VALUING QUALITY

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Abstract

This paper uses revealed preference restrictions and nonparametric statistical methods to bound a quality-constant price series for a good that changes quality over time. Unlike the more usual hedonic regression techniques for estimating quality-adjusted prices, this method does not require us to observe the changing characteristics of the good or to assume a particular functional relationship between these characteristics and quality. To place a bound on quality change using revealed preference conditions we assume that preferences are stable over time, that quality change occurs in one good or group of goods and that the direction of quality change is known.

Key Words: Cost-of-living indices, quality, GARP.

JEL Classification: C43, D11.

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Summary

- This paper suggests a way of using revealed preference restrictions to bound the level of quality change for a good of interest. The model of quality change used is the repackaging model, which assumes that quality change acts as a multiplier on the quantity, and deflator on the price, of the good in question. When consumption data violate revealed preference conditions, it is assumed that this is because the quality of a partcular good is changing. Quality change is calculated as the minimum quality adjustment necessary to this good which ensures that the data are consistent with the axioms of revealed preference. This is consistent with the maintained assumptions that preferences are stable in quality-constant commodity space, that violations can be explained by quality change in one good, and that the quality change is in a known direction.

- The benefit of this technique is that a bound on quality adjustment can be recovered without needing to know about the changing characteristics of a good or to assume a particular functional relationship between characteristics and quality, both of which are necessary for the main types of hedonic approaches to quality measurement.

- We describe how the bound can be tightened using budget expansion paths

- The procedure is applied to simulated data with known quality change to examine its performance. It is also applied to UK micro data on audio-visual equipment over the period 1974 to 1996.

- Audio-visual equipment is a composite commodity which appears in the UK Retail Prices Index. We calculate the minimum quality improvement to audio-visual goods which is necessary to make the data consistent with

stable consumer preferences. We …nd that failure to account for quality improvement in audio-visual equipment causes an upward bias in the RPI of around 1% over the period. This means that the average annual rate of in‡ation (January to January) over the period, which is 8.10%, is biased upwards by 0.05 percentage points and should be 8.05%.

1. Introduction

Economic cost-of-living indices compare the minimum cost of achieving a reference level of economic welfare across di¤erent price regimes. The notion of being able to conduct such a comparison is based on the assumptions that consumers behave as rational utility maximisers and that their preferences remain stable over time. Even in the simplest of circumstances, these are strong requirements. Quality change in goods over time complicates matters further, since a comparison of the cost-of-living at di¤erent times must be based at a reference level of quality. This means it is necessary to be able to calculate the minimum cost of achieving a given level of welfare at a di¤erent level of quality than that which actually prevailed at the time. This requires a theory of how quality change enters the utility function, and hence the cost function.

It is possible to construct price indices without without making any assumptions on the nature of consumer behaviour (the axiomatic approach to index numbers, for example, is concerned with constructing a price index with certain reasonable or desirable empirical properties), but quality change poses no less of a problem for the correct calculation of price indices than it does for cost-of-living indices. Price indices generally compare the price of buying a given basket of goods across di¤erent price regimes. If the quality of some good changes between these two periods, then prices will not re‡ect the cost of buying the same (i.e. quality-constant) good in these periods, and the price index will not be a true re‡ection of how the cost of the …xed basket of goods has changed. To calculate a quality-constant price index, it is necessary to be able to calculate quality-constant prices. The part of the price change in a good which is due to a quality change must be stripped out of its overall price change, so that we are left with prices as if quality had remained unchanged.

There are two main approaches in economic theory to dealing with quality change; the linear characteristics model originally proposed by Gorman (1956) and developed by Lancaster (1966), and the repackaging model of Fisher and Shell (1971) later generalised by Muellbauer (1975). Both interpret quality change as affecting the price of the quality-changing good, so that its price in any period can be calculated at different levels of quality. This means that there is an economic theory of quality change which is logically consistent with the method of dealing with quality change in price indices by quality adjusting prices.

The way in which quality-adjusted prices have often been calculated in the past is by hedonic regression techniques which parametrically estimate prices as some function of observable characteristics of the good in question. We present an alternative method for calculating a bound on quality change which uses the conditions imposed by revealed preference theory. We use the repackaging model of quality change which assumes that quality change can be represented as a multplier on the quantity, and a deflator on the price, of the good in question. When observed data violate the axioms of revealed preference, we calculate the minimum quality adjustment necessary to the good of interest to remove the violations. This does not require us to have information on the changing characteristics of the good or to make any assumptions on the exact form of the relationship between characteristics and price, as is necessary when estimating hedonic price indices. Our identifying assumptions are that preferences in quality-constant commodity space are stable, that observed violations can therefore be explained by quality change in the good of interest, and that this quality change is in a known direction. The assumption of stable preferences is strong, but it is one that is often made, implicitly or explicitly, in applied microeconomic analysis. Moreover, it is a necessary assumption for the concept of a true cost-of-living index to have any meaning. In addition, without the framework of stable preferences, it is diffficult to make sense of the concept of quality change and of consumers' valuation of this quality change. We seek to explain and correct observed violations of the axioms of revealed preference by quality change in one good (or group of goods). This is also a strong assumption, but, given data which is not consistent with the maximisation of a stable, well-behaved utility function, it is interesting to explore whether there are any simple adjustments which will make the data consistent with stable preferences, and to examine the difference that these adjustments make to the calculation of a cost-of-living or price index.

We apply our technique to simulated data with a known path of quality change to assess how well true quality change is bounded. We also apply the technique to the audio-visual goods section index of the Retail Prices Index (RPI) to bound the quality change of this group of goods, and to calculate the impact that using these quality adjusted prices has on the RPI.

The plan of this paper is as follows. In section 2 we set out more formally the problem that quality change presents for the calculation of cost-of-living indices and price indices. We then review the theoretical approaches to modelling quality change in section 3. In section 4 we present a revealed preference method for calculating a bound on quality change, and explain how this bound can be improved by making use of non-parametric estimates of expenditure expansion paths. Section 5 presents the application of our method to simulated data, and section 6 presents the empirical application to audio-visual goods. Section 7 concludes.

2. The problem of quality change for index numbers

The approach to calculating cost-of-living indices using economic theory is based on the hypothesis of consumers as rational utility maximisers. Such a cost-of-living index compares the cost of achieving a reference level of economic welfare across two price regimes. Defining the utility function in period t as

$$u_t = U_t\left(q_t^0; q_t^1; \ldots; q_t^K\right)$$

we have the corresponding cost function

$$c_t(\overline{u}; p_t^0; p_t^1; \ldots p_t^K) = \min\left\{p_t' q_t : U(q_t) \geq \overline{u}\right\}$$

A cost-of-living index between two periods, r and s say, compares the cost of achieving a given level of utility in those periods.

$$C_s^r = \frac{c_r(\overline{u}; p_r^0; p_r^1; \ldots p_r^K)}{c_s(\overline{u}; p_s^0; p_s^1; \ldots p_s^K)}$$

In order for this comparison to be meaningful, preferences must be stable over time, i.e.

$$U_t\left(q_t^0; q_t^1; \ldots; q_t^K\right) \equiv U\left(q_t^0; q_t^1; \ldots; q_t^K\right)$$

Quality change poses a problem since it means that this requirement will be violated. Suppose that good 0 is subject to quality change. Preferences over consumption bundles will not be constant over time since a unit of good 0 in period r is not the same as a unit of good 0 in period s. This means, for example, that

$$U_r\left(q_r^0; q_r^1; \ldots; q_r^K\right) \neq U_s\left(q_s^0; q_s^1; \ldots; q_s^K\right) \text{ for } q_r^i = q_s^i\ \forall i$$

or

$$c_r(\overline{u}; p_r^0; p_r^1; \ldots p_r^K) \neq c_s(\overline{u}; p_s^0; p_s^1; \ldots p_s^K) \text{ for } p_r^i = p_s^i\ \forall i$$

i.e. the utility derived from a consumption bundle which contains identical physical units of all goods in two different periods will not be the same, since the quality of good 0 has changed. Or the cost of achieving a given level of welfare in two periods will not be the same even if unit prices have remained constant.

Because of this, a cost-of-living index that ignores quality change will be comparing the cost of achieving a certain level of utility in different price regimes where those prices represent the cost of a di¤erent quality of good. When the quality of goods is changing over time, the correct procedure should be to compare the cost of achieving a reference level of utility at a reference level of quality across periods. This requires the assumption that preferences over quality-constant goods remains stable over time. We need to know how quality enters the utility function, and hence cost function, to be able to base the cost-of-living index at a given level of quality. Both the linear characteristics model and the repackaging model give the result that quality e¤ects can be expressed purely in terms of price changes, so that the price of good 0 in any period can be adjusted to a di¤erent level of quality.

