Public Sector Wage Policy and Labor Market Equilibrium: A Structural Model*

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Abstract

We develop and estimate a structural model that incorporates a sizeable public sector in a labor market with search frictions. The wage distribution and the employment rate in the public sector are taken as exogenous policy parameters. Overall wage distribution and employment rate are determined within the model, taking into account the private sector's endogenous response to public sector employment policies. Job turnover is sector specific and transitions between sectors depend on the worker's decision to accept alternative employment in the same or different sector by comparing the value of employment in the current and prospective jobs. The model is estimated on British data by a method of moments. We use the model to simulate the impact of various counterfactual public sector wage and employment policies.

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1 Introduction

We formulate a search-theoretic model to explore the impact of public sector wage and hiring policies on labor market performance. The wage offer distribution and job offer rate of the public sector are treated as exogenous policy parameters, and conditional on these, the private sector wage distribution, hiring intensity, and productivity distribution are then derived endogenously. Exploiting data from the British Household Panel Survey (BHPS) the model is then estimated by minimum-distance matching of some key moments from the data. These estimates allow us to make counterfactual policy analysis of different public sector wage policies. We thus propose a structural modeling tool for the ex-ante assessment of public sector wage and hiring policies.

There has been very little done in modeling the public sector explicitly within an equilibrium model of the labor market and nothing to our knowledge that estimates such a model. This is a major oversight when one thinks that in our data 24% of employed individuals were employed by the public sector.[1] It is, of course, naive to believe that with an employment share this large the public sector will not influence wage determination and by extension overall employment.

Instead of modeling the behavior of private sector firms explicitly, the literature thus far has been dominated by reduced form comparisons of the two sectors.[2] The general consensus emerging from the empirical literature is that the public sector wage distribution is more compressed than that in the private sector and workers receive a small public sector wage premium which is more prevalent in low-skilled workers.

With these stylized facts being known for some time, it is fairly surprising that so little has been done in explicitly modeling the interaction between the two sectors. The existing literature that does this has largely focused on assessing the impact of the public sector on the level or volatility of aggregate wages and employment. Papers in that vein include Algan et al. (2002), Quadrini and Trigari (2007), Hörner et al. (2007), and Gomes (2014). All model search as directed to a particular sector and none has direct job-to-job reallocation (within or between sectors). All model

[1] Algan et al. (2002) report that, based on a slightly narrower definition of public sector employment, the OECD finds an average public sector share of total employment of 18.8% in 2000 over a sample of 17 major OECD countries.

[2] For a survey of the literature, see Bender (1998). The literature on the specific topic of fiscal austerity and public sector pay and employment in the UK is still very limited at the time of writing. The Institute for Fiscal Studies has comprehensive descriptive work on the subject (Cribb et al., 2014) and is actively conducting further research.

wages as determined by bargaining over the surplus from a match. Algan et al. (2002) find that the creation of public sector jobs has a massive crowding out effect on private-sector job creation, such that the marginal public sector job may destroy as many as 1.5 private sector jobs in some OECD countries. This crowding out effect is especially strong when public sector wages are high and/or when public and private-sector output are close substitutes. Focusing on the cyclicality of employment, Quadrini and Trigari (2007) examine a public sector wage policy that is acyclical (a single wage) and procyclical (government wage is an increasing function of private sector wages). Calibrating the model for the US economy, they find that public sector employment and wage policy increased employment volatility by four and two times, respectively, over the periods 1945-70 and 1970-2003. They attribute this downward trend in employment volatility to an increasingly procyclical compensation policy adopted by the state. Hörner et al. (2007) model two economies: one where a social planner aims to maximize welfare with public sector wages and employment, the other in the absence of a public sector. The authors conclude that, firstly, that the public sector has an ambiguous effect on overall employment and secondly, that in more turbulent times there will be higher unemployment in the economy with the public sector. The latter result comes from individuals being risk averse and therefore crowding into the safer public sector in more uncertain times. Finally, Gomes (2014) builds a dynamic stochastic general equilibrium model with search and matching frictions calibrated to U.S. data and shows that high public sector wages induce too many unemployed to apply for public sector jobs and raise unemployment. He further argues that the cyclicality of public sector wage policy has a strong impact on unemployment volatility.[3]

Closer to this paper are the recent contributions by Albrecht et al. (2015), Gomes (2015) and Burdett (2011). Both Albrecht et al. (2015) and Gomes (2015) extend, in different ways, the canonical Diamond-Mortensen-Pissarides model (Pissarides, 2000) to incorporate a public sector. In Albrecht et al. (2015), workers search randomly across both sectors, the public sector posts an exogenous number of job vacancies, and has its own wage setting rule. In Gomes (2015), workers direct their search to a particular sector; the public sector has an exogenous wage, but

[3] Another related contribution by Michaillat (2014) stands out as something of an exception. Using a New Keynesian model with random job search by the unemployed, Michaillat finds that the "government multiplier", defined as the additional number of workers hired in the private sector when one public job is created, is positive (in the order of 5-8%) and countercyclical, suggesting that the public sector tends to stabilize labor market fluctuations.

sets its vacancies to minimize the cost of producing a certain amount of public services, subject to technology. Both papers have heterogeneity in workers' human capital, which allows comparison of the impact of various policies across worker categories. Burdett (2011) is closest to us in the sense that firms post wages rather than bargain over the surplus and, crucially, that workers are allowed to search on the job. On-the-job search — and the possibility for workers to switch jobs between sectors — is an essential ingredient of the model as it captures rich and complex aspects of the competition between sectors over labor services. Moreover, as is understood since at least Burdett and Mortensen (1998), on-the-job search begets endogenous wage dispersion. However, in Burdett's model the public sector sets a single wage, leading to the counterfactual prediction that the private sector's response to competition from the public sector is to post a wage distribution with a hole in its support. In our model the public sector's policy is to post wages from a distribution. This assumption, which we further justify in the main text below, allows us to have wage differences in the public sector and a continuous private sector wage distribution with connected support. Unlike the models discussed above we allow for cross-sector differences in job destruction and job offer arrival rates. Crucially, this paper is unique in the literature insofar as the parameters of the model are structurally estimated.

Methodologically, a similar paper to ours is Meghir et al. (2015) who develop an equilibrium wage posting model to analyze interactions between a formal and informal sector in a developing country. Here the two sectors vary in the degree of regulatory tightness, the formal sector firms incurring additional costs to wages in the form of corporation tax, income tax, social security contributions, severance pay and unemployment insurance. While firms in the informal sector are not exposed to these labor market regulations they do face the risk of incurring a non-compliance cost. Similarly to this paper, Meghir et al. estimate their model using indirect inference.[4]

Our estimated model fits observed distributions of wages and transition rates well. Amongst

[4]Apart from the difference in focus, the paper by Meghir et al. (2015) differs from ours in two main respects: selection of firms into sectors and firm entry. In their model, a fixed population of potentially active private sector firms decide wether to be active at all and, conditional on being active, endogenously select into either the formal or informal sector. The equilibrium wage offer distributions of both sectors are determined endogenously. In our model, there is no endogenous selection of firms into a sector and the behavior of the public sector is exogenous. However, the population and productivity distribution of private sector firms is endogenous, determined by a free entry condition that holds at all productivity levels (see Subsection 2.3).

other things, the model captures a positive public premium for low-wage workers, and a zero to negative one for high-wage workers. Yet, a majority of private sector workers would require a large pay increase to motivate them to take up a job in the public sector. The attractiveness of private sector jobs is primarily driven by a faster upward wage mobility (or equivalently higher returns to experience) in the private sector. For low-wage workers, this dominates the value of higher job security offered by the public sector.

In order to demonstrate how the modeling approach can be used to assess public sector wage and employment policies we run simulations that attempt to mimic some of the austerity measures implemented across Europe after the 2008 recession. The specific policies we consider are: a reduction in public sector hiring, an increase in public sector layoffs, and progressive and proportional cuts to the distribution of wages offered by the public sector. All four policies have similar aggregate effects: increased hiring by the private sector; increased turnover within the private sector; decrease in public sector employment which is largely compensated by an increase in private sector employment, summing up to very moderate changes in aggregate unemployment; and finally, a very small impact on mean wages in both sectors and in the aggregate economy. The main difference between those four policies is in their impact on the composition of employment in each sector: while the first three policies (decreased public sector hires, increased layoffs, and across-the-board wage cut) are close to neutral in terms of employment composition, the fourth one, a wage cut targeted to high skilled workers, substantially reduces the share of high skilled workers in public sector employment. While our model offers no way of assessing the full cost of this shift in public workforce composition, its predictions are still a warning against further cuts in high-skill public sector wages, starting from a situation where public sector careers are, because of wage compression in the public sector, already relatively unattractive to high earners.[5]

The paper is organized as follows. In the next section we derive the equilibrium structural model. Section 3 gives an overview of the data and outlines the estimation protocol. Section 4 presents the estimation results. In Section 5 we use the results obtained to run counterfactual

[5]As discussed in the final section of this paper, a similar conclusion is reached, in a somewhat different context, by Gomes (2015).

policy analysis and in Section 6 we conclude.

2 The Model

2.1 Basic Environment

We consider a model of wage-posting akin to Burdett and Mortensen (1998). Time is continuous and the economy is in steady state. A fixed $[0, N]$ continuum of risk neutral, ex-ante homogeneous workers face an endogenous continuum of employers in a frictional labor market. A key aspect of our approach is that the set of employers comprises a continuum of infinitesimally small heterogeneous, profit-maximizing firms which we interpret as Private Sector employers, that coexist with a single, non-infinitesimal, non-profit maximizing employer which we interpret as the Public Sector. Private-sector firms behave in the same way as employers in the standard Burdett and Mortensen (1998) model, while public sector wage offers and labor demand are taken as exogenous. Those modeling choices, discussed in greater detail below, were made to strike a balance between tractability, plausibility, and their ability to capture features of the data and economic mechanisms that are relevant to our objective. Specifically, within the class of models allowing for on-the-job search and cross sector mobility (an essential ingredient of our analysis, as we argued in the Introduction), we had to make a choice of wage determination mechanism. We opted for wage posting over alternatives based on individual bargaining because the latter seemed less plausible to us as a representation of wage setting in the public sector. Ideally, we could have allowed for some degree of bargaining over private sector wages, but having different wage setting mechanisms between the two sectors would have made the model considerably less tractable and transparent.

Because of the public sector's non-infinitesimal size as an employer, changes in public sector employment policies will have a non-trivial impact on labor market equilibrium, both directly and through the private sector's response to said changes in policy. The main objective of this paper is to quantify that impact for various policy changes.

2.2 Workers and Jobs

A worker can be in one of three states, either unemployed or employed in the public or private sector. Throughout the paper we indicate a worker's labor market state using a subscript $s \in \{u, p, g\}$ for unemployment, employment in the private sector and employment in the public sector, respectively. The steady-state numbers of workers in each employment state are denoted as N_u, N_g and N_p.

A job is fully characterized by a constant wage w and the sector it is attached to. Workers receive job offers at a Poisson rate that depends on the worker's state. Jobs are not indefinite and also face a Poisson destruction shock. The notation used to describe all those shocks is largely consistent with the previous literature, δ being used to denote job destruction shocks, λ job offer arrival rates and μ retirement shocks. To explain the two states between which the particular worker transits a two letter index is used. The first letter designates the sector of origin and the second the sector of destination. So for example, λ_{pg} is the arrival rate of public-sector offers to private sector employees, λ_{ug} is the arrival rate of public-sector offers to unemployed workers, and so on. As job destruction always results in the worker becoming unemployed, a single index is used to specify the job destruction shock, δ_p or δ_g. The retirement shock μ is independent of a worker's labor market state and therefore requires no index. After retirement a worker exits the labor market indefinitely and is replaced by another worker who enters into unemployment.

To summarize, a worker employed in sector $s \in \{p, g\}$ faces four random shocks: a retirement shock μ, and it is assumed that retirement is of no value to the worker;[6] a job destruction shock δ_s, after which the worker becomes unemployed and gets flow utility b; a within sector job offer λ_{ss}; and a cross-sector job offer $\lambda_{ss'}$. While the arrival rates from the public sector are exogenous, arrival rates from the private sector and the associated acceptance decisions are determined endogenously.

As in Burdett and Mortensen (1998), a job offer from the private sector consists of a draw from a wage offer distribution $F_p(\cdot)$ which results from uncoordinated wage posting by the set of infinitesimally small private employers, each maximizing its profit taking as given the strategies of other firms and that of the public sector. $F_p(\cdot)$ will be determined endogenously in equilibrium. By contrast, a job offer from the public sector consists of a draw from a (continuous) wage offer distribution $F_g(\cdot)$ which is taken as an exogenous policy tool. We thus assume from the outset that the public sector offers jobs at different wage levels to observationally similar workers. This

[6]We acknowledge that the value of retirement is likely to be determined by the current labor market state and a worker's labor market history. However this introduces employment history as a state variable and would complicate the model significantly.

assumption is both realistic — residual wage dispersion among similar public-sector employees is observed in the data — and needed to avoid the counterfactual prediction of Burdett (2011) that equilibrium features a wage distribution with disconnected support.[7] Furthermore, it makes theoretical sense: because of wage dispersion in the private sector, the public sector seeks to hire workers with dispersed reservation wages.[8] As such, whatever objective one chooses to ascribe to the public sector — this could be cost minimization subject to providing a given amount of public good, or surplus ("profit") maximization, either at the aggregate level, or locally by different independently managed branches and services that compete with one another... In this paper we choose to remain agnostic on the public sector's objective function —, it will generally be in the public sector's interest to post dispersed wage offers.[9]

We finally recognize that public and private sector jobs may differ along other dimensions than just the wage and the transition parameters. There may be, for example, systematic differences in working conditions. Also workers may enjoy a utility surplus ('public service glow') or suffer a utility loss ('public service stigma') from working in the public sector. In order to capture those unobserved differences in a parsimonious way, we assume that the flow utility that workers derive from working in sector $s \in \{p, g\}$ for a wage of w is equal to $w + a_s$, where a_s is a sector-specific 'amenity'. Finally, and without further loss of generality, we normalize a_p to zero, so that a_g reflects the relative utility surplus (or loss) from working in the public sector. This utility surplus is assumed to be the same for all workers.

[7] In Burdett's model, the overall (public + private) wage offer distribution has an atom at the unique public sector wage, say $w_g^\star$. Moreover, private firms are homogeneous, hence achieve equal profits in equilibrium. Now, if a private firm posted a wage $w_g^\star - \varepsilon$, it would make the same profit per worker as a private firm posting a wage $w_g^\star + \varepsilon$, but lose a mass of workers to the public sector compared to the private firm posting $w_g^\star + \varepsilon$, resulting in discretely lower profits going to the firm posting $w_g^\star - \varepsilon$. Therefore, there can be no equilibrium with private firms posting wages arbitrarily close to but below $w_g^\star$, hence the hole in the offer distribution.

