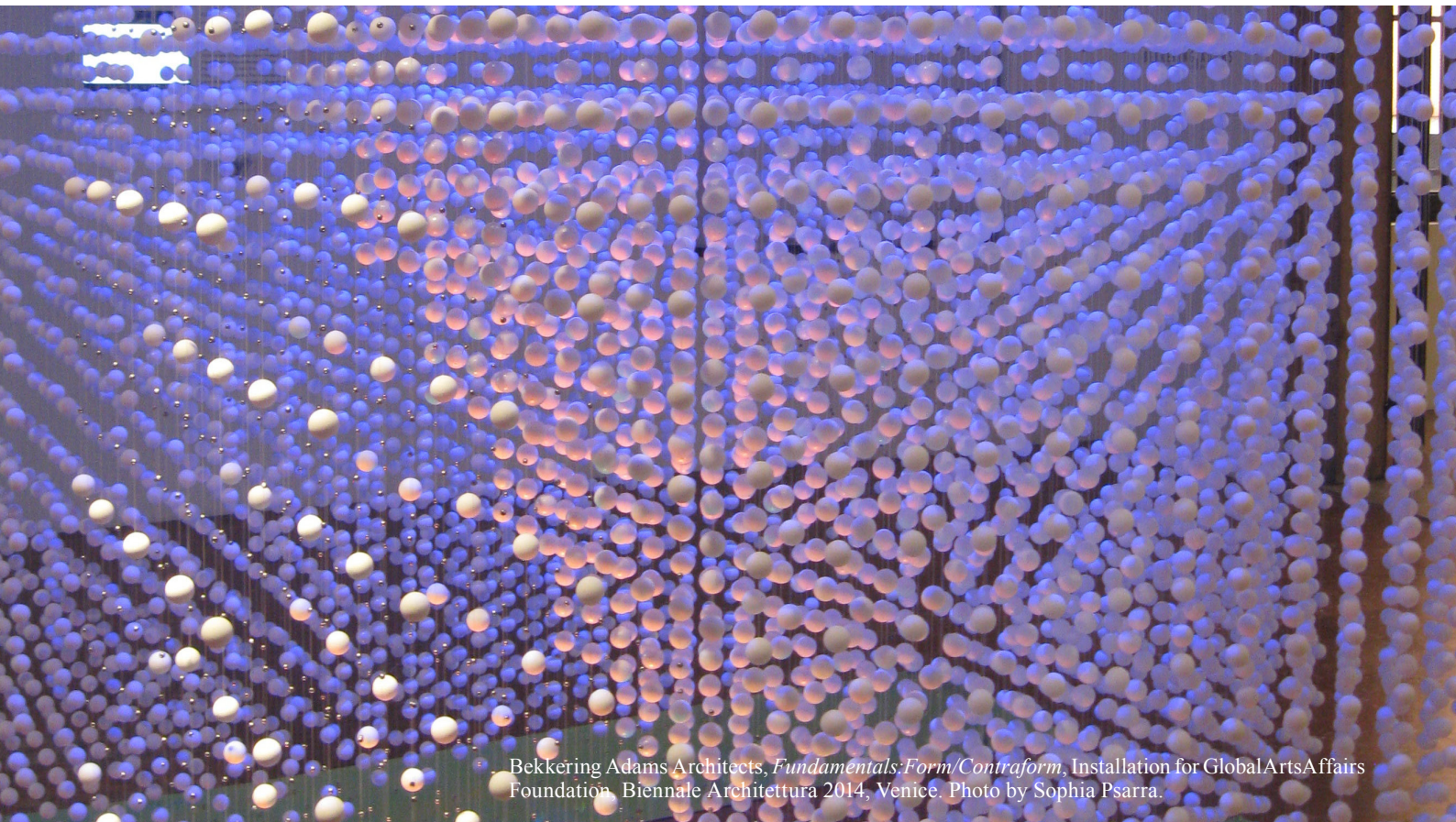


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Bekkering Adams Architects, *Fundamentals: Form/Contraform*, Installation for Global Arts Affairs Foundation, Biennale Architettura 2014, Venice. Photo by Sophia Psarra.

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# Beyond two dimensions: Architecture through three-dimensional visibility graph analysis

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Architecture consists of spatial relations that accommodate functions, afford social relations and create visual interest. Through openings and walls, architects manipulate continuities and discontinuities of visual fields in two and three dimensions. Analytical diagrams and models of these fields have been offered by space syntax, especially through visibility graph analysis (VGA), graphing visual relations in two dimensions. This paper introduces a new approach to VGA that departs from planar restrictions. We show how a graph can be generated of inter-visible locations on a planar surface that incorporates relations among elements in three dimensions. Using this method, we extend the current space syntax analysis of architectural space to a new methodology for diagramming and modelling three-dimensional visual relationships in architecture.

The paper is structured in three parts. The first section provides an overview of the principles of visibility analysis using graphs, and explains the method by which visibility relations of 'accessible' and 'inaccessible' space in two and three dimensions are computed. This leads to a graph representation, which uses a mix of 'directed' and 'undirected' visibility connections, and a new multi-variant spatial categorisation analysis that informs the properties of multi-directional graphs. The second part of the paper tests the three-dimensional visibility model through the analysis of hypothetical and real spatial environments. The third part analyses Giuseppe Terragni's *Casa del Fascio*, describing architectural characteristics that are not captured by two-dimensional analysis, and allowing a comparative understanding of spatial configuration in two and three dimensions. The paper concludes with a discussion about the significance of this new model as an analytical and architectural tool.

*Keywords:*

*Architecture, visibility graph analysis, three-dimensionality, models, diagrams.*

## 1. Introduction

In the article 'From object to field' published at the end of the 1990s, Stan Allen (1997; 1999) argued for a shift away from concepts and models of traditional architectural geometry. Instead of hierarchically conceived top-down ways of representing space, he proposed bottom-up conceptualisations, turning attention from objects to field relations. Theorists and researchers in the space syntax school did not need to perform such a shift, as since the 1970s they had been exploring spaces as spatial relations and visual fields rather than geometrical objects. Yet, despite ground-breaking work throughout this period, the space syntax approach still remains rooted in two-dimensional analysis.

The focus of this paper is three-fold: firstly, to address the reasons for which we need syntactic models that describe visibility properties in three dimensions; secondly, to present a new model for describing and visualising three-dimensional environments; and thirdly, to discuss the first application of this model, and the ways in which it extends space syntax to better account for complexity in architecture.

### ***Theoretical and analytical background***

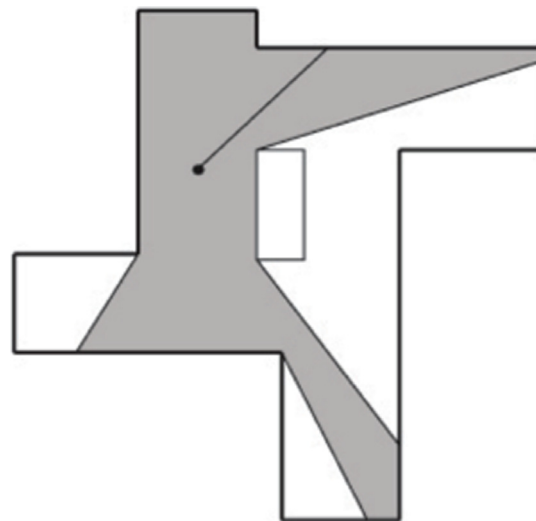
In his book *The Ecological Approach to Visual Perception*, James Gibson posited that visual perception does not arise through inference and interpreta-

tion, but is a direct interaction between people and the setting; the information required for perception is available in the environment (Gibson, 1979). Informed by Gibson's ideas, Benedikt developed 'isovists' and 'isovist fields' as ways of recording and visualising visual information<sup>1</sup> (Benedikt, 1979). An isovist, or 'visibility polygon', is 'the set of all points visible from a given vantage point' (*ibid.*, p.47), while an 'isovist field' is a mapping of geometric values of isovists captured for individual spatial points with respect to a whole environment<sup>2</sup> (Figure 1).