Price indices such as the RPI in the UK are much more commonly constructed than true cost-of-living indices by statistical agencies. A price index measures how the cost of buying a particular basket of goods changes over time

$$P_s^r = \frac{p_r^0 \overline{q}}{p_s^0 \overline{q}}$$

It is easy to see that thinking of quality change as being re‡ected in prices is also an ideal solution to the problem of adjusting price indices to take into account quality changes. For example, take the simple Laspeyres index

$$L_t^{t+1} = \frac{p_{t+1}^0 q_t}{p_t^0 q_t}$$

If the quality of good 0 improves, say, between periods t and t + 1, then p_{t+1}^0 is the price of a higher quality good than was available in period t, so that $p_{t+1}^0 q_t^0$ mis-calculates the cost of buying q_t^0 in period t + 1. If we think that the higher quality of the good is re‡ected in its price level, then the price index can be correctly calculated by stripping out the quality related change in the price of good 0 from its overall price change, i.e. we want to know what the price of good 0 would have been in period t + 1 if its quality had remained at the period t level.

3. Approaches to modelling quality change

Quality change has typically been approached in one of two ways in economic theory:

1. The Linear Characteristics model originally proposed by Gorman (1956) and developed by Lancaster (1966). This has its roots in household production theory, which proposes that households derive utility from non-market goods that are produced from 'inputs' of marketed goods, time etc. The linear characteristics model is a specific example of this theory where households have preferences over various characteristics, and market goods can be described in terms of the units of different characteristics that they embody. Quality change in a market good thus takes the form of a change in the combination of characteristics that the good contains. If the prices of the individual characteristics are known, then the good can be priced at any level of quality (i.e. combination of characteristics).

2. The Repackaging model of Fisher and Shell (1971) and Muellbauer (1975). In this model, quality change is modelled as a multiplier on the quantity of the good in the utility function and a deflator on the price in the expenditure function. Thus, a doubling in quality means that one physical unit of the improved-quality good is equal to two units of the good at its original quality.

Empirical estimation of quality-adjusted prices has tended to apply parametric regression techniques to one of the two models. In the linear characteristics model, it must be assumed that all the relevant characterictics are observable, and then the 'shadow prices' of characteristics each period are estimated by regressing the price of the good on this set of observable characteristics. To estimate the quality multiplier in the repackaging model, it is necessary to decide what this quality mulitplier depends upon. The most natural hypothesis is that it too depends on observable characteristics. The quality multiplier can then be estimated by regressing log prices on a function of observable characteristics of the good plus time dummies to capture the non-quality related part of price changes. The repackaging model does not suggest a particular funtional form for characteristics in the regression, although it is often taken to be log-linear. This process of estimating changes in price that are to do with changes in quality via the estimation of a parametric relationship between prices and observed product specification is generally referred to as hedonics. The assumption that the quality multplier in the repackaging model is a function of observable characteristics means that parametric estimation of the linear characteristics model and the repackaging model tend to look very similar, except for the exact functional relationship between prices and characteristics. Despite this, it can be noted that the two models actually derive from quite different theoretical assumptions about what underlying preferences are defined over. In both cases, once the relationship has been estimated, the good can then be priced at any base set of characteristics.

Both of the hedonic regression techniques require the assumption that the quality of the good depends on a set of observable characteristics. In this paper, we apply the theory of revealed preference to the repackaging model to calculate a bound on quality adjustments to prices. Revealed preference conditions derive from the requirement only that consumers make consistent choices. The attraction of the repackaging model is the way that quality change simply appears as a multiplier on quantities and a deflator on prices. This means that it is possible to use revealed preference conditions to place a bound on quality adjustment without needing to know about the changing characteristics of a good or to assume a particular functional relationship between characteristics and the quality multiplier, as is necessary with parametric estimation of the repackaging model. In section 3.1 we outline the repackaging model in greater detail and then go on to explain how we propose to use revealed preference conditions to calculate a bound on quality change.

3.1. The repackaging model

Suppose that one good (good 0) is subject to quality change (we take this as being over time, but it could equally apply to cross-sectional quality variation by, for example, region), and that ξ_t is an index of the quality of good 0 at time t. The quality parameter enters directly into the utility function

$$u_t = U\left(q_t^0, q_t^1, ..., q_t^K, \xi_t\right)$$

with

$$\frac{\partial U}{\partial \xi} > 0$$

i.e. higher quality yields higher utility all other things equal.

We can choose to base quality in any particular period, for example we can set $\xi_0 = 1$. Then in any subsequent period, the observed quantity of good 0, q_t^0, can always be adjusted to φ_t^0, so that

$$U\left(q_t^0, q_t^1, ..., q_t^K, \xi_t\right) \equiv U\left(\varphi_t^0, q_t^1, ..., q_t^K, 1\right)$$

That is, the quantity of good 0 is quality-adjusted back to some reference level of quality normalised to one. The adjustment to q_t^0 will depend (positively) on ξ_t. Intuitively, we can think about quality improvement, say, as getting a greater quantity of the good at the old, reference quality than the actual higher-quality quantity observed.

Call the quality adjustment a_t, i.e. $\varphi_t^0 = a_t q_t^0$ so

$$U\left(q_t^0, q_t^1, ..., q_t^K, \xi_t\right) \equiv U\left(a_t q_t^0, q_t^1, ..., q_t^K, 1\right)$$

Corresponding to the utility function is a cost function $c\left(u; p_t^0; p_t^1; \ldots; p_t^K; \xi_t\right)$. In a similar fashion, the price p_t^0 can be adjusted (to ρ_t^0) so that

$$c\left(u; p_t^0; p_t^1; \ldots; p_t^K; \xi_t\right) \equiv c\left(u; \rho_t^0; p_t^1; \ldots; p_t^K; 1\right)$$

Now, since

$$\min \sum_{k=0}^{K} p_t^k q_t^k \quad \text{s.t.} \quad u = U\left(a_t q_t^0; q_t^1; \ldots; q_t^K\right)$$

can always be written as

$$\min \frac{p_t^0}{a_t}\left(a_t q_t^0\right) + \sum_{k=1}^{K} p_t^k q_t^k \quad \text{s.t.} \quad u = U\left(a_t q_t^0; q_t^1; \ldots; q_t^K\right)$$

it follows that $\rho_t^0 = p_t^0/a_t$, i.e.

$$c\left(u; \rho_t^0; p_t^1; \ldots; p_t^K\right) \equiv c\left(u; \frac{p_t^0}{a_t}; p_t^1; \ldots; p_t^K\right)$$

Because the observed quantity purchased, q_t^0, is actually like $a_t q_t^0$ units of the good at its period 0 quality, the observed price for a current-quality unit, p_t^0, must similarly translate into a price of p_t^0/a_t for the good at its reference quality. This means that the budget constraint is not violated by this transformation, since

$$\frac{p_t^0}{a_t}\left(a_t q_t^0\right) = p_t^0 q_t^0$$

In its most general form, the quality adjustment a_t may depend on the utility level u_t and/or on the quantities consumed of the different goods q_t (which we can also normalise to one, so that all comparisons are relative to the period 0 base) as well as on the quality index ξ_t. That is to say, a quality improvement is like getting more of the current good at its old quality, but exactly how much more may depend on the consumer's utility level and combination of goods consumed. Muellbauer (1975) shows that if the quality adjustment depends on u_t and q_t as well as on ξ_t, then $a_t = f(u_t; q_t; \xi_t)$, i.e.

$$U\left(q_t^0; q_t^1; \ldots; q_t^K; \xi_t\right) \equiv U\left(f(u_t; q_t; \xi_t)\, q_t^0; q_t^1; \ldots; q_t^K\right)$$

and

$$c\left(u; p_t^0, p_t^1, \ldots, p_t^K; \xi_t\right) \equiv c\left(u; \frac{p_t^0}{f\left(u_t; q_t; \xi_t\right)} p_t^0, p_t^1, \ldots, p_t^K; 1\right)$$

If the quality adjustment depends only on ξ_t, then a_t is a function of ξ_t alone.

In summary, when the good 0 is subject to quality change, and the utility function takes the form

$$u_t = U\left(q_t^0, q_t^1, \ldots, q_t^K; \xi_t\right)$$

then, even when this preference structure is stable over time, preferences over $\left(q_t^0, q_t^1, \ldots, q_t^K\right)$ alone (i.e. ignoring quality change) are not stable over time. The repackaging model allows us to write the utility function in terms of quality-constant goods, $\left(a_t q_t^0, q_t^1, \ldots, q_t^K\right)$, or the cost function in terms of quality-constant prices, and, if we are willing to assume that these preferences are stable over time, then we can, in theory, construct a quality-constant cost-of-living index.

4. A revealed preference approach to calculating quality change

In this section, we outline our proposed method for using the axioms of revealed preference theory for calculating quality-adjusted prices.

Throughout this paper we use the following definitions and notation, following Varian (1982).