[8] Burdett (2011) characterizes the optimal public sector wage *under the assumption that it posts a single wage*, but does not explore the case where the public sector is allowed to post a distribution of wages.

[9] The specific conditions under which this will be the case are difficult to characterize in general and depend not only on the public sector's objective function, but also on the shape of the distribution of wages in the private sector. Yet it is possible to construct simple examples (available upon request) where the public sector optimally posts dispersed wages.

2.2.1 Worker Values and Reservation Wages

An individual's utility is given by the present discounted sum of future wages. For a given worker, the transitional parameters will be unchanged if he moves job within a sector. The acceptance decision for an offer within the worker's current sector is therefore entirely determined by the worker's current wage and the new wage being offered. If the new offer, x, is higher than the worker's current wage, w, he will accept and otherwise reject. However, since a change in sector is not only associated with a different wage but also with a change in transitional parameters, the acceptance decision is not so trivial when the job offer is from another sector. Thus depending on the two sets of transition parameters, an individual may accept a job offer from a different sector with a wage cut, or conversely, require a higher wage in order to accept. These acceptance decisions can be characterized by a set of reservation wages. With the three states we have defined, there will be four corresponding reservation wages, which we define using the same double-index system as for transition parameters: R_{up}, R_{ug}, $R_{pg}(w)$ and $R_{gp}(w)$.

When employed, a worker's reservation wage will be a function of their current wage. The reservation wage applying to private (public) sector offers made to a public (private) sector worker earning w makes said worker indifferent between his current present value and the present value of private (public) sector employment at his reservation wage. Formally, that is $W_p(R_{gp}(w)) = W_g(w)$ and $W_g(R_{pg}(w)) = W_p(w)$, where $W_p(w)$ and $W_g(w)$ are the values of working in the private and public sectors at wage w. It follows from those definitions that the two reservation wages described are reciprocal of each other:

$$R_{pg}(R_{gp}(w)) = w. \tag{1}$$

The reservation wage of an unemployed worker receiving an offer from the public (private) sector is the wage at which they are indifferent between unemployment and the public (private) sector. Formally, the two reservation wages solve the equality, $U = W_p(R_{up}) = W_g(R_{ug})$, where U is the present value of a worker in unemployment. Hence applying (1) to this equality one can derive a second property of the reservation wages:

$$R_{pg}(R_{up}) = R_{ug}. \tag{2}$$

Note that the analogous property for $R_{gp}(\cdot)$ also holds.

2.2.2 Bellman Equations

The value function for an unemployed worker is defined by the following Bellman equation, where r is the rate of time preference and the effective discount rate $(r+\mu)$ is constant across workers:

$$(r+\mu)U = b + \lambda_{up} \int_{R_{up}}^{+\infty} [W_p(x) - U]\, dF_p(x) + \lambda_{ug} \int_{R_{ug}}^{+\infty} [W_g(x) - U]\, dF_g(x), \tag{3}$$

The first term, b is the flow utility an individual gets from being in unemployment. Offers arrive from the public (private) sector at a rate of λ_{ug} (λ_{up}). Wage offers, x are drawn from the private sector from an endogenous distribution, $F_p(w)$, which will be derived from the firm side later. An unemployed worker will accept the job offer if the wage is higher than the worker's reservation wage for that sector, the lower bound of the integral. Inside the integral is the gain the worker makes from switching from unemployment to private sector employment at wage w. The final term is the public sector analogue to the second. The theoretical difference between the two is that the distribution from which public-sector job offers are drawn is an exogenous policy parameter of the model.

Similar value functions define a worker employed in the private and public sectors. Below is the example for a private sector employee:

$$\begin{aligned}(r+\mu)W_p(w) &= w + \delta_p [U - W_p(w)] \\ &\quad + \lambda_{pp} \int_{w}^{+\infty} [W_p(x) - W_p(w)]\, dF_p(x) + \lambda_{pg} \int_{R_{pg}(w)}^{+\infty} [W_g(x) - W_p(w)]\, dF_g(x)\end{aligned} \tag{4}$$

A worker employed in the private sector and earning a wage w has a discounted value from employment given by the right hand side of (4). The first term w is the instantaneous wage paid in the current private sector firm. The next term, is the loss of value an individual would get if he were to transit into unemployment $[U - W_p(w)]$ multiplied by the flow probability of such an event occurring, the private sector job destruction rate, δ_p. At rate λ_{pp} the worker receives an offer from another private sector firm, where the offer is drawn from the distribution $F_p(x)$. If this offer is greater than his current wage w he will accept, hence the lower bound of the integral. Given the offer is received and meets his acceptance criterion, the individual will make a gain in value given by $[W_p(x) - W_p(w)]$. The next term represents the equivalent, except for offers from the public sector. Thus the wage is drawn from a different distribution and the acceptance criteria, the lower bound of the integral, is instead $R_{pg}(w)$. An analogous Bellman equation defines the value function for a worker in the public sector:

$$(r+\mu)W_g(w) = w + a_g + \delta_g [U - W_g(w)] \\ + \lambda_{gp} \int_{R_{gp}(w)}^{+\infty} [W_p(x) - W_g(w)]\, dF_p(x) + \lambda_{gg} \int_{w}^{+\infty} [W_g(x) - W_g(w)]\, dF_g(x) \,. \quad (5)$$

Note the presence of the additional flow utility term a_g, the 'public-sector amenity' discussed above.

The value functions given by (3), (4) and (5) allow us to obtain the reservation wage required to leave the private for the public sector and vice-versa as a function of the transition parameters. This is done using the identity $W_p(R_{pg}(w)) = W_g(w)$ and $W_g(R_{gp}(w)) = W_p(w)$ and assuming differentiability of the value functions. This manipulation is performed in Appendix A and the solution for a private sector worker's reservation wage from the public sector solves the following non-linear ODE:[10]

$$R'_{pg}(w) = \frac{r + \mu + \delta_g + \lambda_{gp}\overline{F}_p(w) + \lambda_{gg}\overline{F}_g(R_{pg}(w))}{r + \mu + \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))}, \quad (6)$$

with initial condition $R_{pg}(R_{up}) = R_{ug}$. It should be noted that R_{up} and R_{ug} themselves depend on the functions $R_{pg}(\cdot)$ and $R_{gp}(\cdot)$ as they are obtained by solving $W_s(R_{us}) = U$ for $s = p$ or g. However, they also depend on unemployment income flows (the bs), which are free parameters, so those reservation wages can themselves be estimated as free parameters.

2.2.3 Flow-Balance Equations and Worker Stocks

The economy being in steady-state, the flows in and out of any given sector, for each class of workers, are equal. Applying this to unemployment, one obtains:

$$(\lambda_{up} + \lambda_{ug}) N_u = \delta_p N_p + \delta_g N_g + \mu (N_p + N_g) \quad (7)$$

The left hand side of (7) is the rate at which workers leave unemployment toward the two sectors of employment. This occurs when a worker receives a job offer from a given employment sector and

[10] Here and throughout the rest of the paper, a bar over a c.d.f. denotes the survivor function, so for example $\overline{F}_p(\cdot) := 1 - F_p(\cdot)$.

the associated wage offer is higher than his appropriate reservation wage. Assuming homogeneous workers, there is no reason why a firm would offer wages below a worker's reservation wage (and if it did, it would employ no worker and therefore become irrelevant to market equilibrium). Therefore we assume without (further) loss of generality that $F_p(R_{up})$ and $F_g(R_{ug})$ are equal to zero. The right hand side is the unemployment inflow, which consists of workers being hit by job destruction shocks in their sector of employment, plus employed workers being hit by retirement shock, who are leaving the labor force and immediately replaced one-for-one by a worker in unemployment. A worker can only be in one of three states, u, p or g so: $N_u + N_p + N_g = N$, where N is the total population of workers, a given number.

Equation (8) is the flow-balance equation for private sector workers, equating the flow into the private sector below a wage w to the flow out, thus imposing that not only is the share of private sector workers constant in the steady state, but so is the distribution of wages amongst them. The left hand side is the flow out of private employment. $N_pG_p(w)$ is the number of private sector workers earning less than a wage w. They exit the labor force at a rate μ and to unemployment through job destruction shocks δ_p. The second and third terms are the exit rates into the public sector and higher paid private sector jobs, respectively, upon receiving a job offer (λ_{pg} and λ_{pp}). The right hand side is the flow into private sector employment, the first term being the flow from unemployment and the second, from the public sector.

$$N_p(\mu+\delta_p)G_p(w) + N_p\lambda_{pg}\int_{R_{up}}^{w}\overline{F}_g(R_{pg}(x))\,dG_p(x) + N_p\lambda_{pp}\overline{F}_p(w)\,G_p(w)$$
$$= N_u\lambda_{up}F_p(w) + N_g\lambda_{gp}\int_{R_{ug}}^{R_{pg}(w)}\left[F_p(w) - F_p(R_{gp}(x))\right]dG_g(x) \quad (8)$$

Rearranging equation (8) and differentiating with respect to the wage rate, w, one obtains:

$$\frac{d}{dw}\left\{\left[\mu+\delta_p+\lambda_{pp}\overline{F}_p(w)\right]N_pG_p(w)\right\} + N_pg_p(w)\,\lambda_{pg}\overline{F}_g(R_{pg}(w))$$
$$- N_g\lambda_{gp}G_g(R_{pg}(w))\,f_p(w) = N_u\lambda_{up}f_p(w)\,. \quad (9)$$

This would be a fairly straightforward ODE in $G_p(w)$ if it was not for the term featuring $G_g(R_{pg}(w))$. This term can be derived by manipulation of the flow balance equation for public sector workers earning less than $R_{pg}(w)$ (instead of w). This manipulation is performed in Appendix A. Plugging this solution into (9), we obtain an ODE that defines $G_p(w)$.

An additional hurdle at this point is the determination of N_p and N_g (with $N_u = N - N_p - N_g$). Those numbers are needed to solve for $G_p(\cdot)$ in the ODE resulting from the combination of (9) and the isolation of $N_g G_p(R_{pg}(w))$, given in the appendix. Now, N_p and N_g are jointly defined by the balance of flows in and out of employment (7), and the flow balance in and out of, say, the private sector, which is given by evaluating the flow-balance equation of private sector workers, equation (8) at $w \to +\infty$:

$$N_p(\mu + \delta_p) + N_p\lambda_{pg} \int_{R_{up}}^{+\infty} \overline{F}_g\left(R_{pg}(x)\right) dG_p(x) \\ = N_g\lambda_{gp} \int_{R_{ug}}^{+\infty} \overline{F}_p\left(R_{gp}(x)\right) dG_g(x) + N_u\lambda_{up}\overline{F}_p(R_{up}) \quad (10)$$

The distribution, $G_g(\cdot)$, can be derived using the identity $R_{pg}(R_{gp}(w)) = w$ applied to the derivation of $G_p(R_{pg}(w))$ in the appendix. The latter equation involves $G_p(\cdot)$, which in turn depends on N_p and N_g, so that those three objects have to be solved for simultaneously. This will be done using an iterative procedure.

2.3 Private Sector Firms

There exists a continuum of private sector firms, the mass and productivity profile of which are determined by a free entry condition. Firms are profit maximizers and select into their heterogeneous level of productivity, $y \in [y_{\min}, y_{\max}]$ upon entry, as will be described momentarily. Firms set their wage w and their search effort (number of vacancies or job adverts) in order to make a number of hires h. The pair (w, h) is chosen so as to maximize steady-state profit flow. A private sector firm choosing to pay w experiences a quit rate of $\Delta(w)$ of its employees, where:

$$\Delta(w) = \mu + \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))$$

As a consequence, the steady-state size of this firm is $\ell(w, h)$:

$$\ell(w, h) = \frac{h}{\Delta(w)} \quad (11)$$

If this firm has productivity y, its steady-state profit flow is:

$$\Pi(w,h;y) = (y-w)\,\ell(w,h) - c(h) - \kappa(y) = (y-w)\,\frac{h}{\Delta(w)} - c(h) - \kappa(y) \tag{12}$$

where $c(h)$ is the cost incurred by the firm to make h hires and $\kappa(y)$ is a fixed flow production cost. It is assumed that both $c(\cdot)$ and $\kappa(\cdot)$ are strictly increasing functions and that $c(\cdot)$ is strictly convex.[11] Because of free entry, firms continue to enter until profit opportunities from doing so are exhausted, so that in equilibrium $\Pi(w,h;y) = 0$ for all firms. The measure of firms at productivity level y and below, determined by this free entry condition, is denoted as $\Gamma(y)$. Optimal wage and search policies $w^\star(y)$ and $h^\star(y)$ can be characterized using the following first-order conditions:

$$y = w^\star - \frac{\Delta(w^\star)}{\Delta'(w^\star)} \tag{13}$$

$$c'(h^\star) = \frac{-1}{\Delta'(w^\star)} \tag{14}$$

The proportion of all offers that are accepted from a private sector firm offering a wage w is $\alpha(w)$.

$$\alpha(w) = \frac{\lambda_{up}N_u + \lambda_{pp}N_pG_p(w) + \lambda_{gp}N_gG_g\left(R_{pg}(w)\right)}{\lambda_{up}N_u + \lambda_{pp}N_p + \lambda_{gp}N_g}$$

It follows that the total number of contacts in the economy is the number of hires divided by the acceptance rate:

$$M = \int_{y_{\min}}^{y_{\max}} \frac{h^\star(y)}{\alpha(w^\star(y))}d\Gamma(y), \tag{15}$$

and that the fraction of these contacts that is attached to a wage lower than a given w, in other words the probability that a wage offer is less than w can be written in the two following manners (the left and right hand sides of equation (16)):

$$F_p(w) = \frac{1}{M}\int_{y_{\min}}^{\tilde{y}} \frac{h^\star(y)}{\alpha(w^\star(y))}d\Gamma(y), \tag{16}$$

where $\tilde{y}$ is such that $w^\star(\tilde{y}) = w$.[12]

[11] We think of $c(h)$ as a training cost. This differs slightly from the standard specification in the literature, which conventionally models recruitment costs as a vacancy posting cost (although there are increasingly many exceptions to this tradition). We adopt this training cost specification both for the algebraic simplicity it affords, and because of the presumption that training, rather than vacancy posting costs make up the bulk of overall hiring costs.

[12] This argument assumes that $y \mapsto w^\star(y)$ is an increasing function. This holds by standard comparative static arguments, given that the firm's objective function as increasing differences: equation (12) implies that $\Pi''_{yh} > 0$, $\Pi''_{yw} > 0$, and $\Pi''_{wh} = 0$ at the firm's optimum.