Isovists have been employed in space syntax research in conjunction with empirical data on how people occupy and move in space. The novelty of Benedikt's approach and syntactic methods of modelling and representation (such as axial lines and convex spaces) was that they discretised the environment and studied spatial and visual relations among discrete elements in ways which situated movement and vision at the centre of the analysis. As a result, Benedikt and space syntax replaced the traditional architectural view of architecture as composition by a view of space 'from inside' (Turner et al., 2001), or from the perspective of the user.

Developing Benedikt's concepts and tools, Turner et al. (2001) extended the isovists' capacity to capture visual relations by graphing their structure. The outcome of this work was visibility graph analysis (VGA), subsequently implemented in Turner's Depthmap software (Turner, 2001) – a method for quantifying and representing visual two-dimensional relations in a built environment. Morphological research has a long history of graph-based approaches (Ore, 1963; March and Steadman, 1971; Steadman, 1973; Kruger, 1979; Hillier and Hanson, 1984), but VGA provides fine resolution diagrams and graphs, assigning values to a continuous field of space discretised into a grid. The method draws from Hillier and Hanson's theory<sup>3</sup> (Hillier and Hanson, 1984), and small-worlds networks (Watts and Strogatz, 1998). It overlays a grid of spatial locations

at a flexible resolution and produces a graph of inter-visibility relations of spatial locations in a layout termed as 'visibility graph'. Turner et al. present a series of meaningful characteristics and relations of visibility to movement and social engagement - a description of the social relationships that spaces afford (Turner et al., 2001).



**Figure 1:**

**Example of an isovist, showing visible space from a single point.**

(Source: Turner et al., 2001)

**Notes:**

<sup>1</sup> Tandy (1967) introduced the concept of viewsheds for the analysis of landscape, but it was Benedikt (1979) who developed the method for the consideration of architectural space. Tandy conceived of isovists as a method to '[take] away from the architectural space a permanent record of what would otherwise be dependent on either memory or upon an unwieldy number of annotated photographs' (Tandy, 1967, p.9). A similar concept has a long history in the form of the 'viewshed' in the field of landscape architecture and planning (Amidon and Elsner, 1968; Lynch, 1976) and in terms of 'intervisibility' in computer topographic models (Gallagher, 1972).

Even though VGA has the ability to describe complex spatial relations, it is limited by the two-dimensional planar nature of this analysis<sup>4</sup>. Architecture on the other hand, is seen and experienced in three dimensions. Three-dimensional visibility and its relationship to movement play a key role in design and the experience of buildings and open spaces. This relationship affects what we can physically reach, what we can observe, and how we comprehend spatial and visual interconnections. Architects often accentuate the difference between what we see and where we can go, not only by controlling movement but also through expanding the three-dimensionality of visual fields. From the point of view of architectonic relations and situated vision, we need VGA models with the capacity to explain how



**Notes:**

<sup>2</sup> Benedikt noted that analysis of multiple isovists is required in order to quantify a whole configuration and suggested that the way in which we experience a space, and how we use it, is related to the interplay of isovists. This led to the development of methods to calculate 'isovist fields' which record an individual isovist's properties for all locations in a configuration by using contours to plot the way those features vary through space. The closeness of the contours shows how quickly the isovist properties are changing.

<sup>3</sup> Whilst visibility graphs provide output which looks similar to that of axial maps (Hillier and Hanson, 1984), it is important to recognise that they are measures of different factors; axial analysis, for example, accounts for movement, while isovists can relate to behaviour that is more dependent on the perception of the environment, and its visible qualities.

<sup>4</sup> VGA is currently performed with the open-source spatial graph analysis software 'depthmapX' (Varoudis, 2012), a fork of the original Depthmap by Alasdair Turner (Turner, 2001).

spaces are perceived as three-dimensional terrains, in which the interplay of visibility and permeability relationships is staged and seen (Hanson, 1998; Koch, 2010).

The question of visibility relations in two and three dimensions has been addressed by many space syntax researchers in the past (Hanson, 1998; Psarra and Grajewski, 2000; Dalton and Dalton, 2009, 2010; Sailer and Penn, 2009; Hillier, 2003; Zamani and Peponis, 2007, 2010; Koch, 2010, 2012). Most of these approaches use vertical links manually joining selected elements that are connected along the third dimension. However, a three-dimensional computer model and its theoretical explication is absent from VGA, and all other methods in space syntax research. This deficit stems from this school's long-standing interest in movement as the major force in explaining the link between social performance and spatial configuration. One explanation for this shortage has been that 'humans don't fly' and therefore, anything that is to be understood about movement in three-dimensional space is contained within two dimensions, if we take into account the vertical links between floors through lifts and staircases. However, in the absence of three-dimensional analysis, this proposition forms a large and untested assumption. People are often attracted towards three-dimensional spaces (Lazaridou, 2013), which in general are considered to provide spatial orientation. Atria and courts are key areas not only for buildings, but also for cities and community life. From large transport interchanges to offices, to museums and galleries, and from assembly buildings and city halls to court-houses and public libraries, such spaces bring natural light and ventilation into the interior, offer large areas of recreation facilities and gathering, connect the inside with the urban space outside, and create a sense of community over conventional space types.

In addition to these factors relating to social performance, the three-dimensional configuration

of buildings covers cultural and aesthetic considerations. It is the medium through which societies often express cultural messages and architects articulate their particular idiom and style. This brings us to another assumption behind the lack of three-dimensional analysis. It is often believed that the third dimension refers solely to aesthetic treatment, as opposed to the two-dimensional which is often associated with capturing the operational and social nature of architecture. This idea is implicitly rooted in Modernism through dictums by such architects as Le Corbusier, who said that the plan is the generator. The implication is that architecture can be divided into two parts, one addressing society and function (two-dimensional), and the other visual appearance (three-dimensional). For architects, who approach design as the *integration* of various relational systems and types of performance - from space and form to function and appearance - this idea is not only absurd, but also breaks architecture into an aesthetic and social practice (Psarra, 2010). If architecture as a discipline has any social significance at all, we cannot afford divorcing aesthetics from society. Architectural aesthetics do not stand apart from society but is shaped by it, and shape society in turn, appropriated by human minds.

Space syntax exposes latent structures in buildings and illuminates characteristics in ways that other diagrams and models (discursive or non-discursive) cannot achieve. However, the split of the architectural discipline into social and aesthetic performance remains ghosted in its conceptual dimensions (Psarra, 2010), and is conditioned by the technological limitations of its models and applications. The role of diagrams and models is especially highlighted here as fundamental, not only in how analysis is conducted, but also regarding how architecture is thought of and conceptualised. If architectural theory has provided conceptual-geometric rather than spatial models of architectural description, this is largely

because it lacks the concept of configuration and its relevant visualisations (Hillier, 1996). Similarly, with specific reference to the contribution of space syntax to architectural theory and analysis, it is the *lack of diagrams and models that can access the third dimension, which leads to partial and incomplete theorisations*. When architecture is conflated with technique due to presuppositions that it exists simply in what can be captured two-dimensionally by the available models, the whole question of architecture is removed and possible innovations in describing its complexity are limited. Arguably, only a small fraction of the built environment is built by architects; not all buildings are architecture, and not all buildings or architecture merit three-dimensional analysis. But the structures that space syntax can reveal in all buildings are architectural. Additionally, the role of space syntax in informing architectural design remains subject to models and techniques that can describe architecture and complex design languages.

In this paper we present a prototype for three-dimensional analysis that departs from planar restrictions. We generate a graph of visibility of locations on a planar surface that incorporates elements of the third dimension. By modelling and diagramming two- and three-dimensional relations, we provide a new methodology for the investigation of configurational and visual relationships in design and architecture.