- q_t is directly revealed preferred to q, written $q_t R^0 q$, if $p_t' q_t \geq p_t' q$
- q_t is directly revealed strictly preferred to q, written $q_t P^0 q$, if $p_t' q_t > p_t' q$
- q_t is revealed preferred to q, written $q_t R q$, if $p_t' q_t \geq p_t' q_s$, $p_s' q_s \geq p_s' q_r$,...,$p_m' q_m \geq p_m' q$, for some sequence of observations $(q_t, q_s, \ldots, q_m)$
- q_t is revealed strictly preferred to q, written $q_t P q$, if there exist observations q_s and q_m such that $q_t R q_s$, $q_s P^0 q_m$, $q_m R q$
- Data is said to satisfy the Generalised Axiom of Revealed Preference (GARP) if $q_t R q \Rightarrow p' q \leq p' q_t$, i.e. $q_t R q$ implies not $q P^0 q_t$

GARP restrictions can only be applied to choices generated from a stable utility function. Therefore, when good 0 is subject to quality change, we cannot apply GARP restrictions to observed choices $\left(q_t^0; q_t^1; ...; q_t^K\right)$ given observed prices $\left(p_t^0; p_t^1; ...; p_t^K\right)$ over time. Indeed, we may expect to find that observed choices violate GARP. The repackaging model supposes that the consumer's maximisation problem can be rewritten as

$$\max U\left(a_t q_t^0; q_t^K\right) \quad s{:}t{:}\ \frac{p_t^0}{a_t}\left(a_t q_t^0\right) + p_t^{K\prime} q_t^K = M_t$$

where q_t^K denotes the vector of all goods except good 0, $q_t^K = \left(q_t^1; ...; q_t^K\right)$, and p_t^K denotes the corresponding prices. Assuming that the utility function $U\left(a_t q_t^0; q_t^K\right)$ is stable over time, then GARP restrictions do apply to quality-adjusted data $\left\{\left(a_t q_t^0; q_t^1; ...; q_t^K\right); \left(p_t^0 / a_t; p_t^1; ...; p_t^K\right)\right\}$.

The question we ask is, given observed choices which violate GARP, can we find a quality-adjusted data set $\left\{\left(a_t q_t^0; q_t^1; ...; q_t^K\right); \left(p_t^0 / a_t; p_t^1; ...; p_t^K\right)\right\}$ which does pass GARP? This method allows us to use the conditions that GARP impose to recover a lower bound for the value of quality change a_t.

Take a two period example to illustrate, where we normalise quality to 1 in period 0, so that $a_0 = 1$. Suppose we observe a GARP violation

$$p_0^{\prime} q_0 > p_0^{\prime} q_1 \text{ and } p_1^{\prime} q_1 > p_1^{\prime} q_0 \tag{4.1}$$

What we really want to know is whether the quality-adjusted data pass GARP, i.e. whether

$$p_0^{\prime} q_0 \lessgtr p_0^0\left(a_1 q_1^0\right) + p_0^{K\prime} q_1^K \tag{4.2}$$

and

$$\frac{p_1^0}{a_1}\left(a_1 q_1^0\right) + p_1^{K\prime} q_1^K \lessgtr \frac{p_1^0}{a_1} q_0^0 + p_1^{K\prime} q_0^K \tag{4.3}$$

Suppose we assume that quality is increasing, so $a_1 > 1$ (naturally, the same principles can be applied if we instead assume quality has deteriorated, so that $0 < a_1 < 1$). This gives the following relationship between equations 4.1, 4.2 and 4.3:

$$p_0' q_0 \quad > \quad p_0' q_1 \tag{4.4}$$

$$\Downarrow$$

$$p_0' q_0 \quad ? \quad p_0^{0\prime} \left(a_1 q_1^0 \right) + p_0^{K\prime} q_1^K$$

since

$$p_0^{0\prime} \left(a_1 q_1^0 \right) + p_0^{K\prime} q_1^K > p_0' q_1$$

i.e. nothing is revealed about preferences over q_0 and $\left(a_1 q_1^0, q_1^K \right)$ from this equation.

In addition

$$p_1' q_1 \quad > \quad p_1' q_0 \tag{4.5}$$

$$\Downarrow$$

$$\frac{p_1^{0\prime}}{a_1} \left(a_1 q_1^0 \right) + p_1^{K\prime} q_1^K \quad > \quad \frac{p_1^{0\prime}}{a_1} q_0^0 + p_1^{K\prime} q_0^K$$

since

$$\frac{p_1^{0\prime}}{a_1} q_0^0 + p_1^{K\prime} q_0^K < p_1' q_0$$

and, of course

$$\frac{p_1^{0\prime}}{a_1} \left(a_1 q_1^0 \right) + p_1^{K} q_1^K = p_1' q_1$$

i.e. assuming that quality is improving, then this equation does tell us something about preferences over q_0 and $\left(a_1 q_1^0, q_1^K \right)$.

This is intuitively obvious. Since we are assuming a quality improvement so that $a_1 q_1^0 > q_1^0$, then if q_1 is revealed preferred to q_0 even in the quality unadjusted data, we would expect it to remain so when quality improvement is taken into account. But if $q_0 P^0 q_1$, then we might expect this relationship to change when we account for the fact that good 0 is of a higher quality in period 1 than period 0.

Since we know from equation 4.5 that $\left(a_1 q_1^0; q_1^K\right) P^0 q_0$, in order to make the data pass GARP, we need to find an a_1 large enough to give

$$p_0^0 q_0 < p_0^0 \left(a_1 q_1^0\right) + p_0^{K0} q_1^K$$

So the lower bound on a_1 is[1]

$$a_1 \geq \frac{p_0^0 q_0 - p_0^{K0} q_1^K}{p_0^0 q_1^0}$$

This method of quality adjustment is illustrated in figure 4.1 for a two good case. The observed choices q_0 and q_1 violate GARP. We need to redraw the period 1 choice in period 0 quality space. This multiplies the quantity and reduces the price of the 0th good in period 1. The points where the period 1 choices in period 0 quality terms can lie are along a horizontal line through q_1, as indicated by the horizontal dotted line. The dashed line to the right of the original period 1 budget constraint indicates the minimum amount of quality improvement necessary for these choices to satisfy GARP. This just places the adjusted period 1 bundle on the period 0 budget constraint.

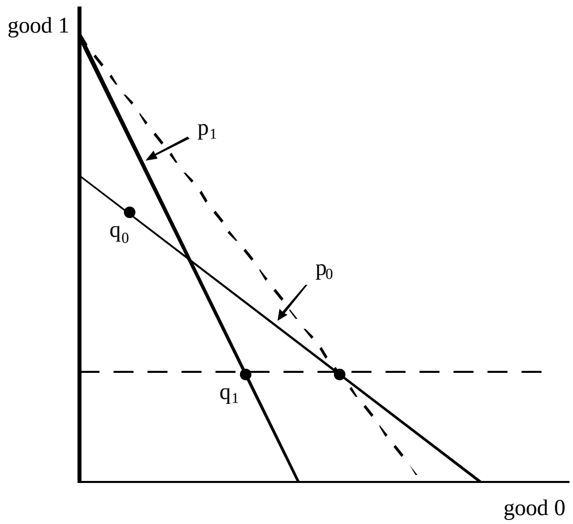
The problem with this technique, as pointed out, for example, by Varian (1982), is that observed data often lacks the power to invoke GARP. This is because real incomes tend to grow over time while relative prices exhibit small variations, and so budget lines can fail to cross. This is illustrated below in figure

[1] Note that the choice of time base is irrelevant. If we instead based quality in period 1 and looked for an $0 < a_0 < 1$, we will again find that $p_1^0 q_1 > p_1^0 q_0 \Rightarrow q_1 P^0 \left(a_0 q_0^0; q_0^K\right)$ so we need to find an a_0 sufficiently small to give

$$p_0^0 q_0 < \frac{p_0^0}{a_0} q_1^0 + p_0^{K0} q_1^K \tag{4.6}$$

Comparing this to equation 4.6 yields $a_0 = 1/a_1$.

Figure 4.1: Quality adjusting data using GARP — a two good, two period example.



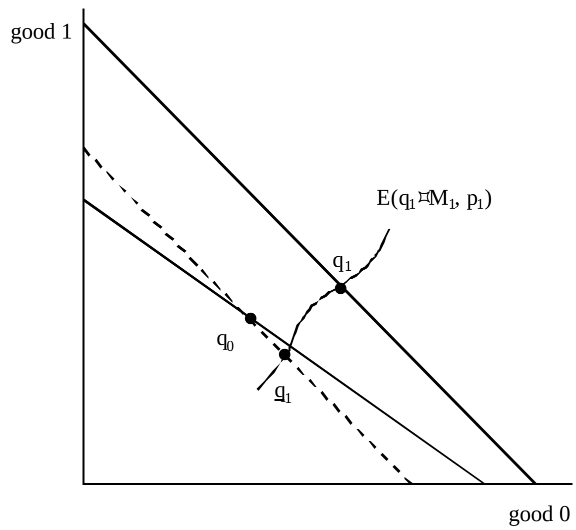
4.2 by the consumption choices q_0 and q_1 on the solid budget lines. With data that looks like this, there is no possibility of a GARP violation, and any amount of quality improvement would still allow the data to pass GARP.

