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MATHEMATICS AND DIGITAL TECHNOLOGY: CHALLENGES AND EXAMPLES FROM DESIGN RESEARCH

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Mathematics is a ubiquitous and vital substrate on which our culture is built. Yet this fact is seldom fully exploited in educational contexts. The first step must, in our view, be to open the black box of invisible mathematics to more people, (see Hoyles, 2015). A key challenge for task design and an organising design principle is to exploit digital technology to reveal more of what mathematics actually is; first, by offering a glimpse of the mathematical models underlying a given (and carefully chosen) phenomenon; and second, by fostering an approach to mathematical tasks that transcends the purely procedural. We describe in this paper how we have attempted to address these challenges.

BACKGROUND

A common theme for research in task design with digital technologies is that learning evolves in ways that are contingent on design. We follow a programme of design research (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003) with a theoretical framework underpinning the design of the activities presented to the students, and the fine-grain HCI of the software. Our designs were driven by the theoretical framework of constructionism, which argues for learning by building with computer tools (Harel & Papert, 1991). But as we shall see, there is so much to elaborate about the complexities of the design process. In Noss & Hoyles, 1996, we argued that there is a complex relationship and mutual influence of tool and knowledge. We noted that digital tools – particularly those symbolically represented - shape mathematical learning as students 'think with and through the tool', constructing what we termed "situated abstractions". Reciprocally, the tools are themselves shaped by the context of mindful use (for a related discussion of the idea of mindful engagement with technologies, see Salomon, Perkins & Globerson, 1991). It is not only that digital technologies add new representations or link old ones; research is increasingly coming to recognise that digital representations change the epistemological map of what it is intended to teach and learn (Kaput & Roschelle, 1998). In a complementary strand of research, this process has been described as one of "instrumental genesis", whereby artefacts are transformed into "instruments" - systems with which the user gains fluency and expressive competence (see Vérillon & Rabardel, 1995, Drijvers & Trouche, 2008). Our common vision is that computational tools are a means by which new mathematical meanings can be developed but in so doing the role of the tools in shaping the meanings must be acknowledged. Building on this framework, Olive et al. (2010) propose technology as a fourth vertex for Steinbring's (2005) "didactic triangle", in order to illustrate how the interactions among student, teacher, task and technology form the 'space within which new mathematical knowledge and practices may emerge' (ibid. p.169).

Significant progress has been made in designing sets of digital tools (DTs) or "microworlds" embedded in activities through which to pursue mathematical learning goals, taking on board the framework of DR, where the iterative development of the microworld is considered as a piece of

DR in itself (see Hoyles & Noss, 2015). In this sense, microworld design is an incubator for developing and researching radical approaches to innovative mathematical learning. As Hoyles (1993) put it, a powerful way to think about the microworld idea is a vision in which "software tools and knowledge would grow together interactively in the pursuit of epistemologically rich goals" (ibid. p. 3).

As well as the evolution of design research there has been a parallel evolution of *task* design research. In her editorial for the recent ICMI book, (Watson & Ohtani, 2015), Watson makes the point that few studies justify task choice or identify what features of a task are essential and what features are irrelevant to the study. We agree. This is what, we presume, Papert had in mind when he criticised the more general field of mathematics education research for not allocating sufficient energy to consider the 'what' rather than merely the 'how' of teaching (keynote address to ICMI study group, Hoyles & Lagrange, 2009).

By contrast, we note that in the domain of mathematics and DT, the task, its design and the software are all at the forefront of the collective design research effort, and highly visible. This is hardly surprising as the enterprise of the design of digital tools focuses closely on identifying and expressing mathematical concepts in novel ways, e.g. dynamically rather than statically. In this paper, we present some theoretical and practical exemplars arising from two design research projects that illustrate our approach. The first explores the role of programming in mathematical learning, based on our on-going experience of a large-scale design research study in England, the ScratchMaths[1] project. The second derives the Cornerstone Maths project[2], which seeks to exploit the dynamic and visual nature of DT to stimulate engagement with mathematical ways of thinking among students aged 11-14 years. In both projects, we designated some tasks as "landmark activities" to be used as a 'framework for action" in the DR (Cobb *et al*. (2003) and as a focus for our data collection in the implementation phases of the DR.