## 2. Graphing visual relations in three dimensions

We begin by providing an overview of the underlying principles of planar VGA, and move next to introducing three-dimensional relations through the classifications of 'accessible' and 'inaccessible' space. The proposed approach is based on Turner et al.'s method for 2D VGA, but extends this through a multi-directional graph representation (Varoudis, 2014). This representation uses the above classifications, and a combination of 'directed' and

'undirected' visual graph relationships, to account for three-dimensional space through a multi-variant categorisation of three-dimensional spatial locations.

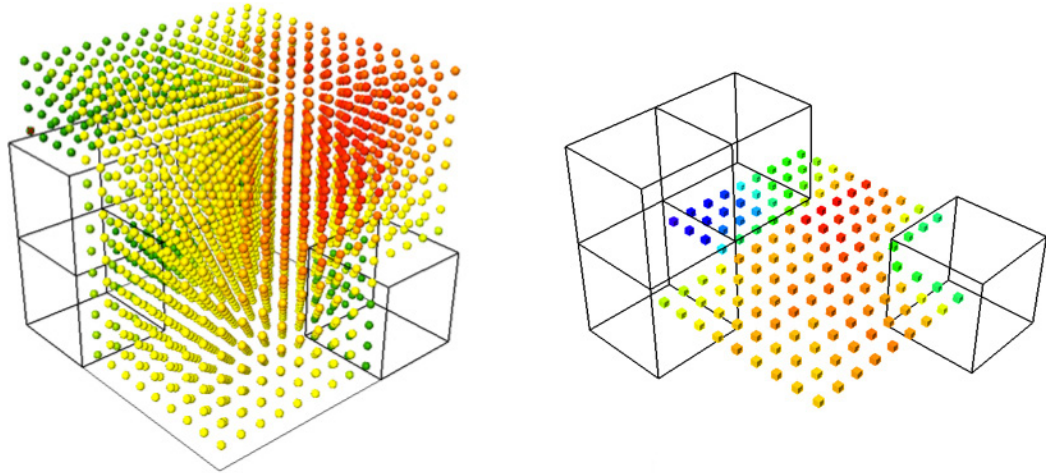
Two-dimensional VGA, as defined by Turner et al. (2001), requires a two-step procedure: firstly, defining an appropriate set of locations on the plan, which form the vertices of the visibility graph; secondly, constructing graph edges between nodes to express direct visibility relations. The most obvious approach to construct nodes and edges is through a grid of locations covering a spatial system at regularly spaced intervals. Ideally, we should select the appropriate grid resolution that 'adequately describes' the spatial system. In practice though, we must try to define a set of locations that offers an acceptable 'near-full' description of the space. For an edge to be added to the graph, the two locations it connects must be directly inter-visible. In Turner's et al.'s (2001) VGA the graph edges are all 'undirected'. This means that mutually visible points in a two-dimensional plane have a symmetrical relationship with each other, in the sense that if 'A' sees 'B', 'B' also sees 'A'. In graph terms, an 'undirected' graph is one in which the edges have no orientation; that is, the edge ('A', 'B') is identical to the edge ('B', 'A').

At a basic level, a simple 3D VGA analysis can be achieved based on Turner et al.'s principles. This means graphing relations among locations in a three-dimensional grid, and taking into account all-to-all visual inter-connections (as depicted in Figure 2a). Such an interpretation of visual relations in space is conceptual and mathematical. It considers all spatial points as nodes in a graph, even though certain points cannot be the locus of human-centred visibility, as they represent 'inaccessible' positions. This is because all-to-all three-dimensional relations are inherently independent of human occupation. In real world environments bound to limitations of gravity and accessibility, we are not able to access

Figure 2:

(a) VGA with 3D 'all-to-all' relations - visual integration.

(b) 3D VGA for comparison.



## Notes:

<sup>5</sup> Directed graph analysis using 'directed' edges has been employed by Peponis and Conroy Dalton in analyzing cross visibility relationships among exhibits in exhibition layouts. The analysis was conducted using Pajek (Peponis and Conroy Dalton, 2004). In this case we use both 'directed' and 'undirected' edges in a 'mixed' graph.

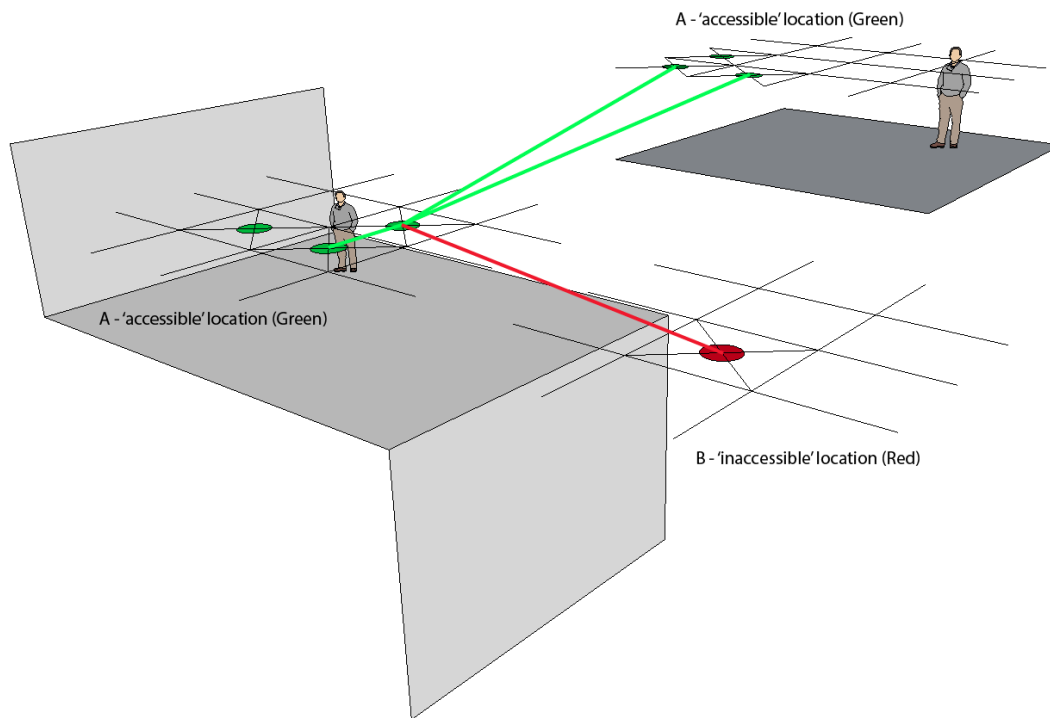
<sup>6</sup> If we need to consider furniture and other low obstacles in the analysis, the plane of spatial points can be located at floor level.

all points in three-dimensional space. We might project our body to other locations and hypothesise what a space looks like from habitable and non-habitable positions (a point inside a void or in a ceiling, Psarra and McElhinney (2014) in this Issue), but from the viewpoint of human occupation, three-dimensional relations are meaningful with reference to embodied vision. It is acknowledged that all-to-all visibility relations in three dimensions are of particular interest, capturing architectonic relationships that govern the conceptual volumetric organisation of space. However, in this paper we focus on three-dimensional space that is knowable through embodied movement and vision (Figure 2b depicts a three-dimensional analysis that focuses on the latter concept).

Therefore, constructing a graph in three dimensions involves differentiating between 'directed' and 'undirected' relationships in graphs, and developing a new strategy for a multi-variant categorisation and sub-setting of 'locations to be analysed' in space. More particularly, forming the graph involves three distinct and interrelated sets of decisions: first, we must select an appropriate set of locations

that fill the three-dimensional volume of the building or space to be analysed; second, from these locations we must choose the appropriate sets of visible locations that are 'accessible' and those that are 'inaccessible' to form the vertices of the graph; and third, construct the graph consisting of 'directed' and 'undirected' edges<sup>5</sup>. To illustrate how categorisations of space and vertices are made in this process we refer to Figure 3.

