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THE DYNAMICS OF CAR SALES:
A DISCRETE CHOICE APPROACH

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ABSTRACT

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The Dynamics of Car Sales: A Discrete Choice Approach *

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May 3, 2006

Abstract

This paper studies the joint dynamics of aggregate car sales, prices and income. We analyze these series using a dynamic discrete choice model which is consistent with microeconomic evidence on the infrequency of durable purchases. We estimate the parameters of this choice problem at the household level. Through aggregation we show that the model can reproduce the dynamics of demand captured by an ARMA model, as in Mankiw (1982), and the joint dynamics summarized through a VAR representation of car sales, income and prices. We find that most of the variation in car sales is due to shocks which influence the replacement probability rather than the cross sectional distribution of car vintages.

1 Introduction

This paper focuses on understanding the behavior of durable consumption expenditures. Spending on durables is an important component of aggregate spending and one that fluctuates considerably over the business cycle. At the household level, spending on durables, such as cars, is lumpy: purchases are relatively infrequent and large. Modeling the time series behavior of durable purchases, such as cars, in a framework consistent with household evidence remains an open challenge.

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From the aggregate perspective, Mankiw (1982) presents evidence that the permanent income hypothesis (PIH) model of durable expenditures is inconsistent with observed data. In particular, he argues that in a single agent choice problem in which utility is a quadratic function of the stock of durables, the optimal choice of the agent implies that expenditures on durables will follow an ARMA(1,1) process, where the MA component is parameterized by the rate of durable goods depreciation. Mankiw estimates an ARMA(1,1) time series representation of quarterly durable goods expenditures for the US. In contrast to the predictions of the theory, he finds that durable goods expenditures follow an AR(1) process. Put differently, Mankiw estimates the rate of depreciation of durable goods to be 100%. We call this finding the "Mankiw puzzle".

From the household's perspective, Lam (1991) reports that households only occasionally adjust their stock of durables. Consistent with this finding, Bar-Ilan and Blinder (1988,1992), Bertola and Caballero (1990) and Caballero (1990,1993) view aggregate observations on durable purchases as the outcome of the aggregation over heterogeneous microeconomic agents. Taken together, these papers certainly suggest that a model of heterogeneity and discrete adjustment can qualitatively match relevant parts of the data. [1]

This paper studies the determinants of the time series representation of durable expenditures in an explicit dynamic, discrete choice framework: can a dynamic discrete choice representation of household durable purchases produce the observed time series behavior of durable expenditures? We address this question by looking specifically at two distinct features of spending on an important component of consumer durables, aggregate car sales. First, we confirm the ARMA(1,1) representations that underlie the "Mankiw puzzle" for our various measures of automobile sales. Second, we estimate and study a VAR representation of automobile sales, prices and income. Here we find that the impulse response function displays dampened oscillations in response to an innovation in income. So, besides confronting the Mankiw puzzle for car sales, we ask whether an aggregated discrete choice model can match and explain this rich time series response to an income shock.

This paper builds upon the framework of Adda and Cooper (2000a) who investigate the

[1] However, there is no characterization of the time series properties of durable purchases offered in these papers and thus the "Mankiw puzzle" remains open. For example, the final section of Caballero (1993), entitled "ARMA Representation and Impulse Responses" displays impulse response functions for Cars and Furniture and states that "The shapes are broadly consistent with the description given in the paper." Whether or not the estimated model can produce an ARMA representation close to that reported by Mankiw is not specifically addressed.

In a related, independent study, Attanasio (2000) estimates (S,s) rules for automobile purchases using microeconomic data. After estimating the model, he undertakes an evaluation of the aggregate time series implications of the model, as we do in this paper. He finds that if there is more persistence in the shocks to the target relative to the persistence in the shocks to the band, then the model is able to match observed aggregate behavior.

effects of scrapping subsidies on car purchases. An important difference between this paper and the existing literature is that the empirical implications are drawn directly from the dynamic optimization problem without imposing any structure directly on agents' decision rules. In particular, while our model of durable replacement is of the optimal stopping variety, we do not specify (S,s) bands directly nor do we find it necessary to specify a "desired" stock of durables in our estimation. We do this for two reasons. First, we find that empirically the PIH assumptions which underlies this "desired stock" approach are not supported by the data.[2] Second, deriving the optimal durable expenditure policy from a dynamic optimization framework and then using this same structure for estimation is more consistent theoretically.[3]

We find that the aggregate model based upon the dynamic optimization of heterogeneous microeconomic units can "explain" both the AR and MA parts of Mankiw's regression results. Further, a comparison of the impulse response functions generated by the models with that obtained through an unrestricted VAR reveals why a PIH model has difficulty matching the data. Suppose that there is an income shock. In the data, the initial burst of sales is followed by a reduction in sales and then dampened oscillations (relative to the initial level). It is precisely these endogenous fluctuations in sales that the estimated PIH model misses. It is captured in our model by the interaction of a state dependent hazard function and the evolution of the cross sectional distribution of car vintages.

We then use our structure to uncover the sources of these dynamics. In general, the dynamic discrete choice structure generates variations in aggregate sales from two sources: fluctuations in the replacement probability and the evolution of the cross sectional distribution of car vintages. We find that most of the variation in car sales is due to shocks which influence the probability of replacement. Put differently, the endogenous evolution of the cross sectional distribution contributes surprisingly little to the time series variation of car sales.[4]

[2] In Adda-Cooper (2000b) we argue that the finding, reported in Caballero (1990), that an ARMA(1,q) representation of durable expenditures reconciles the evidence and the PIH model is not robust across samples and the choice of q.

[3] These two points are, of course, related. Since we do not have convincing evidence that the PIH prediction holds even in the "long-run", linking the estimation to a target seems unwarranted. Our approach to estimation through a characterization of the complete household dynamic optimization overcomes this problem as we do not require the specification of a target.

[4] This is similar to the findings reported in Cooper, Haltiwanger and Power (1999) which studied the implications of machine replacement for aggregate investment. This finding is not inconsistent with the emphasis placed on the movement of the cross sectional distribution in Adda and Cooper (2000a) since in that exercise there was a policy action that had a significant effect on the cross sectional distribution of vintages.

2 Evidence on Aggregate Car Purchases

This section presents evidence on the behavior of aggregate car purchases. We first show the raw data on sales for the US and France and also the cross sectional distribution of cars by age over the sample period.

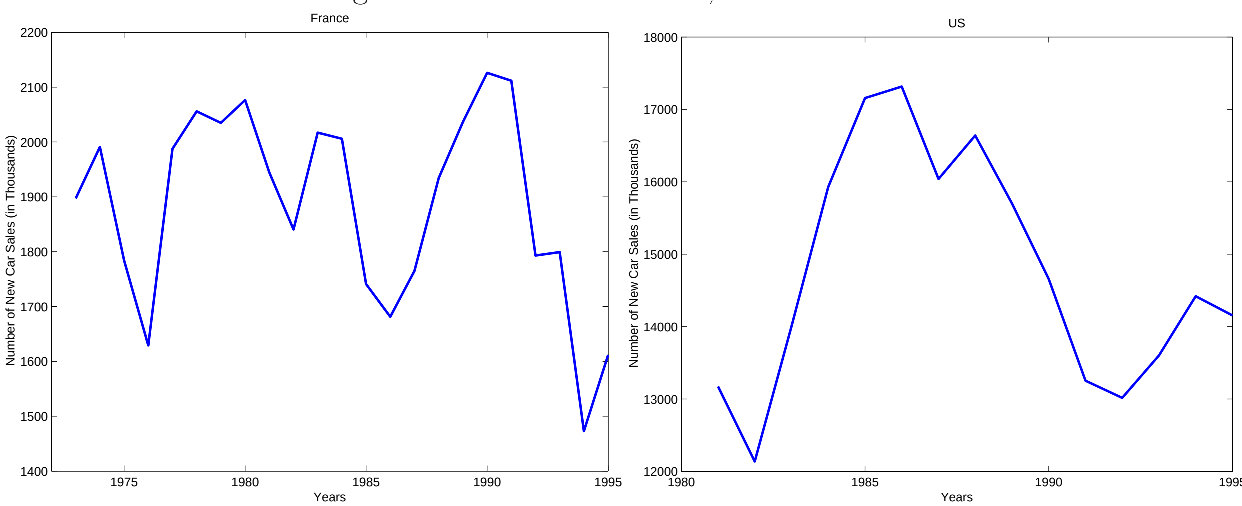
We then extend the ARMA(1,1) representation stressed by Mankiw in three ways. First our sample period is longer. Second we study both the US and France. Third, we focus on both total durables and cars.

Finally, we present the impulse response functions (IRFs) from a VAR on car purchases, income and prices. We use these IRFs to illustrate why the ARMA(1,1) representation is inadequate. These IRFs are also used to evaluate the time series implications of our estimated model.

2.1 Facts: Car Sales and the Cross Sectional Distribution

The time series data for car sales in the US and France are displayed in Figure 1. The data are annual sales of new cars (measured as registrations of new cars) which is consistent with the focus of the model on the extensive choice of either purchasing a new car or retaining an existing one. As with other durable goods, these series display considerable volatility.