Similarly, the fraction of employees earning a wage less than $w^{\star}(y)$ do so because they are employed by firms with a productivity lower than y. Thus:

$$H\left[\ell\left(w^{\star}(y), h^{\star}(y)\right)\right]=G_p\left[w^{\star}(y)\right], \tag{17}$$

where $H(\cdot)$ is the distribution of firm sizes among employed workers.

We are now in a position to close the model given public sector policy choices and endogenize private sector job offer arrival rates. To this end, we need to make one final assumption — that the relative search intensities of workers in the three labor market states, i.e. unemployment, employment in the private sector and employment in the public sector are constant. These will be denoted s_{up} (normalized to 1 without loss of generality), s_{pp} and s_{gp} respectively. The arrival rates of private sector offers hence have the following expressions:

$$\lambda_{up}=\lambda_p \qquad \lambda_{pp}=s_{pp}\cdot\lambda_p \qquad \lambda_{gp}=s_{gp}\cdot\lambda_p.$$

The private sector job offer arrival rate λ_p (per search efficiency unit) can then be recovered from the following and equation (15), for a given distribution of productivity amongst firms $\Gamma(y)$:

$$M=\lambda_p\left(N_u+s_{pp}N_p+s_{gp}N_g\right). \tag{18}$$

This and equation (16) illustrate the private sector firms' response to changes in public sector policy in terms of search effort (or number of offers) and wage offer distribution. Those equations determine λ_p and $F_p(\cdot)$ respectively, given public sector hiring policy, embodied in the $\{\lambda_{ug}, \lambda_{pg}, \lambda_{gg}\}$ rates, wage offer policy, embodied in the distribution $F_g(\cdot)$ and "job security" policy, embodied in the public sector layoff rate δ_g. Note however that we do not consider any response of the private sector in terms of its layoff rate δ_p.

3 Data and Estimation

We now outline our estimation protocol, which is based on minimum-distance matching of certain descriptive moments of the data.[13] We set the discount rate r ex-ante at 0.004 (where one unit of time is a month), implying an annual rate of approximately 5%. The Poisson retirement rate is set

[13] For a comprehensive overview of related simulation-based methods, see Gouriéroux et al. (1993).

at 0.002, meaning the average working life is 40 years. Θ, given below is the exogenous parameter vector which we intend to estimate:

$$\Theta = [b, a_g, \delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg}, F_p(\cdot), F_g(\cdot), \Gamma(\cdot), c(\cdot), \kappa(\cdot)]$$

Note that the two offer distributions (F_p and F_g), the distribution of firm types (Γ), the hiring cost function $c(\cdot)$ and the fixed production cost $\kappa(\cdot)$ all feature in the list of parameters. As will become clear below, those distributions and cost functions are non-parametrically identified. However, for numerical tractability, we will make parametric assumptions on F_p and F_g as outlined later.

The rest of this section focuses on obtaining estimates for the vector Θ. We begin by describing the moments we match and how we obtain them from the data, and we then describe in detail the estimation procedure. Results are presented in the next section.

3.1 The Sample

The data used in the analysis are taken from the BHPS, a longitudinal data set of British households. Data were first collected in 1991 and the households selected were determined by an equal probability sampling mechanism.[14] Since then, there have been 18 further waves, collected annually. The model outlined is derived under a steady state assumption. Therefore it is necessary that the time period used is short and has approximately constant shares in each of the three states across time. We choose data from 2004 to 2008 to satisfy this assumption, allowing long enough time after the Conservatives' drive toward privatization in the 80s and 90s but before the Great Recession of 2008.

Using retrospective accounts of employment history we construct a panel dataset of respondents at a monthly frequency. We include in our data those who across our panel reach at least 25 years of age and don't exceed 60. Wages are CPI-adjusted and we trim the wage distributions in each sector, treating data as missing if it is below the 1st or above the 99th percentile in either employment sector. We also exclude individuals with holes in their employment history and once someone becomes inactive they are from then on excluded. Thus, consistent with our model, an agent can

[14] From wave 9 the BHPS was extended to include Scotland and Wales and from wave 11, Northern Ireland. All three regions are over represented in the sample and therefore we weight the data accordingly.

Table 1: Descriptive Statistics

	Private Sector	Public Sector	Unemployment
size of each sector:			
all	72.0 %	25.6%	2.4%
Male university graduates	75.1%	23.6%	1.3%
Male < university	84.3%	12.0%	3.8%
Female university graduates	49.5%	49.2%	1.3%
Female < university	67.4%	30.7%	1.9%
mean hourly earnings			
all	11.99	12.72	-
Male university graduates	18.52	17.97	-
Male < university	11.76	12.11	-
Female university graduates	13.94	15.21	-
Female < university	8.73	9.70	-
standard deviation of hourly wages:			
all	6.91	5.73	-
Male university graduates	10.36	7.96	-
Male < university	5.60	4.54	-
Female university graduates	7.66	5.82	-
Female < university	4.07	3.80	-

Source: Data comes from the BHPS, moments are based on 4,442 individuals between 2004 and 2008, inclusive.

be in one of three states, unemployment or employment in the private or public sectors. We define private sector employment as anyone who declares themselves as employed in a private sector firm, non-profit organization or in self-employment and public sector employment as in the civil service, central or local government, the NHS, higher education, a nationalized industry, the armed forces or a government training scheme.

The two sectors vary in their composition of workers, particularly in gender and human capital (see Table 1). We therefore divide our sample into four strata, defined by gender and education. After stratifying for different levels of education we find that university degree results in the largest difference in the employment levels and wage distributions. We provide estimates separately for four subsamples comprising, respectively, 661 university educated males, 1,568 males without a

Table 2: Job mobility within and between sectors

University Educated Men:	Private Sector	Public Sector	Unemployment
Private Sector	0.0125	0.0011	0.0022
Public Sector	0.0033	0.0089	0.0011
Unemployment	0.1541	0.0302	—
Less than University Men:	**Private Sector**	**Public Sector**	**Unemployment**
Private Sector	0.0123	0.0003	0.0037
Public Sector	0.0030	0.0068	0.0019
Unemployment	0.0882	0.0076	—
University Educated Women:	**Private Sector**	**Public Sector**	**Unemployment**
Private Sector	0.0138	0.0021	0.0029
Public Sector	0.0024	0.0078	0.0007
Unemployment	0.1102	0.0424	—
Less than University Women:	**Private Sector**	**Public Sector**	**Unemployment**
Private Sector	0.0115	0.0017	0.0025
Public Sector	0.0026	0.0063	0.0012
Unemployment	0.1072	0.0308	—

Note: Transition rates are monthly. Rows do not add up to one. The two entries on the main diagonal are the fractions of workers *changing jobs* within the private and the public sector, respectively.
Source: Data comes from the BHPS, rates are based on 4,442 individuals between 2004 and 2008, inclusive.

university education, 649 university educated females, and 1,564 females without a university education, all of whom we follow for a maximum of 5 years. There is some attrition which we assume to be exogenous. Table 1 shows some basic descriptive statistics. Consistent with the literature on the public-private sector relationship, we find the British public sector is better educated, predominantly female, on average receive higher wages for which there is less dispersion within the sector. Also, consistent with the gender inequality literature we find that males earn more than their female counterparts and their earnings are also more dispersed.

Table 2 conveys information about the extent of job mobility, both within and between sectors. Counting in each month the number of people making each type of transition and the number in each state, we construct monthly cross-sector transition matrices. Averaging these across our time period, we obtain the transition matrix shown in Table 2. A transition within employment is defined if a worker reports a change of job and begins work in a new establishment, without reporting an intervening spell of unemployment. To avoid spurious cross sector mobility, for movements across sectors, we further condition on there also being a change in employer.[15]

Private sector workers are, on the whole, more mobile than public sector workers. The cross sector difference in mobility rate is particularly stark for women. Both high skill and low skill private sector female workers are approximately 50% more likely to change jobs in a given month than their public sector counterparts. A closer look reveals that private sector workers experience much more frequent *within-sector* job changes than their public-sector counterparts. Mobility *between* employment sectors, however, is dominated by public sector employees moving to the private sector, cross-sector mobility in the other direction being a comparatively rare event. The separation rate into unemployment is significantly smaller in the public sector than in the private sector. Finally, perhaps the most important conclusion to be drawn from Table 2 is that direct, job-to-job reallocation *between employment sectors* is substantial: given the transition rates in Table 2 and the various sectors' relative sizes given in Table 1, one can infer that about 20 percent of the employment inflow into the private sector comes from the public sector, and that about 30 percent of the private sector employment outflow goes into the public sector. High skill workers seem to have the best of both worlds, with higher rates of job movement and lower job destruction.

In addition, we have data on the distribution of firm sizes in the population of employed workers in the private sector. These data are taken from the Inter Departmental Business Register (IDBR) which contains information on VAT traders and PAYE employers in a statistical register representing nearly 99% of economic activity. A caveat that applies to this particular data is that it refers to employers' sizes in terms of all employees' skills combined, whereas our estimations are carried out on subsets of the data stratified by skills. Given our stylized modeling of the firm as a single-input constant-returns-to-scale production unit, we shall ignore this issue by assuming that either the distribution of firms' sizes among employees is the same across skill groups, or that the optimal number of hires h derived by a firm is shared in constant proportions between the different skill groups.

[15] A large fraction of reported public/private sector changes in the BHPS are *not* accompanied by a reported change of employer. We take those as spurious transitions caused by either recall error or misinterpretation of the survey question. We thank an anonymous referee for drawing our attention to this important data issue.

3.2 Estimation

Identification of the model's parameters Θ comes from two data sources: observed transitions between labor market states and observed wage distributions. Data on the distribution of firms' sizes allow us to retrieve estimates of $\Gamma(\cdot)$, $c(\cdot)$ and $\kappa(\cdot)$.

Observed sector-specific wage distributions are direct empirical counterparts to $G_p(\cdot)$ and $G_g(\cdot)$ in the model. While neither has a closed-form solution, both can be simulated given parameter values. In order to map the wage distribution well, we take as moments to be matched 50 quantiles of each distribution, giving 100 moments in total: $\{w_{s,j}\}_{s=1,2,j=1,\cdots,50}$.

Turning to transition moments, we match the eight transition rates reported in Table 2. Denoting these as $\pi_{ss'}$ where s is the state of origin and s' the state of destination, we thus add eight moments to match: $(\pi_{up}, \pi_{ug}, \pi_{pu}, \pi_{pp}, \pi_{pg}, \pi_{gu}, \pi_{gp}, \pi_{gg})$. The theoretical counterparts of those monthly transition rates are given by the probabilities of a certain type of transition occurring within a one-month period. The theoretical counterparts of π_{pu}, π_{pp}, π_{pg}, π_{gu}, π_{gp} and π_{gg} all have similar expressions: $\pi_{ss'}^{\text{model}}$ is constructed by taking the probability that an exit from state $s = p$ or g, given wage w, occurs before one month has elapsed, multiplying it by the conditional probability of exiting toward s', given that an exit occurs and given initial wage w, then finally integrating out w using the relevant initial wage distribution, $dG_s(w)$. For example:

$$\pi_{pp}^{\text{model}} = \int_{R_{up}}^{+\infty} \frac{\lambda_{pp}\overline{F}_p(w)\left(1 - e^{-\left(\mu+\delta_p+\lambda_{pp}\overline{F}_p(w)+\lambda_{pg}\overline{F}_g(R_{pg}(w))\right)\times 1}\right)}{\mu + \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))} dG_p(w),$$

where the "$\times 1$" term in the exponential is there as a reminder that π_{pp} is a monthly transition probability and that all the flow parameters (δ_p, λ_{pp}, etc.) are monthly. The theoretical transition rates from unemployment are simpler (as there is no wage to integrate out):

$$\pi_{up}^{\text{model}} = \frac{\lambda_{up}\left(1 - e^{-(\mu+\lambda_{up}+\lambda_{ug})\times 1}\right)}{\mu + \lambda_{up} + \lambda_{ug}},$$

and symmetrically for π_{ug}^{model}.

As for the estimation of the function $c(h)$, i.e. the cost of making h hires for a private sector firm, we will be using the 12 cutoffs of the distribution of firms' sizes within private sector employment, denoted $\{H(\ell_i^c)\}_{i=1,\cdots,12}$, where $H(\cdot)$ is the cumulative distribution function of firm sizes among private sector employees and the ℓ_i^c's are the 12 size cutoffs for which employment sizes are grouped into in the IDBR. As will be discussed in the next section, the distribution of firm productivities in the population of firms, $\Gamma(y)$, will then be estimated by matching 50 points of the private sector wage *offer* distribution corresponding to the observed wage quantiles seen above, $\{F_p(w_{p,j})\}_{j=1,\cdots,50}$. Finally, the fixed production cost $\kappa(\cdot)$ is given by the free entry condition $\Pi(w,h;y)=0$.

3.3 Estimation Procedure

We first estimate the first twelve components of Θ by matching the 108 moments described above, leaving $\Gamma(\cdot)$ and $\kappa(\cdot)$ out. $\Gamma(\cdot)$ is backed out in a final step as the underlying private firm productivity distribution that rationalizes the estimates of $F_p(\cdot)$ and $F_g(\cdot)$ obtained in previous steps, and $\kappa(\cdot)$ is backed out as the production cost that rationalizes the distribution of firm sizes given free entry, i.e. such that $\Pi(w,h;y)=0$. We also make the following parametric assumptions about $F_p(\cdot)$ and $F_g(\cdot)$.[16] First we assume that the wages offered to private sector workers follow a generalized Pareto distribution with scale parameter $\sigma_p \in (0,\infty)$ and shape parameter $\xi_p \in (-\infty,\infty)$. The support of the distribution is adjusted such that the infimum is at R_{up} and the supremum is $\overline{w}_p$ which is set equal to the top percentile in the observed wage distribution.

$$F_p(w) = \begin{cases} 1-\left(1+\frac{\xi_p(w-R_{up})}{\sigma_p}\right)^{-\frac{1}{\xi_p}} & \text{if } w \in [R_{up},\overline{w}_p] \cap \xi_p \neq 0 \\ 1-\exp\left(-\frac{w-R_{up}}{\sigma_p}\right) & \text{if } w \in [R_{up},\overline{w}_p] \cap \xi_p = 0 \\ 0 & \text{if } w < R_{up} \\ 1 & \text{if } w > \overline{w}_p, \end{cases}$$

It then proves convenient to parameterize $F_g(\cdot)$ as equal to $F_p(\cdot)$ transformed through the CDF of a Beta distribution:

$$F_g\left(R_{pg}\left[F_p^{-1}(x)\right]\right) = B(x;\alpha_g,\beta_g), \qquad \text{for } x \in [0,1] \tag{19}$$

where $B(\cdot;\alpha_g,\beta_g)$ is the incomplete regularized beta function with parameters α_g and β_g, both strictly positive. Those parameters determine the shape of the distribution $F_g(\cdot)$. Note that the latter parameterization carries the implicit assumption that the lower support of $F_g(\cdot)$ is precisely R_{ug}. This assumption, although not implausible, has no real theoretical justification as the public

[16] In principle, $F_p(\cdot)$ and $F_g(\cdot)$ are non-parametrically identified: we could estimate as many quantiles of $F_s(\cdot)$ as we observe for $G_s(\cdot)$ $(s=p,g)$. We use parametric assumptions for numerical convenience.

sector is not assumed to be profit maximizing and as such may offer wages that are all strictly greater than the workers' common reservation wage. Experimenting with richer specifications, allowing for the lower support of $F_g(\cdot)$ to be strictly above R_{ug}, led to the conclusion that (19) is a valid approximation. The reservation wages are estimated as the minimum accepted wage in either sector.[17]

In order to match the moments described we implement a two-step algorithm. In a first step, we use the eight flow parameters $(\delta_p, \delta_g, \lambda_{up}, \lambda_{ug}, \lambda_{pp}, \lambda_{pg}, \lambda_{gp}, \lambda_{gg})$ to fit the eight transition rates derived from our model to those observed in the data, conditional on initial guesses about the offer distributions $F_p(\cdot)$ and $F_g(\cdot)$. This first, just-identified step produces a perfect fit to observed transition rates. Then, in a second step, conditional on the transition rates obtained from the first step, we derive the offer distributions that minimize the distance between the vector of quantiles of the empirical and theoretical wage distributions $G_p(\cdot)$ and $G_g(\cdot)$ (we use equal weights on all quantiles when computing this distance). The process is repeated until convergence. We find that, while admittedly inefficient, this iterative two-step protocol performs better a than one-step procedure in terms of speed of convergence and avoidance of local maxima.

