Tragedy of the Spectrum Commons — a myth or reality?

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Abstract

The tragedy of the spectrum commons is a topical subject in licence-exempt wireless communication. The UK government has recently asked Ofcom to open up bandwidths for short-term spectrum access for research and development initiatives to increase the efficiency of spectrum utilization.The collection and analysis of data to assess congestion as a function of pricing, investment in research and development, manufacturing incentives and the distribution of devices is not within the boundaries of this present study because it is very difficult and costly. For this reason companies, such as *ABIresearch*, have emerged to provide such data collection services, which include taking devices apart and pricing each component (called tear downs). In view of these obstacles, we present a mathematical model whose arguments stem from well-known market trends. We build a model from a game theoretic point of view of the the tragedy of the commons that represents the unmanaged wireless communications environment. The primary purpose of this investigation is to determine what forces are likely to lead to or prevent a tragedy in the utilization of licence-exempt spectrum.

Background

Garret Hardin, a prominent biologist introduced the expression "Tragedy of the Commons" to describe the inevitable overuse of a free but finite shared resource (Hardin, 1968). The solutions he proposed were privatization and government intervention to prevent the irreversible depletion of non-renewable resources, such as fish populations or forests, by extracting or by introducing (i.e. pollution) too much of something. Hardin's thesis was a strict contradiction to Adam Smith's assumption of an invisible hand (Hardin, 1998). Smith argued that a selfish individual "neither intends to promote the public interest, nor knows how much he is promoting it"yet as a result of each individual's revenue maximizing behavior, each "directs that industry in a manner as its produce might be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention." (Smith, 1776: 4.2) Hardin had conversely concluded that "Ruin is the destination toward which all men rush, each pursuing his own best interest in a society that believes in the freedom of the commons. Freedom in a commons brings ruin to all."(Hardin, 1968: 1244).

Elinor Ostrom and those who followed in her footsteps showed that in fact Hardin's thesis was limited to cases of open access (rather than to managed commons) that assumed little or no communication, in an environment where some individuals were never allowed to take joint benefits into account. (Ostrom, 1990; Ostrom, Gardner, and Walker, 1994; Gibson, Andersson, Ostrom and Shivakumar, 2005) These discussions revolve largely around non-renewable scarce natural resources which are bound only by location. What about renewable resources where exploitation is bound by space and also time? Wireless spectrum is a renewable resource that if overused today is still available tomorrow, but unlike fish or pasture, unused Wi-Fi access today cannot be used tomorrow. This form of tragedy is momentary, or is it?

The ancient Greek historian Thucydides in the 5th century BC describes tragedy as a process that emerges from a collective action problem. Archidamos, the King of Sparta argues that the vote of the Peloponnesian confederacy (*ksympantas*) to go to war is a result of the private interests of individual states (*heneka ton idion*, 1.82.6). Thucydides described the problem of collective action in terms of voters who are unable to communicate with one another in order to devise mechanisms to manage their common interest (1.3.4, in Trojan war; 8.66.3, in Athens' stasis, 8.1 tragedy). In a confederacy states individually pursue their private interests since they lack the ability to communicate and coordinate with one another, and thus they destroy the common interest. Thucydides adds that this is the result of a fundamental ignorance with regard to the cause of the collective's weakening (1.3.4; 141, if states, the cause is the physical distance from one another; 8.66.3, if citizens, because it is "impossible for everyone to know everyone else", 8.1 or everyone's physical distance from the object of judgment).

"Each thinks that their inertia [private-interest] will do no harm, and that it is someone else's responsibility rather than theirs to make some provision for the future: the result is that with all individually sharing this same notion they fail as a body to see their common interest going to ruin." (1.141, trans. M. Hammond)

Each state in the Peloponnesian confederacy has equal voting power. A vote represents a state's investment of resources and commitment to the common interest, much like a manufacturer's investment in new technology when there is still demand for his inferior technology. Assume an individual state voting in the common interest today comes at a cost c to its private interest. Nonetheless, when a state votes in its private interest there exists a collective invisible cost to the common interest, which is divided among the states. Over a repeated number of meetings to vote, the common interest becomes less valuable every time a state votes in its private interest.

Assume the value of the confederacy's common interest is v and that equal voting power is translated as an equal division of the common interest among the states (1.120.1, 1.141). I now use common and public interchangeably. Following the description, Thucydides first assumes there are two states. According to Thucydides, each state *individually* believes that it is some *other* state's responsibility to use its vote in the public interest.[1] When one state votes in its private interest, its payoff is $v/2$, if the other state votes in the public interest. Then the benefit to the other state when it votes in the public interest is $(v/2) - c$. Even if the value of the common interest is shared $(v/2)$, a state which votes in the public interest will always incur a cost c to itself. If both states vote in the public interest, they both get $(v/2) - c$, whereas if they both vote in their private interest they ruin the common interest, which we value as 0. This 2x2 game describes the free-rider problem (Ober (2008), (2009), (2010)), where one player piggy-backs on the efforts of another. In this case a state obtains the benefits of the confederacy without contributing to its provision. The payoff matrix below summarizes the problem.