4.1. Improving the bound

We can improve the information we have in data at arbitrary total expenditure levels by using a method similar to one proposed by Blundell, Browning and Crawford (1998) designed to help improve the power of tests of GARP. This involves the non-parametric estimation, from micro data, of budget expansion paths, which show how consumer demand changes as total expenditure changes in a given price regime. E¤ectively, the budget constraint can be moved in or out to any desired point.

This technique allows us to make use of the following proposition both for

Figure 4.2: Failure of GARP to provide quality-change bounds from raw data, and using the expansion path to improve the bound.



testing whether the unadjusted data pass GARP and for calculating a quality adjustment to ensure that the data does pass GARP.

Proposition 4.1. If $p_t^0 q_t = p_t^0 q_r$ and $p_r^0 q_r \cdot p_r^0 q_t$ and all goods are normal, then anywhere on the period t expansion path passes GARP with the choice q_r.

Proof.
(1) Suppose we move out along the period t expansion path from q_t to $\mathbf{q}_t$, so $p_t^0 \mathbf{q}_t > p_t^0 q_t$. Since all goods are normal, $p_r^0 \mathbf{q}_t > p_r^0 q_t$, so

$$p_t^0 \mathbf{q}_t > p_t^0 q_r; \quad p_r^0 q_r < p_r^0 \mathbf{q}_t$$

which passes GARP.
(2) Similarly, if we move in from q_t to $\overline{q}_t$ then $p_t^0 \overline{q}_t < p_t^0 q_t$ and, given normality, $p_r^0 \overline{q}_t < p_r^0 q_t$, so

$$p_t^0 \overline{q}_t < p_t^0 q_r; \quad p_r^0 q_r \ ? \ p_r^0 \overline{q}_t$$

which also passes GARP.
■

What this means, returning to the two period example, is that if we set the budget constraint in period 1 equal to the level which would just make q_0

affordable, so that

$$M_1 \left(= p_1' q_1 \right) = p_1' q_0$$

and we find that these choices satisfy GARP, then, if we are willing to assume that all goods are normal (which we can test for a given data set), we know that anywhere on the period 1 expansion path passes with q_0.

This technique is illustrated in figure 4.2. This shows the expansion path giving choices of consumption bundle under price regime p_1 for any level of total expenditure M_1, denoted $E(q_1 \mid M_1; p_1)$. The dotted line indicates the level of total expenditure in period 1 which would just make q_0 affordable. The bundle $\underline{q}_1$ shows the period 1 choice at this level of expenditure. In this illustration, the choices q_0 and $\underline{q}_1$ constitute a violation of GARP.

Proposition 1 helps us to quality-correct the data in the following way. Suppose we set $M_1 = p_1' q_0$ and find that the data violate GARP, so that

$$\begin{aligned} M_1 &= p_1' q_0 \\ M_0 &> p_0' q_1 \end{aligned}$$

We want to find the minimum quality improvement, say, that ensures the data pass GARP. By increasing a_1 and moving in along period 1 expansion path we can keep

$$\tilde{M}_1 = \frac{p_1^0}{a_1} q_0^0 + p_1^{K\prime} q_0^K$$

so we know that if we can find a point where

$$M_0 \leq p_0^0 \left(a_1 \tilde{q}_1^0 \right) + p_0^{K\prime} \tilde{q}_1^K$$

(where $\tilde{q}_1^0$ and $\tilde{q}_1^K$ denote choices given total budget $\tilde{M}_1$) then this value of a_1 will ensure that everywhere along the period 1 expansion path passes with q_0.

Proposition 4.2. If all goods are normal, the smallest such value for a_1 will be where $\tilde{M}_1 = \frac{p_1^0}{a_1} q_0^0 + p_1^{K\prime} q_0^K$ and $M_0 = p_0^0 \left(a_1 \tilde{q}_1^0 \right) + p_0^{K\prime} \tilde{q}_1^K$.

Proof.
(1) Suppose we find an $\bar{a}_1$ such that

$$\overline{M}_1 = \frac{p_1^0}{\bar{a}_1} q_0^0 + p_1^{K\prime} q_0^K; \quad M_0 < p_0^0 \left(\bar{a}_1 \bar{q}_1^0 \right) + p_0^{K\prime} \bar{q}_1^K$$

(2) We can then reduce a_1 to the point where

$$\overline{M}_1 < \frac{p_1^0}{\tilde{a}_1} q_0^0 + p_1^{K\prime} q_0^K; \quad M_0 = p_0^0 \left(\tilde{a}_1 \bar{q}_1^0 \right) + p_0^{K\prime} \bar{q}_1^K$$

(3) From this point we can move out along the period 1 expansion path to $\tilde{M}_1$ where

$$\tilde{M}_1 = \frac{p_1^0}{\tilde{a}_1} q_0^0 + p_1^{K\prime} q_0^K; \quad M_0 < p_0^0 \left(\tilde{a}_1 \tilde{q}_1^0 \right) + p_0^{K\prime} \tilde{q}_1^K$$

since $\tilde{M}_1 > \overline{M}_1$ and therefore, assuming normality, $\tilde{q}_1^i \geq \bar{q}_1^i \; \forall i$ with the inequality being strict for at least one good. Therefore we have another $\tilde{a}_1 < \bar{a}_1$ which still ensures non-violation of GARP.

■

The next question is whether we can be sure that such a value of a_1 will always exist. The answer to this is yes, and is explained below.

Proposition 4.3. When $M_1 = p_1^0 q_0$ and $M_0 > p_0^0 q_1$ there will always exist a value of a_1 such that $\hat{M}_1 = \frac{p_1^0}{a_1} q_0^0 + p_1^{K\prime} q_0^K$ and $M_0 = p_0^0 \left(a_1 \hat{q}_1^0 \right) + p_0^{K\prime} \hat{q}_1^K$.

Proof.
(1) Everywhere on the period 1 expansion path is associated with a value of a_1 which comes from setting

$$M_1 = \frac{p_1^0}{a_1} q_0^0 + p_1^{K\prime} q_0^K$$

Call this a_{1X}, so

$$a_{1X} = \frac{p_1^0 q_0^0}{M_1 - p_1^{K\prime} q_0^K} \tag{4.7}$$

(2) Similarly, there is a value for a_1 given M_1 which comes from setting

$$M_0 = p_0^0 \left(a_1 q_1^0 \right) + p_0^{K\prime} q_1^K$$

call this a_{1Y}, so

$$a_{1Y} = \frac{M_0 - p_0^K q_1^K}{p_0^0 q_1^0} \tag{4.8}$$

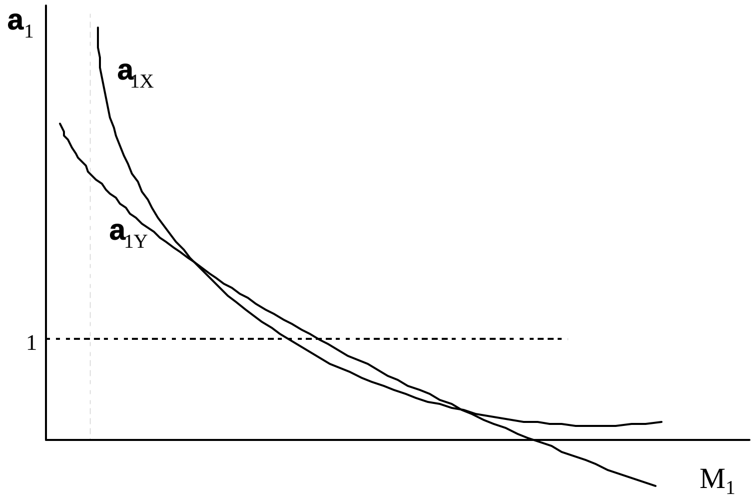
(3) Therefore, we are looking for the value of M_1 at which $a_{1X} = a_{1Y}$.
(4) Since the raw data violates GARP, we know that when $a_{1X} = 1$, $M_0 > p_0^0 q_1^0 + p_0^{K\prime} q_1^K$ so, from equation 4.8, $a_{1Y} > 1$, i.e. a_{1X} lies below a_{1Y} at this point. Call this point on the period 1 expansion path $\tilde{M}_1$ (= $p_1^{\prime} q_0$).
(5) Inspection of equation 4.7 shows that $a_{1X} \to \infty$ as $M_1 \to p_1^{K\prime} q_0^K$ (call this value of the period 1 budget $\underline{M}_1$). Similarly, equation 4.8 shows that $a_{1Y} \to \infty$ as $M_1 \to 0$
(6) Equation 4.7 also shows that $a_{1X} \to 0$ as $M_1 \to \infty$. Equation 4.8 shows that $a_{1Y} = 0$ when $p_0^{K\prime} q_1^K = M_0$ (call the value of M_1 where this occurs $\overline{M}_1$).
(7) Steps (4) and (5) imply that a_{1X} and a_{1Y} must intersect somewhere between $\underline{M}_1$ and $\tilde{M}_1$ where $a_{1X} = a_{1Y} > 1$, and steps (4) and (6) imply that a_{1X} and a_{1Y} must intersect somewhere between $\tilde{M}_1$ and $\overline{M}_1$ where $0 < a_{1X} = a_{1Y} < 1$.
■

Therefore, when the data violate GARP there will be both a quality improvement and a quality deterioration adjustment that will fix the rejection. Although, in the examples, we have only illustrated quality improvements, exactly the same principles apply to quality deteriorations since they are, after all, simply different values for a. Because, when the data violate GARP, the nature of the functions a_X and a_Y means that there will always be a choice of quality adjustment, we will have to make an a priori assumption on the direction of quality change that we are expecting, for example in the case of computers it would be reasonable to say we think quality has been improving.