We provide a brief outline to the idea of landmark activity, and how it plays out in the context of design in terms of:

- The anticipated learning goals;
- How the task design is planned to exploit the affordances of the digital tools embedded in the activity;
- Some preliminary observations on the degree of fidelity of resulting classroom implementations.

THE RATIONALE FOR LANDMARK ACTIVITIES

We define landmark activities as those designed to trigger a rethinking of mathematical ideas or an extension of previously held ideas – the 'aha' moments that indicate surprise. They can provide evidence of particular mathematical understandings of the concept, the anticipated learning goal.

[1] ScratchMaths is a 3-year research project funded by Education Endowment Foundation from Sep 2014: researchers Laura Benton, Ivan Kalas, Piers Saunders.

[2] The research reported in this paper was funded by the Nuffield Foundation (Award reference 91909): researcher Alison Clark-Wilson. We gratefully acknowledge funding by the Li Ka Shing Foundation for the prior research developing Cornerstone Maths 2010-13, a collaboration between the London Knowledge Lab, UCL Institute of Education and the Center for Technology in Learning, SRI International, Menlo Park, USA.

We surmise that disruptive but carefully designed technologies can lead to a 'situation of non-obviousness' (Winograd & Flores 1986, p. 165), where established routines are 'replaced by conflict, disagreement or doubt'. These moments, we conjecture, are particularly conducive to learning. Others have studied how underlying theories on how unanticipated classroom events can be instrumental in developing teachers' epistemology and some have elaborated the underlying role of technology in such 'disrupted' processes, (or example, the notions of 'hiccups', Clark-Wilson, & Noss, 2015, and of 'critical incidents', Aldon, 2011).

Our landmark activities by contrast, are planned for optimal engagement with the concepts at stake by means of the innovative mediational affordances of the embedded DT. Thus we take as read that in technology-enhanced mathematics classrooms, the use of DT can disrupt routine practices in a *transformative* sense, and ensuing breakdowns can promote further reflection and thinking again. Thus the selection and design of the landmark activities with this process in mind are the first stages of our design research. The next stage is one of implementation of the landmark activities in classrooms, with observations of teacher moves and student responses along with 'post-lesson' teacher and student interviews, with analyses of these data feeding into the next phase of the design in an iterative way.

EXEMPLAR 1: THE SCRATCHMATHS PROJECT

ScratchMaths (SM) is a 3-year research project involving a one-year iterative design phase followed by a 2-year implementation phase with students aged 9-11 years. The SM intervention is intended to comprise approximately 20 hours teaching time across each of the two school years, with the first year focusing on computational thinking (see, Wing, 2008) with an implicit mathematical component, and the second year foregrounding explicit investigations of key mathematical concepts using the programing tools. Thus the ambitious vision of ScratchMaths is to introduce students and teachers in the first year to a new representational infrastructure (based on Scratch) with which to express mathematical concepts and procedures, with the intention that these skills will be exploited the following year to explore key concepts through mathematical reasoning and problem solving. The intervention has been subject to cycles of iterative design research with the final quantitative outcome measure being the national standardised mathematics test scores, taken by all students in England at the end of primary school. Here we focus on the early phases of design research.

We designed tasks with clear learning outcomes and explicit guidance for implementation in written form and as part of professional development support for the teachers (face-to-face and online). One early outcome of the design research § was the emergence of the need for an explicit framework of pedagogy to help successful implementation of the different aspects of the SM intervention We devised a framework consisting of five unordered constructs, the 5Es, clearly based on a host of research into good practice in teaching mathematics, but also framed by findings emerging from early design workshops. The 5Es are: **Explore:** Investigate ideas, try things out for yourself and debug in response to feedback. **Envisage:** Have a goal in mind and predict what the outcome might be before trying out. **Explain:** Explain what you have done and articulate the reasons behind your approach to yourself, to peers and to the teacher. **Exchange:** Share different approaches, try to see a problem from another's perspective as well as defend your own approach in comparison with others. **bridgE:** Make links between the programming work and the language of 'official' mathematics and explore commonalities and differences.