For the purposes of this work, the classification of 'accessible' nodes defines locations that we can physically access with our body. These locations are situated on a horizontal plane positioned at eye-level<sup>6</sup>. The set of locations in three-dimensional space, that is, those we cannot reach and inhabit, are considered as 'inaccessible' points. Let us suppose that we graph visual relations in an environment consisting of two horizontal planes (Figure 3), configured in a way in which the two planes are occupiable and visual links exist between them through a three-dimensional void. An observer at point 'A' can see and access all other points within the horizontal plane located at eye-level (170cm above ground). He or she can also see point 'B',



**Figure 3:**

*Green signifies 'accessible' locations; red expresses 'inaccessible' locations inside the void. Green lines are 'undirected' graph edges, while red ones are 'directed' edges in a graph.*

which is vertically separated from the horizontal surface he or she stands on (or more specifically, the surface parallel to the floor plate at the height of the observer's eyes), but which is visually connected with this surface.

Point 'A' and all points on the horizontal plane to which 'A' belongs are defined as 'accessible' locations. In contrast, 'B' and all points inside the void are considered as 'inaccessible'. Further, the relationships of 'A' to all 'accessible' points are represented by 'undirected' graph connections. In contrast, the relationship between 'A' and 'B' is a 'directed' relationship. 'Undirected' connections (or 'undirected' graph edges) link points that are both origins and destinations of sight ('accessible' points); 'directed' connections on the other hand, are those that originate from 'accessible' locations and connect with 'inaccessible' ones. In other words, a graph describing three-dimensional

space is a 'mixed graph' with 'undirected' edges between mutually visible 'accessible' vertices, and 'undirected' edges between 'accessible' and 'inaccessible' ones.

We employ a multi-variant categorisation algorithm, which produces two sets of locations in space based upon the criteria described above. To accomplish this, a mechanism for selecting sub-sets according to many variables directly linked to spatial and architectural features was developed. For the purpose of this paper, these multiple variables were linked with 'accessible' locations determined by horizontal planes at eye-level, and 'inaccessible' spatial points situated in voids, balconies, bridges, etc. found below and above eye-level. Variables can be adapted in future models so as to include any spatial or architectural feature that distinguishes between 'accessible' and 'inaccessible' space, such as transparent, translucent or reflective surfaces

**Notes:**

<sup>7</sup> Connectivity is the mathematical graph measure of the degree of a vertex and it is usually denoted as “deg(v)”.

Visual Integration is calculated with the sum of one over the distances between

$$\text{all } x, y: C_H(x) = \sum_{y \neq x} \frac{1}{d(y,x)}$$

(Psarra, 2009; Psarra and McElhinney, 2014, in this Issue), including monitors, ambient screens and projections that construct hybrid topologies (Fatah gen. Schieck, 2005; Schnädelbach, 2007, 2010; Schnädelbach et al., 2006, 2007, 2010; Varoudis, 2011, 2012; Varoudis et al., 2011).

**3. Analysing the graph**

After constructing the 3D visibility graph, two values, ‘3D connectivity’ and ‘3D visual integration’ (Figures 5, 6) are computed<sup>7</sup>. Before presenting these measures and the results of the analysis, it is important to clarify a key concept in 3D VGA in relation to computation and representation of measures. Starting with 3D connectivity, this measure in the mixed 3D visibility graph is similar to the ‘neighbourhood’ measure of a vertex in graph theory and is calculated as the number of ‘accessible’ vertices immediately connected through a ‘directed’ edge plus the number of ‘inaccessible’ vertices connected through an ‘undirected’ edge

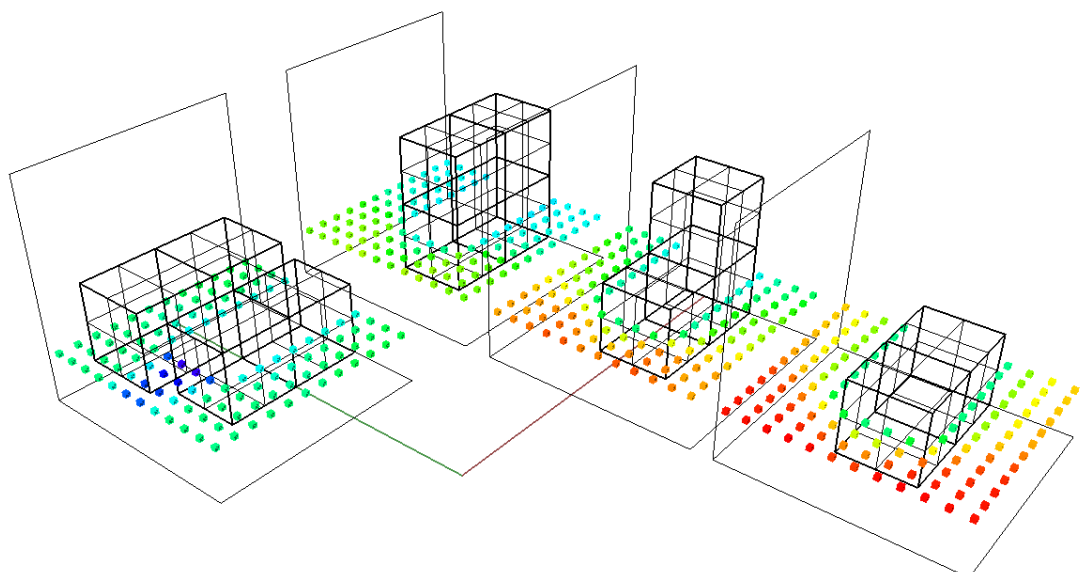
(‘mixed connectivity’). Although in the calculation of 3D connectivity we take into account directed and non-directed edges, we represent the value of each vertex solely at an ‘accessible’ location that represents three-dimensional values on a horizontal plane, as seen in Figure 4. This technique might at first seem unusual for showing three-dimensional relations. However, this convention is adopted simply because values can be assigned only to those locations that are accessible. Those points that feature as visibility destinations (‘inaccessible’) contribute values to the accessible spatial points, but have no values in themselves (and no node count).

Mixed graphs provide an initial indication of the increase in visual information, which can be provided through the introduction of 3D ‘inaccessible’ locations. Figure 4 presents the distribution of 3D connectivity in four hypothetical environments, produced by inserting volumes of different size and shape inside a larger volume, (Figures 4, 5). We use these environments in order to understand how

**Figure 4:**

**3D connectivity.**

Min/blue: 278,  
max/red: 1390.