In addition, in our application, we always find that there is a unique value for $a_{1X} = a_{1Y} > 1$ and a unique value for $0 < a_{1X} = a_{1Y} < 1$. A sufficient condition for uniqueness is that both a_{1X} and a_{1Y} are convex to the origin with respect to M_1 (for positive values of a_1, which is the only area we are interested in). This is illustrated in figure 4.3.

Equation 4.7 shows that, if all goods are normal, then a_{1X} is convex (i.e. $\partial a_{1X}/\partial M_1 < 0$ and $\partial^2 a_{1X}/\partial M_1^2 > 0$). If all goods are normal, then we can be sure that $\partial a_{1Y}/\partial M_1 < 0$, but the sign of $\partial^2 a_{1Y}/\partial M_1^2$ depends on the vector of

Figure 4.3: Graphical representation of calculation of quality adjustment when the raw data fail GARP.



the second derivatives of q_1 with respect to M_1. For example, if these are all zero, then that is sufficient for convexity (but it is not necessary).

4.2. More than two periods

So far, we have illustrated our technique of placing a bound on quality change using only two periods. In practice, we will probably want to calculate the quality change of a good over a longer time period. When there are more than two periods, we calculate a chronological set of quality adjustments in the following way. First, we decide on the direction of quality change we are looking for, so that whenever there is a violation of GARP, we always choose the quality-improving adjustment or always choose the quality-deteriorating adjustment. Suppose we have decided on quality improvement for example, then we assume that once quality has improved it must remain at at least that level from that time onwards. The process is given in the following algorithm:

Inputs:
A set of total expenditures $\{M_0; M_1; ...; M_T\}$.

A set of quality inclusive prices $\{p_0; p_1; \ldots; p_T\}$.
A set of budget expansion paths $\{E(q_0|p_0; M); E(q_1|p_1; M); \ldots; E(q_T|p_T; M)\}$ where q_t is $(K+1 \times 1)$.

Output:
A set of quality adjustment factors $A = \{1; a_1; \ldots; a_T\}$.

Algorithm:
1) Set $B = \{0; 1; \ldots; T\}; A = \emptyset; D = \emptyset$
2) Set $b = \inf\{B\}; C = B \backslash b$
3) Compute $q_t = E(q_t|p_t; M_b)$ and $q_b = E(q_b|p_b; M_b)\ \forall t \in C$
4) If $p_b' q_b > p_b' q_t$ goto (5), otherwise $\tilde{\alpha}_t = 1$, goto (6).
5) Compute $\tilde{\alpha}_t$ and $\tilde{M}_t$ such that

$$\tilde{\alpha}_t = \left(p_t^{0} q_b^{0}\right) / \left(\tilde{M}_t - p_t^{K'} q_b^{K}\right) = \left(M_0 - p_b^{K'} \tilde{q}_t^{K}\right) / \left(p_b^{0} \tilde{q}_t^{0}\right)$$

where $\tilde{q}_t = E\left(q_t | p_t; \tilde{M}_t\right)$:

6) Set $\tilde{\alpha}_t = \sup\{\tilde{\alpha}_t; \tilde{\alpha}_s\}\ \forall s < t$
7) Set $\tilde{A}_b = \tilde{\alpha}_t$: Set $D = D \cup b$: Set $B = B \backslash D$:
8) If $b \neq T$ goto (2), otherwise goto (9).
9) Reset $B = \{0; 1; \ldots; T\}$
10) $\tilde{A}_b = \left\{\inf\left\{\tilde{A}_{b-1}\right\} \tilde{\alpha}_i | \forall \tilde{\alpha}_i \in \tilde{A}_b\right\}$
11) $A = \left\{\inf\left\{\tilde{A}_b\right\} | \forall \tilde{A}_b\right\} \cup 1$
12) Stop.

Let us assume we are looking for quality improvements. The algorithm begins by comparing periods 1 to T in turn to period 0 (our reference quality level period in which quality is normalised to 1) and calculates the quality minimum adjustment whenever GARP is violated. Since we are assuming that a quality impovement can never be reversed, we then calculate a path of quality adjustments from these results which sets each period's quality adjustment equal to the maximum of what the comparison with period 0 tells us, and the highest quality adjustment calculated for preceeding periods. For example, suppose that we have periods 0 to 4, and the comparison of periods 1 to 4 with period 0 gives us this set of values for a_t/a_0 of 1.5, 1.2, 5, 1. Then the path of quality change is reset to 1.5, 1.5, 5, 5. This is saved as the set $\tilde{A}_0$ and the algorithm moves on to use year 1 as the base year and compares periods 2 to T to year 1. This gives us a path of quality improvement relative to period 1's quality which is stored in $\hat{A}_1$, and so on until the final comparison between periods $T-1$ and T. At the end, all of these quality adjustment paths are re-based, recursively to the period 1 reference quality. For example, using our illustration, suppose we found that, compared to period 1, quality in periods 2, 3 and 4 was 2, 3, 3. Then, since we know from the previous comparison of period 1 to period 0, that period 1 is at least 1.5 times better than period 0, this translates into quality in periods 2 to 4 compared to period 0 of 3, 4.5, 4.5. Then, we must take the maximum of the necessary adjustments from the comparisons with period 0 and with period 1 — i.e the comparison with period 1 told us that period 3 is at least 4.5 times better than period 0, but the direct comparison with period 0 told us that period 3 must be at least 5 times better than period 0, so we must take 5 as the value of quality improvement. This redefines $\hat{A}_1$ as $\{1.5, 3, 5, 5\}$. The algorithm carries out this process for all periods to give us the set A, which, when ordered from the smallest to the largest element gives the final path of quality change.

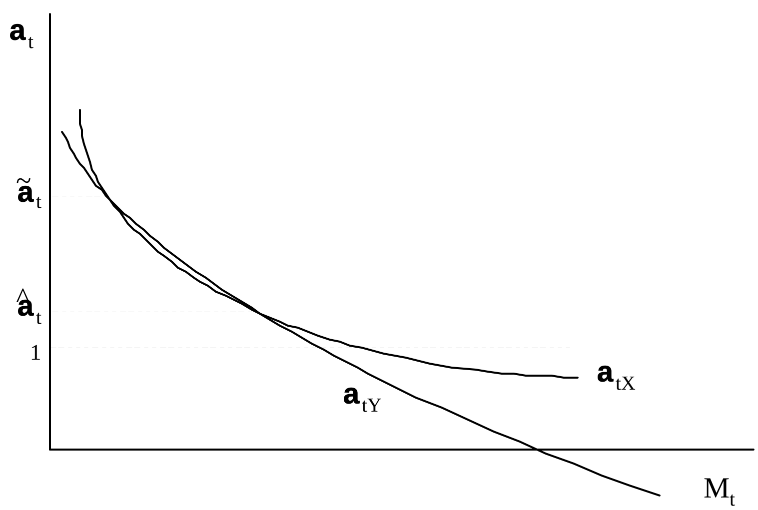
This gives the intuition behind the quality-adjusting process, but it needs a slight modification for the following reason. At the end of the first iteration, for example, we know that everywhere on the expansion paths for periods 1 to T passes GARP with q_0. We also know that for a period that initially failed GARP with period 0, we calculated the minimum quality improvement necessary to make that period pass with period 0. Therefore, we know that any further quality adjustments to that period later on in the process will not introduce violations with period 0. However, if a period initially passed with period 0, its price may be adjusted later on in the process, and we cannot be sure that this will not introduce a violation between this period and period 0. To see this, consider what the functions a_{tX} and a_{tY} look like for period t which satisfies GARP with period 0. Since the data passes GARP, we have