Now turning to the estimation of the last two components of Θ, namely $c(h)$ and $\Gamma(y)$, we use the fact the larger firms pay higher wages, i.e. ℓ is increasing in w, which is consistent with our model (see appendix). This and data on $H(\ell_i^c)$ allow us to infer the wage rates w_i^c paid at each cutoff size ℓ_i^c:

$$G_p(w_i^c) = H(\ell_i^c) \qquad \text{for } i = 1, \cdots, 10 \tag{20}$$

where $\ell_i^c = \ell(w_i^c, h_i^c) = \ell(w^\star(y_i^c), h^\star(y_i^c))$. Now, with the pair (ℓ_i^c, w_i^c) at the 12 cutoff sizes, we are able to infer both h_i^c and $c'(h_i^c)$ at these 12 points thanks to equations (11) and (14):

$$h_i^c = \ell_i^c \Delta(w_i^c) \qquad \text{and} \qquad c'(h_i^c) = \frac{-1}{\Delta'(w_i^c)}$$

We thereby obtain a non-parametric estimate of the shape of $c'(\cdot)$ over a set of 12 points, from which we extrapolate the derivative of the cost function over the whole range of wages. We retrieve

[17]Since the first and last percentile of the two wage distributions are trimmed, the minimum wage in each sector corresponds with the first percentile of the earnings distribution. When the data are resampled for a bootstrap procedure this will allow for two sided variation in the reservation wages.

the cost function itself by integration, assuming $c(0) = 0$.

All that is left to estimate now are the distribution of productivities in the population of firms, $\Gamma(y)$, and the production cost, $\kappa(y)$. Productivity levels are derived from the first-order condition (13), which gives us a relationship $\tilde{y}(w)$, where $\tilde{y}$ is such that $w^\star(\tilde{y}) = w$. The number of hires $h^\star(y)$ is estimated by inverting the derivative of the cost of hire function in equation (14): $h^\star(\tilde{y}(w)) = c'^{-1}(-1/\Delta'(w))$. Manipulation of the expression for the wage offer distribution, equation (16) gives us an expression for $\Gamma(\tilde{y}(w))$ (this performed in the appendices). Thus we obtain the $\Gamma(\cdot)$ distribution that matches 50 points of the F_p distribution previously estimated. Finally, the fixed production cost $\kappa(\cdot)$ is obtained at the same set of productivity values directly from the free entry condition $\Pi(w, h; y) = 0$, which from (12) implies that $\kappa(\tilde{y}(w)) = (y(w) - w)\, h^\star(\tilde{y}(w))/\Delta(\tilde{y}(w)) - c(h^\star(\tilde{y}(w)))$.

4 Results

4.1 Labor Market Transitions

Parameter estimates of the model are given in Table 3,[18] of which the top two panels contain all transition parameters. The unit of time associated with the transition/offer arrival rates is a month. Again, given our estimates of the rest of the parameter vector, those transition rate values produce a perfect fit to the observed monthly transition rates reported in Table 2.

A striking feature of our parameter estimates is the large on-the-job offer arrival rates for workers in both sectors. Comparison of our estimates of λ_{pp}, λ_{pg}, λ_{gp}, and λ_{gg} with the corresponding monthly transition rates π_{pp}, π_{pg}, π_{gp}, and π_{gg} (see Table 2) suggests that in aggregate employed workers only accept approximately 6 percent of all the offers received. Moreover, employed workers, regardless of sector, receive offers substantially more frequently than unemployed workers do. Those results contrast with standard findings from simpler, one-sector wage-posting model. However, the same pattern arises in estimates obtained by Meghir et al. (2015) in a different two-sector wage

[18] Given in parentheses are the 95% confidence intervals. These are obtained by resampling the data, allowing for repetition and running the estimation protocol outlined previously on repeated redraws of the data. Transition rates are non-negative by construction across all redraws, so that their distributions across redraws is non-symmetric, as can be seen in Table 3. We therefore find displaying confidence intervals more informative than standard errors.

Table 3: Parameter Estimates

Parameters	Males		Females	
	University	< University	University	< University
δ_p	0.0022 (0.0016,0.0030)	0.0038 (0.0032,0.0044)	0.0030 (0.0019,0.0041)	0.0025 (0.0020,0.0030)
λ_{up}	0.1705 (0.1245,0.2339)	0.0928 (0.0761,0.1118)	0.1197 (0.0849,0.1699)	0.1155 (0.0920,0.1437)
λ_{pp}	0.2416 (0.1477,0.4271)	0.1085 (0.0838,0.1437)	0.2273 (0.1258,0.4957)	0.1423 (0.1037,0.1974)
λ_{pg}	0.0173 (0.0085,0.0401)	0.0023 (0.0011,0.0059)	0.0716 (0.0243,0.1808)	0.0261 (0.0160,0.0469)
δ_g	0.0011 (0.0005,0.0018)	0.0019 (0.0010,0.0030)	0.0007 (0.0003,0.0012)	0.0012 (0.008,0.0017)
λ_{ug}	0.0334 (0.0132,0.0576)	0.0080 (0.0041,0.0125)	0.0460 (0.0264,0.0671)	0.0331 (0.0235,0.0444)
λ_{gp}	0.0765 (0.0365,0.1407)	0.0364 (0.0170,0.0573)	0.0402 (0.0215,0.0949)	0.0398 (0.0253,0.0598)
λ_{gg}	0.1759 (0.0925,0.3847)	0.0629 (0.0372,0.1204)	0.2611 (0.1164,0.6751)	0.1188 (0.0741,0.2148)
σ_p	1.0198 (0.6738,1.6976)	2.1484 (1.4858,2.5325)	1.5390 (0.7605,0.24045)	1.2003 (0.8055,1.5213)
ξ_p	0.3925 (0.2937,0.4592)	0.1034 (0.1000,0.2145)	0.2239 (0.1132,0.3533)	0.1365 (0.1000,0.2336)
α_g	1.9032 (0.7050,4.1312)	1.4358 (0.3652,3.2602)	0.3366 (0.1000,1.5902)	0.5144 (0.2075,1.1003)
β_g	1.2259 (1.0620,1.3715)	0.9985 (0.7833,1.2024)	0.9047 (0.7979,1.0655)	0.8389 (0.7578,0.9315)
b	6.7143 (0.2526,10.1513)	3.1939 (1.1248,5.2511)	5.8239 (1.7171,8.3802)	2.8698 (0.9607,4.5116)
a	−0.7812 (−2.4541,0.7041)	−0.3257 (−1.2654,0.3808)	−1.2900 (−2.3973,−0.4464)	−0.4568 (−0.9149,−0.1481)

Note: 95% confidence intervals are given in the parenthesis.

posting model, with a formal and an informal sector estimated on Brazilian data.[19]

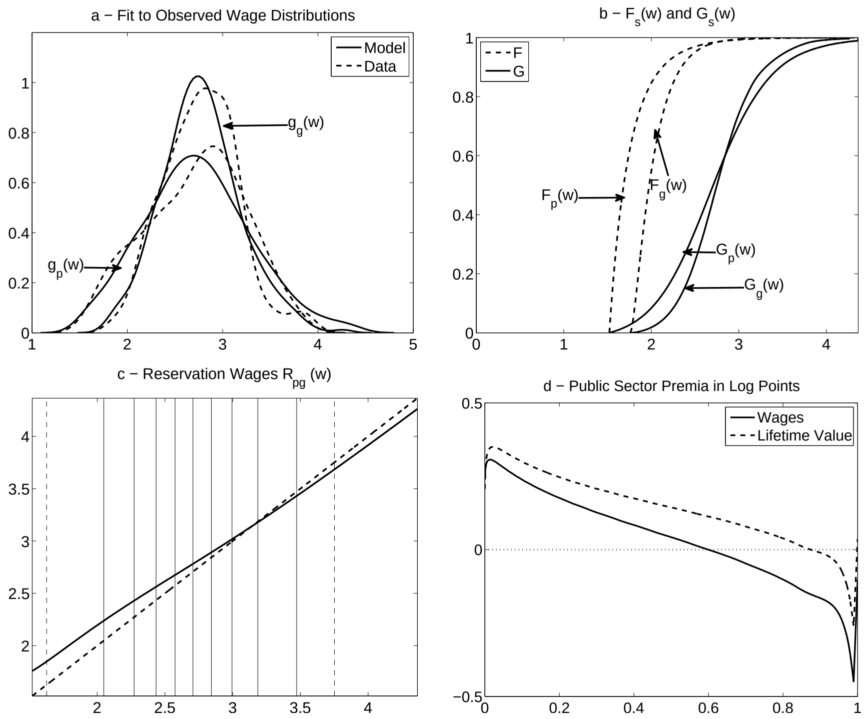
4.2 Wages and Worker Values

Table 3 also reports estimates of the wage offer distribution parameters, σ_p, ξ_p, α_g and β_g, and an estimate of the flow value of unemployment, b, which depending on sex and skill varies from about £2.87/hr for low skill women up to around £6.71/hr for high skill men. The values of b are estimated to imply an unemployed workers' private sector reservation wage R_{up} of £4.57/hr (high skill men), £2.72/hr (low skill men), £1.73/hr (high skill women), and £2.27/hr (low skill women), all taken directly from the data as explained in Subsection 3.3. The corresponding reservation wages for the public sector are: £5.80/hr (high skill men), £2.53/hr (low skill men), £6.13/hr (high skill women), and £3.35/hr (low skill women). The fact that R_{up} and R_{ug} can both be lower than b, which is unusual in empirical wage posting models, is a consequence of the relative values of offer arrival rates on- and off-the job: unemployed workers are prepared to sacrifice some income to benefit from the more efficient on-the-job search technology. Finally, there is a small public sector *stigma*, but it is small relative to wages and is only statistically significant for women.

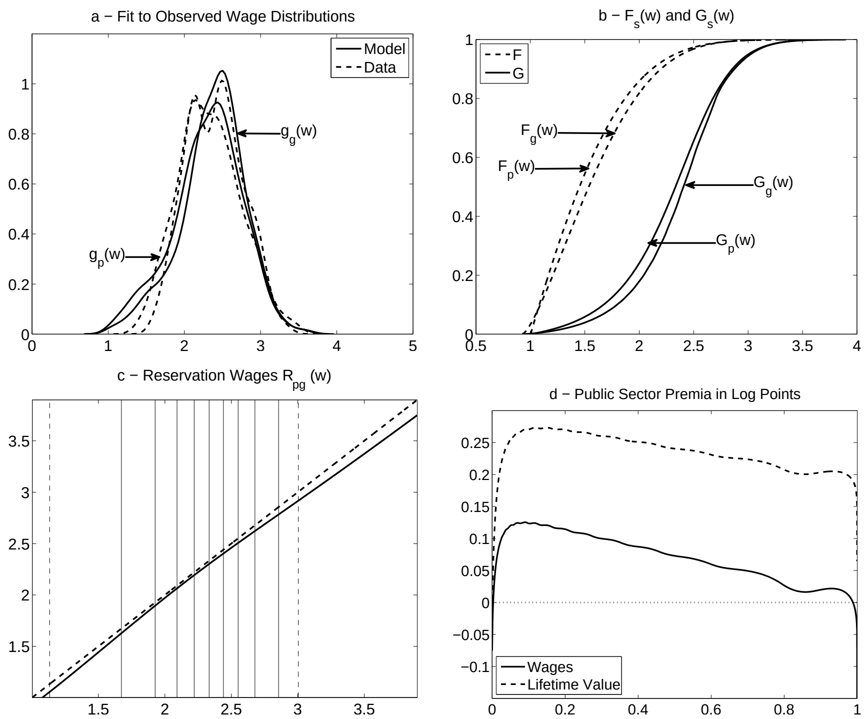
We now turn to an analysis of wage distributions. Panels (a) in Figures 1 and Figures 2 show the model's fit to observed cross-section (log-) wage distributions in both sectors of employment. The fit is reasonably good in both sectors. In passing, we note that, as is well documented elsewhere in the literature, the public-sector wage distribution dominates the private-sector one except in the top two deciles. There is also markedly less wage dispersion in the public sector.

Panels (b) in Figures 1 and 2 show estimated log-wage offer distributions in both sectors, $F_p(\cdot)$ and $F_g(\cdot)$, together with the distributions of accepted log-wage offers, $G_p(\cdot)$ and $G_g(\cdot)$. Both offer distributions are fairly concentrated, much more so than the corresponding accepted offer distributions. Indeed the large estimated offer arrival rates imply a large extent of stochastic dominance of $G_s(\cdot)$ over $F_s(\cdot)$ for $s = p$ or g. We also see that the public-sector offer distribution

[19]It is indeed striking that the two attempts at estimating two-sector versions of the wage-posting model that we are aware of (namely, Meghir et al., 2015 and our own paper) both find a higher contact rate for employed workers, which we interpret as employed workers having access to a different search technology than unemployed workers. Aside from this interpretation, those consistent findings may be taken to suggest that the model has complex and as yet not well understood aggregation properties, which ought to be explored given the popularity of that model. Such an exploration is clearly beyond the purview of this paper and is left for future research.