$$\text{Free-rider} = \begin{array}{c} \\ \textit{Public} \\ \textit{Private} \end{array} \begin{array}{cc} \textit{Public} & \textit{Private} \\ \left(\begin{array}{c} (v/2) - c, (v/2) - c \\ (v/2)^*, (v/2) - c^* \end{array} \right. & \left. \begin{array}{c} (v/2) - c^*, (v/2)^* \\ 0, 0 \end{array} \right) \end{array}$$

[1]The notation follows that set out in Maynard-Smith (1982). Two animals are contesting a resource of value v. In biology, v stands for "value" that translates as "the Darwinian fitness of an individual obtaining the resource would be increased by v". The resource in this case can be territory for favorable breeding. v is the gain in fitness to the winner, or equivalently, the additional offspring from mating there rather than elsewhere. Among an infinite population, the contest between two players is assumed to be determined by a pairing off at random. With respect to Thucydides, this is analogous to conceptualizing what *one* player thinks some *other* player should be doing. In this way players cannot condition their play on whether they are "player 1" or "player 2". This is a game which is symmetric in strategies and payoffs. (Gintis 2009)

There are two equilibrium solutions: (Private, Public) and (Public, Private). That is to say that each state considers that another state will vote sometimes in the public interest and sometimes in its private interest. This strategic environment has an evolutionary stable strategy because $(v/2) > c$, such that it is worth risking one's private interests to ensure the common interest. (Maynard-Smith, Price, 1973; Maynard-Smith, 1982: 15, 181-2.) Although this may be the game that the states believe to be in, the reality is that both are always choosing to vote independently in their private interest. [2] A state is unable to observe another state acting in the public interest or even what concerns the public interest "due to infrequent meetings", and so independently assumes that the other state knows it is acting in its private interest. When all the states share the same notion, over time (i.e. a series of votes) the invisible cost to the common interest forces v to decrease in value. Each state's share of the common interest $(v/2)$ must always be greater than the cost to its private interest c. Otherwise there is no incentive to choose to vote in the public interest. If $(v/2) < c$,[3] the problem has dynamically evolved into a Prisoner's Dilemma.

$$\text{Prisoner's Dilemma} = \begin{array}{c} \\ \textit{Public} \\ \textit{Private} \end{array} \begin{array}{c} \begin{array}{cc} \textit{Public} & \textit{Private} \end{array} \\ \begin{pmatrix} (v/2)-c, (v/2)-c & (v/2)-c, (v/2)^{*} \\ (v/2)^{*}, (v/2)-c & 0^{*}, 0^{*} \end{pmatrix} \end{array}$$

The equilibrium of this interaction is (Private, Private), such that states do not benefit from voting in the common interest, no matter what. The case above is for a population of two states, where one state considers how another state actually behaves. Thucydides takes it one step further and generalizes the two state interaction to the case of many states. [4]

[2] Thucydides describes interaction comprehensively, as a player's version of the interaction, in contrast to the actual interaction. The mismatch between the modeling and the real is repeatedly discussed in the application of game theory to literature. Rapoport (1960) 238, in Shakespeare's *Othello*, "if [Othello] believes Desdemona, he may as well believe her version of the game ... and decide which game is in fact being played."; Melhmann (2000) 77, in Goethe's *Faust*, Mephisto "realizes that his view of the game was false".

[3] In the case $(v/2) \leq c$, then there are three equilibria and this environment describes the gradual shift toward defection: before defection there is indifference.

[4] By a *constructio ad sensum*, "all" becomes "each" (1.141.6.4-6) and to reiterate the point he proceeds with the inverse meaning, where "each" becomes "all" (1.141.7.5-7). In between these statements, there are two *gnomai* (i.e. assumptions): 1. The states vote at a great common cost for private revenge and also vote to minimize private cost 2. Meetings are infrequent whereby they spend little time on the common interest and for

The multiplayer case where "all individually share this same notion" is called a tragedy of the (unmanaged) commons.

Thucydides saw the ruin of interstate relations as dependent upon the intangible value of the unmanaged "common interest". [5] The matrix formulations above are the simplest representation of an evolutionary tragedy of the commons.

This interpretation of tragedy is founded upon the notion that the cost of one's behavior is not factored into a benefit at which a choice can be valued. In the case of licence-exempt wireless spectrum, the user will benefit at the moment of choice and simultaneously incur an invisible cost. In agreement with Smith, Coase argued that "the value of what is obtained as well as the value of what is sacrificed to obtain it" are externalities that are internalized by property rights and bargaining (Coase 1960:2, esp. 1959; Herzel 1951). In the case of spectrum, Aftab has shown that even if a token economy is introduced, which essentially allocates the resource to those who value it most, "high-priority" packets have access "at the expense of the low-value packets" (Aftab, 2002). The core problem in the allocation of spectrum is pricing: how does the market, and especially users, value spectrum? (Hazlett, 2001) McAfee and Miller proposed a way of valuing the free renewable resource of spectrum. Through an analogy of picnic tables at a park, they concluded that unrestrained free access leads to a more efficient use of the spectrum resource (McAfee and Miller, 2012). However this holds primarily for cases where transportation costs to the park are low, which implies that the user's valuation of the