The SM intervention comprises a host of investigations and exercises on and off the computer, to be undertaken individually or in pairs. We now turn to describe one landmark activity. The learning objectives were to explore how to move a sprite without dragging it, snap blocks together to create a script, and explain the script, debugging if necessary. In addition, the mathematical goals included reasoning in steps, abstracting from immediate action, exploring angle as turn, and a total turn of 360 degrees.

As preliminary work, students were given five existing Scratch blocks, (see Fig. 1), thus constraining activity merely to turning a given number of degrees, to 'stamping' the original tile, and 'moving' the tile in a straight line The students can click them together to build a script and observe the outcome. This simple scenario hides a number of deep mathematical as well as computational concepts. From a computational point of view, the key concept is that a single block can have a repeatable outcome: and that putting blocks together leads to predictable results. This latter point, we found was surprisingly difficult for some students, and its mathematical corollary was a major stumbling block for many. The idea that mathematics is a game played with constrained rules, that algorithms have a rationale, that little pieces of knowledge can be brought together to represent larger ones, and that mathematical statements have consequences are all in some sense, deep. Furthermore, there is a major conceptual challenge that involves recognising the structure of the intended outcome, and predicting running the script in the future – in mathematics an analogy would be to envisage the output of a function for different values of the input.

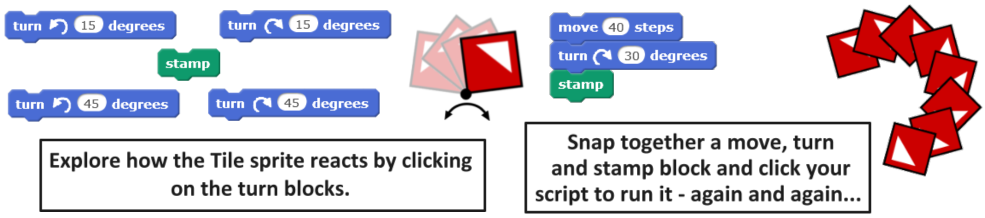

Figure 1. ScrachMaths: Direct drive activity (left) and building simple scripts (right)

The tension that had to be resolved was first between the tool and learning. Once students were familiar with the tool, it was observed that the ease of building scripts tended to encourage students to build extremely long scripts, by simply clicking blocks together: without, first envisaging the outcome. Super long scripts appeared to have status as demonstrating a lot of 'work'! In fact, in some classes, pupils went to great lengths to ensure that their scripts were longer than others'. The challenge is to establish a norm in which the aesthetic and pragmatic value of short scripts is recognised along with the appreciation that long scripts are hard to explain and to predict what they would do. Thus a key 'rethinking' promoted in SM classrooms entailed using definitions instantiated in 'build your own block', BYOB, in order to reduce complexity and aid readability. One last point highlights the relationship between learning outcomes and the affordances of the digital tools available. An earlier version of Scratch (1.4) did not allow the 'user' to build your own block (BYOB), and it is perhaps here that we find the reason for the ubiquity of the 'longer is better' preferences on the part of the pupils. At the very least, the advent of BYOB in version 2.0 gave us as researchers a different and more powerful tool with which to promote mathematical description: write a program, give it a name, and reuse it.

Similarly, introducing the repeat block into the landmark activity alongside the constraint in the activity of 'no overlaps', provoked the need for further reflection while opening the opportunity to build connections between the computational and mathematical ideas. Again progress was varied in implementation with some students 'seeing' no connections, while others were observed calculating the value of the repeat block by dividing 360 by any chosen value in the turn block and iterating. Sometimes this resulted in a decimal number, e.g. 5.5, which they then inputted into the repeat block. Teachers used the 5E pedagogical framework along with "unplugged" activities (away from the computer) to promote and consolidate this new window through which to think about angle and turn.