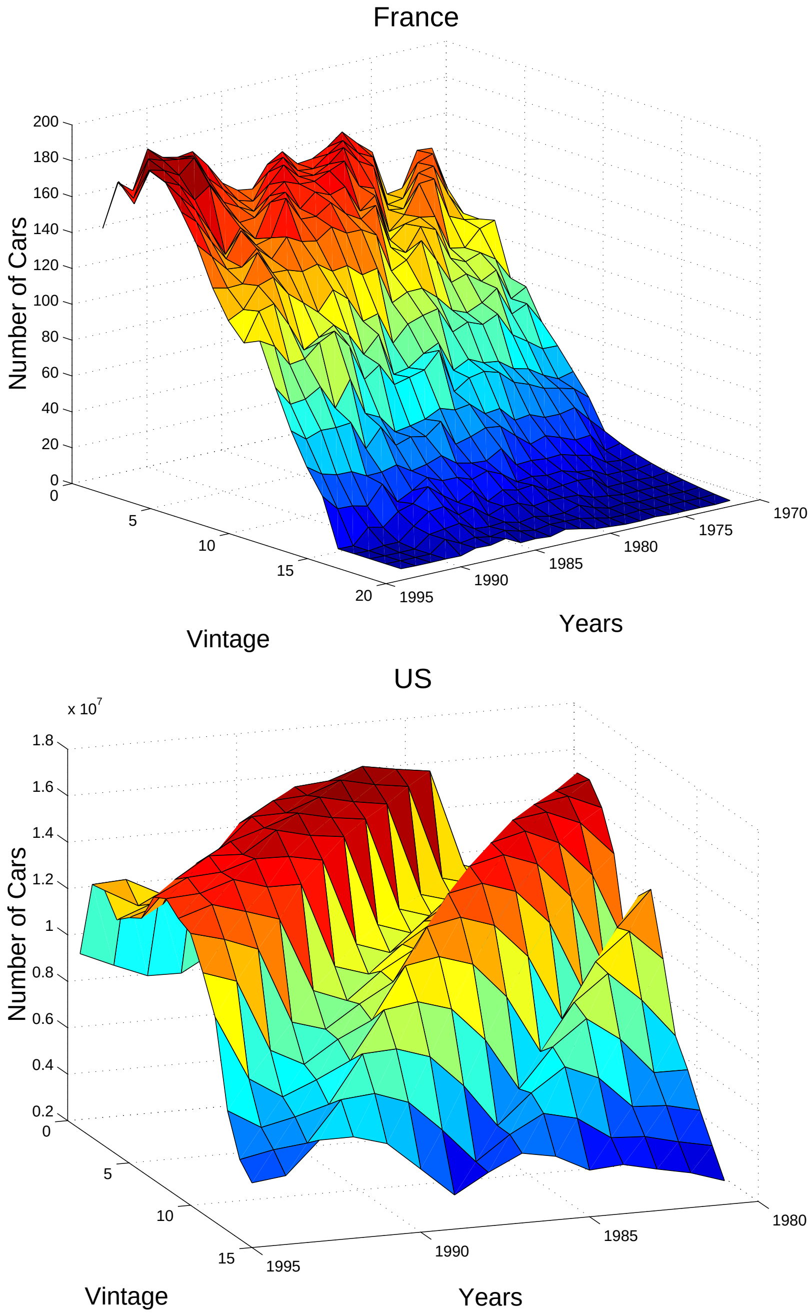
Figure 1: Sales of New Cars, France and US



For our estimation, these data are supplemented by information on the cross sectional distribution of car vintages, in Figure 2. These show the fraction of cars by age for each year of the sample.[5]

[5]For the US data on the cross sectional distribution comes from issues of Ward's Automotive. For France, this information comes from the CCFA. The French data stops in 1995 to exclude the scrapping subsidies

Figure 2: CDF of Cars by Vintage and Year, France and US



There are a couple of points to make clear about the behavior of these cross sectional distribution functions (CDFs). First, the new car sales appear each year along the top ridge of the figures. As time passes, these cars age and appear as older cars in subsequent CDFs. This explains the "ripples" that appear over time in these figures. As time passes, some cars are scrapped and others are destroyed in wrecks etc. This is seen in the figure by following the evolution of cars as they age over time.

studied in Adda and Cooper (2000a).

Using these CDFs, one can also visualize echo effects. Suppose that in some period, such as 1989 in France, there is a burst of sales. In subsequent years, as this cars age, there is a bulge in the CDF. Eventually, these cars reach the optimal (state dependent) scrapping age and there is a new burst of sales. Of course, this process is tempered by the endogenous and exogenous scrapping that occurs at earlier ages.

In our estimation, the CDFs come into the analysis in two ways. First, we match moments from the CDF in the estimation of parameters. Second, variations in the CDF plan a role in explaining time series variation in sales. Part of our analysis is to decompose sales into variations induced by the CDF and those induced by variations in prices and income.

2.2 Time Series Representations

In this subsection, we explore the dynamics of car purchases in French and US data. We use two tools. The first is an ARMA(1,1) representation which extends the analysis of Mankiw. Second, we consider a VAR in sales, income and prices to broaden the characterization of the data. After the estimation of the model, we revisit these characterizations of the time series.

2.2.1 ARMA(1,1) Representation

Following Hall (1978), Mankiw (1982) extended the permanent income hypothesis model to account for durability. In this model, the agent only consumes a durable good and faces an uncertain income. If the utility function is quadratic, then expenditures on durables e_t by the representative household follow:

$$e_{t+1} = \delta\alpha_0 + \alpha_1 e_t + \varepsilon_{t+1} - (1-\delta)\varepsilon_t. \tag{1}$$

where δ is the depreciation rate of the stock of durables and ε_t is the innovation to income. Using aggregate quarterly US data on durables, Mankiw (1982) shows that the series are better described by an AR(1) process than an ARMA(1,1).

Working with annual series for France and the US, we report very similar results in Table 1. The rows pertain to both aggregated durable expenditures and estimates based on cars. For the latter, we have data on both total expenditures on cars (for France) and new car registrations. The columns refer to estimates without removing a trend and then with the removal of a linear trend.

For both countries, the estimated rate of depreciation is quite high. Clearly the hypothesis that the rate of depreciation is close to 100% per year would not be rejected for most of the specifications. Further, the results are robust to the detrending method (we have also tried exponential trends and obtained very similar results). For France, the data exhibit a trend so

Table 1: ARMA(1,1) Estimates on US and French Data

Specification	No trend		Linear trend Removed	
	α_1	δ	α_1	δ
US durable expenditures	1.00(.03)	1.5 (.15)	0.76 (0.12)	1.42 (0.17)
US car registration	0.36(.29)	1.34 (.30)	0.33 (0.30)	1.35(0.31)
France durable expenditures	0.98 (0.04)	1.20 (0.2)	0.56 (0.24)	1.2 (0.36)
France car expenditures	0.97(0.06)	1.3 (0.2)	0.49 (0.28)	1.20 (0.32)
France car registrations	0.85 (0.13)	1.00 (0.26)	0.41 (0.4)	1.20 (0.41)

Notes: Annual data. For the US, source FRED database, 1959-1997. French data: source INSEE, 1970-1997. US registration: 1968-1995.

we get different values for the AR coefficient.[6] Thus Mankiw's "puzzle" seems to be robust across categories of durables, countries, time periods and the method of detrending. That is, under the null hypothesis of the PIH model, the estimated rate of depreciation is quite high.[7]

2.2.2 Impulse Response Functions

A second and more general way of representing durable expenditures is through a VAR. While focusing on ARMA representations, much of the literature on durable expenditures has ignored the joint dynamics of durables, income and prices over time. Here we present results using a VAR composed of automobile sales, automobile prices relative to the CPI and income. While this representation has no structural interpretation, it provides a better characterization of the dynamics of sales.

The impulse responses for new car registrations, prices and income, both for France and the US are reported in Figure 3. The variables were ordered as income, prices and sales. With this order, innovations to income are exogenous, prices respond to both price and income innovations and sales respond to innovations in all three variables. This ordering was imposed on actual data as well as the simulated data from the model, described later. At this point, we do not provide a structural interpretation of these impulse responses. This interpretation is provided below through our model of car purchases.

[6] Contrary to the argument in Bar-Ilan and Blinder (1988,1992), sales and expenditures have very similar time series properties. This may reflect the fact that variations in expenditures are largely a consequence of the extensive margin (to buy a car or not) rather than the intensive margin (how much to spend on a given car). Leahy and Zeira (2005) contains a model in which income variations alone lead solely to changes in decisions on the extensive margin.

[7] Adda-Cooper (2000b) provided additional discussion of richer representations of the data. This included the specification in Bernanke (1985) which includes convex adjustment costs for the stock of durables and price variations as well as the specification in Caballero (1990) who considers an ARMA(1,q) models. We focus on the ARMA(1,1) representation both because of its prominence in the literature and, as described in Adda-Cooper (2000b), its robustness.

The first two graphs displays the response of sales to an orthogonal income shock.[8] In both countries, after an initial increase, the sales are characterized by dampened oscillation around the baseline. These oscillations could arise for two reasons. First, as emphasized in the literature on non-convex adjustment costs with heterogenous agents, the endogenous evolution of the stock of cars can potentially produce replacement cycles and thus oscillations in sales.[9] A second explanation is that income and prices are serially correlated and have some cross dynamics. Indeed, from the impulse-response above, prices and income also oscillate around the baseline following a shock on price or on income (figure not shown). We return to an evaluation of the relative importance of these two sources of dynamics later.

The response of sales to price differs across the two countries. For the US, the price innovation leads to a reduction in sales. The dynamic response through oscillations in sales then appears. For France, the response to a price innovation is that sales increase. We return to a discussion of this finding below.