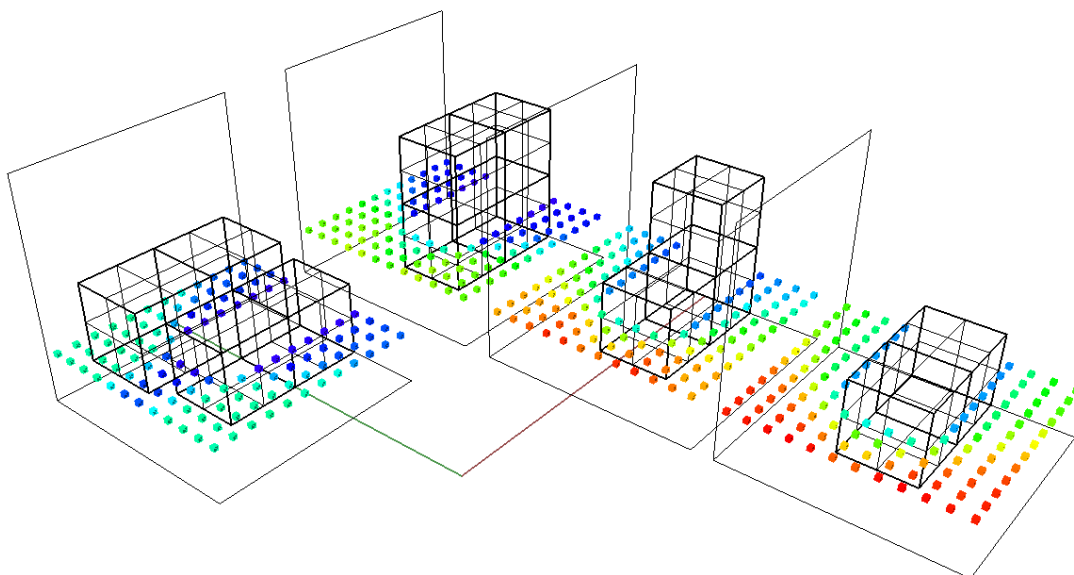


geometric variations of the volumes in two and three dimensions affect global and local visibility patterns of 'accessible' and 'inaccessible' relations. As space is divided into a grid of locations, the connectivity values give a clear indication of the '3D isovist size' at that location with small deviation due to the grid quantisation.

Figure 4 shows how differences in the height of the inserted volume affect 3D connectivity values around it. More specifically, values increase as the height of this volume decreases clockwise from the first figure on the left to the middle and last figure on the right. As we lower the height of the volume we affect the number of 'inaccessible' points that are seen by the 'accessible' ones located at eye level. An analysis of these configurations using traditional VGA methods (Turner et al., 2001) would have produced four identical outcomes. The 3D VGA model in Figure 4 clearly shows the impact of the variations brought by the three-dimensional configuration of space, such as the use of double height voids upon visibility relations.

In order to introduce '3D integration' we need to consider integration in 2D VGA. This measure accounts for the shortest path between two vertices in the graph; that is, the least number of edges that need to be traversed in order to get from one vertex to the other. The mean shortest path length for a vertex is simply the average of the shortest path lengths from that vertex to every other vertex in the system, and so represents an average of the number of turns (plus one) required for any journey within the space. Turner et al. (2001) explain that the mean shortest path in the original implementation of VGA is related to Hillier and Hanson's 'integration' (1984). Hillier and Hanson relate the 'visual accessibility' of spaces to the number of changes in direction, whilst a visibility graph can describe the visual accessibility of every location in the spatial system through the number of edges traversed to get from each to all others.

'3D integration' (Figure 5), builds on integration as defined by Hillier and Hanson (1984) and Turner et al. (2001). It is calculated by taking into account all



**Figure 5:**

**3D integration.**

Min/blue: 0.54,  
max/red: 0.89.

**Notes:**

<sup>8</sup> The Casa del Fascio is one of the most characteristic expressions of the Rationalist movement in structure and form. Rational architecture was popular in Italy between the 1920s and 1940s, and incorporated the styles of Classicism and Futurism. The architecture of this movement was typified by the stripping of applied decoration, and a focus on the intrinsic characteristics of the building's materials in the design.

shortest paths with origin at 'accessible' locations. Visual mean shortest path in a three-dimensional environment is heavily influenced by the third dimension. As the structure of the multi-directional graph includes all 'inaccessible' locations in the third dimension, which are visible from an 'accessible' location, the measure is not only influenced by the addition of 'inaccessible' locations (i.e. the steps from a vertex to all other vertices in the system), but also by the increase in total number of vertices in the system (since this is part of the calculation of mean shortest paths). The mixed graph constructed for the purpose of this analysis includes a high number of points that are 'dead-ends', because of 'inaccessible' locations, but also a greater number of nodes. Figure 5 depicts 3D Integration values in the same hypothetical environment as seen in Figure 4. Similarly to 3D connectivity, 3D integration increases clockwise from left to right with the decrease in the height of the inserted volumes.

The following example shown in Figure 6 examines a more complex environment built from combinations of cubic volumes. While the previous experiments are abstract representations of

architectural spaces, this one could be considered as an approximation of an urban setting. Figure 6 illustrates how building heights and under-building passages influence 3D integration. Even though we focus on the analysis of buildings, the three-dimensional architectural analysis model we discuss here can be descriptive of different types of spaces across different scales.

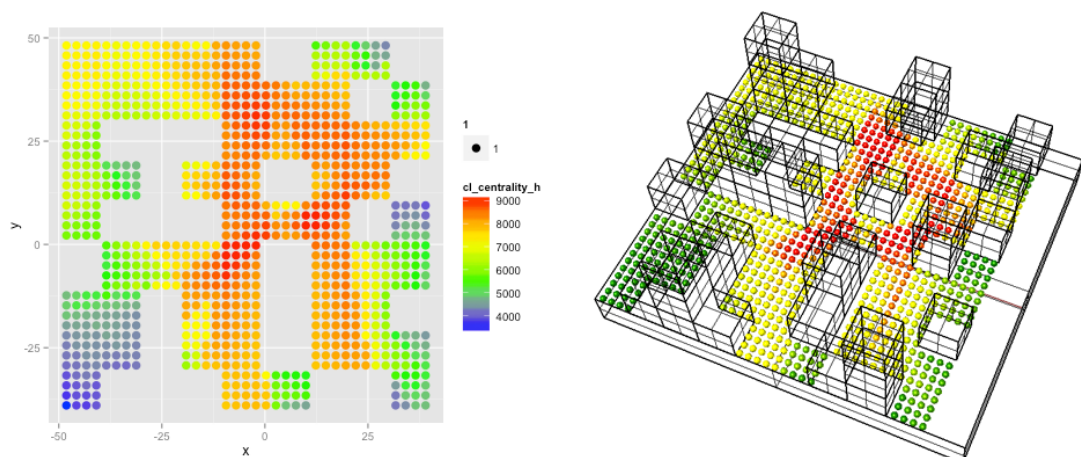
Having demonstrated the principles and application of 3D VGA in hypothetical spaces, we now move to explore the potential of this model in describing real three-dimensional architectural environments through an analysis of Giuseppe Terragni's Casa del Fascio.

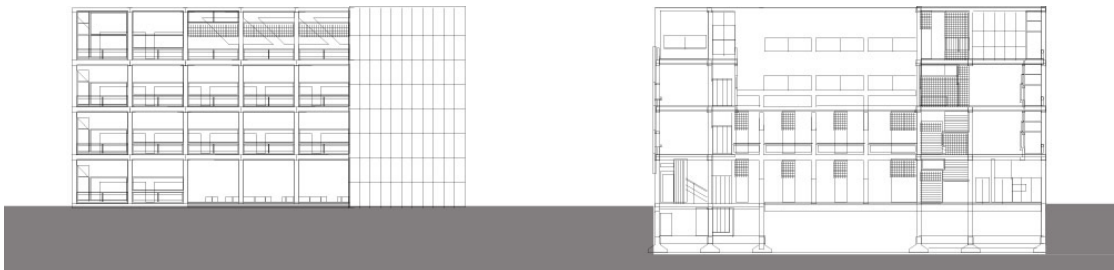
**4. Casa Del Fascio**

Casa del Fascio is Terragni's 'most patently modern building' (Schumacher, 1991, p.66), generally considered as the best known example of Italian Rationalism<sup>8</sup>, and his masterpiece. It was commissioned by the National Fascist Party and built on the site behind the Cathedral in Como, Italy between 1936 and 1939. Terragni was a follower of Modernist architecture and became the leader of *Gruppo 7*,

**Figure 6:****3D integration.**

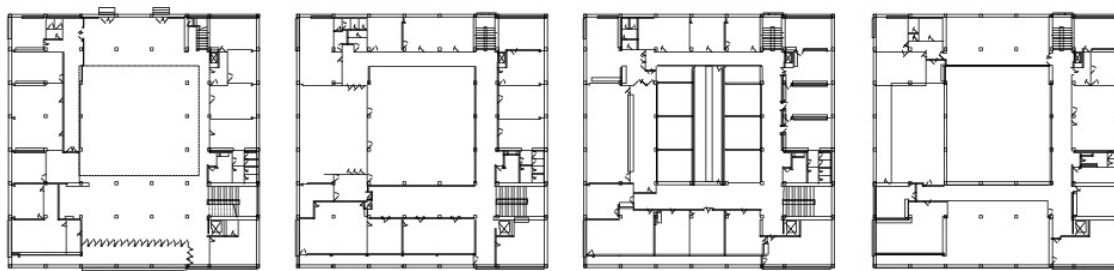
Min/blue: 0.14,  
max/red: 0.56.