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the most part carry on with their own business. For a different interpretation of the voting free-rider problem as a n-person Prisoner's Dilemma see Brams (2011) 111-126, Brams, Kilgour (2009). See Axelrod (1984) for the most famous experimental study of the repeated prisoner's dilemma, but "Axelrod's theoretical results are not robust". Axelrod's process is problematic because he does not allow mutation and also any limit point is a Nash Equilibrium. Nonetheless, the theory of the "evolutionary stable strategy cannot be straightforwardly applied to infinitely repeated games", (Osborne, 2004: 439-441). Kim (1994) reviews the literature and the extensions of the PD as random pairwise matching among a population.

[5] Thuc. 2.60 for citizens and the state; Note that Plato discusses the collective action problem with respect to citizen voters and the incentive of sanctions to obey the law. This differs from Thucydides because their is no law code to regulate interstate relations (*Rep.*2.360b-c). See generally Morrison (1994) on the topos of comparing cities and individuals in Thucydides, and specifically that all are equal under Athenian law and this is called a model (*paradeigma*, 2.37). Also see Aristotle's *Politics* 1261b for the point on the assumption that someone else will act as if they could observe your actions. Modern authors who discuss the problem Smith (1776) 4.2; Hume (1739-40) 3.2.8 for a very similar description to Thucydides', who also considers the incentive of two people and then generalizes for many people. There exists a problem in moving from individual to group motivation which I do not discuss here, see also Arist.*Pol.*1.1.; Mill (1848) 5.11.12; Pareto (1935) 3.1496 pp.946-7.

spectrum is in most cases low: "providing unlicensed spectrum is advantageous over licensing when there is a large chance of very low value use." In other words, a user shows a willingness to pay for the resource, but would not be greatly damaged if he did not have access. Would this be the case for users in executive meetings on tablets, in a stampeding crowd at a concert, in a terrorist attack at an airport, or even watching Usain Bolt win an Olympic Gold on live broadband television? All these cases are places and times at which the resource has the highest intrinsic value and access is lowest. These are the instances that require focus.[6]

Licence-exempt wireless spectrum is an open access resource. It is consumed by users who are unable to communicate with each other and are unable to devise joint benefits (like car pooling to reduce congestion) because of a fundamental ignorance regarding the costs incurred by device density. The culpability for interference in wireless networks is usually assigned to device manufacturers, i.e. technology. Technology in point of fact has been improving at phenomenal rates to keep up with the demand for the resource, and devices are built to squeeze as much out of the available band as possible.[7]

Manufacturers, however, are interested in their individual revenue maximization, and in turn flood the market with as many devices as possible (Ting et al. 2004). Since interference is dependent upon the density of devices, the resource becomes steadily less accessible. This is of particular interest to wireless providers who rely on technological innovation to remain competitive. It is then plausible to first model the tragedy as density dependent, then assess the relationship of technology to density dependent interference, and finally to build a model using this spring board. We begin with an elementary model that looks at revenue maximizing firms in competition with each other to show the market mechanism that leads to overuse (Frank, 1998; Gibbons, 1990). Demand for the resource is increasing, thus firms supply as many devices as they are able to produce to buyers. The model demonstrates that the tragedy

[6]Rifkin (2000), in "The Age of Access" he argues for the increasing scarcity of human attention (i.e. time) in comparison to location as a result of "the shift in the structuring of human relationships from ownership to access". The model below is concerned primarily with the spectrum utilization of small-cell infrastructures that provide access to a myriad of special purpose systems. These general purpose digital devices, such as commercial cell phones, provide users with greater spectral capacity and also government with public safety. (CSMAC 2008) Our study does not concern those devices whose spectral band usage is underused.

[7]There exists"the spectrum challenge posed by the rapidly increasing demand for mobile broadband capacity. The need for additional spectrum to meet the anticipated increase in demand for wireless and mobile data has been widely recognized internationally", Ofcom report 29 March 2012"Securing long term benefits from scarce spectrum resources"; Real Wireless report on techniques for increasing the capacity of wireless broadband networks: urlhttp://www.ofcom.org.uk/static/uhf/real-wireless-report.pdf.

is a consequence of each seller's self-interest. We then incorporate a model from evolutionary biology (Rankin, 2007) to identify other variables that could influence density. This model of the spectrum commons describes the dynamics of social evolution. It describes the tension between conflict and cooperation over time (Frank, 1998).