2.2.3 Can the ARMA model match the IRFs?

Figure 4 shows the response of sales from the estimated ARMA model for the US and France.[10] As shown in Figure 3, the actual impulse response functions from the estimated VAR exhibit oscillations in sales following an income shock. The ARMA model is incapable of matching this feature of the data. It is instructive to understand the differences in these responses.

Using an ARMA(1,1), the impact of a shock on sales is $\rho^{t-1}(\rho-\alpha)$, t periods after the shock, where ρ is the AR coefficient and α is the MA one. First, as ρ is positive in the estimation, there is no way the ARMA(1,1) can reproduce the oscillations in the impulse response function. Second, in order to match the patterns of response over time, $(\rho-\alpha)$ has to be positive. As ρ is estimated between 0.4 and 0.8 on aggregate data (see Table 1), this means that the implied "depreciation rate" δ cannot be lower than 0.2 to 0.6. We see here an important point about the ARMA(1,1) model: it is structurally unable to deliver a "depreciation rate" low enough to be credible. We see this as the origin of the "Mankiw puzzle".

[8] Error bands are included as well. The impulse response functions for France are tightly estimated. Those for the US are not because parameters of the lagged price, sales, income are not very precisely estimated. The series is shorter than for France.

[9] Discussions of these oscillations, in different theoretical environments, appear in the Cooper, Haltiwanger and Power (1999) study of investment dynamics and in the work on durables in Adda and Cooper (2000a) and Leahy and Zeira (2005). Stacchetti and Stolyarov (2006) emphasize the role of product innovation.

[10] Here, as in the Mankiw (1982) model, the impulse in the ARMA is viewed as an income shock.

3 Dynamic Discrete Choice Model

Using these facts as motivation, we now turn to our model which we ultimately use to understand these observations. The next subsection discusses the theoretical specification of the discrete choice model at the household level. We then turn to aggregation of the model.

3.1 Household Behavior

The starting point for this analysis is a variant of the dynamic programming problem explored in Adda and Cooper (2000a).[11] Consider an agent with a car of age $i = 0, 1, ...$ in state (z, Z) where z is a vector of household specific taste shocks and $Z \equiv (p, Y, \varepsilon)$ is a vector of aggregate state variables. As above, p is the relative price of the durable good, Y represents aggregate income and ε is an aggregate taste shock. Throughout, $i = 1$ denotes a new car and $i = 0$ refers to a household with no car.

In state (z, Z), the household decides whether to retain a car of age i or scrap it. If the household decides to scrap the car, then it receives the scrap value of π and has the option to purchase a new car. The new car is available immediately. If the household retains the car, then it receives the flow of services from that car and cannot, by assumption, purchase another car within the period. Thus the household is constrained to own at most a single car. Each of these choices is influenced by a choice specific realization of an *iid* shock z_j for the choices $j = k, r$.[12] In addition, we allow for a constant utility gain, α_k, from the option of keeping the car.

For tractability, we **initially** place a number of restrictions on this household optimization problem. First, there is no second-hand market: cars are either kept or scrapped. Second, the household is forced to finance the durable purchase from current income: there is no borrowing or lending in the model.[13] This restrictions are imposed for purposes of tractability and to focus on new car sales. We explore the implications of allowing used car trades and savings in Section 6 on robustness.

Formally, let $V_i(z, Z)$ represent the value of having a car of age i to a household in state (z, Z). Further, let $V_i^k(z, Z)$ and $V_i^r(z, Z)$ represent the values from keeping and scrapping an age i car in state (z, Z), respectively. Then,

[11] In this formulation, we have altered the form of the household specific shock. It is now assumed to represent choice specific random component in utility with an extreme value distribution. As in Rust (1987), this structure is quite useful for the solution and estimation of the dynamic discrete choice model.

[12] This shock has an extreme value distribution. Here $j = k$ refers to "keep the car" and $j = r$ refers to "replace the car".

[13] In contrast, Leahy and Zeira (2005) allow for perfect capital markets but restrict the household to make a single purchase over its lifetime.

$$V_i(z, Z) = \max[V_i^k(Z) + \alpha_k + z_k, V^r(Z) + z_r] \tag{2}$$

where

$$\begin{aligned} V_i^k(Z) &= u(s_i, Y, \varepsilon) + \beta(1-\delta)E_{(Z',z'|Z,z)}V_{i+1}(z', Z') + \beta\delta E_{(Z'|Z)}V^r(Z') \\ &\text{and} \\ V^r(Z) &= u(s_1, Y - p + \pi, \varepsilon) + \beta(1-\delta)E_{(Z',z'|Z,z)}V_2(z', Z') + \beta\delta E_{(Z'|Z)}V^r(Z'). \end{aligned}$$

In the definitions of $V_i^k(Z)$ and $V^r(Z)$, the car is assumed to be destroyed (from accidents and breakdowns) with probability δ.[14] The cost of a new car in numeraire terms is $p' - \pi$, which is stochastic since the price of a new car in the next period is random. The scrap value of a car is independent of its age: the value of replacement is independent of i.

Define the utility function to be additively separable between durables and nondurables:

$$u(s_i, c) = \left[i^{-\gamma} + \varepsilon \frac{(c/\lambda)^{1-\zeta}}{1-\zeta} \right] \tag{3}$$

where c is the consumption of non-durable goods, γ is the curvature for the service flow of car ownership, ζ is the curvature for consumption and λ is a scale factor. In this specification, the taste shock, ε, influences the contemporaneous marginal rate of substitution between car services and nondurables.[15]

In order for the agent's optimization problem to be solved, a stochastic process for income, prices and the aggregate taste shocks must be specified. We assume that aggregate income, prices and the unobserved preference shock follow a VAR(1) process given by[16]

$$\begin{aligned} Y_t &= \mu_Y + \rho_{YY}Y_{t-1} + \rho_{Yp}p_{t-1} + u_{Yt} \\ p_t &= \mu_p + \rho_{pY}Y_{t-1} + \rho_{pp}p_{t-1} + u_{pt} \\ \varepsilon_t &= \mu_\varepsilon + \rho_{\varepsilon Y}Y_{t-1} + \rho_{\varepsilon p}p_{t-1} + u_{\varepsilon t}. \end{aligned}$$

The covariance matrix of the innovations $u = \{u_{Yt}, u_{pt}, u_{\varepsilon t}\}$ is

$$\Omega = \begin{bmatrix} \omega_Y & \omega_{Yp} & 0 \\ \omega_{pY} & \omega_p & 0 \\ 0 & 0 & \omega_\varepsilon \end{bmatrix}$$

As the aggregate taste shock is unobserved, we impose a block diagonal structure on the VAR, which enables us to identify all the parameters involving prices and aggregate income

[14] The car wreck occurs, by assumption, at the end of the period so that the household receives the service flow of the car during the period, prior to the breakdown.

[15] This taste shock is included in the vector Z.

[16] We have only a single lag to economize on the state space of the agents' problem.

in a simple first step regression. This considerably reduces the number of parameters to be estimated in the structural model. We allow prices and income to depend on lagged income and lagged prices.

The aggregate taste shock potentially depends on lagged prices and income. The coefficients of this process along with ω_ε are estimated within the structural model. By allowing a positive correlation between the aggregate taste shock and lagged prices, given that prices are serially correlated, we can reconcile the model with the fact that sales and prices are positively correlated in the data. This allows us to better capture some additional dynamics of sales and prices in the structural estimation. An alternative way would be to model jointly the producer and consumer side of the economy, to get an upward sloping supply curve. However, solving for the equilibrium is computationally very demanding.

The policy functions generated from this optimization problem are of an optimal stopping variety. That is, given the state of the household, the car is scrapped and replaced iff the car is older than a critical age. Letting $h_k(z, Z; \theta)$ represent the probability an age k car is scrapped, the policy functions imply that $h_k(z, Z; \theta) = 0$ if $1 \leq k < J(z, Z; \theta)$ and $h_k(z, Z; \theta) = 1$ otherwise. Here $J(z, Z; \theta)$ is the optimal scrapping age in state (z, Z) when θ is the vector of parameters describing the economic environment.

The remaining part of the model is firm behavior. As in Adda and Cooper (2000a), we assume that the costs of production are independent of the level of production. Combined with an assumption of constant mark-ups, this implies that the product price is independent of the cross sectional distribution of car vintages.

This assumption of an exogenous price process greatly simplifies the empirical implementation of the model since we do not have to solve an equilibrium problem. In fact, we have found that adding information on the moments of the cross sectional distribution of car vintages has no explanatory power in forecasting car prices in the French case. Results are mixed for the US case, as the average age of cars significantly predicts future prices.

Before proceeding further, note that the underlying model stresses the replacement of older cars with new ones. Actual data presumably includes a component of car sales to agents who do not scrap a car before buying a new one. Further, there are surely instances where an agent scraps a car but does not buy another. Clearly movement of this type on the "extensive margin" creates a variation in sales not included in our model. To deal with this issue, we have detrended the data to remove the effects of population growth on sales. Further, from our investigation of some additional panel data on French households, we find that less than 2% of the sales are to households which have no cars.

3.2 Aggregate Implications

Aggregating over individual households (distinguished by z) leads to a prediction of the aggregate demand for cars and a prediction of the cross section distribution of car vintages.[17] Denote the aggregate probability of scrapping a car of age k in aggregate state Z by $H_k(Z)$ where

$$H_k(Z;\theta) \equiv \int h_k(z, Z; \theta) dG(z) \tag{4}$$

where $G(z)$ is the distribution of the household specific shock.