**Figure 7:**

**Casa del Fascio.**  
Front elevation (left)  
and section (right).



**Figure 8:**

**Casa del Fascio,**  
**plans.**  
From left to right:  
ground floor, 1st floor,  
2nd floor, 3rd floor.



a group of architects known as the Italian Rationalists. His architecture is known for mixing modern principles with Italian local characteristics. This mix of styles is reflected in the organisation of the Casa, reminiscent of a variety of typological sources spanning the Roman compound, the Venetian Palazzo, the Palladian en suite plan, the Beaux-Art plaid grid, and the Como courtyard house (Schumacher, 1991; Eisenman, 2003). Further, this is reflected in the proportions of the Casa, measuring 33m at the base and rising 16.5m tall, creating half a perfect cube and recalling the emphasis that Classical architecture placed on harmonic ratios. The footprint of the building is a perfect square, with a covered rectangular court functioning as the main assembly hall – ‘the spiritual and celebratory centre of the whole building’ (Terragni in Schumacher, 1991, p.157) – located slightly off centre. The main floor

contained the offices for the provincial party leaders and secretaries of the party; the second floor was used by the administration; while the top floor housed offices and offered two large ‘loggias’ (terraces) situated along the front back axis opposite each other.

The building was intended as ‘the Glass House of Fascism’, exemplifying Mussolini’s concept of Fascism as a house without obstacles between the political leaders and the people. This idea presupposed continuity between inside and outside, so that the public could comfortably approach the building and circulate throughout it, establishing immediate relationships with the directors, commanders and their offices (Terragni in Schumacher, 1991). The intention was also for the leader to speak to his followers assembled inside, and still be heard by people gathered in the piazza outside (*ibid.*).

**Notes:**

<sup>9</sup> This concerns the widespread typology of the atrium or courtyard building that is encountered in many different cultures independently of architectural or regional styles, materials and surface characteristics.

<sup>10</sup> It is essential to emphasise that knowledge of architectural history and theory in this case comes to the aid of the analysis of architecture that chooses building samples informed by the scientific method. This is not a case of adopting either from science or the humanities for architectural research but of complementarity in epistemologies and methods.

We have chosen Casa del Fascio for a number of reasons. Firstly, for its capacity to transcend styles, displaying Classical, Modernist and regional characteristics. Moving beyond historical and stylistic contingencies, the Casa forms a *universal type*<sup>9</sup>. It thus exceeds the limitations posed by a single example, enabling more generalisable observations about architecture to be drawn through the use of three-dimensional analysis. Secondly, the Casa's configuration as a court building means it functions as a *generic type* able to accommodate different functional types, from large houses to public buildings<sup>10</sup>. Thirdly, Terragni's concept of 'visibility' between the public and the officials (*ibid.*), supports the hypothesis that an architectonic expression of this concept might exist through visibility relations in the building. And finally, the Casa was selected due to its contribution to the history of formal description in architecture through diagrams, and specifically through the work of Peter Eisenman (2003) who has

extensively analysed it. To this aspect we return in the last section, where we discuss the role of diagrams and models in analysis and design.

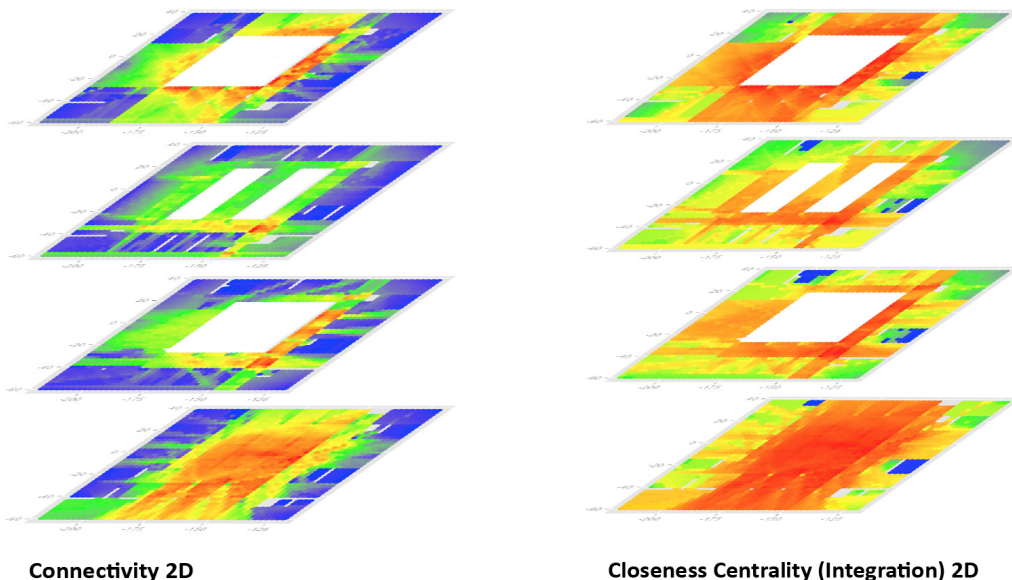
We analysed Casa Del Fascio by identifying approximately 30,000 visibility-locations within the four floors of the building, 5,000 of which were categorised as 'accessible'. Figure 9 shows 2D and Figure 10 shows 3D connectivity (left) and integration (right) of the entire building. Similarly to the analysis of the hypothetical environments described in the previous section, these figures present the values of the 'accessible' locations, with blue indicating low values and red expressing high values for each measure.

Starting with the two-dimensional analysis of the ground floor, Figure 9 shows that high connectivity and integration values are located in the central space, stretching from the front to the back of the building. Although the ground floor is extensively covered by high values of both measures, integration has stronger coverage than connectivity, linking

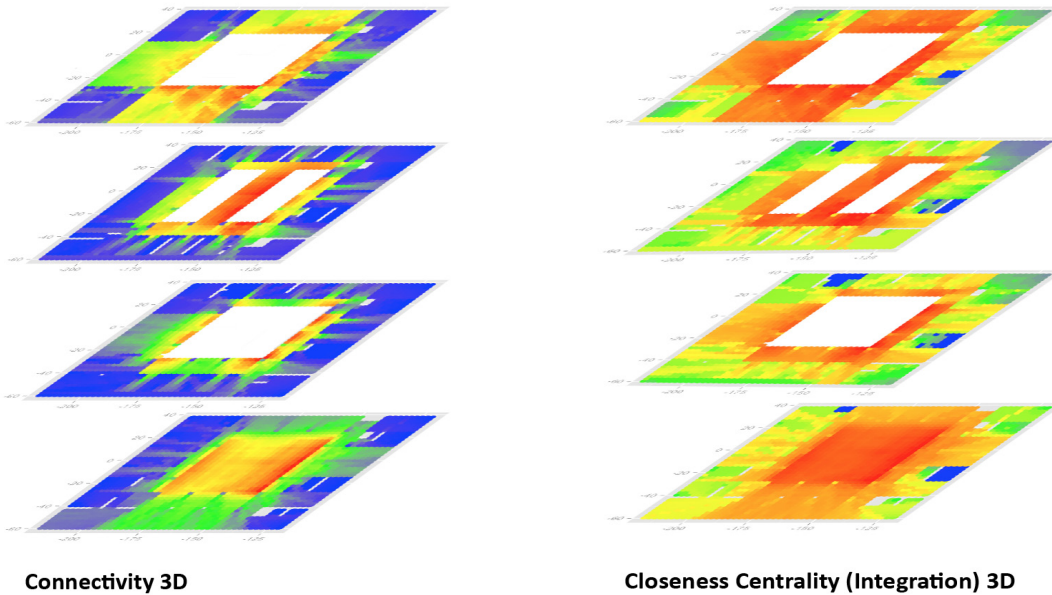
**Figure 9:**

**2D connectivity (left) and 2D integration (right).**

The colour range is normalised across each floor individually.