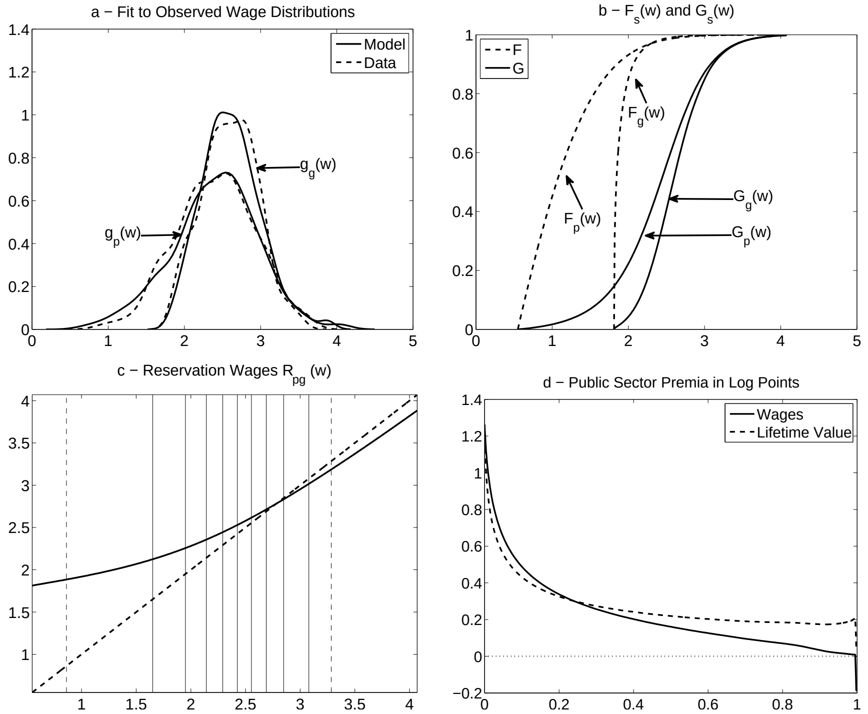
Male University Graduates



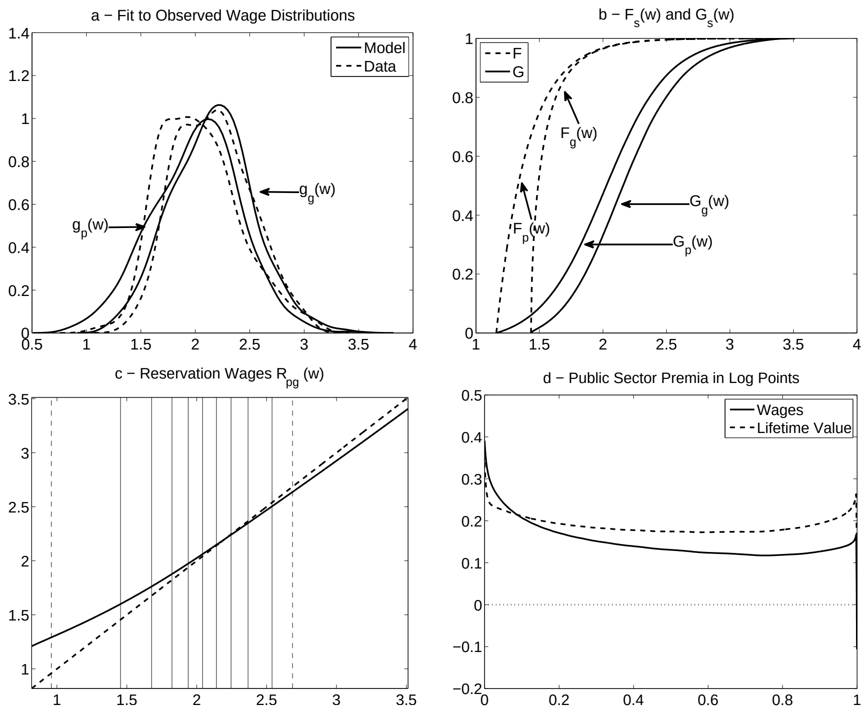
Males < University Education

Note: In panels (c) the dashed line on that graph is the main diagonal and the vertical lines materialize the deciles of the private-sector wage distribution.

Figure 1: Wage distributions and reservation wages



Female University Graduates



Females < University Education

Note: In panels (c) the dashed line on that graph is the main diagonal and the vertical lines materialize the deciles of the private-sector wage distribution.

Figure 2: Wage distributions and reservation wages

dominates its private-sector equivalent. However part of that dominance is "undone" by later reallocation within and between sectors: the dominance of $G_g(\cdot)$ over $G_p(\cdot)$ is less marked than the one of $F_g(\cdot)$ over $F_p(\cdot)$. The main driver here is the rate at which workers receive offers from the private sector (measured by the sum of λ_{pp} and λ_{gp}) which is higher than that at which they receive public sector offers. This implies quicker upward wage mobility in the private than in the public sector. As a consequence, the distribution of private sector wages $G_p(\cdot)$ dominates the private sector offer distribution $F_p(\cdot)$ by more than $G_g(\cdot)$ dominates $F_g(\cdot)$.

Panels (c) in Figures 1 and 2 plots $R_{pg}(w)$, the reservation wage of private-sector employees presented with offers from the public sector. The dashed line on that graph is the main diagonal and the vertical lines materialize the deciles of the private-sector wage distribution, $G_p(\cdot)$. It appears on this plot that, for lower wages, $R_{pg}(w) > w$, i.e. private sector employees will only accept public sector employment with an associated wage increase. The likely reason is again that upward wage mobility is quicker in the private sector. However, as one moves up the distribution, $R_{pg}(w) < w$, as when one earns a higher wage better job security in the public sector, $\delta_g < \delta_p$, dominates the higher mobility rates of the private sector. Note this pattern does not quite follow for low skilled men where $R_{pg}(w)$ tracks the 45 degree line fairly tightly. Inspection of this particular panel and (a) and (b) suggest the two sectors are fairly similar for this stratum.

Finally, our model allows us to examine the public-private sector pay gap. While this pay gap is conventionally assessed in terms of wages, in our model workers care not only about their wages but also about future wages, which depend on transition rates and expected future wage progression patterns that differ between sectors. Following Postel-Vinay and Turon (2007), we thus assess the public-private pay gap in terms of lifetime values of employment, as well as raw wages. Specifically, panels (d) in Figures 1 and 2 display the public sector premium in terms of wages and lifetime value across the quantiles of their respective distributions. How the public sector premium, measured by wages or lifetime value, changes as one moves up the earnings distribution seems to depend on the skill level of the worker. For university educated men and women the premium shrinks from large and positive to negative as one moves up the distribution. For those without a university degree the premium associated with public sector employment seems to be persistent across the whole distribution. This is broadly consistent with the findings of Postel-Vinay and Turon (2007) who estimate a descriptive model of wages and job mobility across the public and private sectors, also on BHPS data, but with a much richer representation of worker heterogeneity and wage dynamics: they suggest that as one moves up the aggregate earnings distribution the premium by either measure declines and can be negative for the highest earners.

Postel-Vinay and Turon further find (for males) that measuring the premium in terms of wages rather than lifetime values leads one to overestimate the benefit of public sector employment. Our results differ from theirs on this particular dimension: here, we find that the premia measured in lifetime values and in wages are roughly similar across the entire distribution for low-skill men, low-skill women, and high-skill women, while the lifetime-value premium is larger than the wage premium for high-skill men.[20]

5 Counterfactual Policy Analysis

Using the estimated parameters of the structural model we simulate the effects of various changes in public sector wage and employment policy. While the model allows the simulation of many possible policies, from a topical perspective, an assessment, by simulations, of the various public sector austerity measures being enacted across Europe seems to be a sensible subject to pursue.

Before beginning, it is important to make clear that this paper has nothing to say on whether or not austerity is good economic policy. Rather, conditional on wanting to implement it, this model can inform policy makers on the likely impact of different ways to go about it. Moreover, our results only concern public spending on the wages of public sector employees. Wages for public sector workers cost the UK government £174 billion in 2008. This accounts for around 30% of total expenditure and 50% of non-investment expenditures (ONS, 2009).

Policies we consider are categorized into employment and wage policies. Employment policies are reducing hiring in the public sector and increasing firing, respectively modeled as changes to δ_g

[20] We suspect that the source of this inconsistency is the way in which this paper attempts to avoid spurious mobility across sectors. One of the main equalizing forces of lifetime values between the public and private sectors is cross-sector mobility. By discounting a large fraction of reported cross-sector moves as spurious (see Sub-section 3.1), we cut average transition rates between sectors by about a half compared to the raw data reported in Postel-Vinay and Turon (2007).

and λ_{sg}, where $s \in \{u, p, g\}$. Wage policies are treated as changes to the distribution $F_g(\cdot)$.

5.1 Counterfactual Policies

Formally, the policies we simulate are represented by equation (21), where new policies are denoted by a * and the indicator function equals one if a worker has a university degree. The parameter π is the degree by which the parameters change (see Appendix B for the simulation protocol).

$$\begin{aligned}
\text{Increase in fires: } & \delta_g^\star = (1+\pi) \times \delta_g \\
\text{Decrease in hires: } & \lambda_{sg}^\star = (1-\pi) \times \lambda_{sg} \text{ where } s \in \{u, p, g\} \\
\text{Proportional reduction in wages: } & w_g^\star = (1-\pi) \times w_g \\
\text{Progressive reduction in wages: } & w_g^\star = (1-\pi) \times w_g \times \mathbf{1}\{\text{high skilled}\}
\end{aligned} \tag{21}$$

These policies are aimed at approximating the types of policies implemented across Europe during the "*age of austerity*".[21] Specifically, the policies documented in (21) are intended to replicate policies implemented in Italy, UK, Spain and Portugal, respectively. While within each country a variety of policies have been undertaken, these four countries all adopted different principal tactics in reducing the wage bill of public sector employees. Italy froze all new recruitment; the UK actively cut public sector jobs; Spain froze public sector pay across the board; Portugal implemented a 10% average pay cut on the higher earners in the public sector. While Portugal did not impose wage cuts only on those with a university degree, this policy mimics a cut in high wages fairly well as the high earners in the economy are concentrated in this subgroup. In our data over 60% of the top 10% of earners have a university degree, compared to less than 30% in the overall population of workers.

It is difficult to compare policies without a clearly defined metric for assessment. For each policy we compute the savings the public sector makes as a proportion of its initial expenditure. This savings rate is given by the following formula, where the superscript $\star$ denotes a simulated

[21] The phrase was coined by Prime Minister David Cameron at the Conservative party forum in Cheltenham, 26th April 2009.

policy and s_i is the share of the total economy comprised in strata i:[22]

$$\text{savings} = \frac{\sum_{\text{strata } i} s_i \left(N_{g,i} \int_{R_{ug,i}}^{\infty} w dG_{g,i}(w) - N^{\star}_{g,i} \int_{R^{\star}_{ug,i}}^{\infty} w dG^{\star}_{g,i}(w) \right)}{\sum_{\text{strata } i} s_i N_{g,i} \int_{R_{ug,i}}^{\infty} w dG_{g,i}(w)}.$$

Conveniently, for each of the policies, this savings rate is monotonically increasing in π. This means it is possible to plot a variety of labor market outcomes on the degree of savings made, rather than an arbitrary parameter π. This is what is done in all of the remaining figures, where we consider a range for the savings rate between 0 and 15% of the initial public sector wage bill. To put things into perspective, the UK public sector wage bill fell by about 7% (in real terms) between the years 2010-2014, while the share of the public sector in total employment fell by about 10% over that same period.[23]

5.2 Private Sector Response

We first consider how collectively private sector firms respond to the policies described along the following dimensions: hiring, wages, total employment, and productivity.

5.2.1 Private Sector Hiring

Figure 3 highlights the change in total hires by private sector firms across all policies and strata, after entry and exit of new private firms has occurred.[24] With varying magnitudes, all policies increase private sector hiring. The policies have greater implication for high skilled men for whom the public sector constitutes a relatively large share of total employment. For example, the results on Figure 3 assert that total private sector hires of high skilled men will increase by 8 to 16% (depending on the particular policy considered) if the government aims for a 10% cut in the public sector wage bill.

[22] This formula is only valid, strictly speaking, for the cases of an increase in public job destruction and of a cut in public wages. Indeed, a reduction in public hiring is likely associated with extra savings on (unmodeled) hiring costs — the public-sector counterparts to our private-sector $c(h)$. We should bear this caveat in mind when interpreting the simulation results.

[23] 2010 is the year when the UK government changed from Labour, which had been in power since 1997, to a coalition of the Conservatives and Liberal Democrats, and marked the beginning of the "age of austerity" in the UK.

[24] All of the series plotted in this section have been normalized to one at the initial point (i.e. at the equilibrium predicted by our estimated model, before any new policy is implemented). As such, all these plots should be read as reporting changes relative to that initial reference point. Also, to keep the figures readable we refrained from plotting confidence bands around the various series plotted in this section. However, to get a sense of precision, in Appendix C we report standard errors of some predicted statistics at one level of savings on the public sector wage bill (10%).

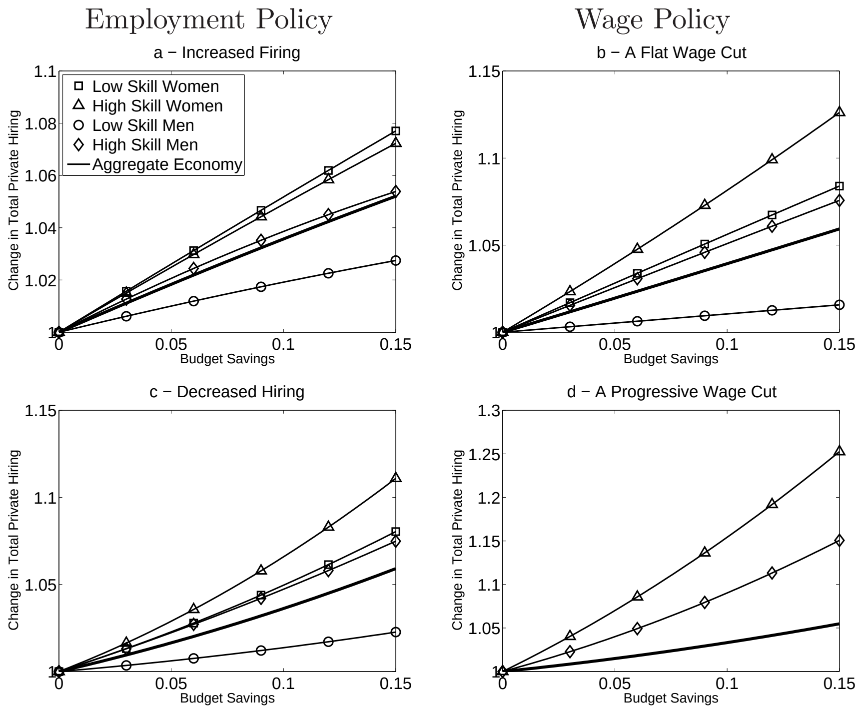


Figure 3: Total private sector hiring

While these numbers may seem high, one should recall that they apply to *total* (or gross) private sector hiring, including the poaching of workers from other private sector firms. In other words, some of this increase in private hiring reflects an increase in job-to-job turnover within the private sector. To gauge the extent to which that is the case, we plot on Figure 4 the policy response of *net* hiring by the private sector, i.e. the inflow of workers into the private sector that come from either unemployment or the public sector.[25] While net private hiring increases in response to all four policies considered, the amount by which it does so is comparatively modest: for example, net hires of high skilled men (the category most affected by all policies) increase by 2.5 to 4.5% (depending on the particular policy) if the government is to achieve a level of savings of 10%.