The Model

We assume there is a finite number N of manufacturers selling devices in a market, or likewise selling Wi-Fi products to users. The i^{th} firm sells a_i number of devices, such that $A = a_1, a_2, ..., a_n$. Therefore, the total number of devices sold to users is $\alpha = \sum_{i=1}^{n} a_i$. The term α represents an environment that is already saturated and therefore whose network value is at its highest. The user benefit of each device is determined by a function $v(\alpha)$ that depends on the total number of devices at a set location and time (e.g. the area of a stadium during a football match). This new function v represents interference and the value of access. On the one hand, the function represents the signal-to-interference ratio - as device density increases, interference increases. Conversely, the fewer devices, the less interference. On the other hand, the function also represents the intrinsic value of access at a specific location and time. This is a standard definition for the value of spectrum. "An analysis based on a single parameter such as bit/sec/hertz will lead to erroneous conclusions ... Simply stated the objective of any spectrum policy should be to maximize the utility and benefit of the radio spectrum which is a broader objective than just technical efficiency." (CSMAC 2008) Both density and intrinsic value, the CSMAC's utility and benefit, respectively, are assumed here to represent user benefit. An arbitrary signal-to-interference ratio level is set where user benefit is considered to be 0, since there is no access. This limit is determined by two assumptions:

$$\alpha_{\max} : v(\alpha) > 0 \text{ for } \alpha < \alpha_{\max} \tag{1}$$

The resource benefits the users as long as the number of devices is less than $\alpha_{\max}$. If α should exceed $\alpha_{\max}$, then the user has no access to the resource and thus no benefit. The function is discontinuous from this point onward. The benefit of each device as a function of all the devices in use is monotonically decreasing as a result of interference (concave to the origin).

$$v(\alpha) = 0 \text{ for } \alpha \geq \alpha_{\max} \tag{2}$$

The strategy of each manufacturer i is to maximize his revenue by selling $a_i \in [0, A_{\max}]$.

The revenue of manufacturer i from selling a_i devices, when the number of devices sold by other manufacturers is $a_1, ..., a_{i-1}, a_{i+1}, ..., a_n$, is:

$$\arg\max a_i\ v(a_1, ..., a_{i-1}, a_i, a_{i+1}, ..., a_n) \tag{3}$$

If all manufacturers are assumed to act in their own self-interest, then all are behaving symmetrically. No manufacturer has an incentive to deviate from his chosen action, and the equilibrium is $a^* = (a_1^*, ..., a_n^*)$ as long as the user is still benefiting from the device. Thus, each a_i maximizes the revenue function such that $v(a_i + \alpha_{-i}^*) + a_i v'(a_i + \alpha_{-i}^*) = 0$ and if we add all N firms together $\alpha^* = (a_i^* + \alpha_{-i}^*)$ simplified is:

$$v(\alpha^*) + \frac{\alpha^*}{N} v'(\alpha^*) = 0 \tag{4}$$

We now compare this Nash equilibrium solution to that of the social optimum, i.e. where all manufacturers maximize jointly α rather than individually a_i.

$$\arg\max \underset{0 \leq \alpha < \alpha_{max}}{\alpha}\ v(\alpha) \tag{5}$$

Solving for α^{**} we find $v(\alpha^{**}) + \alpha^{**} v'(\alpha^{**}) = 0$ which represents the equitable division of sales among manufacturers at the congestion limit of the resource. What we need to compare is the total quantity in the market and thus α's. Holding all other terms equal we must verify that α^* is less or equal to α^{**}, according to Smith's assumption of the invisible hand. Contra Smith, these terms show how selfish behavior leads to a depletion of the resource; when each of the manufacturers maximizes individually, the joint density of devices exceeds α_{max}.

$$\frac{\alpha^*}{N} = \alpha^{**} \text{ and therefore } \alpha^* > \alpha^{**} \tag{6}$$

This formally expresses the result that both private and public benefits cease once the tragedy is reached. On the war between Athens and Syracuse, which Athens lost as a result of its collectively inefficient decision making due to the distance from the object under scrutiny, Thucydides concludes: " The burden of loss lay on each privately and the city as a whole" (8.1)

Although this appears to be conclusive, and that doom is imminent, historically another variable has allowed device numbers to go up while keeping the resource accessible to users, perhaps prolonging the inevitable demise of licence-exempt spectral bands. Some in fact do believe in the force of competition to stimulate innovation (Shepard, 2002; Reed, 2002; Benkler, 2003; Werbach, 2004), while others believe the commons is doomed to tragedy if left unregulated (Kwerel and Williams, 2002; Hazlett, 2001).

It is important to note that the spectrum commons is not entirely free of regulation. It abides by technical etiquettes built into the devices called politeness rules and polite protocol. These impose power limits, communication protocols e.g. listen-before-talk, or specify a maximum level of interference i.e. a minimum signal-to-interference tolerance.[8] Technological advancements such as opportunistic spectrum utilization in cognitive radio, intelligent beamforming, multiple intelligence interaction, in addition to the likely self-enforcing principle of Moore's Law, allow inferior devices to survive competition. The newer technologies both increase processing power per chip and make spectrum sharing more efficient.

Both sides of the argument have valid points and need to be represented in the model. The function v depends not only on the density of devices α, but also on different types of technological innovation that determine a threshold value. The wireless environment is fundamentally characterized by different types of interacting spectrum technologies that are constantly increasing in efficiency (this includes sharing).