**Figure 10:**

**3D connectivity (left) and 3D integration (right).**

*The colour range is normalised across each floor individually.*

the inside and the outside, as Terragni envisioned. The right edge of the void above ground level is picked up by the distribution of high connectivity values, while both edges feature in the distribution of integration. However, by and large, the void is not clearly distinguishable through this analysis from the rest of the highly connected areas; that is, the entrance and lobby area on the ground floor. The distribution of connectivity on the first and second floors captures the strong effect of axially introduced by the two balconies on the front and right sides, which are linked with the two staircases in the building. While these circulation elements are clearly highlighted by both measures, the bridge on the second floor has low levels of connectivity. Finally, all four balconies on the fourth floor seem to be well integrated and well connected at the local level. Values are strong along the edge of the atrium void, and this spread extends into the front and back loggias.

Looking at the patterns of 3D connectivity in Figure 10, we see the impact of the atrium as a space through which many other spaces can be seen located at different heights inside the building. Clearly demarcating the void on the ground floor, the warm connectivity colours demonstrate that this area stands in contrast to the rest of the spaces on this level, and particularly the entrance, due to the lower ceiling of these spaces. On entering, visitors have long horizontal views to the far end of the building, unveiling the interior as demonstrated by the expansive coverage of warm colours on the ground floor (Figure 9). As visitors walk towards the central area, the atrium's void affects the distribution of visual connections along the vertical direction, 'impressing' with the volume of spatial points its footprint on the entrance level (Figure 10).

While high values of 2D connectivity, and the highest values of 2D integration on the first and second floor are located at the front and right

**Notes:**

<sup>11</sup> This was clearly intended as we read in Terragni's text on the Casa:

*'The large hall is not symmetrical to the axis of the entrance because of the passage to the right leading to the two staircases (one for the public and a service stair for federation workers). This asymmetry emphasizes the functionalism of the building, directing public circulation to stairs, elevators, services, etc.'*

*(Terragni, in Schumacher, 1991, p.157).*

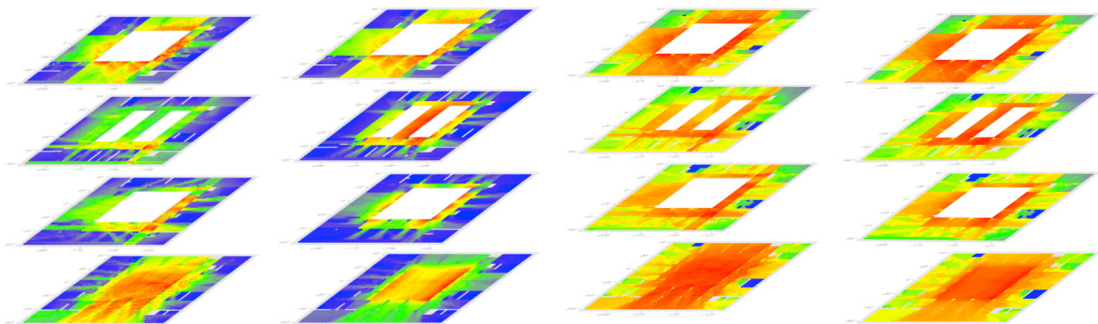
balconies (Figure 9), those of 3D connectivity and 3D integration are almost equally distributed on all balconies around the void (Figure 10). Further, a strong concentration of both 3D measures occurs on the second floor bridge. Therefore, the visibility structure in two dimensions guides the viewer towards the balconies that are directly linked with the staircases and the vertical organisation of circulation<sup>11</sup> (Figure 9). In contrast, the visibility properties in three dimensions emphasise the horizontal structure of circulation surrounding or crossing the void. 2D and 3D visibility therefore (Figure 11) play a complementary role in guiding the visitor to the global structure of movement. This is clearly illustrated in Figures 12 and 13 which show normalised values of 2D and 3D integration across the whole building. We see in these figures that 2D integration in the balconies around the atrium and the bridge is much lower than 3D integration in these areas. The Casa offers a clear demonstration that three-dimensional articulation is not simply about aesthetics and architectural expression, but also intelligibility and cognitive considerations related to the global structure of circulation.

However, the relationship between the 2D and 3D properties (Figure 11) is more complex than this, as already explained in relation to the ground

floor. Strongly influenced by the atrium, high values of 3D connectivity and 3D integration on the top floor spread throughout the terraces (Figure 10), compared with 2D values in the same areas (Figure 9). Similarly to the entrance level, the interior is 'drawn outwards' to the loggias that excavate the front and back elevations of the building (Figure 7). The focus of this paper is on visibility relations from the viewpoint of the observer, but worthy of particular reference is the way in which the analysis illuminates conceptual architectonic relations and their association with the architect's intentions. The two-dimensional and three-dimensional configuration of the Casa captures Terragni's dual intention to remove boundaries between the interior and the exterior, whilst simultaneously constructing heightened interiority through a strong vertical core. In addition, the penetration of warm colours of 2D and 3D integration through the offices on most levels shows a direct interface between the public and the officials, and social co-presence throughout the building. More generally, the analysis helps demonstrate that Terragni translated Musollini's vision for the Glass House of Fascism – into which 'all may look' – through the visibility structure of the building. This translation is much subtler than the one implied by the metaphor of a glass building, or

**Figure 11:**

*Left, 2D and 3D connectivity; right, 2D and 3D integration, side-by-side comparison.*



by the literal use of transparent materials and glass characterising many assembly buildings.

Returning to the interplay between 2D and 3D configuration, it is useful to consider an element where this interplay takes a characteristic form and discuss it in the light of the new analytical model. This refers to the visual-spatial interplay of the second floor bridge, which defines a favourite typology of 'catwalk over the void' for many architects. In terms of situated visibility, the difference between 2D and 3D values as experienced from this element creates a tension between the horizontal organisation of movement and the hollowing out of the volume in three dimensions. In graph theoretic terms, the catwalk is a place where the number of 'accessible' nodes and 'directed' edges is minimised, as opposed to that of 'inaccessible' nodes and 'un-directed edges', which is maximised. The multi-directional graph typologies and classification of graph nodes we have produced begin to provide more rigorous descriptions of architectural complexity, and of the cognitive relationship between the building and the viewer, in ways which were previously unavailable. In addition, by pairing the two types of analysis (2D and 3D), we can clearly bring out the interplay between architectural organisation in two dimensions and three-dimensional articulation, capturing the complex spatiality of buildings.

Concluding this section, it is important to explain that when faced with an iconic and diachronic building of Modernism such as Casa del Fascio, that exceeds both context and style, the three-dimensional analysis triggers a more generalisable discussion. This firmly suggests that rather than a mere distribution of movement flows and programmes, architecture is a cognitive system encompassing a complex language of formal, spatial and programmatic relations. In addition, the interface between inside and outside, and between the visitor and the officials in the Casa suggests that architecture is not an uncritical celebration of spatial complexity, but

a political project<sup>12</sup> in which this complexity shapes relationships among citizens, and those of buildings with the city (Aureli, 2013).