Let $f_t^-(k)$ be the period t cross sectional distribution of k **prior** to any decisions by households and $f_t^+(k)$ be the period t cross sectional distribution of k **after** the period t scrapping decisions by households. The difference between $f_{t-1}^+(k)$ and $f_t^-(k)$ reflects car wrecks that occur at the end of period $t-1$.

Using this notation, aggregate sales in period t are given by

$$S_t = \sum_{k \geq 0} H_k(Z_t;\theta) f_t^-(k) \tag{5}$$

where θ is a vector of parameters. New car sales in period t come from two sources. The first is the scrapping of cars by households in period t. The fraction of cars of age k is $f_t^-(k)$ and the period t hazard is $H_k(Z_t$. The second source is car wrecks. These wrecks are included in $f_t^-(0)$ with $H_0(Z_t;\theta) = 1$ as $k = 0$ indicates a household without a car.

The evolution of these cross sectional distributions are given by:

$$\begin{aligned} f_t^+(k) &= [1 - H_k(Z_t;\theta)] f_t^-(k) \text{ for } k \geq 2 \\ f_{t+1}^-(k) &= f_t^+(k-1)(1-\delta) \text{ for } k \geq 2 \end{aligned}$$

and

$$f_t^+(1) = S_t, \qquad f_t^-(0) = \delta, \qquad f_t^-(1) = 0.$$

These processes can be summarized as

$$f_t^+(k) = [1 - H_k(Z_t;\theta)](1-\delta) f_{t-1}^+(k-1) \text{ for } k \geq 2 \tag{6}$$

so that

$$S_t = f_t^+(1) = \sum_{k \geq 2} H_k(Z_t;\theta)(1-\delta) f_{t-1}^+(k-1) + \delta \tag{7}$$

which conforms to (5).

From an initial condition on the cross sectional distribution, it is possible, using (7), to generate a time series for the cross sectional distribution given a sequence of hazard functions.

[17] As we have assumed that z is iid, the distribution of this shock is independent of the distribution of car ages.

These hazard functions, in turn, depend on the parameters, θ, and the realizations of the shocks. Thus given the parameters, an initial cross sectional distribution and a distribution for the shocks to income, prices and tastes, we can simulate both sales and the time series of cross sectional distributions.

4 Estimation

This section states our approach to estimation. We then present parameter estimates and evaluate the results. Using the estimates, we return to the ARMA representation of the time series.

4.1 Method

The parameters describing the joint process of aggregate income and prices are estimated in a first step, to reduce the number of parameters in the structural estimation. The estimation results for this first stage are displayed in Appendix A. The remaining parameters, $\theta = \{\gamma, \lambda, \xi, \sigma_y, \alpha_k, \delta, \rho_{\varepsilon p}, \rho_{\varepsilon Y}, \omega_\varepsilon\}$, are estimated from the policy functions generated by the solution of the households' optimization problem.

A natural estimation strategy is to find the parameters that bring data from the simulated model as close as possible to the data. In our estimation, we make use of different types of observations. First, we use time series observations on sales, prices and income to match the sales predicted by our model. Concentrating on sales exclusively does not identify the parameters of the model such as the depreciation rate of car services, γ. Sales are the results of the interaction of a hazard function and the cross section distribution, both to be estimated. If the shape of the cross section distribution is not pinned down, there are several sets of parameters that would produce the same level of sales. Therefore, we also match three moments characterizing the cross sectional distribution as well as three moments characterizing the probability of scrapping a car, obtained from the yearly change in the cross-sectional distribution of cars by vintage. The price, income and sales series were linearly detrended prior to the structural estimation since the model itself has no trends.

Given a vector of parameters θ and a realized path of preference shocks ε, the model predicts aggregate sales and the evolution of the cross sectional distribution. These simulated series can be compared to their empirical counterparts. The estimation strategy is to find θ to minimize the "distance" between the actual and simulated data.

Formally, the criterion we minimized, via the simplex algorithm, was a weighted average of the difference between actual and predicted sales and between actual and simulated moments characterizing the cross sectional distribution of car ages, weighted by their actual variance. The parameters were estimated by minimizing the overall criterion:

$$\mathcal{L}(\theta) = \phi \mathcal{L}^1(\theta) + \mathcal{L}^2(\theta)$$

where ϕ is a scale parameter defined to be equal to the inverse of the variance of sales. The minimization of the overall criterion yields a root-T consistent estimate for any fixed number of simulations. We discuss the two components of this objective function in turn.

The first component of the criterion is

$$\mathcal{L}^1(\theta) = \frac{1}{T}\sum_{t=1}^{T}\left[(S_t - \bar{S}_t(\theta))^2 - \frac{1}{N(N-1)}\sum_{n=1}^{N}(S_{tn}(\theta) - \bar{S}_t(\theta))^2\right].$$

The first term is the standard nonlinear least square criterion which measures the squared distance between observed and average predicted values of the variables. For the sales series, the estimation is conditional on the realization of aggregated income and prices at each date, as well as on the initial cross sectional distribution. Given these realizations and the initial condition, for each value of the parameters, we can see how close our simulated sales is to observed sales.

Specifically, let S_t be the observed aggregate sales for the year t. Let $S_{tn}(\theta) = S(Y_t, p_t, \varepsilon_{t,n}, \theta)$ be the predicted sales for year t and for the draw $n = \{1, \dots, N\}$ of the unobserved aggregate shock, $\varepsilon_{t,n}$. The first component of the objective is essentially the squared distance between S_t and an average measure (over the taste shocks) of $\bar{S}_t(\theta) = \sum_n S_{tn}(\theta)/N$. However, such a criterion produces an inconsistent estimator for a fixed number of simulations.

To overcome this problem, we follow Laffont, Ossard and Vuong (1995) by including the second term which is a second order correction for the inconsistency bias introduced by the random draws of the preference shocks. Under standard regularity conditions, the asymptotic distribution of the estimators is normal and root-T consistent, for any fixed number of simulations N, (see Laffont, Ossard and Vuong (1995)). In practice, we fix the number of random draws to 50. We find that the correction for simulation error is then negligible.

The second piece of the objective contained additional moments and is specified as

$$\mathcal{L}^2(\theta) = \sum_{i=\{5,10,15\}}\left(\phi_i^F(\bar{F}^i - \bar{F}^i(\theta))^2 + \phi_i^H(\bar{H}^i - \bar{H}^i(\theta))^2\right)$$

where ϕ_i^j is a weight equal to the empirical inverse of the variance of each moment, $j = F, H$. For these moments, the cross sectional distribution is referenced by F and the hazard function by H.

The moments of the cross sectional distribution we match are the average (over the 1981-95 sample for the US, over the 1972-94 sample period for France) fraction of cars of ages 5, 10 and 15. The idea is to use these critical ages to characterize the average cross sectional distribution. The predicted counterparts were obtained on simulated data from the model, for a similar sample size.

Formally, let $\bar{F}^i = (1/T)\sum_t F_t^i, i = 5, 10, 15$ be the average fraction of cars of age 5, 10 or 15 during the sample period. Similarly let $F_{t,n}^i(\theta), i = 5, 10, 15$ be the predicted fraction of cars of age 5, 10 or 15, in period t and given draw n of the unobserved taste shock. Let $\bar{F}^i = \sum_{t,n} F_{t,n}^i(\theta)/(TN), i = 5, 10, 15$ be the average predicted fraction of cars of age 5, 10 or 15.

In addition, the moments include the average hazard rates at ages 5, 10 and 15. These moments are defined from the sample as $\bar{H}^i = (1/T)\sum_t H_t^i, i = 5, 10, 15$ where H_t^i is the period t hazard for age i cars.

4.2 Estimation Results

Table 2 provides a summary of our estimated parameters for both countries.[18] From this table, the rate of depreciation of the service flow is about 34 percent on an annual basis for France and 41 percent for the US These parameters are precisely estimated and are significantly different from zero. To get a sense of magnitude, at $\gamma = 0.4$, it takes about five years to lose 50% of the value of the car. Further, we find that there is some curvature in the utility function for both countries, with a parameter around $1.7 - 1.8$, which is in the usual range for curvature estimates from nondurable consumption using Euler equation estimation.

Further evidence on the estimation model is provided in Table 3 which indicates the actual and predicted moments. The model captures fairly well the cross sectional distribution of car vintages.

For both France and the US, the probability of car breakdown is estimated at one to two percent per year. This corresponds to the empirical hazard for very young cars. From Table 3, the hazard rate is considerable higher for 5 and 10 year-old cars in both countries. This is captured in the simulated data as well, reflecting, *inter alia*, the estimated value of γ.

The models are able to match rather closely the aggregate sales as the R^2 vary from 0.72 to 0.93. By comparison, the R^2 obtained from an OLS regression of sales on lag sales, prices and income is 0.46 for France and 0.60 for the US.

When testing the over-identifying restrictions of the model, we reject them for both France and the US. For France, the sales predictions are not as close to the actual data as in the US. For the US, the model fails to match aspects of the CDF and the hazard functions, particular the hazard for 15 year-old cars.

[18] The standard errors are in parentheses.