This leads us to an important remark. However implemented, the first-order effect of "austerity" (defined in the narrow sense of the four different policies described in Subsection 5.1) is to reduce the competitive pressure exerted by the public sector on the private sector. This happens either because the public sector simply hires less, or because it makes itself less attractive by offering less

[25]Formally, net private sector hires are equal to $\lambda_{up} N_u + \lambda_{gp} N_g \int_{R_{ug}}^{+\infty} \overline{F}\left(R_{gp}(x)\right) dG_g(x)$.

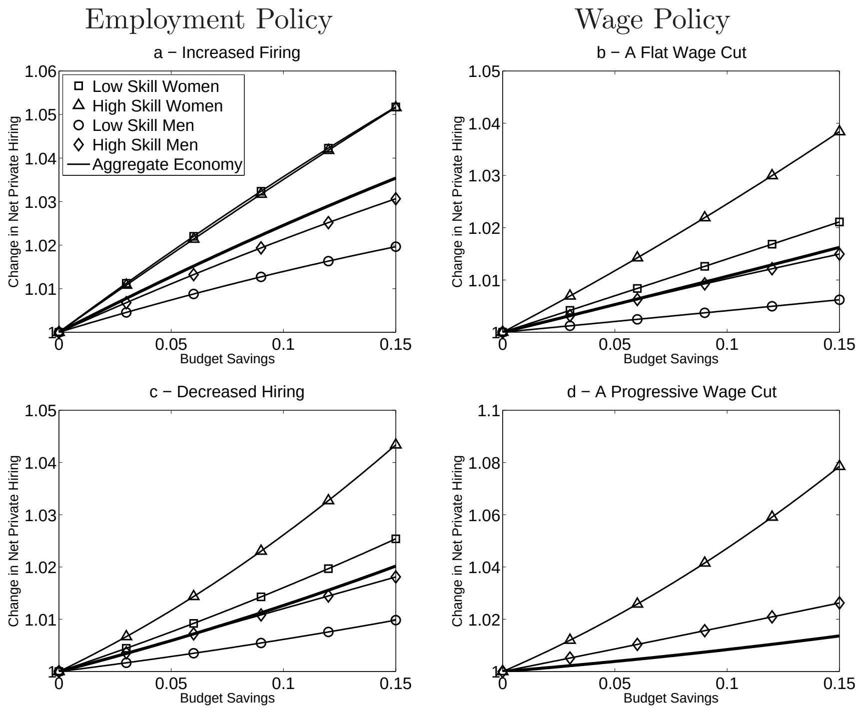


Figure 4: Net private sector hiring

durable jobs or lower wages. Private sector firms respond to this reduced competition by opening more jobs. However, many of those added job openings are taken up by workers who already hold a private sector job: a perhaps overlooked consequence of those austerity policies is to intensify job-to-job turnover in the private sector. Our model suggests that this effect is quantitatively substantial.

Further to the previous remark, it is clear that austerity encourages more private sector firms to enter the market. For the reasons just explained, all austerity measures reduce the rate at which workers quit private employment into the public sector, giving instantaneous positive option value of opening a private firm. This induces entry, until the free entry condition holds again. To assess the importance of this margin, we consider the policy responses of gross and net private sector hiring, this time keeping the population of firms the same as it was before the policy. Those are plotted on Figures 5 and 6, which parallel Figures 3 and 4, only shutting down entry and exit.

Those "intensive margin" responses are markedly more modest than their counterparts from

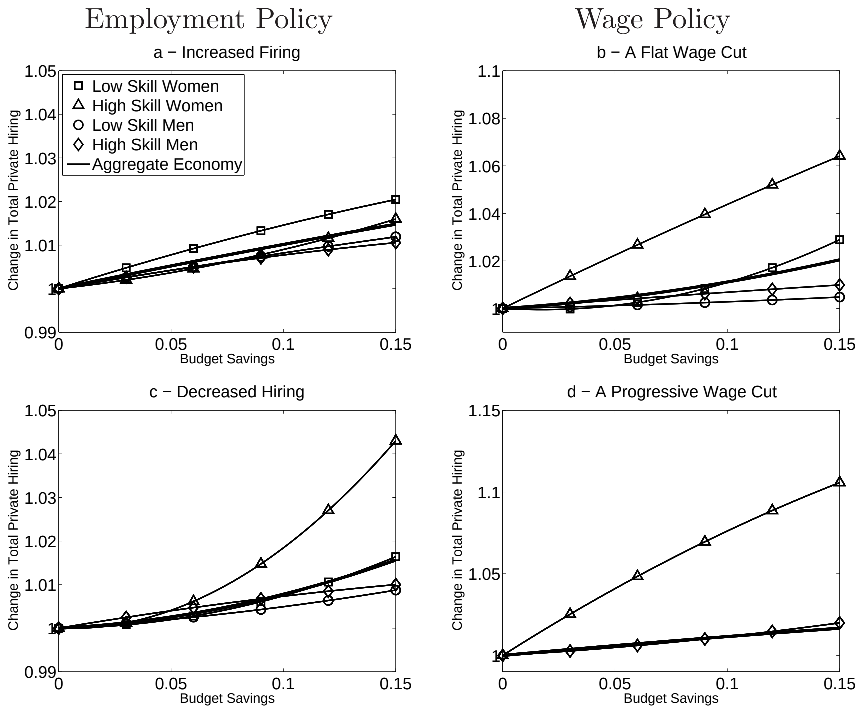


Figure 5: Total private sector hiring (without free entry)

Figures 3 and 4.[26] This implies that a large share of the increase in private sector hiring following the implementation of austerity policies comes from the entry of new private sector firms into the market rather than from the expansion of existing firms. This can be traced back to the convexity of the estimated hiring cost $c(h)$, which makes it costly for firms to increase their size too much.[27]

5.2.2 Private Sector Wages

Figure 7 shows the change in the mean wage offered by private sector firms (the mean of $F_p(\cdot)$) across all policies and strata. All policies decrease the mean wage offered, the effect being, once again, greatest for university educated men, who under a progressive wage cut aimed at cutting the public wage bill by 10% see a reduction in the mean wage offered by private firms of about 3%. Therefore, the increase in private sector hires analyzed in the previous subsection comes about through increases in job openings — or higher recruiting intensity — rather than through more

[26]Note from Figure 6 that net private sector hires can sometimes *decline* in response to austerity when entry of new firms is shut down (this is the case for high skilled men in panels b and d of Figure 6). The reason is that the source population of net private sector hires (the total of N_u unemployed workers and N_g public sector workers) declines by more than the private sector's net hiring rate increases.

[27]We thank an anonymous referee for suggesting this to us.

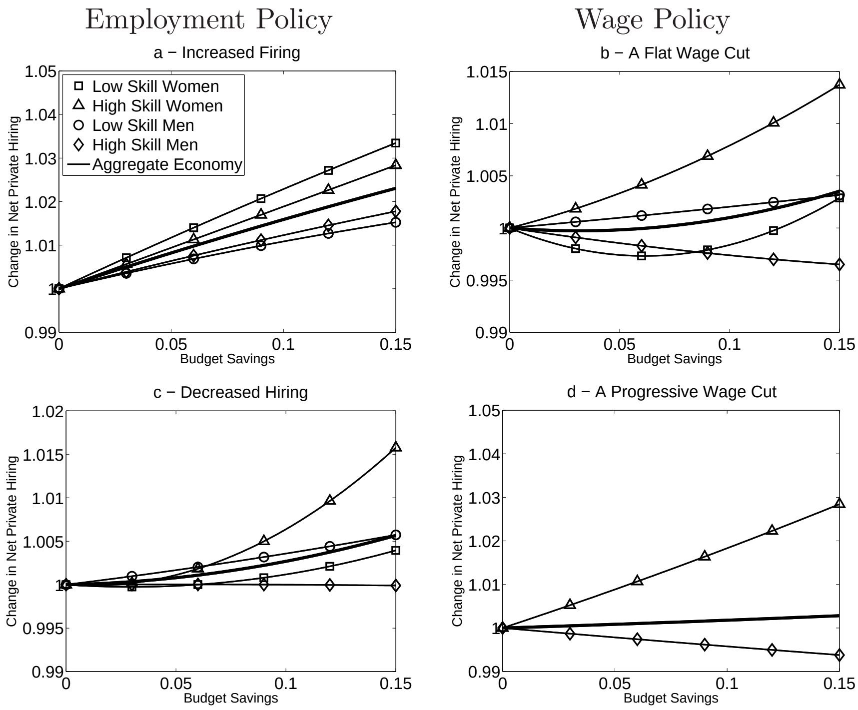


Figure 6: Net private sector hiring (without free entry)

attractive wage offers.

This decline in wage offers, however, does not necessarily translate into lower average wages in the private sector. Indeed, the increase in private sector turnover indirectly increases wages as workers climb the (private-sector) job ladder at a faster rate. This effect will counteract the lower average wage offered by private firms. The extent to which it does so is measured on Figure 8, which plots the cross-sectional mean private sector wage (the mean of $G_p(\cdot)$) as a function of the savings achieved by our four policies. Interestingly, the overall impact of those policies on private sector wages is minimal (depending on stratum and policy the impact can be positive or negative, but it is always small), implying that the increased speed of job upgrading balances the lower wage offers almost exactly.

5.2.3 Private Sector Employment

The private sector's responses to all four austerity policies are to intensify its hiring effort while offering lower wages. The combined effect of these two responses on total private sector employment is *a priori* ambiguous: the increase in private sector hiring tends to increase private sector

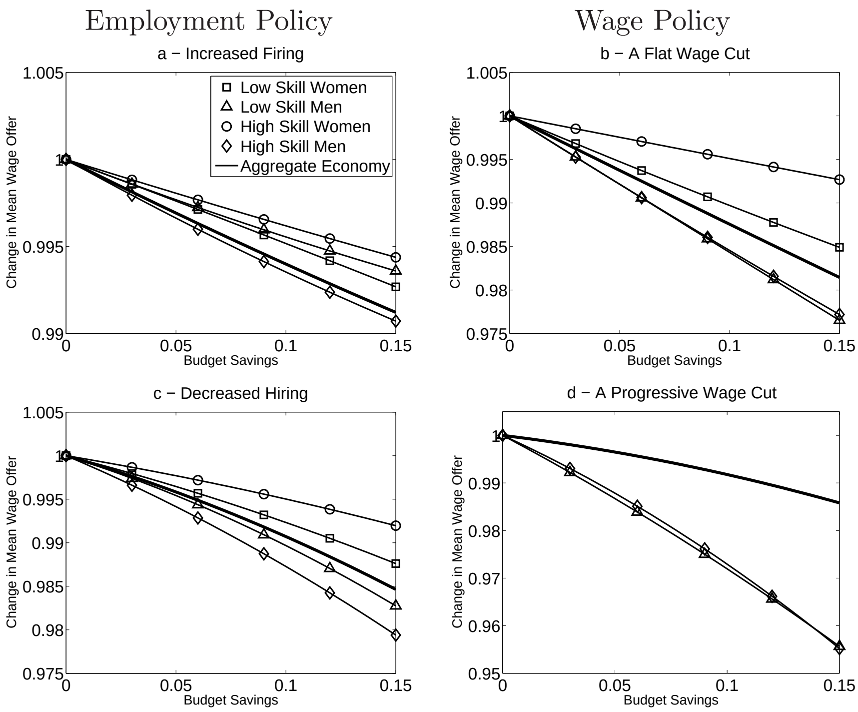


Figure 7: Private sector wage offers

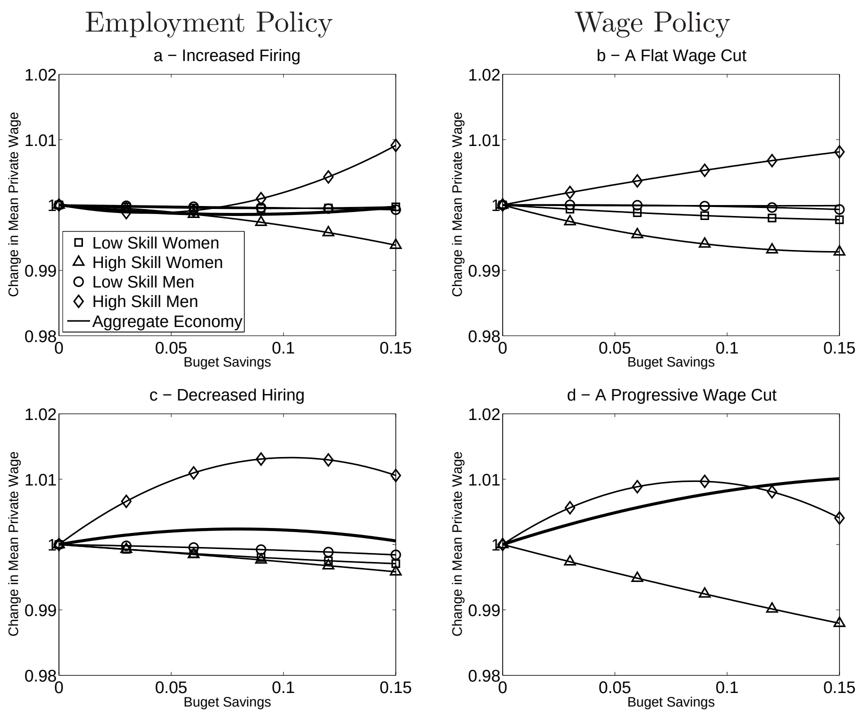


Figure 8: Private sector mean wage

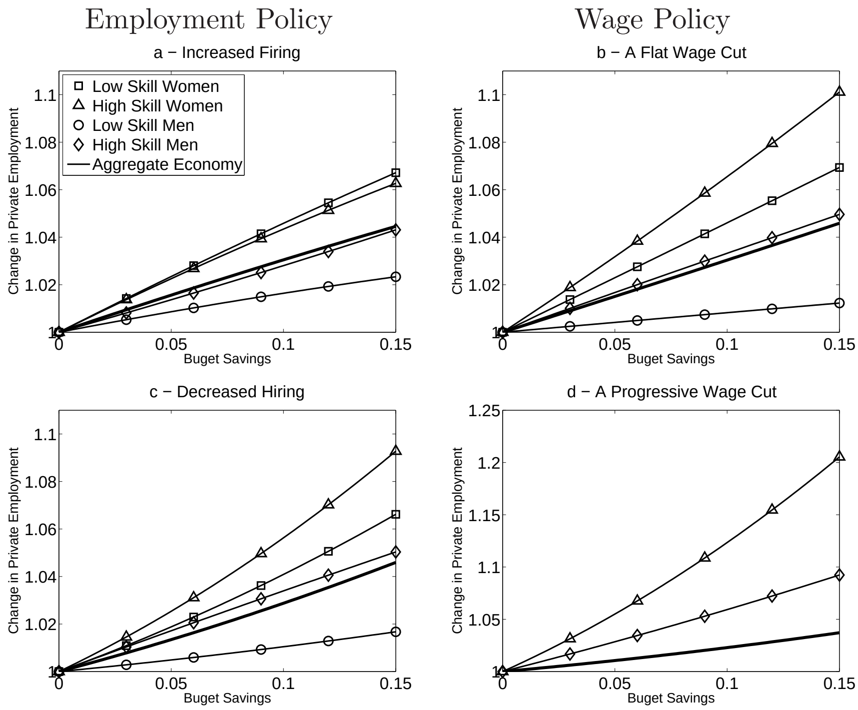


Figure 9: Private sector employment

employment while the decrease in wages offered tends to decrease employment as fewer workers will be poached from the public sector and more workers will quit into that sector. This ambiguity is resolved in Figure 9, which shows total private sector employment, for each separate worker category and for all four categories combined, as a function of the savings achieved by each of our four policies.