## 5. Conclusion

In this paper we have presented a new approach, developing and applying VGA in three-dimensional environments. Rather than investigating visibility relations in two dimensions and producing a model for an entire building through vertical links, we have constructed a new and unique way of categorising and sub-setting locations in space based on multiple variables and constraints. This involves a mixed graph of 'directed' and 'undirected' visual connections as graph edges. The mixed graph addresses visibility from the perspective of situated vision, differentiating between vertices that can be physically accessed ('accessible') and those that are seen but cannot be reached through movement ('inaccessible'). Further, we have developed a new method of diagramming 3D visibility analysis, which records values in two dimensions.

Much work on visualisation and application of this analysis, already available in 2D VGA, is still needed; such as three-dimensional point isovists, additional 3D visibility measures, agent simulations in three-dimensional environments, and comparisons of the values across the 3D visibility graph with empirical data of movement and occupation. Considering these future directions for work, it is important to emphasise that the preliminary findings of this analysis show it to be the combination of 2D and 3D analysis that generates new and encompassing ways of capturing architectural complexity, and places space syntax at the centre of analysis and critique of architectural design.

We now return to the role of models and diagrams in enabling a re-examination of syntactic knowledge in buildings, and the way in which this is available to situated vision. Faced with a large series of diagrams that explore the formal principles

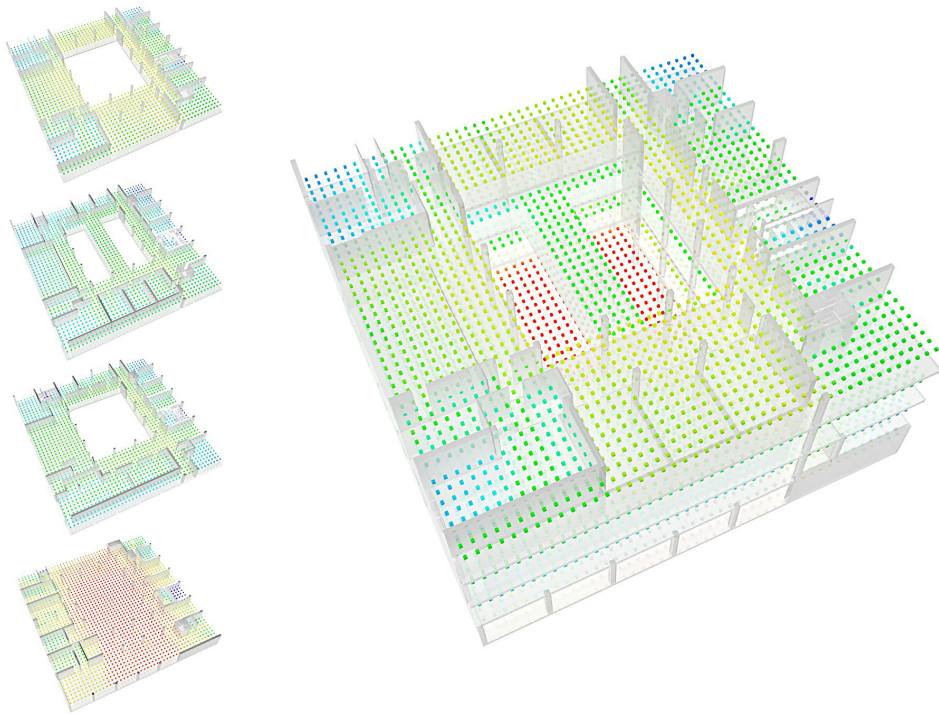
## Notes:

<sup>12</sup> We must be clear in distinguishing the notion of a 'political project' from the specific political ideology that was behind this building.

**Figure 12:**

**2D integration.**

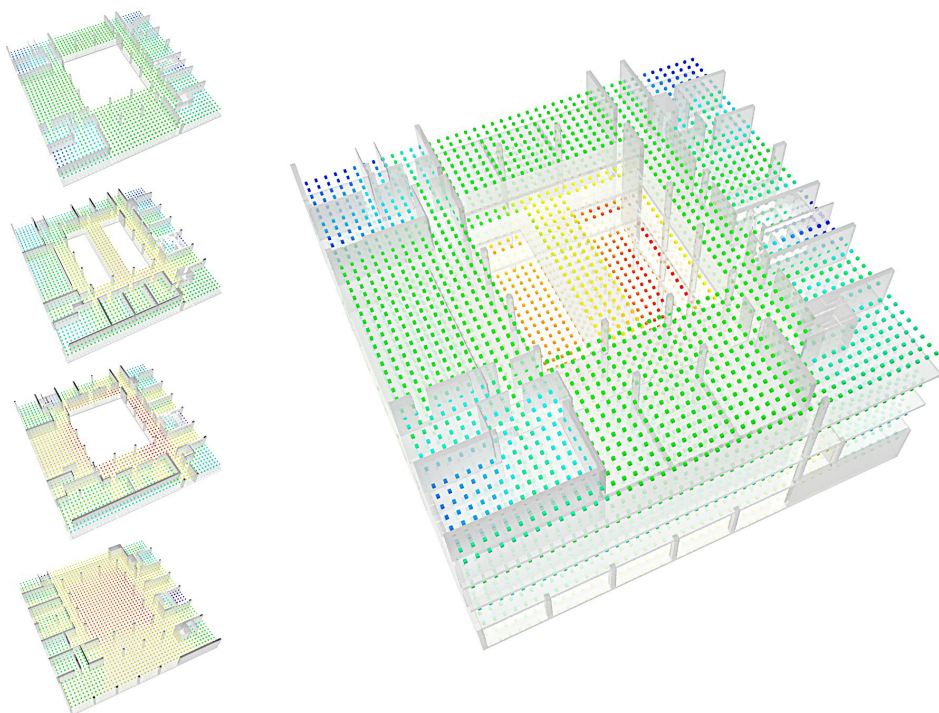
Values are normalised from 0 to 1 across the whole building.



**Figure 13:**

**3D integration.**

Values are normalised from 0 to 1 across the whole building.





of Terragni's architecture (drawn over a period of 40 years), Eisenman concludes that certain kinds of readings inside the Casa cannot be known from its representation on paper or screen; that 'there is an oscillation between the knowledge offered by drawings and diagrams and that available through experience' (Eisenman, 2003, p.301). He suggests that certain types of architecture are more open than others to complex readings and both conditions of interpretation are partial. Eisenman's observation touches upon not only the relationship between architectural composition and situated vision, but also the powers and limitations of models and diagrams in capturing architectural complexity.

We have proposed in the introduction that the separation between *instrumentality* and *aesthetic expression* in architecture is often related to constraints imposed by a structured set of concepts, language and tools on what can be described and represented. Like architectural languages, analytical models and diagrams often become 'languages' in themselves, producing canons of analytical and intellectual activity that illuminate but also constrain thinking. *Engaging with architecture rather than simply with instrumentality in architecture is not only essential for better informing architectural design, but also advantageous in exceeding the limitations imposed by existing languages of description and representation.* This is because architecture possesses levels of complexity that will always challenge description, and help re-establish the boundaries of what can and cannot be described.

The proposed model and diagrams are used to derive a graph that has the potential to set analysis and theory free from previous planar restrictions. This can better enable us to understand the play of combinations between spaces, elements and forms, with their multi-dimensional typologies of vertices and edges in two and three dimensions. Yet, a whole set of geometrical and spatial properties of composition and situated vision are still absent from this

approach. The all-to-all graph we have developed for investigating three-dimensional relations of visibility (Figure 2) has not been explored in the context of this work, but presents an opportunity for future exploration. More particularly, in combination with the mixed graph, it may provide a useful method to account for geometrical organisation – external to human experience – and the spatial organisation of visibility locked to the human perspective (Peponis et al., 1997; Psarra, 1997; Psarra, 2009).

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