Table 2: Estimated Parameters for Discrete Choice Model

Parameters	Baseline	
	France	US
γ	0.34 (0.03)	0.41 (0.08)
δ	0.02 (0.006)	0.01 (0.003)
ζ	1.71 (0.01)	1.79 (0.05)
λ	5732 (3e-6)	1.3e5 (1e-6)
σ_{ε}	0.10 (0.14)	0.006 (0.01)
$\rho_{\varepsilon,p}$	-5.7e-4 (4e-5)	1.1e-4 (4e-5)
$\rho_{\varepsilon,Y}$	1.3e-4 (2e-5)	1.9e-5 (7e-6)
α_K	2.6 (0.01)	2.06 (0.40)
ρ_{ε}	0.36 (0.06)	0.75 (0.03)
cor(Obs. Sales,Pred Sales)	0.72	0.93
P(over ident)	0.16	0.54

4.3 Time Series Representations

Using the estimated parameters, we can now explore the time series properties of the model. As in the motivation, we focus first on the ARMA representation and then on the impulse response functions.

4.3.1 ARMA Representation

Our interest is partly in the aggregate time series of new car sales produced by our estimated model compared to those obtained from the data. ARMA(1,1) representations of the sales series are reported in Table 3. The ARMA(1,1) results come from 100 simulations of 100 periods. The table reports the averages and the standard deviations of the coefficients.

Our principle finding is that our model is able to reproduce the abnormally high value for the "rate of depreciation" inferred from the MA(1) coefficient, when the estimation is done through an ARMA(1,1) representation as in Mankiw (1982). To this extent, our model is able to reconcile a low physical depreciation rate at the micro level (2%) with a coefficient close to one at the macro level, as viewed through the PIH model. The autoregressive coefficient is also estimated quite close to its value in the annual data once the time series is detrended.

Table 3: Observed and Predicted Moments from CDF

France		
Parameters	Observed	Baseline
Fraction of cars less 5	0.48 (0.04)	0.57
Fraction of cars less 10	0.83 (0.03)	0.85
Fraction of cars less 15	0.96 (0.01)	0.96
Hazard rate at age 5	0.04 (0.04)	0.09
Hazard rate at age 10	0.17 (0.09)	0.17
Hazard rate at age 15	0.25 (0.16)	0.19
AR for Sales	0.26 (0.38)	0.28 (0.29)
MA for Sales	0.44 (0.57)	0.09 (0.34)
US		
Parameters	Observed	Baseline
Fraction of cars less 5	0.36 (0.03)	0.40
Fraction of cars less 10	0.71 (0.03)	0.70
Fraction of cars less 15	0.91 (0.02)	0.88
Hazard rate at age 5	0.02 (0.01)	0.03
Hazard rate at age 10	0.11 (0.03)	0.08
Hazard rate at age 15	0.17 (0.03)	0.10
AR for Sales	0.68 (0.24)	0.56 (0.08)
MA for Sales	0.33 (0.30)	0.25 (0.12)

4.3.2 Impulse Response Functions

Figure 5 displays the results of one standard deviation shocks to income, sales and prices on sales in the three panels, for both actual data and also from simulated data. All data are in logs.[19] The first row is the response of France automobile (log) sales to these shocks and the response in the US.

For France, the model predicts a burst in sales following an income shock and then an ultimate fall in sales by year 5. This prediction is also found in the actual data though the oscillation in sales are more dampened. For the US, a similar pattern emerges with a fall in sales in year 8 for both the observed and simulated data. The oscillations predicted by the model are somewhat larger than those found in the data.

Figure 5, row 2 shows the impact of a shock of sales on sales. For both countries, the model is able to track the response over time. Within our model, we interpret a shock on

[19]The impulse response functions from the simulated data come from the estimation of a VAR on simulated data from the estimated model.

sales as a shift in the aggregate taste shock, ε, and its effect through time on sales.

The bottom row of the figure shows predicted and actual responses to a price increase. In the data from France, sales rise in response to a price increase while the model has the opposite prediction. This reflects the positive correlation between the innovations in prices and sales. [20] Although the sales response to prices is negative in US data, this response is much larger in the simulated data.

Overall, our model is able to reproduce a number of patterns pertaining to the dynamics of sales. The next section investigates the sources of fluctuations in sales.

5 Decomposing the Results: What lies behind the Oscillations?

The estimated model has the ability to match an ARMA(1,1) time series representation of car expenditures, as well as impulse response functions of sales. Given this "empirical success", we now turn to a more intuitive discussion/evaluation of the model by looking further at impulse response functions. Here we try to go inside the results to better understand the source of the dynamics, particularly the oscillations in the impulse responses.

Caballero (1993) explains why a dynamic discrete choice model might explain the response of durable expenditures to an income shock. The key point is that a shock to income produces a dynamic in durable expenditures as agents respond differentially. While agents may differ along a number of dimensions, our analysis focuses on the cross sectional distribution of car vintages. Essentially agents with relatively old cars will respond to the income shock by replacing their car first and then agents with younger cars will respond later. The delayed response simply reflects the upward sloping adjustment hazard: all else the same, agents with younger cars are less likely to respond to income variations than are agents with older cars. The evolution of the cross section distribution through time can be a source of fluctuations, which are picked up by the impulse responses. As the distribution evolves, following (6), sales will respond. In fact, the magnitude of this response depends on the slope of the hazard function: the flatter is the hazard, the less responsive will sales be to the evolution of the cross sectional distribution.

A second source of movement is the dynamics induced by prices and income as these processes are serially correlated. Movements in these variables are represented by shifts in the probability of adjustment (hazard).

[20] From the covariance matrix of the VAR residuals, the covariance of the log sales and log price innovation is positive. Consequently, in the Cholesky decomposition: the response of log sales to a price innovation is also positive. These features of the VAR representation underlie the reported impulse responses.

We study the relative importance of these two influences (hazard and CDF shifts) in two ways. First, we decompose the time series of sales into these two influences. Second, we recompute the impulse response functions either by holding the CDF fixed or by limiting the shifts in the hazard function.

5.1 Sales

For the time series of sales, using (5), the change in aggregate sales can be decomposed into two terms. The first term is the change due to shifts in the hazard functions, such as price or income movements against a fixed cross-section distribution. The second term is the contribution of the shifts in the cross-section distribution, holding the hazard function fixed:

$$S_t - S_{t-1} = \sum_k [H_k(z_t;\theta) - H_k(z_{t-1};\theta)] f_{t-1}(k) + \sum_k H_k(z_t;\theta)[f_t(k) - f_{t-1}(k)] + u_t \qquad (8)$$

For our simulated data, this decomposition is exact. The error term reflects the fact that, in the actual data, there are measurement problems and not all sales variations are a consequence of replacement. Inspection of the time series of the error process indicates little structure to this error supporting the view that it is mainly due to measurement problems.

Given data on the cross section distribution, we can compute the contributions to the change in sales of the fixed hazard and fixed cross-section distribution components. The fixed CDF component tracks the change in sales very closely both for the US and for France, whereas the other component has a much smaller variance and have a low correlation with the change in sales. In particular, the R^2 associated with shifts in hazards is equal to 0.93 for the US and 0.75 for France and the one associated with shifts in the cross section distribution is only 0.05 for the US and 0.23 for France. Thus in the actual data, hazard shifts are the main source of fluctuations.

From the simulated data of our estimated model, we can also evaluate the contribution of each term to the variability in aggregate sales. From a simulated sample of length 400, we find very similar results to the real data: shifts in hazards are the most important determinant of sales. The R^2 associated with shifts in hazard is equal to 0.93 for France and the US, whereas the R^2 associated with shifts in the cross sectional distribution is only 0.01 for France and 0.02 for the US.

5.2 Decomposing the IRFs

Figure 6 shows the impulse response functions for the US and France from two simulation exercises. In the first, we hold the CDF fixed and consider the effects of an innovation to income. In the second case, we allow the CDF to evolve but impose that the income variation be temporary. Thus the hazard function shifts out for one period only.

We find that with a fixed CDF, the impulse response functions are close to the global impulse response. Thus the dynamic is mainly due to the evolution of the prices and income. The dynamics induced by the evolution of the cross section distribution contributes surprisingly little. Evidently, the depreciation of cars along with the household specific shocks are significant enough to eliminate replacement cycles.

Narrowing down our search for an explanation of these oscillations, we simulate the model with a fixed CDF, but we eliminate the cross effect of prices and income (we set $\rho_{Yp} = \rho_{pY} = 0$). Figure 7 displays the impulse response functions for the US.[21] We then find that all the oscillations are gone. From this we conclude that the oscillations in sales are mainly due to the dynamics of prices and income, and more particularly to the cross effects.

6 Robustness

The results in the previous section have been obtained from a model which restricted the trades of agents: (i) there was no market for the sale of used cars and (ii) agents were not allowed to borrow and lend. We now study how our results depend on these restrictions. We extend our model in those two directions and evaluate the robustness of the dynamics of sales using both an ARMA(1,1) representation and impulse response functions from a linear VAR model with sales, income and the relative price of cars.

We first start with a brief description of the extensions to the baseline model. A more formal description of these extended models can be found in Appendix B.

6.1 Used Car Markets

Our first modification is to allow agents to sell their cars in a second-hand market.[22] In the baseline model, an agent receiving, say, an adverse income shock was unable to smooth consumption by trading a relatively new car for an older one. This restriction reduces the value of car ownership and influences the optimal scrapping decision for the individual.

In this section, we extend model (2) to include a resale option. Individuals have the choice between keeping their car for one more period, purchasing a new car or purchasing a

[21] The French case, not shown, is similar.

