3 \quad (79)$$

$$z_4(\boldsymbol{\theta}) = 1261.29 + 0.0239\theta_1 \quad \text{if} \quad [\theta_1, \theta_2] \in \mathrm{CR}_4 \quad (80)$$

The explicit solutions of the mp-MO$^2$U$^2$ demonstrate that less strict environmental policies result in reduction of the optimal cost whereas more power demand tends to increase the hourly operational cost. This can also be envisaged in Fig. 7, where the optimum hourly operational cost varies in the parametric space, i.e. $\theta_1$ is the uncertainty in power demand and $\theta_2$ is the environmental penalty. It should be noted again that since the problem is of multi-objective nature the notion of optimality is referred to in the context of *Pareto optimality*.

## 4. Conclusions

Decision making in process and energy systems is becoming more complex than ever. This is driven by factors such as increase in demand for energy, stricter environmental restrictions and the requirement to maintain economic competitiveness in the market. The decision making problem therefore naturally becomes a multi-objective optimisation problem, where trade-offs between the conflicting objectives, e.g., cost and environmental impact, must be systematically evaluated. While this problem is complex enough, presence of uncertain parameters, such as fluctuations in the quality of the feedstock and market prices, renders decision making process even more complicated. Traditionally and except for a few papers, issues pertaining to multiple objectives and problems arising due to the presence of uncertainty, have been dealt with separately. A few papers that deal with multi-objective optimisation and uncertainty simultaneously, usually employ an optimisation technique to address multi-objective aspects and another technique to address the uncertainty aspects. There is a clear lack of an optimisation framework which considers the formulation and solution of multi-objective optimisation under uncertainty for energy systems in an integrated manner. The key novelty and contribution of the work presented in this work is that multiple objectives are reformulated as constraints bounded by varying parameters and then these parameters are augmented with the uncertain parameters, resulting in a unified modelling and optimisation framework. The optimisation problem thus obtained is formulated and solved as a multi-parametric program – an optimisation technique that provides the optimal solution as a complete map of the parameters without exhaustively enumerating the entire space of the parameters. The key advantage of this approach is that the decision maker can systematically analyse the interactions between multiple-objectives and uncertain parameters, and examine their relative sensitivities to the main objective function. Such a unified modelling and optimisation tool and the information obtained by solving multi-objective optimisation problems using this tool, are invaluable for the decision making process. This has been demonstrated through two case studies on power generation. Future work will focus on application of the proposed methodology on other problem areas including energy conversion and storage, sustainable cities and climate change mitigation. A systematic analysis of multiple objectives and uncertain parameters is also expected to help energy policy makers in shaping future of the energy road-map.

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