$$M_t = p_t^0 q_0$$

$$M_0 \leq p_0^0 q_t$$

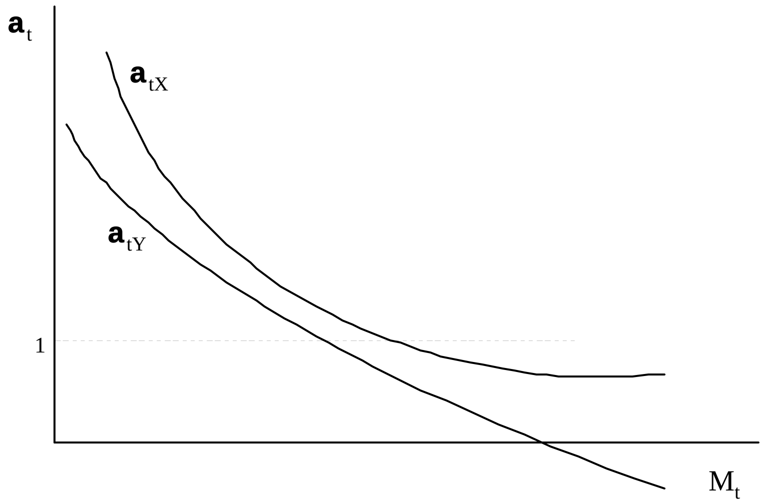
with, therefore, a_{tY} lying below a_{tX} when $a_{tX} = 1$ and when $M_0 < p_0^0 q_t$ (when $M_0 = p_0^0 q_t$ then $a_{tY} = 1$ when $a_{tX} = 1$). We know that the adjusted data will violate GARP whenever $a_{tY} > a_{tX}$, and we cannot be sure that this will not happen if period t is quality adjusted so that $a_{tX} > 1$. The data could look like that in figure 4.4, so that setting a_t anywhere between $\hat{\mathbf{a}}_t$ and $\tilde{\mathbf{a}}_t$ would introduce a GARP violation. Or it could look like that in figure 4.5 (or a_{tX} could just be tangent to a_{tY} at some point, or could cut a_{tY} when $a_{tX} < 1$) in which case any amount of quality improvement would still leave the data satisfying GARP.

Figure 4.4: The case where quality adjustments can introduce GARP violations when the raw data pass GARP.



The same point obviously applies to each iteration of the procedure as each successive year becomes the base year. This means that at the end of the quality

Figure 4.5: The case where any amount of quality adjustment still leaves the data passing GARP.



adjustment procedure, we need to recheck the data to see whether any such violations have been introduced. If they have, then we need to reiterate the quality adjustment procedure until all such violations are eliminated. We know that this cannot continue inde…nitely, since there are a …nite number of bilateral comparisons between periods, and, once a violation between two periods has been corrected, violations will never be reintroduced by further quality improvements.

4.3. The nature of the quality adjustment function

If the quality adjustment to prices is a function of u_t and q_t as well as of the quality parameter $»_t$, then we might expect to …nd a di¤erent set of quality adjustments depending on which part of the expenditure distribution is chosen for the base bundle, since u_t and q_t will vary with total expenditure (if we assume everyone has identical preferences, then variations in u_t and q_t come only from variations in M_t). In our empirical application, we calculate the quality adjustment using di¤erent points of the expenditure distribution as the base bundle to see whether we get di¤erent paths for quality change.

5. Simulations

To get some idea of the power of these techniques, we first simulate demand data with known quality change and apply the algorithm described above. In general the bound on the quality change we are able to recover will, for a given true quality change, depend upon the evolution of relative prices over time. It is, for example, relatively easy to construct an example — typically with improving quality and an even more rapidly increasing relative price for the affected good — in which GARP restrictions can be used to bound the true quality change quite closely. Equally it is easy to construct an example in which GARP restrictions provide no information. In this section we report the results of three different simulations in which we apply the ideas described above to randomised processes for relative prices for a given quality change scenario. We then assess how well the procedure does compared to the known quality change. In each we use the following CES model

$$\max u_t = \left[\left(a_t q_t^0 \right)^b + \sum_{k=1}^{K} \left(q_t^k \right)^b \right]^{\frac{1}{b}}$$

$$s.t. \; M_t = \sum_{k=0}^{K} p_t^k q_t^k$$

where we set $b = 0.5$ and where the quality parameter on the 0th good in period t is a_t. We set $K + 1 = 10$ and $T + 1 = 21$ and we alter the quality of the 0th good in each period according to three scenarios:

1. Exponential quality improvement.
2. Logistic quality improvement.
3. Discrete quality improvement.

For each scenario we calculate demands given prices and compute the revealed preference lower bound on the quality adjustment term. We repeat this 1000 times, each time generating a (10×21) matrix of relative prices according to

$$p_t = \alpha_t p_{t-1} + u_t$$

where the starting values are ones, the errors are uniformly distributed on the unit interval and the vector α_t is uniformly distributed on the range 0.9 to 1. We then take the simulated prices for the quality-improving good, let us relabel them as $\tilde{p}_t^0$, and replace them with the quality-inclusive prices that would be observed in practice. We allow for some randomness in the relationship between quality and price by calculating the quality-inclusive price according to the following relationship

$$\ln p_t = \ln \tilde{p}_t^0 + \ln(a_t \gamma_t)$$

where γ_t is uniformly distributed over the range 0.95 to 1.05. The quality improvement held constant over repetitions. The following figures illustrate the true quality time series for the 0th good and the revealed preference bound at the 5th, 50th and 95th percentiles of its distribution. The idea is to see how well we do 90% of the time.

In each figure the upper line is the time series for the true quality change. Compared to the true quality change, in all cases the (median) lower bound provided by GARP restrictions is probably quite poor. The GARP-based method manages to pick up at least some of the quality variation for all scenarios, and for the logistic and discrete models, where the quality change levels off over time, 90% of the simulations recover up to around one third of the improvement by the end of the period. Nevertheless this can only be interpreted as the cost of abandoning parametric methods if we assume that the chosen parametric method recovered the true quality change accurately. In the absence of data on characteristics (as

Figure 5.1: Exponential quality improvement

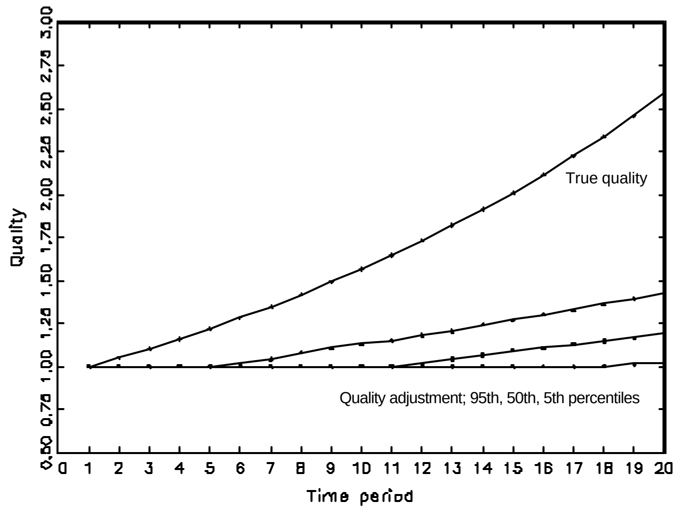


Figure 5.2: Logistic quality improvement

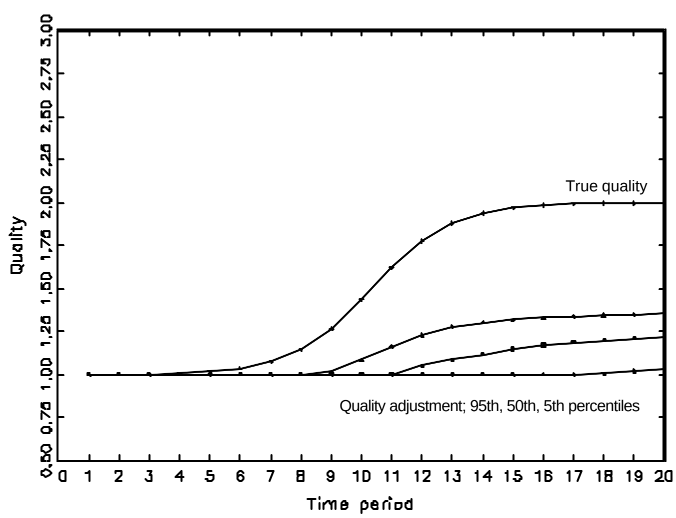
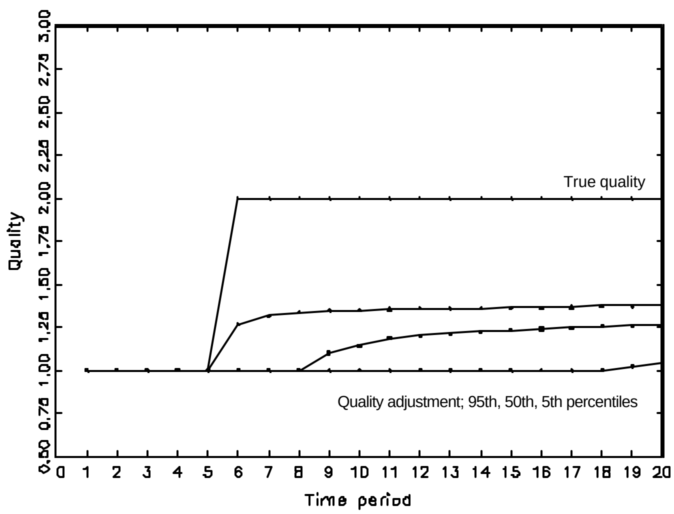


Figure 5.3: Discrete quality improvement



here), or complete knowledge of the underlying data generating process (other than the unknown quality parameter in each period), it is not clear how a usable parametric method could be formulated. A numerical approach to calculating the quality adjustment factor, for example, could only be implemented if the investigator knew that preferences were CES with parameter $b = 0{:}5$. Without this sort of insight the usual practice of using Marshallian demand curves to make inferences regarding the appropriate functional form cannot work in this context since if the observed data $(p; q)$ violate GARP there exists no functional form which is consistent with a well-behaved utility function which takes observed q as its argument.