[22] Rust (1985) uncovers the mapping between the solution of an optimal scrapping problem for a planner and the equilibrium of a group of agents with a second hand market in an economy with no aggregate shocks. From that perspective, the preferences estimated in the model without secondary markets are not those of individual households but may be closer to the preferences of the planner. In our exercise, we add resale as an option for private agents.

second-hand car, while selling their own as a second-hand car. The price of a second hand car is a decreasing function of vintage and varies with the price of new cars. We estimate the parameters describing the price function from micro data. That is, the price for used cars is not determined within the model. Rather, the price of used cars as a function of their age is taken from the data and imposed on the individual choice problem. [23]

The model is estimated as in Section 4 but we also impose that markets clear, i.e. that for any vintage, demand is (approximately) equal to supply. This is done within the estimation by adding market clearing as an additional moment. We also add as moments the probability of keeping a car from one period to another and the probability of buying a new car (as opposed to a second hand car).

6.2 Capital Markets

The absence of capital markets in model (2) implies that the cost of buying a durable good cannot be spread over time, thus implicitly increasing the cost of such expenditures. With complete contingent markets, this variation in marginal utility would be smoothed out over time: consumption would be lower over all time periods due to car ownership but variations in marginal utility would not only arise at the time of car purchase.

To evaluate the implications of capital market imperfection, we consider a variant of the model in which there is borrowing and lending.[24] To do so, we introduce wealth as a state variable in (2) and thus allow borrowing and lending. We estimate the model as in section 4. Given that we do not have data on the joint distribution of cars (by vintage) and household savings over time, we initialize our simulations using the ergodic joint distribution of savings and cars, obtained by prior simulations.

6.3 Robustness of Implied Dynamics of Car Sales

Details about the models described in sections 6.1 and 6.2, the estimation procedure and estimation results can be found in Appendix B. In this section, we focus on the time series implications for the sales of new cars from these two extended models.

The first results in section 4.3 was the ability of our baseline model (2) to replicate the observed ARMA(1,1) pattern of aggregate sales. Table 4 displays the ARMA(1,1) implication for the two extended models, for both countries. These results are obtained by simulating the model over 30 years and estimating an ARMA(1,1) model. We used 100 replications to get the standard errors. Each replication consists of a new set of draws for the taste shock ε.

[23] We use the CEX for the US and the Enquete de Conjoncture for France.

[24] An earlier version of the paper also considered the case of linear utility. Our results on the ARMA representation of the time series of car sales held there as well.

Table 4: Robustness Check. Second Hand Market and Perfect Capital Markets

	AR(1)		MA(1)	
Parameter	Estimate	st. err.	Estimate	st. err.
US Observed	0.68	(0.24)	0.33	(0.30)
US Second hand markets Model	0.59	(0.15)	0.32	(0.21)
US Capital markets Model	0.47	(0.17)	0.19	(0.28)
France Observed	0.26	(0.38)	0.44	(0.57)
France Second hand markets Model	0.68	(0.20)	-0.14	(0.40)
France Capital markets Model	0.73	(0.15)	0.38	(0.43)

Note: Monte Carlo results obtained over 100 replications with sample length 30. French estimates on annual car registration, 1972-1994. US estimates on annual car registrations 1981-1995. All ARMA models included a linear trend.

For the US, both models are able to replicate closely (in a statistical sense) both the autoregressive and the moving average components. In particular, we cannot reject that the MA coefficient is zero, which would imply a 100% depreciation rate in the Mankiw (1982) model. For France, both models predict an autoregressive components which is higher than the one estimated on observed data. The moving average component is also lower than in the actual data. This means that the “implied” depreciation rate is even closer to 100%.

We now turn to the impulse response functions obtained from a VAR(1) of sales, income and prices as presented in section 4.3.2. We simulate each model over 30 years with 100 replications to obtain simulated series for sales, income and prices. We then compute the (orthogonalized) IRFs by estimating a VAR(1). As previously, the ordering is: income, prices and sales. The impulse response functions of sales to shocks to income, prices or sales for all models are presented in Figure 8. In all cases and for both countries, the extended models generate impulse response functions which are quite close to the one obtained for the baseline model. For France, the IRFs are closer to the baseline one than for the US. In particular, the response of sales from the model with second-hand markets is not particular close to the baseline, even though the general shape is similar.

From these results, we conclude that the introduction of either capital markets or second hand markets do not alter in any significant way the results for our baseline model in section 4.2 in explaining the dynamics of *aggregate* sales. Clearly though, these extensions of the baseline model have rich implications in other dimensions, such as savings or the type of cars that individuals hold.

7 Conclusion

We have studied the aggregate time series implications of a model of consumption of both durables and nondurables at the household level. Our model is a model of dynamic discrete choice where agents make infrequent purchases of durables. We extend our model to incorporate second-hand markets and borrowing and savings. These models, once aggregated across heterogenous households are able to reproduce some of the main features of the aggregate data. In particular, our model matches up with impulse response functions from the data, particularly the oscillations in car sales. We also have found that the estimated model goes a considerable way towards solving the "durables puzzle" of Mankiw (1982). [25]

Our approach to the problem follows a methodology that is quite different from that put forth by Bar-Ilan and Blinder (1988) and utilized in much of the subsequent work. We specifically avoid the specification of individual optimization in terms of (S,s) bands and instead focus on the underlying parameters of the individual's dynamic discrete choice problem. Further, our model was estimated using data which also emphasized properties of the cross sectional distribution of car ages. Still, we find that this modeling approach delivers time series implications that match certain features of the data.

In trying to understand our finding, we are naturally led to a decomposition of the movements in sales into two components: shifts in the hazard function and the evolution of the cross sectional distribution. We report that most of the variation in the change in sales can be attributed to shifts in the hazard function, though the evolution of the cross sectional distribution is present to some extent. This contrasts with the conclusion inferred in Caballero (1993), which stresses the importance of the evolution of the cross section distribution of durables.

We show that our main conclusions are robust to two main assumptions we make in solving our model: the absence of second hand markets and the absence of borrowing and lending. Although these extended models are richer in many dimensions they have similar explanatory power for the dynamics of car sales.

In terms of further work, there are two elements of our basic model that deserve additional attention. First, there are undoubtedly additional insights to be gained from allowing endogenous price variations due to upward sloping supply to explain some of the results. The assumption of exogenous prices dramatically simplifies our numerical analysis: the cross sectional distribution of car vintages would be an element in households' state vector if prices were determined endogenously.

Second, the models used in this study have implications at the household as well as

[25] To our knowledge, Bar-Ilan and Blinder (1988) deserve credit for drawing the attention of the profession to this point. Our results show the link quantitatively.

the aggregate level. We have not exploited the household implications of the model. Supplementing the time series observations with household data on durable and non-durable purchases provide a natural area for further empirical research.

Appendix

A Estimation Results for joint Process of Income and Prices

Table 5 displays the estimation results for the joint process of income and prices, used in the structural model.

Table 5: VAR for prices and Income

	US		France	
Parameter	Estimate	standard error	Estimate	standard error
ρ_{YY}	0.75	0.12	0.67	0.12
ρ_{Yp}	1.25	0.47	0.14	0.22
ρ_{pp}	0.68	0.14	0.65	0.17
ρ_{pY}	-0.10	0.03	-0.04	0.09
ω_Y	3.0e6	-	2.6e6	-
ω_p	2.7e5	-	1.5e6	-
ω_{Yp}	2.6e5	-	-1.6e5	-

Note: Regression done on detrended annual series.

B Extensions to Our Baseline Model

B.1 Used Car Markets

To include resale in our model, we study the following dynamic programming problem which modifies (2) to include a resale option. As earlier, let $V_i(z, Z)$ be the value of an age i car in state (z, Z). This value is given by

$$V_i(z, Z) = \max[V_i^k(Z) + \alpha_k + z_k, V_i^r(Z)] \tag{9}$$

where

$$\begin{aligned} V_i^k(Z) &= u(s_i, Y, \varepsilon) + \beta(1 - \delta_i)EV_{i+1}(z', Z') + \beta\delta_i EV^r(Z') \\ &\text{and} \\ V_i^r(Z) &= \max[V_i^n(Z) + z_n, V_i^u(Z) + \alpha_U + z_u]. \end{aligned}$$

In this case, replacement can either be through a new car, denoted $V_i^n(Z)+z_n$ or the purchase of a used car, denoted $V_i^u(Z)+\alpha_U+z_u$. These options are defined by

$$\begin{aligned} V_i^n(Z) &= u(s_1, Y+\max(\phi p_i(Z),\pi)-p_1(Z),\varepsilon) \\ &\quad +\beta(1-\delta_i)EV_2(Z')+\beta\delta_i EV_0^r(Z') \end{aligned} \tag{10}$$

$$\begin{aligned} V_i^u(Z) &= \max_j \left[u(s_j, Y+\max(\phi p_i(Z),\pi)-p_j(Z),\varepsilon)\right. \\ &\quad \left.+\beta(1-\delta)EV_{j+1}(Z')+\beta\delta EV_0^r(Z')\ \right] \end{aligned} \tag{11}$$

In these expressions, the value of replacing a car may depend on its age through the resale market. Accordingly, $V_0^r(Z)$ refers to the value of replacement following a wreck with the restriction that $\max(\phi p_0(Z),\pi)=\pi$.