While one of our 5Es (Exchange) points to the pedagogical advantages of collaboration, it proved (perhaps unsurprisingly) to be challenging in SM classroom. It seemed to operate most effectively when teachers encouraged the 'more able' students to support the less able by 'teaching' them what they had already discovered for themselves and where individual discoveries could spread around the whole class with rather little teacher intervention, as students collectively monitored what their peers were working on. This is an interesting example of fidelity achieved in tandem with the evolution of the intervention; the intervention aligning itself with the 'natural' ecology of the classroom. More work is underway to explore this phenomenon further.

EXEMPLAR 2: THE CORNERSTONE PROJECT

The design foci of the Cornerstone Mathematics (CM) Project are several core mathematical concepts to be explored by middle school students (11-14 years) in ways that exploit the affordances of dynamic digital tools that can make links between key representations. The CM project began in 2010 as collaboration between research teams at SRI International, USA, and UCL Institute of Education, London. The project adopted a "design-based research" approach to increase student use of bespoke dynamic mathematical technology in lower secondary English mathematics classrooms (see, Hoyles, Noss, Vahey, & Roschelle, 2013; Clark-Wilson, Hoyles, Noss, Vahey, & Roschelle, 2015). The resulting web-based software, student materials, teacher support materials and mandatory professional development focus on topics known to be hard to teach and where the DT can clearly offer new ways to expore themathteamtics: linear functions, geometric similarity and algebraic patterns and expressions. In this project (and in an ongoing project with Alison Clarke-Wilson), we again used the construct landmark activity to provide a focus not only for task design, but also to tease out the extent to which classroom practice aligns with the epistemic and learning goals of the CM materials and sheds light on learning (of teachers as well as students) that follows engagement with the activity. (Clark-Wilson, Hoyles, Noss, Vahey & Roschelle, 2015).

In CM the process of identification of landmark activities went through several stages. First, the research team made their own selections from the student workbook based on past experience and theoretical concerns. Then they discussed their selections and agreed a list of activities that were highly aligned to the design principles of the CM curriculum unit under discussion, and which could reasonably function as landmarks, in relation to the three criteria outlined earlier. This process was repeated face-to-face with a focus group of three teachers, selected as they had provided thoughtful reflections to online surveys, and who provided their rationale for their choices.

The following activity was selected as one landmark activity in the unit around linear functions. Fig 2 shows the software environment (the software was derived from Simcalc): it comprises a simulation (top right) performed by Shakey, a timer, play and editing (top left) and three 'standard' mathematical representations of how distance varies with time; a graph, a table of values and an algebraic function. The anticipated learning goals were to identify speed as the gradient of the graph and link that to the coefficient of the function, and also to identify the starting point of Shakey with the intercept of the graph on the 'distance' axis and with the constant in the function.

The aim of this activity was to explore the software, play the simulation, and watch the effects on the graph, the table of values and the function. Then Shakey's 'journey' could be edited, either by changing the graph (making it steeper or adjusting its starting point,), by changing the function, or by manipulating the simulation itself or any combination of these, and then reflecting on the effects on the journey, while trying to tease out and explain how the different representations were linked. Note that the graph and the narrative – but not the table – can 'drive' the simulation in contrast to the usual situation in which a graph is a read-only representation of it.