6. Empirical Application

We now apply the procedure to UK data on audio-visual equipment. This is a section index of the Retail Prices Index and contains television sets, radios, audio and video cassette recorders, musical instruments, and repairs to these goods.

The bulk of the goods in this category have probably been subject to quality improvement over the period studied, which is 1974 to 1996. The aim is to use the techniques we have outlined to place a lower bound on the average quality improvement of this bundle of goods over the period which will ensure that the entire data set passes GARP. We assume that the direction of quality change is upwards.

6.1. Data

The indices calculated in this paper use information on price movements from the section indices of the retail price index for the period 1974 to 1996, and correspondingly grouped household expenditure data from the Family Expenditure Surveys (FES) from 1974 to 1996. The FES is an annual random cross section survey of around 7,000 households (this represents a response rate of around 70%). The FES records data on household structure, employment, income and the spending over the course of a two week diary period. All members of participating households over the age of 16 are asked to complete a spending diary. In the FES the information is aggregated to the household level and averaged across the two week period to give weekly expenditure …gures for over 300 di¤erent goods and services.

We group the data into the thirteen non-housing RPI group index level: food, catering, alcoholic drink, tobacco, fuel and light, household goods, household services, clothing and footwear, personal goods and services, motoring expenditure, fares and other travel costs, leisure goods and leisure services. Finally we remove audio-visual equipment from leisure services and create a fourteenth category for it. The prices we use are the corresponding RPI group price indices for the other twelve groups, the RPI section index for audio-visual equipment and we recompute the leisure goods group index without audio-visual goods using the published weight data and section indices for that group. Since the demand data from each year of FES is collected throughout the year (except for a couple of weeks around Christmas) we also average the prices within the year. Audio-visual equipment carries a weight of 7/1000 in 1996.

6.2. Estimation

We estimate the budget expansion paths we require to improve the quality bound available from the raw data by nonparametric smoothing across the cross section of households within each month/price regime. That is, within each month, prices are assumed constant across households and we use the cross-section variation in total expenditure to identify the expansion path. To be more explicit denote log expenditure for the ith household by $\ln M_i$ and budget share for the ith household by w_{ij} for the jth good. For each commodity j and each household i; we assume a Piglog structure

$$w_{ij} = f_j(\ln M_i) + \varepsilon_{ij}. \tag{6.1}$$

Since $\ln M$ is endogenous then $E(\varepsilon_{ij}|\ln M_i) \neq 0$ or $E(w_{ij}|\ln M_i) \neq f_j(\ln M_i)$ and the nonparametric estimator will not be consistent for the function of interest. To adjust for endogeneity in $\ln M$ we use the augmented regression technique in a semiparametric estimation framework due to Robinson (1988). We use log income $(\ln y)$ as an instrumental variable such that

$$\ln M_i = \pi \ln y_i + v_i \tag{6.2}$$

with $E(v|\ln y) = 0$, and we assume that the following linear model holds

$$w_{ij} = f_j(\ln M_i) + v_i\rho_j + \varepsilon_{ij}. \tag{6.3}$$

We assume

$$E(\varepsilon_{ij}|v;\ln M) = 0 \text{ and } Var(\varepsilon_{ij}|v;\ln M) = \sigma_j^2(v;\ln Mx). \tag{6.4}$$

Following Robinson (1988), a simple transformation of the model can be used to give an estimator for the parameter ρ_j. Taking expectations of (6.3) conditional on $\ln M_i$, and subtracting from (6.3) yields

$$w_{ij} - E(w_{ij}|\ln M) = (v_i - E(v_i|\ln M))\rho_j + \varepsilon_{ij}. \tag{6.5}$$

Replacing $E(w_{ij}|\ln M)$ and $E(v_i|\ln M)$ by their nonparametric estimators, the parameter ρ_j can be estimated by ordinary least squares and is $\sqrt{n}$ consistent and asymptotically normal. The estimator for $f_j^h(\ln M)$ with bandwidth h is then

$$\hat{f}_j^h(\ln M) = E^h(w_{ij} - v_i\hat{\rho}_j|\ln M) \tag{6.6}$$

In place of the unobservable error component v we use the first stage residuals

$$\hat{v} = \ln M - \hat{\pi}\ln y \tag{6.7}$$

where $\hat{\pi}$ is the least squares estimator of π. Since $\hat{\pi}$ and $\hat{\rho}$ converge at $\sqrt{n}$ the asymptotic distribution for $\hat{f}_h(\ln M)$ follows the distribution of $E^h(w_{ij} - v_i\rho_j|\ln M)$.

In our empirical application we use a Nadaraya-Watson kernel regression estimator of the jth share equation with bandwidth h,

$$\hat{f}_j^h(\ln M) = \frac{N^{-1}\sum_l K_h(\ln M - \ln M_l)w_{ij}}{N^{-1}\sum_l K_h(\ln M - \ln M_l)} \tag{6.8}$$

with sample size N, where $K_h(\cdot) = h^{-1}K(\cdot/h)$ is chosen to be a Gaussian kernel weight function $K(\cdot)$, and $\ln M_l$ is the l'th point in the $\ln M$ distribution at which we evaluate the kernel. Using the same bandwidth to estimate each $f_j^h(\ln M)$ guarantees adding up across equations.

To compute demand bundles at some given total expenditure level $\left(\ln \tilde{M}\right)$ from these semiparametric Engel curves, we utilise our common price regime assumption (dropping the bandwidth)

$$E\left(q_j|\ln\tilde{M};\pi_j\right) = \hat{f}_j(\ln\tilde{M})\left(\frac{\tilde{M}}{\pi_j}\right).$$

Since the nonparametric Engel curve has a pointwise asymptotic standard error we can evaluate the distribution of each $\hat{f}_j(\ln M)$ at a point: Briefly, for bandwidth choice h and sample size N the variance can be well approximated at the point $\ln \hat{M}$ for large samples by

$$\mathrm{var}(f_h(\ln \hat{M})) \simeq \frac{\sigma_j^2(\ln \hat{M})c_K}{Nh\hat{f}_h(\ln \hat{M})}$$

where c_K is a known constant and $f_h(\ln M)$ is an (estimate) of the density of $\ln M$ and

$$\sigma_j^2(\ln \hat{M}) = N^{-1}\sum_i \left(\frac{K_h(\ln M_i - \ln \hat{M})}{N^{-1}\sum_i K_h(\ln M_i - \ln \hat{M})} \right) (w_{ij} - \hat{f}_j(\ln \hat{M}))^2.$$

This allows us to compute the variance-covariance matrix for the expansion paths and hence to compute standard errors for the quality adjustment by the delta method using the prices and expenditure levels as known weights.

6.3. Results

We calculate the quality adjustment for different points in the within-year log total expenditure distribution and report the results in table 6.1. The column referring to the 1st decile, for example, reports the minimum quality adjustment necessary for the dataset consisting of demands calculated at the 1st decile point of the within year log total expenditure, and the group prices indices, to pass GARP. All of the points in the total spending distribution which we have examined require some quality adjustment in order to pass GARP and hence to be consistent with the existence of a stable set of preferences over the period. In general the minimum necessary quality adjustment is greatest in the middle of the distribution. By the end of the period, for example, the dataset consisting of prices and demands at mean log total expenditure, requires a minimum quality adjustment to audio-visual goods of nearly 2.4. That is, by the end of this period, the observed price index for audio-visual equipment needs to be adjusted downwards to at least around 40% of its level to be consistent with quality-constant preferences. For the set of demands at the first decile point in each within-year log total expenditure distribution to be consistent with stable preference over the period, the required quality adjustment is about 1.8 – i.e. the price by the end must be reduced to about 55% (at least) of its level to allow for quality changes. These differences across the spending distribution are to do with the way the expansion paths spread out as total outlay changes and are, in general, driven by the nonhomotheticity of the data and compositional differences between the deciles. Note that, as discussed above, even if quality change was exogenous and common in the sense that the choice of goods and the quality improvement facing all households was identical, the welfare derived from that change in quality (which is essentially what we aim to bound) will vary with income, and the demands for other goods.