Here there are three options: keep the car (V^k), sell the car and purchase a new one (V^n) or sell the car and purchase a used car (V^u). Associated with these three options are individual specific shocks, denoted (z_k, z_n, z_u) which have an extreme value distribution.

In this specification, $p_j(Z)$ denotes the price of age j cars in state Z and ϕ parameterizes the fraction of the car value recovered by the consumer. Since agent's have an option of scrapping their cars, agents who sell their cars receive $\max(\pi, \phi p_j(z))$. Note that we have introduced in model (9) a depreciation rate δ_i which is a function of the vintage. Contrary to our baseline model (2), new sales and scraps are disconnected as individuals who buy a new car can resale the old one to another agent. Hence, to take into account the upward slopping hazard of scrapping as cars age, we have to allow for a non constant rate of depreciation.

The introduction of a second-hand market introduces interesting complications into the model associated with the determination of the price vector for used cars. Our approach is to use the empirical price function estimated from French and US data. These state-contingent price functions are taken as given by the households.

A concern with this approach is that it does not include a market clearing condition. That is, there is no requirement that at the estimated parameters, the assumed prices actually clear markets for all car vintages. We impose the market clearing condition by incorporating as a moment that supply equals demand for each vintage at the observed price. Given that we have more moments than parameters, we can only minimize the imbalance on the second-hand market, so that market clearing holds approximately. In addition, as our model is richer than the baseline model, we add two moments, the probability of keeping a car for (at least) one year and the proportion of individuals who purchase a new car, conditional on replacing a car. The observed moments are taken from the CEX for the US and the Enquete de Conjoncture for France. [26]

[26] The probability of keeping a car is estimated at 0.82 (France) and 0.87 (US) annually. The probability

The estimation proceeds basically as described in section 4 with the added feature that the price functions are estimated separately. From data obtained for both countries, we estimate a price function $p_i(Z) = p_0(Z)e^{-\tau i}$. For both countries, the price functions were best described with $\tau = 0.2$.

The results are presented in Table 6 under the columns "Second Hand Markets". Relative to the baseline model, the estimated value of γ is much higher for both the France and the US. The estimated value of ζ is slightly lower, at around 1.4. The fraction of the car value recovered by the consumer, ϕ, is estimated at 0.95 for France and 0.84 for the US. This parameter is difficult to pin down as it represents a transaction cost over and above the psychological cost α_U. The cost introduced from $\phi < 1$ represent a vintage specific cost as it interacts with the price function.

This model fits the US time series of sales better than the baseline specification and the overidentifying restrictions cannot be rejected for this country. For France, the fit is worse than the baseline case in part due to the difficulty to match observed sales.

The row labeled "misallocation" measures the percent of market disequilibrium across all the vintages. From these figures we see that markets are quite close to clearing.

Table 6: Estimated Parameters for Discrete Choice Model, Model 2 & 3

Parameters	Baseline		Second Hand Markets		Capital Markets	
	France	US	France	US	France	US
γ	0.34 (0.03)	0.41 (0.08)	0.54 (0.04)	0.36 (0.12)	0.42 (0.10)	0.52 (0.39)
δ	0.02 (0.006)	0.01 (0.003)	0.09 (0.005)	0.01 (0.001)	0.08 (0.01)	0.006 (0.006)
ζ	1.71 (0.01)	1.79 (0.05)	1.42 (0.01)	1.41 (0.04)	1.55 (0.08)	3.31 (0.22)
λ	5732 (3e-6)	1.3e5 (1e-6)	4.3e5 (1e-8)	9.8e5 (2e-7)	1.4e6 (6e-6)	56868(1e-5)
σ_ε	0.10 (0.14)	0.006 (0.01)	0.006 (0.000	0.01 (0.05)	0.08 (0.02)	0.08 (0.05)
$\rho_{\varepsilon,p}$	-5.7e-4 (4e-5)	1.1e-4 (4e-5)	-3e-4 (2e-5)	5.9e-5 (2e-5)	-2.1e-4 (2.2e-05)	3.2e-5 (3.4e-5)
$\rho_{\varepsilon,Y}$	1.3e-4 (2e-5)	1.9e-5 (7e-6)	1.3e-4 (1e-5)	2.2e-5 (5e-6)	3.1e-5 (8e-06)	-3e-7 (9e-6)
α_K	2.6 (0.01)	2.06 (0.40)	1.45 (0.05)	2.7 (0.07)	1.39 (0.47)	1.51 (3e-12)
α_U	-	-	-0.91 (0.01)	-0.72 (0.04)		
ρ_ε	0.36 (0.06)	0.75 (0.03)	-0.37 (0.06)	0.90 (0.15)	0.04 (0.08)	0.32 (0.49)
τ	-	-	0.2	0.2		
ϕ	-	-	0.95 (0.05)	0.84 (0.17)		
R^2 for Sales	0.72	0.93	0.46	0.96	0.73	0.79
P overident.	0.16	0.54	0.65	0.02	0.21	0.19
% Mis-allocated	-	-	0.16	0.08	-	-

of buying a new car is respectively 0.48 for France and 0.45 for the US.

B.2 Capital Markets

We extend model (2) to include savings. Denote by A the savings at the start of the period. The choice of the agent is now:

$$V_i(Z,A,z) = \max[V_i^k(Z,A) + z_k, V^r(Z,A) + z_R] \tag{12}$$

where

$$\begin{aligned} V_i^k(Z,A) &= max_{A'} u(s_i, A + Y - \frac{A'}{R}, \varepsilon) + \beta(1-\delta) EV_{i+1}(Z', A', z') \\ &+ \beta\delta EV^r(Z', A') \end{aligned}$$

and

$$\begin{aligned} V^r(Z,A) &= max_{A'} u(s_1, A + Y - p + \pi - \frac{A'}{R}, \varepsilon) + \beta(1-\delta) EV_2(Z', A', z') \\ &+ \beta\delta EV^r(Z', A'). \end{aligned}$$

where A' is constrained to be positive. Here R is the assumed constant gross rate of return and set at 1.05. With this modification of the dynamic programming problem, we can simulate the individual choices using the estimated parameters. The model implies a probability of scrapping a car which depends on current income, prices, the taste shock *and* assets. To simulate the sales of cars through time, we need to know the joint density of cars and assets for the first period. Given that we do not have it from the data, we use the ergodic distribution obtained through a simulation of the model for a large number of periods.

Our findings for this case are summarized in Table 6 under the column "Capital Markets". The estimation of the utility depreciation of cars is roughly in of the same magnitude for both countries, slightly lower for France and slightly higher for the US. The curvature of the utility function is larger for both countries, especially for the US. Now that individuals have can smooth their purchase over many periods, the estimation can pick a utility function with more curvature. The fit of the model is worse for France and better for the US.

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Figure 3: Impulse Response Functions