It is important to consider time and space as limits. Nowak and May (1993) model the spatial evolutionary n-person prisoner's dilemma so that individuals play with their neighbors. If we keep population numbers fixed, we can observe over time how a variable threshold of interference resistance (y-axis) copes with increasing numbers of interference resistant technologies with comparatively low thresholds (x-axis). Spatial effects were shown to promote the survival of cooperation by means of a frequency dependent selection. Here (Figure 1), computer modeling was used to simulate a very simple interaction between interference resistance and different device technologies. The players are devices that are placed in a two-dimensional spatial array of 50x50 where each individual has 8 neighbors (up, down, left, right plus those on the diagonals). There is a preferential rule which determines whether a device with access to spectrum (some value above the signal-to-interference threshold - required or self-imposed) will experience interference. Inferior technology devices are red, while superior technology devices

[8] "Where possible, any spectrum released in the future for licence-exempt devices should be used based on the spectrum commons model, wherein devices for multiple applications share the same frequencies, subject to politeness rules [5] and polite protocols [6]. While spectrum commons will be the default approach, exclusive licence-exempt use of spectrum by a specific application will be considered on a case-by-case basis where technical constraints, international obligations, or safety issues require such use."[5]"Politeness rules are constraints on radiated power profiles as functions of frequency, time, and space (e.g. spectral density, duty cycle, and beamwidth)"[6]"Polite protocols are interference avoidance mechanisms implemented at the physical layer (PHY) and/or medium access control (MAC) layer of the radio protocol stack, that enable multiple autonomous devices to share the radio resource. An example is the listen-before-talk protocol of Wi-Fi." Ofcom, Office of Communications (2007) 7.

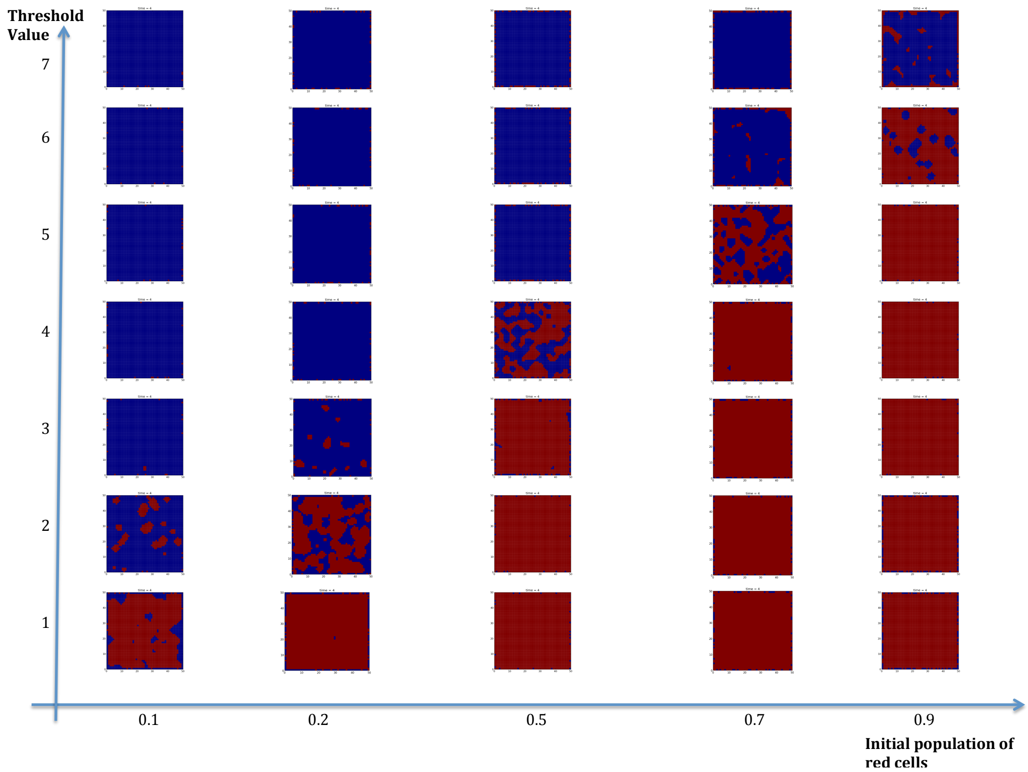

Figure 1: Phase Diagram

are blue. A blue device will remain blue depending on how many of his neighbors are of an inferior technology and have themselves already lost reception. A blue device surrounded by some threshold number of red devices will turn red, signaling it has lost access. This congestion model also has the inverse effect on red devices. If a red device is surrounded by some threshold number of blue devices, the red device turns blue, signaling it has regained access. This aspect helps to model the effect of spectrum sharing among blue devices. (The simulation is based on Thomas Schelling's segregation model (1971),[9] but instead of replacing a red for a blue, we

[9]The Phase Diagram used here is taken from the course material of "Quantitative Methods" (2013) a first year core module in interdisciplinary mathematics taught by Dr. Hannah Fry (UCL, Mathematics) for the Department of Arts and Sciences, who both coded the model based on Schelling (1971) adding the element of contagion, and designed the diagram. Whilst concluding the present research project on the spectrum commons, I both helped to teach plenary lectures and was involved in developing course material, which inevitably incorporated evolutionary spatial dynamics, the tragedy of the commons and of course the repeated Prisoner's Dilemma.

have a contagion effect. His tolerance threshold is our interference threshold.)