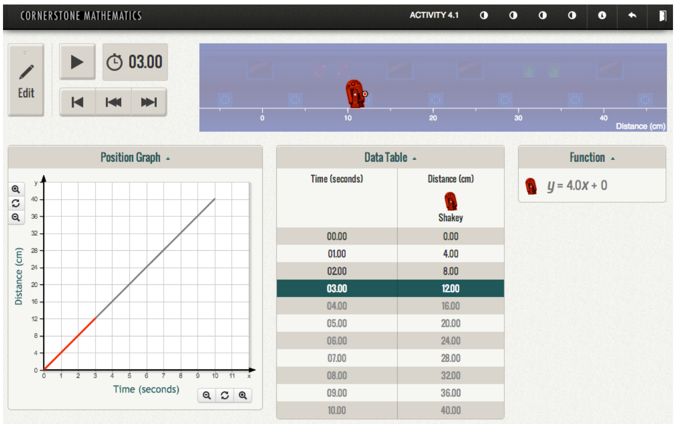

Figure 2. Cornerstone Maths Linear Functions: The software environment

We anticipated that the focus on the dynamic representations and the links between them would be sufficiently novel to engage the students and teachers in rethinking what they knew about linear functions: all our prior work gave support to this, hence its selection as a landmark activity. Our early analyses indicated that the dynamic approach was successful in provoking a rethinking of the meaning of the graph and its relationship with the algebra although there was considerable variation in implementation. Here we report what might be a common mutation of the innovation in some classrooms.

The teacher had fully bought in to the idea of establishing the link between the equation and the graph. But we noticed the generic manner of his approach, with little or no exploitation of the dynamic affordances of the digital tools. For example, the task was presented on the interactive whiteboard and the teacher simply talked about it and described the different 'windows', reminding the class of their functionality, but only verbally – not by demonstrating. We noticed when students worked on the activity themselves (mainly in pairs), the teacher circulated giving advice, but again general advice: such as 'try different things' 'you should be exploring', and even 'you need to establish the link between graph and equation'. He did not play with the simulation at any point. The researcher asked him if he would kindly do this and demonstrate, say, how to edit the graph. He was resistant as 'he wanted them to explore', but eventually agreed to do this and changed the steepness of the graph asking what "would happen to Shakey?" Many could not find the words to describe their ideas – some made vague references to going faster, many did not know what to refer to. But again, what was notable was the teacher still did not play the simulation to illustrate what happened after editing or point to the key changes and the links between them. Unsurprisingly, rather few of the students could articulate the connections between the representations.

We draw similarities with early research with computers, in which it was reported that teachers often felt they had no role: they wanted pupils to explore and sought to restrain themselves from telling answers (or funnelling towards them), but this was interpreted as an injunction against telling pupils anything at all!

CONCLUDING REMARKS

In this paper, we have presented the background and rationale for landmark activities, and illustrated how tasks were designed to exploit the affordances of the digital tools to achieve specific learning goals. We also sketched some observations from classroom implementations. At this early stage we can simply note how these were crucially shaped by the teachers' appreciation of the new affordances for learning mathematics. It is not just a matter of expertise in the use of the software but rather the conscious exploitation of the tools to promote a new window on the mathematical ideas at stake.

While it is too early to draw generalised conclusions from these data, we might simply note the fragility of innovation fidelity, maybe especially for a computationally based innovation, and we conjecture for the following reasons. First the close tie between affordance – what the system invites the learner to do – and the relationship between this to what the teacher feels inclined to focus on. Second, the landmark construct gives teachers the opportunity to operationalise the notion of a window through which to gain insight into student meanings, but this may not be exploited. Third, and perhaps most significantly, the idea of landmarks brings some systematicity to the difficult and enduring methodological challenge of identifying what matters to teachers and students in the context of classrooms. By focusing on task design, we acknowledge the role of a learning ecology (Cobb et al., 2003) which depends centrally on 'the tasks or problems that students are asked to solve", as well as the tools and materials in use. From a methodological point of view, the landmark idea may help to tame the complexity of inter-relationships between the different elements that shape an intervention, all the more complex, of course, where digital technologies are involved. One possible new strand of the design research methodology might be to strengthen the 'mixed-methods' research framework by building an even stronger complementarity between qualitative and quantitative data analyses that harness emerging techniques of big data and learning analytics.

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