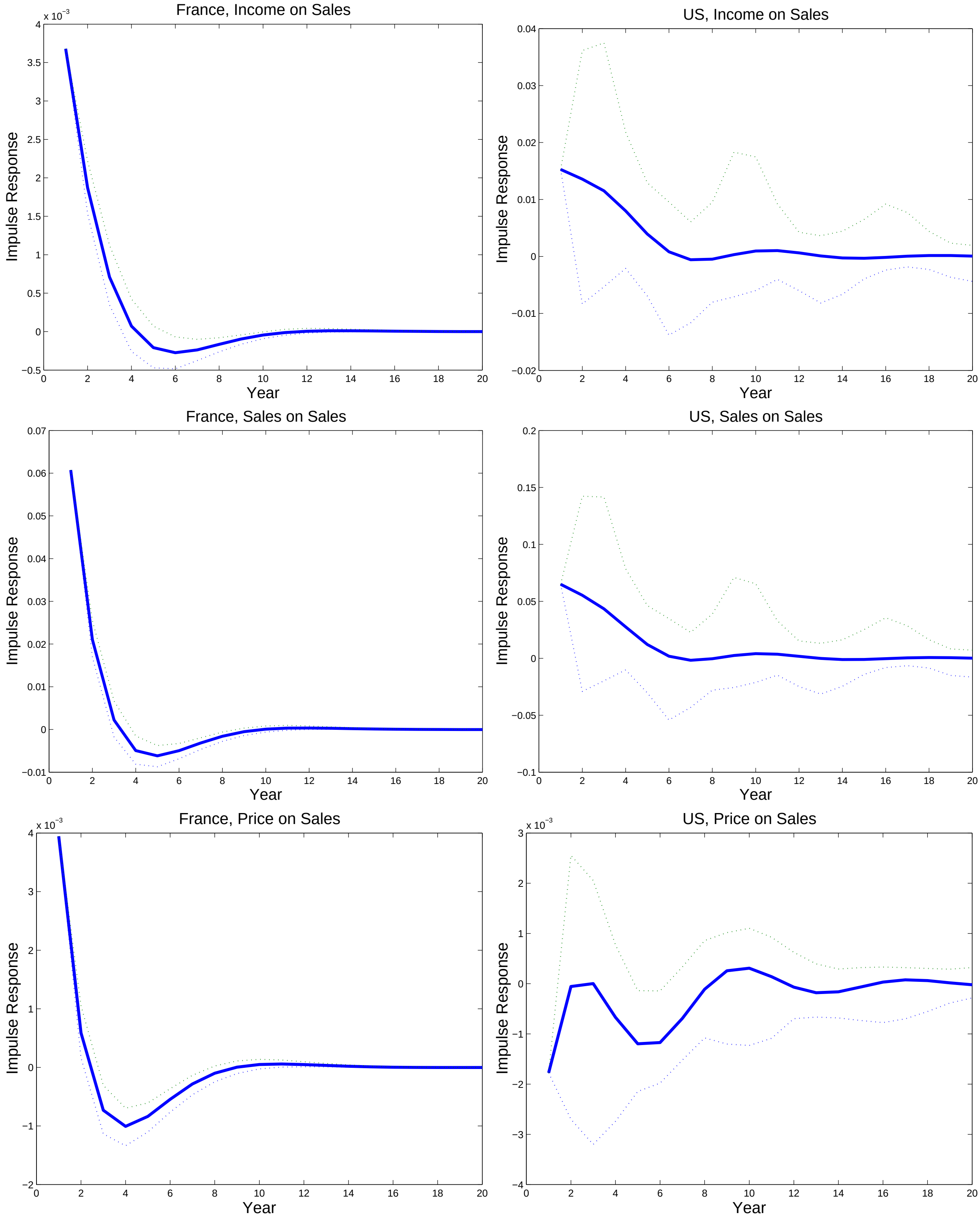

Figure 4: ARMA and IRF Comparison

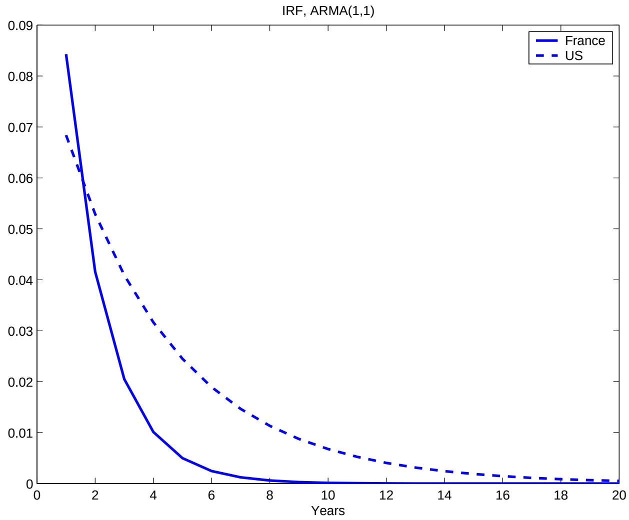

Figure 5: Impulse Response Function for Sales, Simulated and Actual Data

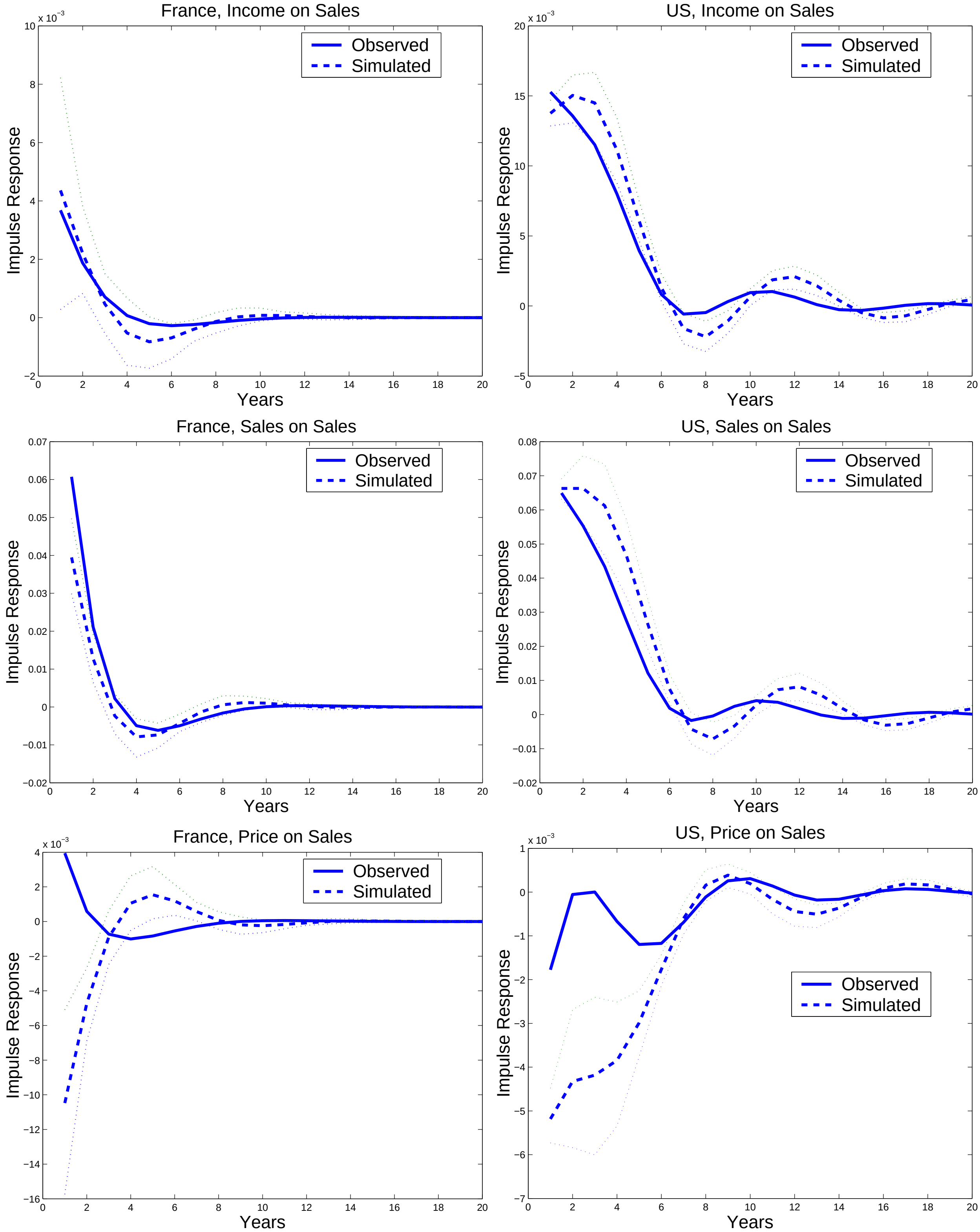

Figure 6: Impulse Response Functions: Decompositions

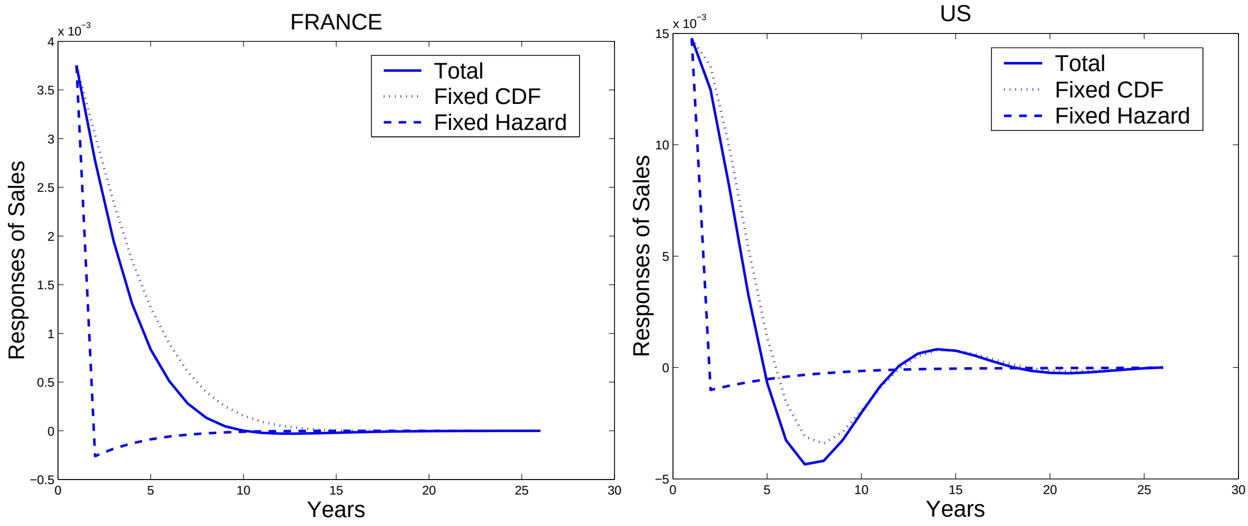

Figure 7: Impulse Response Functions: No Price Interactions

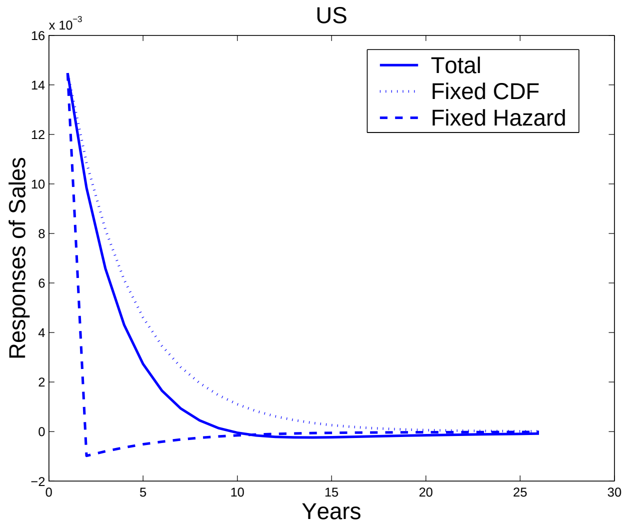

Figure 8: Impulse Response Function, All Models