The devices or the individuals carrying devices do not move about. People find themselves in a certain state at which point devices begin to cause each other interference. The limit is reached when the thresholds are at a stable state. The percentage of the population that will be red is determined at the initial state. Each map in the Phase Diagram iterates over five generations and shows how the threshold value and the stable state of the population of red devices are apparently linearly related. Initial states with larger numbers of inferior devices devolve into tragedies, whereas smaller numbers of inferior devices require lower threshold values in order for access to stabilize. A conclusion that rings of common sense.

An important aspect of this simulation is that the location of a red or blue device in the initial state is random, which represents that the tragedy of the spectrum commons occurs without notice. From the examples given at the beginning of the paper we noted that we wish to study points of congestion which are of the greatest value to the user. In the case of terrorist attacks, the element of surprise is the strategy, which is similar to unpredictable catastrophes (natural or human) as well. In the case of broadband access or music concerts, telecommunication device manufacturers, and wireless service providers in general know, with high probability, that users will experience congestion. Access should be a required provision, as it was for users in the Olympic park, but not at the concert in Germany. Researchers have found a solution to excited/panicked crowds in small areas. They are developing smart phone applications that notify users where congestion points are forming, to redirect and thus to relieve exit congestion at highly populated events. But how will users have access to the apps, if they have neither cellphone reception nor wireless access?

The question of why a manufacturer would invest in competition surfaces: is it altruism? is it reciprocal altruism? or is it that cooperators tend to survive? The population dynamics of bacteria in Rankin (2007) can help to conceptually grasp the process of evolution when it is determined by a variety of variables that also affect population growth. The evolutionary spatial model of device production is an analogical model, as opposed to other production functions deduced from experimental evidence, like the Cobb-Douglas (1928), which does not imply any analogy. (Gilbert, 2008: 4-5; see Ostrom, Gardner, Walker (1994): passim, esp.114ff.)

Since it is difficult to capture live interacting technology at specific times, we can best measure the impact of technological innovation on the benefit to the user v through each firm's investment r in research and development, or more generally as investment in competition. [10]

[10]FCC regulation is particularly explicit regarding it's belief in the ability of licence-exemption to create incentives that drive competition and innovation. US Electronic Code of Federal Regulation. Title 47 "Telecom-

$$\frac{r_i}{\bar{r}} \tag{7}$$

Devices that are more interference robust are more costly to manufacture, such that "robustness is a choice variable". (Ting et al. 2004) Manufacturer i's investment level is denoted r_i. The average investment level is denoted $\bar{r}$. This fraction allows us to represent successful advances in technology as a result of quantifiable investment. If $\bar{r} = r_i$ the expression equals 1, which mathematically expresses the idea that there is no benefit to the user from investment in technology. If $\bar{r} < r_i$, then the manufacturer invests more than average and by assumption has a breakthrough in technology. (Frank, 2005, 2008) If however $\bar{r} > r_i$, then the manufacturer invests less than average and there is no breakthrough in technology. Both the first and last assumptions characterize a less competitive manufacturer. It is assumed that these manufacturers lose market share to manufacturers with better devices (Rankin, 2006: 174),[11] making more of the bandwidth available.

This term requires some refinement. Not all device manufacturers compete with one another. Some manufacturers specialize in ISM bands (voice and control wireless communication) whereas other manufacturers specialize in Wi-Fi devices and wireless data transfers. In the case of intraspecific competition rather than interspecific, fairly accurate knowledge of one's own market is assumed to be accessible to manufacturers of particular devices (Lotka-Votlerra model for interspecific versus intraspecific competition). We do not model this interaction in the r terms above, and therefore the statements herein are directed at a manufacturer who is part of a market that produces a similar range of devices. The basic algorithm suggests that each manufacturer must outperform those manufacturers within his product market in order to survive. Manufacturers of inferior technology devices compete against one another, regardless of those competing with superior technology. A manufacturer who produces a mixture of technologies low and high should then do the same and then compare his results against aggregate average investment in technology.

munication", Part 8 "Preserving the Open Internet", Section 8.1 "Purpose": "The purpose of this part is to preserve the Internet as an open platform enabling consumer choice, freedom of expression, end-user control, competition, and the freedom to innovate without permission." This part addresses broadband internet access services that exclude "dial-up internet access", and include fixed wireless services, satellite services and mobile stations. See Ofcom, Spectrum Sharing Consultation (9 Aug. 2013) "The future role of spectrum sharing for mobile and wireless data services" p.4, 36ff, and https://www.gov.uk/government/publications/information-economy-strategy.