In order to calculate an adjustment in a way consistent with the RPI we concentrate on the adjustment at the mean of the distribution. Figure 6.1 shows the quality adjustment parameter for audio-visual equipment over the period with 95% confidence bounds. The confidence bounds widen over time because (with the exception of the first quality adjustment) the adjustment in period t is dependent on the adjustment carried out in some earlier year (as described in the algorithm). As a result variances become compounded.

Figure 6.2 shows the annual average price index for audio-visual equipment before and after the quality adjustment is carried out. The dashed line around the adjusted price index is the 95% confidence interval. If we take the published annual index as nonstochastic (this is necessary as the sampling variance of the RPI and its components are not published) then the difference is everywhere statistically significant. Figures 6.3 and 6.4 show, respectively, the bounds on the absolute difference in index points and the percentage difference between the (non-housing) RPI with and without the quality adjustment. The bounds refer

Table 6.1: Quality adjustment, by fractile of the within-year log total expenditure distribution, 1974=1.

Year	Percentile of ln M distribution					
	10th	25th	50th	75th	90th	mean
1974	1.000	1.000	1.000	1.000	1.000	1.000
1975	1.000	1.000	1.000	1.000	1.000	1.000
1976	1.000	1.000	1.000	1.000	1.000	1.000
1977	1.000	1.000	1.000	1.000	1.000	1.000
1978	1.000	1.000	1.332	1.411	1.000	1.266
1979	1.000	1.000	1.332	1.411	1.000	1.273
1980	1.000	1.000	1.332	1.411	1.000	1.273
1981	1.000	1.000	1.332	1.411	1.000	1.273
1982	1.404	1.000	1.332	1.411	1.000	1.273
1983	1.404	1.000	1.335	1.411	1.024	1.276
1984	1.404	1.000	1.335	1.411	1.024	1.276
1985	1.404	1.000	1.706	1.411	1.024	1.648
1986	1.404	1.000	1.706	1.585	1.199	1.648
1987	1.404	1.000	1.706	1.585	1.248	1.648
1988	1.404	1.207	1.737	1.585	1.248	1.690
1989	1.789	1.358	1.832	1.840	1.248	1.793
1990	1.789	1.406	1.894	1.840	1.248	1.894
1991	1.789	1.406	1.894	1.840	1.248	1.894
1992	1.789	1.406	1.894	1.840	1.248	1.894
1993	1.789	1.406	1.894	1.840	1.248	1.894
1994	1.789	1.718	2.290	1.840	1.248	2.381
1995	1.840	1.718	2.290	1.928	1.886	2.381
1996	1.840	1.718	2.290	1.928	1.886	2.381

Figure 6.1: Quality adjustment and 95% condifence bounds by year, 1974=0

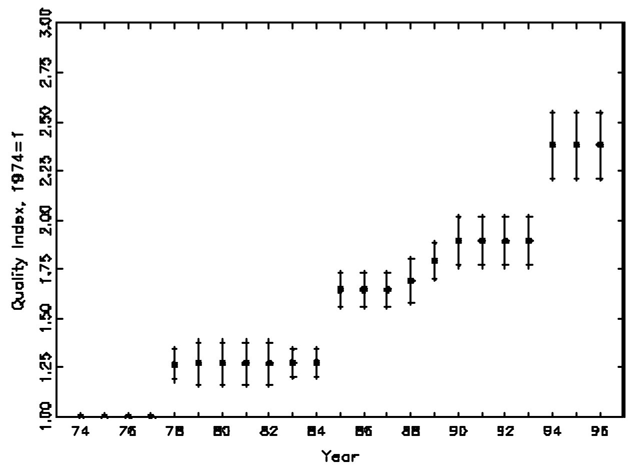


Figure 6.2: Mean price index for audio visual equipment, adjusted and unadjusted, 1974=100

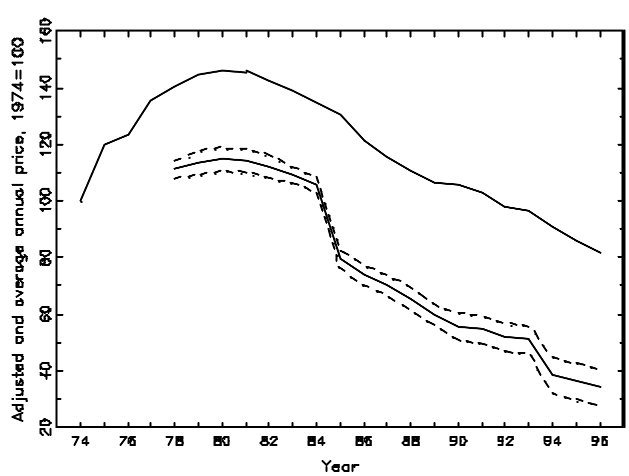


Figure 6.3: Di¤erence between non-housing RPI with and without quality adjustment, index points, 1974=100

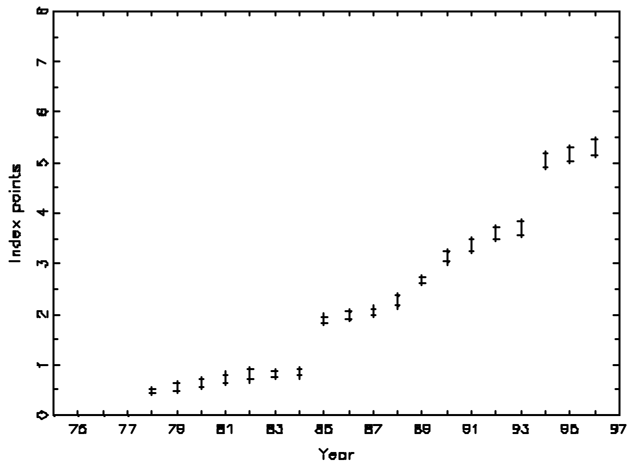
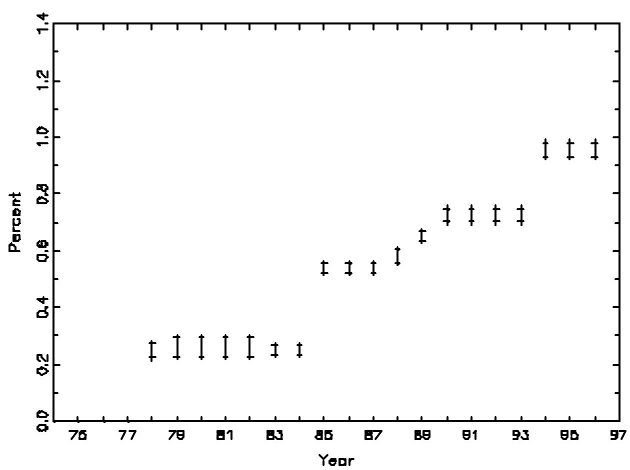


Figure 6.4: Percentage di¤erence between non-housing RPI with and without quality adjustment



to the calculation of the audio-visual equipment quality-constant RPI at the top and bottom of the adjusted price series 95% confidence interval. This indicates that by the end of the period failure to account for quality change in audio-visual goods alone caused an upward bias of around 6 index points, or around 1%. Over the period the annual average January to January non-housing rate of inflation was 8.1%. The rate was 8.05% once the minimum adjustment for quality improvement in audio-visual equipment was made.

7. Conclusions

This paper has suggested a way of using revealed preference restrictions to bound the level of quality change for a good. The theoretical model used is the repackaging model, which hypothesises that quality change is reflected by a multiplier on the quantity of the good and a deflator on its price. The method used requires the maintained assumptions that preferences in quality-constant commodity space are stable and that the quality change is in a known direction. We explore whether violations of revealed preference conditions can be explained by quality change in one good (or group of goods). The main benefit of this technique is that a bound on quality adjustment can be recovered without needing to know about the changing characteristics of a good or to assume a particular functional relationship between characteristics and quality — both of which are necessary for the main types of hedonic approaches to quality measurement. We describe how the bound can be tightened using expansion paths. The procedure is simulated under conditions of known quality change to examine its performance. It is also applied to UK micro data over the period 1974 to 1996, assuming that audio-visual goods have improved in quality. Audio-visual equipment is a composite commodity which appears in the UK Retail Prices Index. We find that failure to quality-adjust audio-visual equipment to correct the data for violations of revealed preference conditions causes an upward bias in the RPI of around 1% over the period. This reduces the annual average January to January non-housing rate of in‡ation over the period from 8.1% to 8.05%.

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