Clearly, austerity boosts private employment in all cases. The results are most profound for women, who pre reform are more than twice as likely to be employed by the public sector as their male counterparts. Once again the progressive wage policy has the largest impact on affected workers: a 10% cut in the public sector wage bill would increase private sector employment of university educated women by 12%.

5.2.4 Private Sector Productivity

Having established that austerity policies will cause the private sector to employ more workers in all strata, largely through the entry of new firms into the market, it is natural to ask about the "quality" (i.e. the productivity) of those newly created private jobs. To address this question,

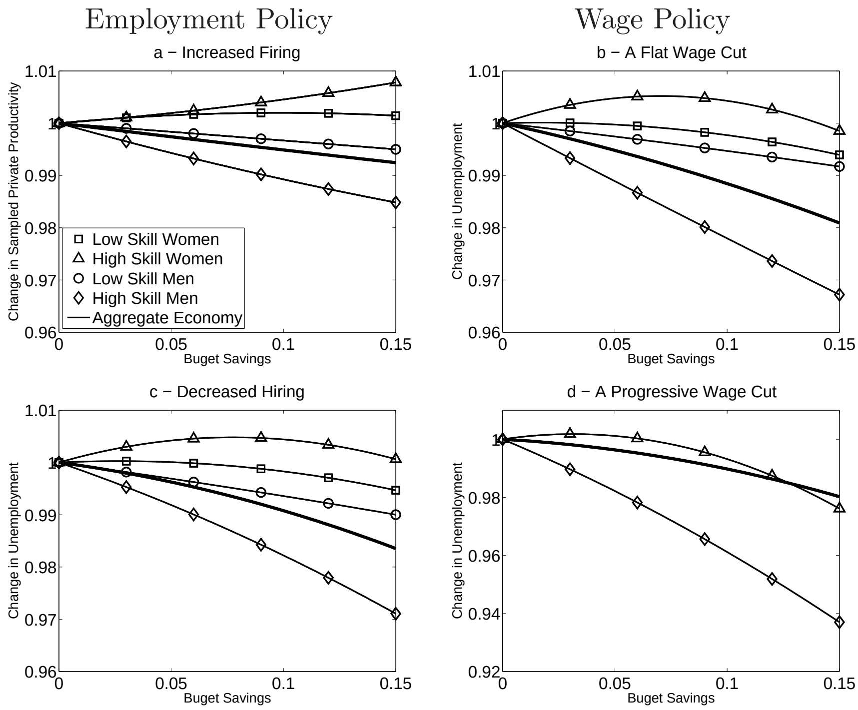


Figure 10: Mean sampled private sector productivity

Figure 10 plots the productivity (y) of the average job sampled by a worker upon receiving a private sector job offer. It shows a sizable decline in the quality of jobs drawn by most strata, particularly the university educated males.

Next, cross-sectional average private sector productivity is plotted on Figure 11. As for wages, the cross-sectional average productivity differs from the mean sampled productivity because of the gradual selection of workers up the job ladder: although productivity is being drawn from a distribution with a lower mean, workers are making more frequent draws from said distribution after implementation of policy, which has been shown to boost private sector labor demand. Thus much of the declines seen in Figure 10 are "undone" in the cross-section. Indeed the majority of strata see very modest changes in the mean match productivity. Aggregate private sector productivity is barely affected by any of the policies.

5.3 Public Sector Response

We now briefly turn to the public sector's response to the four policies considered, focusing, for brevity's sake, on employment and wages. By design of the policies, the public sector wage bill will

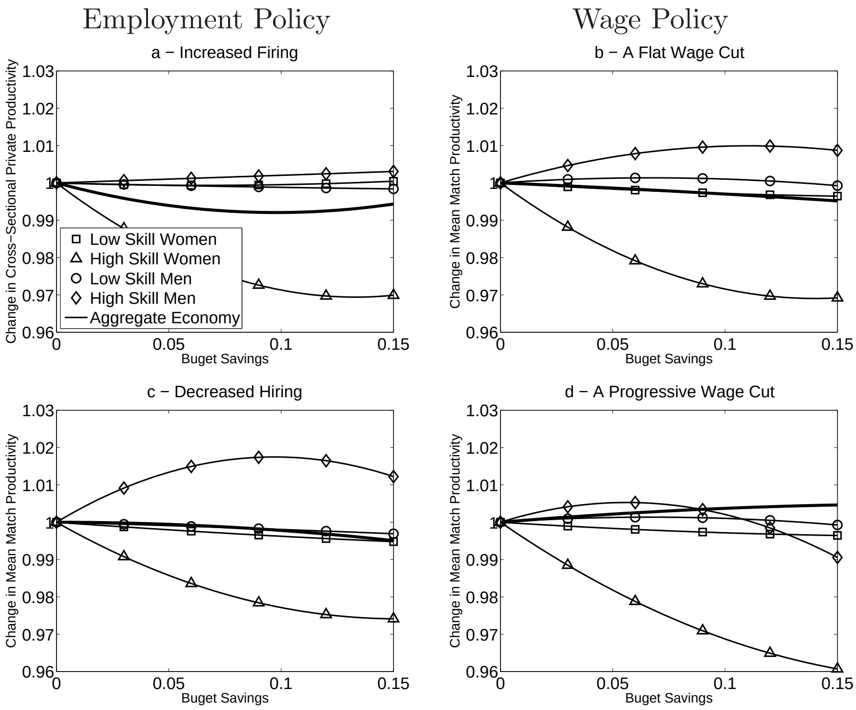


Figure 11: Cross-sectional mean private sector productivity

be cut by a given percentage amount. But how do these savings come about? Is it through lower average wages in the public sector, or through a smaller public sector workforce?

Answers to these questions are given by Figures 12 and 13, which plot, respectively, total public sector employment and the mean public sector wage for each worker category separately and for all four categories combined, as a function of the reduction in the public sector wage bill. All four austerity measures are associated with a sharp fall in public sector employment and a comparatively small, although still substantial decline in the mean wage of public sector employees. Inspection of the two figures thus reveals that the bulk of government savings results from a decline in public sector employment, rather than a decrease in the wage rate. That said, the four policies all implement cuts in slightly different ways; we return to this question below.

5.4 Aggregate Employment Response

We finally combine the model's predictions about public and private sector employment to assess the response of aggregate (un)employment to the four policies considered in this section. Those are plotted, again for each separate worker category and for all categories together, on Figure 14.

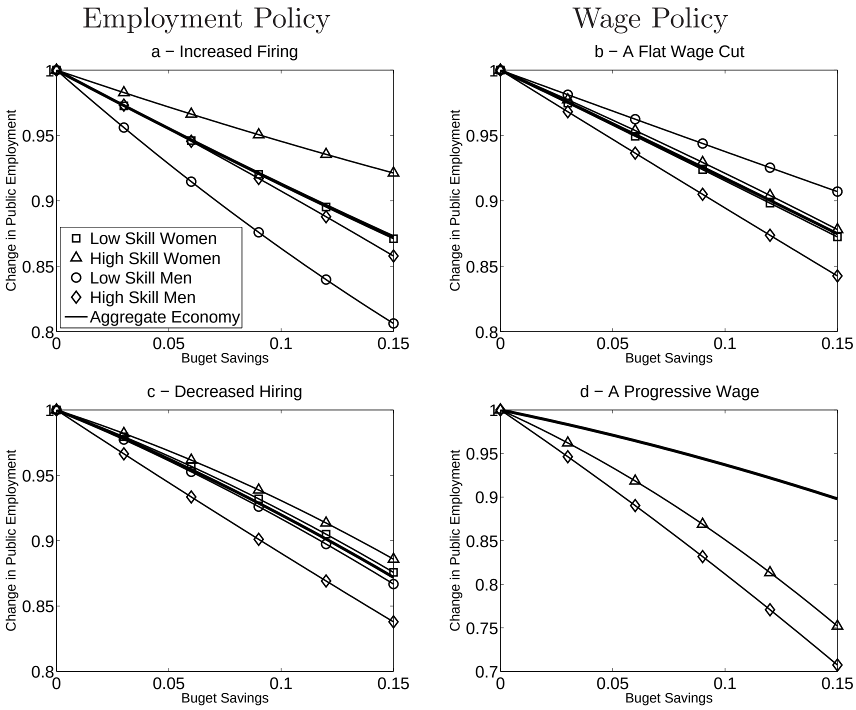


Figure 12: Public sector employment

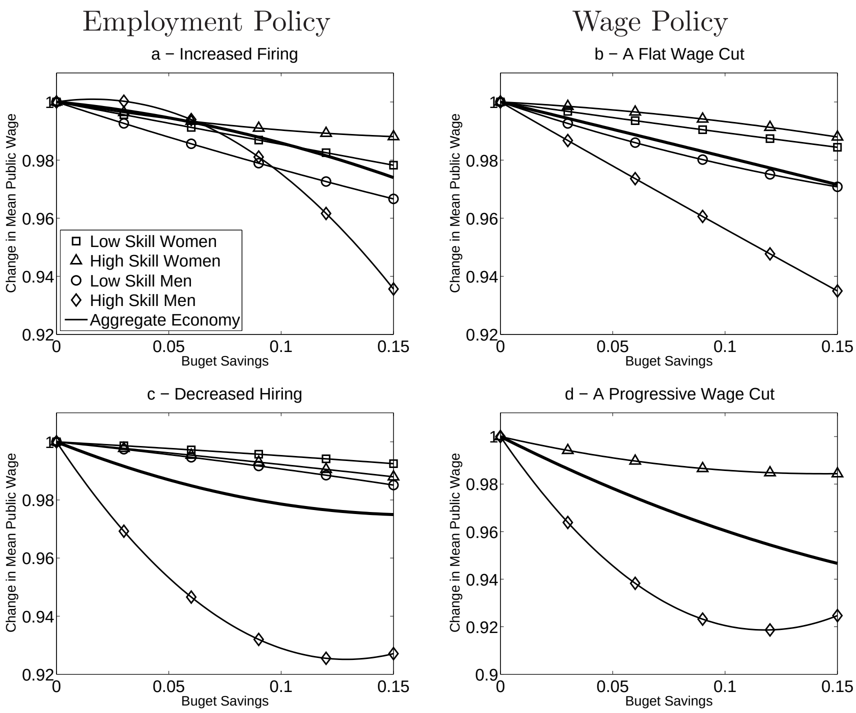


Figure 13: Public sector mean wage

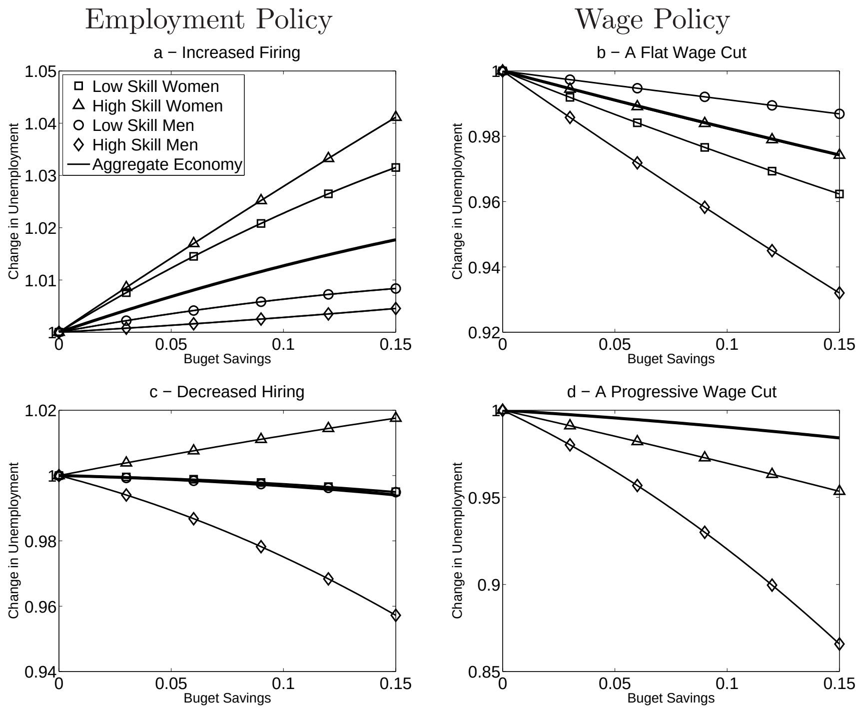


Figure 14: Aggregate (un)employment effects

The sign of the aggregate unemployment rate's response to policy differs between policies: unemployment increases in response to increased public sector firing, stays constant in response to a cut in public sector hiring, and declines in response to a flat or progressive cut in public sector wage offers (meaning that the private sector creates more jobs than are being cut in the public sector under those wage policies). In all cases, however, the magnitude of the change in aggregate unemployment is small in either direction (for example, the unemployment rate of high-skilled women rises from 2.38% to 2.48% in response to 15% government spending cut achieved through increased firing in the public sector — Figure 14a).