[11] "If a mutant individual invests slightly more in competition, they will produce more offspring than other individuals in the population."

$$v(A, r_i, \bar{r}) = \left(\frac{r_i}{\bar{r}}\right)^{f(\alpha)} \tag{8}$$

The number of devices in the market is assumed to be increasing at a rate defined by a function $f(\alpha)$, where $f(\alpha)$ is always greater than 1. We assign the magnitude of the relative benefit of a manufacturer's investment in technology $f(\alpha)$ as dependent upon the density of devices α. Contra Hardin, this ratio models the growth of a manufacturer's output with respect to the density of the other manufacturers' outputs. Every manufacturer also has a fitness, or some level of interference resistant technology. This environment represents the density-dependent intraspecific competition among device manufacturers, and by extension, among the devices themselves.

$$f(\alpha) = \left(\frac{p\alpha}{\alpha + \gamma}\right) \tag{9}$$

The parameter γ describes how the benefits from competition v depend on the intensity of density dependence. It is a spatial parameter that represents the local interference effect, simulating how many devices are competing for the same spectrum. If $\left(\frac{r_i}{\bar{r}}\right) > 1$, then $v > 1$. Thus, v continues to increase over time as a result of $\gamma > 1$ which represents that this manufacturer's efficient technology is pushing out inferior technologies [12] and simultaneously decreasing the population of inefficient devices. If instead $\left(\frac{r_i}{\bar{r}}\right) < 1$, then $v < 1$. Thus, v continues to decrease over time because $\gamma = 0$ and this manufacturer's technology is inefficient and is not competitive, likely flooding the market with more of his own devices. If $\gamma = 0$ the benefit function is density independent. The latter case describes a growing population with poor competition.

The growth of the population of devices is linked to increasing demand for devices as a result of falling prices,[13] such that $p \geq 0$ scales the benefits gained from investing in technology. As technology tends to improve, the prices of devices tend to fall. Innovation, outmodes devices and reduces manufacturing costs, allowing manufacturers to offer cheaper and better devices. This implies that $f(\alpha)$ depends also on falling prices p for the end user. [14] Therefore p is

[12] Thuc. 1.71.3 "It is necessary, as is the case in any craft (techne) for innovation always to win out".

[13] For a tangible example of this trend, in the USA the average revenue per minute for wireless mobile telephone services was 0.44 dollars in 1993 and 0.07 dollars in 2005. Data taken from the FCC, Commercial Mobile Radio Services (CMRS) Competition Report (11th Annual), Division presentation.

[14] An important side note regarding the parameters p and γ, the former is a magnitude and thus a fixed

the change in price trends of devices, which allows us to associate access/user benefit to price $f(\alpha, p)$. The expression $p\alpha$ allows us to assume that any change below or above 1 increases benefit gained from technological innovation, such that for $p < 0$ there is no benefit. A price increase is modeled as any value $p \in [0, 1)$ and a price decrease as $p \geq 1$. This is because the relative benefit from investing in competition $f(\alpha)$ must increase as population density increases, since more competition leads to fewer devices in operation. Alternatively, as the number of devices increase, v necessarily decreases.

If α is increasing over time, and v remains constant, it can be assumed that this manufacturer's efficient technology (or increased sharing) is pushing out inferior technologies and simultaneously decreasing the population of inefficient devices, and so we define a mortality rate $m(\alpha)$. Mortality represents average device replacement. For example, smart phones are replaced on average every 18 months and laptops every 3-4 years. So that as α increases mortality rises, and as α decreases mortality falls. In the case mortality falls, the assumption is that the technology of some of the manufacturers' is inefficient and not competitive, likely flooding the market with more of his own devices. This is the case that describes a growing population with poor competition.

In the same way that technologies die off, they are also born or rather developed. The birth rate b is a constant which expresses that technologies appear regardless of competition and represents historical demand. There is also a cost in investing in competition, such that the more one invests in technology the risk of that technology not yielding enough profit increases. The function $h(r) = r^\beta$ is a simple function that represents an accelerating or decelerating risk, if $\beta \geq 1$ or $\beta < 1$, respectively. Putting all these elements together, we have a model that represents the average per manufacturer benefits and costs of investing in competition dependent on the density of devices in the licence-exempt spectrum market: his fitness.

$$v(A, r_i, \bar{r})b - m(\alpha) - h(r) \tag{10}$$

We can then model the growth rate of α, given one manufacturer's investment choice.

$$\dot{\alpha} = v(A, r_i, \bar{r})b\alpha - m(\alpha)\alpha - h(r)\alpha \tag{11}$$

By way of an example, in a pair-wise invasibility plot we can model investment in competition r_i on the y-axis and average investment in competition $\bar{r}$ on the x-axis (Rankin, 2007:

measurable value which is not dependent on the distance from the epicenter of interference. The latter is a value of intensity in which the responses from the local populace are taken into consideration when calculating the intensity of the the signal.

176). This corresponds to the simple agent-based model simulation: the more investment the higher the threshold value (interference resistance), and the more devices have low interference resistance the more interference (the less blue devices survive). We can now see clearly that the relationship is a straight line and that it is up to the other parameters to determine the regions of growth and decay and ultimately the ESS.