One policy implication of those small aggregate employment effects is that a government implementing the policies assessed need not worry about large increases in unemployment. Much of the fall in public sector employment will be soaked up by the private sector. However, Figure 14 also reveals, together with Figures 9 and 12, that the burden of austerity will be distributed unevenly across worker categories, and may change the composition of employment, both in the public and in the private sectors. We discuss this issue in the next and final subsection.

5.5 Taking Stock: Comparing Between Austerity Policies

The previous subsections have established that the four austerity policies considered — increased public sector job destruction, decreased public sector hires, across-the-board cut in public sector wages, and targeted cut in the public sector wages of high skilled workers (a.k.a. "progressive" wage cut) — all have qualitatively similar aggregate impacts: increased hiring by the private sector, increased turnover within the private sector, decrease in public sector employment which is largely compensated by an increase in private sector employment, summing up to very small changes in aggregate unemployment. In all four cases, the impact on private sector mean wages is small, while the average public sector worker sees their wage cut by 1 to 8%.

Quantitatively, the "increased firing", "decreased hiring", and "flat wage cut" policies have roughly similar impacts (at least if one focuses on wages and employment effects). In particular, none of these three policies causes drastic changes in the composition of employment: to a rough approximation, those three policies shift labor away from the public sector into the private sector in proportions that are comparable across the four worker categories considered in this paper.

Yet the fourth policy — the "progressive wage cut" — stands out as an exception. The fact that this policy is targeted to only two out of our four worker categories (high skilled men and women, which together sum up to about 30% of the total workforce) has an important consequence: the actual wage cut that needs to be implemented in order to achieve a given level of total savings on the public sector wage bill is larger under a progressive wage cut than under a flat one. As a result, the impact on affected workers (i.e. high skilled workers) of a progressive wage cut is bigger than that of a flat wage cut that achieves the same overall savings. Therefore, contrary to the other three policies considered, the progressive wage cut substantially reduces the share of high skilled workers in the public sector workforce (while increasing that same share in private sector employment).

Absent a public sector production function, the cost of this shift in workforce composition is impossible to assess quantitatively. Yet it is likely to be costly, as the public sector likely produces skill-intensive goods and services that have no close substitutes in the private sector. Indeed, Gomes (2015) estimates, in a somewhat different model with an explicit public sector production function where skilled and unskilled workers are imperfect substitutes, calibrated to the UK economy, that a cut of more than 6% of high-skill wages in the public sector reduces aggregate welfare.[28] The main reason behind Gomes's finding is that, like in our model, cutting high-skill wages in the public sector makes it very difficult for the public sector to hire skilled workers, who are an essential input into government production.

6 Conclusion and Discussion

This paper explores the impact of public sector wage and hiring policy on labor market equilibrium within an estimated structural model. The rates at which the public sector hires and lays off workers and the distribution of wages offered by the public sector are exogenous policy tools. The rates at which workers receive offers from the private sector are endogenous as is the distribution from which those offers are drawn. The model allows for mobility between and within three labor market states and the model is estimated using British data.

With this modeling approach, policies relating to wages and employment in the public sector can be assessed prior to implementation. We apply the model to see what the effects on the British economy would have been under a variety of policies implemented across Europe aimed at cutting the wage bill of the public sector. We find surprisingly little impact on private sector wages and on aggregate employment. If one cuts public sector wages/employment, the private sector soaks up the majority of the fall in employment and wages remain relatively stable. Yet some of these policies have a strong impact on the composition of employment in both sectors. In particular, a progressive wage cut in the public sector which primarily affects high skilled workers substantially reduces the share of skilled workers in public sector employment. While our model cannot assess the cost of this impact on public sector production, other contributions suggest that it may be high. We should further point out that our approach abstracts from possible short-run effects of aggregate demand and/or wage rigidities, and as such can only inform us of long term changes

[28] Gomes's model is simpler than ours along certain dimensions (chiefly, it does not allow for on-the-job search, so that the public and private sectors only compete in the recruitment of unemployed workers), but richer in others, such as the explicit modeling of a public sector production function, and the presence of a government budget constraint. Incidentally, it is worth noting that Gomes (2015) also predicts that cutting the wages of high skilled public sector workers will reduce aggregate unemployment, by an amount in the same ballpark as our own prediction.

in wages and employment. This is not to downplay the importance of short-run demand effects, which have received a fair amount of attention both in the policy debate and the macro literature. Rather, we acknowledge that our contribution speaks to the longer run impact of policy.

This model can inform policy makers and sits in a small subset of equilibrium search models of the labor market that are designed with policy primarily in mind. We therefore tried to keep the model simple, transparent, and "user-friendly" enough, while still capturing the main interactions between the public and private sectors. As such, our modeling has some limitations. Chief among those are the lack of explicit modeling of public sector behavior (which makes the model silent about optimality or the welfare effects of policy), and the maintained assumption of ex-ante homogeneous workers (while in reality some workers may have qualifications that tie them to a particular sector, thus affecting their mobility between the public and private sectors).[29] Future research in this area should address those limitations.

[29] A more substantive reason why we maintained the assumption of worker homogeneity (within observable categories) is that the separate identification of worker and firm heterogeneity without recourse to matched employer-employee data would rely heavily on the model's structure, and as such its credibility would be subject to question.

APPENDIX

A Theory: Intermediate Derivations

A.1 Derivation of the Reservation Wage, Equation (6)

The value function for a private sector worker earning a wage w, is given in equation (4). Assuming differentiability:

$$W_p'(w) = \left[r + \mu + \delta_p + \lambda_{pp}\overline{F}_p(w) + \lambda_{pg}\overline{F}_g(R_{pg}(w))\right]^{-1} \tag{22}$$

This also gives $W_p'(w)$ by analogy. Integrating by parts in (4) yields:

$$(r + \mu + \delta_p) W_p(w) = w + \delta_p U + \lambda_{pp} \int_w^{+\infty} W_p'(x) \overline{F}_p(x) dx + \lambda_{pg} \int_{R_{pg}(w)}^{+\infty} W_g'(x) \overline{F}_g(x) dx \tag{23}$$

Plugging the various value functions into the definition of $R_{pg}(w)$ given in the paper, one obtains the following, fairly complicated expression:

$$\begin{aligned} R_{pg}(w) = -a_g + \frac{r+\mu+\delta_g}{r+\mu+\delta_p} w + \left\{\frac{r+\mu+\delta_g}{r+\mu+\delta_p}\delta_p - \delta_g\right\} U \\ + \left\{\frac{r+\mu+\delta_g}{r+\mu+\delta_p}\lambda_{pp} - \lambda_{gp}\right\} \int_w^{+\infty} W_p'(x)\overline{F}_p(x) dx \\ + \left\{\frac{r+\mu+\delta_g}{r+\mu+\delta_p}\lambda_{pg} - \lambda_{gg}\right\} \int_{R_{pg}(w)}^{+\infty} W_g'(x)\overline{F}_g(x) dx \end{aligned}$$

Differentiating yields (6).

A.2 Derivation of the Private-Sector Wage Distribution, Equation (9)

Equation (9) would be a simple ODE if it was not for the term featuring $G_g(R_{pg}(w))$. We now show how to express that term as a function of w and $G_p(w)$. Writing the flow-balance equation for the public sector yields:

$$\begin{aligned} \left\{\mu + \delta_g + \lambda_{gg}\overline{F}_g(w)\right\} N_g G_g(w) + N_g \lambda_{gp} \int_{R_{ug}}^{w} \overline{F}_p(R_{gp}(x)) dG_g(x) \\ - N_p \lambda_{pg} \int_{R_{up}}^{R_{gp}(w)} \left[F_g(w) - F_g(R_{pg}(x))\right] dG_p(x) = N_u \lambda_{ug} \left[F_g(w) - F_g(R_{ug})\right]. \end{aligned}$$

Now applying the latter equation at $R_{pg}(w)$ (instead of w), we get:

$$\begin{aligned} \left\{\mu + \delta_g + \lambda_{gg}\overline{F}_g(w)\right\} N_g G_g(R_{pg}(w)) + N_g \lambda_{gp} \int_{R_{ug}}^{R_{pg}(w)} \overline{F}_p(R_{gp}(x)) dG_g(x) \\ - N_p \lambda_{pg} \int_{R_{up}}^{w} \left[F_g(R_{pg}(w)) - F_g(R_{pg}(x))\right] dG_p(x) = N_u \lambda_{ug} \left[F_g(R_{pg}(w)) - F_g(R_{ug})\right]. \end{aligned} \tag{24}$$

Adding (24) to (8):

$$N_pG_p(w)\left\{\mu+\delta_p+\lambda_{pp}\overline{F}_p(w)+\lambda_{pg}\overline{F}_g(R_{pg}(w))\right\}$$
$$+N_gG_g(R_{pg}(w))\left\{\mu+\delta_g+\lambda_{gp}\overline{F}_p(w)+\lambda_{gg}\overline{F}_g(R_{pg}(w))\right\}$$
$$=N_u\lambda_{ug}\left[F_g(R_{pg}(w))-F_g(R_{ug})\right]+N_u\lambda_{up}\left[F_p(w)-F_p(R_{up})\right], \quad (25)$$

which can be solved for $N_gG_g(R_{pg}(w))$. Plugging the solution into (9), we obtain an ODE defining $G_p(w)$. Note that by considering $w\to+\infty$ in the latter equation, one obtains (7).

A.3 Productivity Distribution

Using a change of variable and assuming the that $w^\star(y_{min})=R_{up}$, equation (16) is equivalent to:

$$\begin{aligned}
F_p(w) &= \frac{1}{M}\int_{R_{up}}^{w}\frac{h(z)}{\alpha(z)}\gamma(y(z))\,y'(z)\,dz \qquad (26)\\
f_p(w) &= \frac{1}{M}\frac{h(w)}{\alpha(w)}\gamma(y(w))\,y'(w)\\
\frac{d}{dw}\{\Gamma(y(w))\} &= \frac{Mf_p(w)\,\alpha(w)}{h(w)}\\
\Gamma(y(w)) &= M\int_{R_{up}}^{w}\frac{\alpha(x)}{h(x)}dF_p(x)\\
\Gamma(y(w)) &= M\int_{0}^{F_p(w)}\frac{\tilde{\alpha}(x)}{\tilde{h}(x)}dx
\end{aligned}$$

Where, $\alpha(x)=\tilde{\alpha}(F_p(x))$, $h(x)=\tilde{h}(F_p(x))$ and $h(w)=c'^{-1}\left(\frac{y-w}{\Delta(w)}\right)$

B Simulation Protocol

Previously the wage offer distribution of the public sector ($F_g(w)$) was parameterized as a function of the wage offer distribution of the private sector ($F_p(w)$), equation (19). As $F_p(w)$ is an endogenous object it will change with changes to public policy, and we therefore need to fix $F_g(w)$ ex-ante. Taking the point estimates reported in Section 4 we fix the distribution derived from the Beta transform of equation (19).

We implement policy changes as changes to the job offer arrival rates of public sector jobs, public sector job destruction and the wage offer distribution. Simulating the new equilibrium is performed using an iterative procedure:

1. We start with guesses for the values of the parameters (σ_p, ξ_p) of the $F_p(\cdot)$ distribution and of the private sector job offer arrival rate λ_p. The initial guesses $(\sigma_p^0, \xi_p^0, \lambda_p^0)$ are the estimated values of these parameters before the implementation of any policy.

2. The worker side is re-evaluated as before: given current guesses of $(\sigma_p, \xi_p, \lambda_p)$, equations (6) through (9) are solved as before, giving us new values of N_u, N_p, N_g, $G_p(w)$, $G_g(w)$, R_{up}, R_{ug} and $R_{pg}(w)$.

3. Turning to the firm side, we first solve the first-order conditions (13) and (14) for $h^\star(y)$ and $w^\star(y)$, the firms' optimal policies, given current guesses of $(\sigma_p, \xi_p, \lambda_p)$.

4. Finally, the free entry condition $\Pi(w, h; y) = 0$ is imposed to update the guesses of $(\sigma_p, \xi_p, \lambda_p)$. Steps 1–4 are repeated until convergence.

C Standard Errors for the Predicted Impact of Policy

As indicated in the main text (Footnote 24), Table 4 reports the estimated effect of the four austerity policies considered on our main statistics of interest, together with their standard errors, given a target level of savings on the public sector wage bill of 10%. The standard errors, given in parentheses, are computed from repeated simulations based on bootstrapped parameter estimates.

Table 4: Percentage change in outcome after a 10% cut to the public wage bill

	Inc. firing	Dec. hiring	Flat cut	Progr. cut
University Educated Men				
Mean public wage	−2.47	−7.11	−4.37	−7.95
	(1.31)	(2.29)	(2.75)	(3.93)
Mean private wage	0.19	1.33	0.58	0.94
	(0.36)	(0.59)	(0.67)	(0.91)
Unemployment Rate	0.28	−2.49	−4.62	−7.98
	(1.04)	(1.81)	(1.51)	(3.48)
Private Employment	2.80	3.40	3.32	5.93
	(1.00)	(0.81)	(0.82)	(1.47)
Men, < University				
Mean public wage	−2.32	−0.93	−2.16	—
	(0.46)	(0.46)	(0.50)	
Mean private wage	−0.05	−0.09	−0.02	—
	(0.06)	(0.08)	(0.05)	
Unemployment Rate	0.63	−0.30	−0.88	—
	(0.75)	(1.01)	(0.81)	
Private Employment	1.64	1.04	0.82	—
	(0.41)	(0.29)	(0.19)	
University Educated Women				
Mean public wage	−0.97	−0.78	−0.68	−1.42
	(0.59)	(0.52)	(0.62)	(1.03)
Mean private wage	−0.31	−0.26	−0.63	−0.83
	(0.33)	(0.41)	(0.39)	(0.80)
Unemployment Rate	2.79	1.22	−1.77	−3.04
	(0.70)	(1.89)	(2.65)	(5.80)
Private Employment	4.34	5.62	6.55	12.34
	(1.10)	(1.11)	(1.46)	(4.11)
Women, < University				
Mean public wage	−1.46	−0.48	−1.06	—
	(0.28)	(0.20)	(0.30)	
Mean private wage	−0.05	−0.22	−0.18	—
	(0.12)	(0.21)	(0.30)	
Unemployment Rate	2.28	−0.25	2.58	—
	(1.16)	(2.08)	(2.34)	
Private Employment	4.58	4.09	4.61	—
	(1.39)	(0.98)	(1.02)	

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