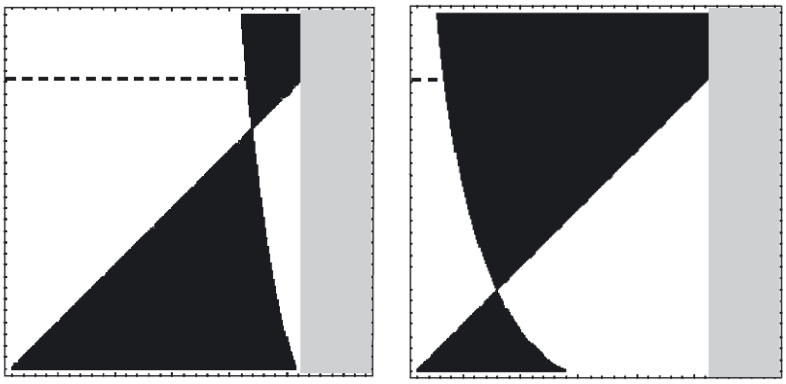

Figure 2: y-axis r_i, x-axis $\bar{r}$, Figures 2a and 2b

White areas represent that manufacturer i increases his production, while black represents manufacturer i decreases his production. The ESS is the point at which the black and white areas meet. First assume a manufacturer chooses an above average investment in technology $\left(\frac{r_i}{\bar{r}}\right) > 1$ in an environment with high population density (and with $m(\alpha) = 1$).Then assume there is price stagnation $p = 1$ and accelerating risk $\beta = 1.5$. If there is low intensity of density-dependence of the benefits from competition $\gamma = 0.01$ and the population average strategy $\bar{r}$ is high, the ESS is close to the extinction threshold. (Figure 2a) Any higher investment by manufacturer i above the dashed bar would lead all other devices to extinction, since the average investment would have to rise in order to compete (i.e. no one would be able to compete leading to exits and bankruptcy). Rankin posits that as a result the i^{th} individual would also go extinct. (This need not be the case if we think in cycles.) Conversely, if there should be a high intensity of density-dependence of the benefits $\gamma = 100$, a lower average investment in competition $\bar{r}$ pushes the i^{th} manufacturer's equilibrium investment strategy further away from the extinction threshold. (Figure 2b)

What we are trying to interpret is not whether technology will improve but how fast must the market push for technological innovation as a result of the increasing demand for wireless devices. An idea stated but not modeled by McAfee and Miller "Unlicenced bands are preferable in the presence of innovation". An interesting next step would be to see whether their valuation insights could be combined with density dependent intraspecific competition. They themselves admit that "It is not clear whether innovations in telecommunications hardware occur at a fast enough rate to affect values in the manner described in the paper." (McAfee, Miller, 2012: 351-352) In our model innovation is a variable not an assumption.

There are four scenarios in the research on spectrum that need evaluation. Devices under the licence-exempt regime can either be limited by an institution or can be set by manufacturers who are maximizing profits. A manufacturer's investment in competition can likewise be chosen by himself or required by a regulator. A graphical analysis of these four environments may lead to interesting conclusions on how parameters and variables interact. An interesting case study would be to see the evolution of the digital communication standard from 900MHz CT1 and CT2 to DECT (Digital Enhanced Cordless Communication), where DECT almost universally replaced the former standard in most countries except for North America on account of regulation. Then from DECT to Wi-Fi, where DECT was replaced in terms of data while still maintaining double the coverage of Wi-Fi. The replacement of one technology (DECT) for the other (Wi-Fi) is costly. Both DECT and Wi-Fi in turn compete with 3G and 4G on voice and data.

As the literature attests, the market mechanism alone does not have a strong enough invisible hand to ensure the efficient allocation of the open access resource, because manufacturers will sell as many inefficient devices as they can to users, if the average investment strategy is too low (or too high). The oft proposed solution is to change demand by either raising prices or increasing the allocation of licence-exempt spectrum. An alternative, however, would be for regulators to also inform users that it is in their best interest to actively demand that device manufacturers demonstrate awareness of the spectrum commons. This would force the average investment strategy to rise, and push away from the threshold (or close the gap if the average was already high). Consequently, through market pressure manufacturers would strive to sell products with greater robustness and better methods of sharing. This not only sees improvements in device robustness, but also an investment in geographic sharing in the form of MAS (multi-antenna signal processing). Users, now aware of the cause, can put pressure on product manufacturers to be user conscious and thereby increasing competition, i.e. help γ and $h(r)$ remain high. This is equivalent to manufacturers forming what Thomas Schelling called a "self-restraining coalition" and what Steven Brams called "a public outcry" (Schelling, 1978: 214, 223-4; Brams, 2011: 119ft.7), in order to turn "permission" to innovate into an "obligation" to innovate (Ostrom, et al., 1994: 38). Marketing campaigns should chide the industry for their general negligence and demand that industry communicate to users their initiatives (i.e. their investment strategies to find technological solutions) to combat the tragedy of the spectrum commons. This should be part of every firm's corporate social responsibility. On the other hand, alternatively, regulators can require a minimum level of robustness. Institutional management can set a threshold level r_i. The latter regulatory solution, in contrast to the former public awareness solution, requires a stupendous amount of statistical analysis and a number of predictive assumptions, whereas the former is a solution that would allow the market to self-regulate freely.

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