IMA Presidential Address: The potential and challenges for mathematics teaching & learning in the digital age

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The underlying theme of my Presidential address was to outline the evidence of the importance of mathematics and some of the challenges that need to be faced if we are to increase participation in the subject. The first, overarching and enduring challenge is the invisibility of the subject. Most people just do not understand what the subject is about and why they are studying it for so many hours over so many years. I will point to some dangers if we do not rise to the challenge. The second challenge is to acknowledge, and then to address the complexities of teaching and learning mathematics: it is not a matter of mere exposition of procedures. It is perhaps appropriate for my IMA Presidential address to quote from a message from HRH The Duke of Edinburgh, President of the IMA in 1976-77, who acknowledged in the proceedings of the Second International Congress on Mathematical Education (ICME2) held in Exeter, 1974, that mathematics was 'vitally important for everyone in this technological age, and then went on to argue that 'any advance in the techniques of teaching is to be welcomed' since: *"those fortunate beings who find mathematics a joy and a fascination will probably get on, whatever the standard of teaching* ***. It requires real genius to light a flicker of understanding in the minds of those to whom mathematics is a clouded mystery."*** (ICME2 proceedings, 1974, emphasis added). This remark seems even more relevant now over 40 years later, not only concerning the importance of mathematics, but also the explicit recognition of the challenge of teaching for engagement. My claim is that using computers in carefully designed ways can help us address these challenges.

The Importance of mathematics

There is widespread acceptance that mathematics is important, even vital, for an individual and for society. Thus in most countries, the high status of mathematics and mathematics education is rarely contested. It is presumed to be "a vehicle toward social and political progress" (Gates & Vistro-Yu, 2003, p. 62), and central to the development of a well-trained workforce that can advance the economic standing of a country. Thus governments face a range of distinct but interrelated policy challenges, which include providing universal mathematical literacy for all, ensuring a mathematical foundation to support the study of other subjects that are increasingly demanding higher levels of mathematics, and stimulating the most able to continue with mathematics study after it is no longer compulsory and on into university (see, for example, Hoyles & Ferrini-Mundy, 2013). It is thus almost universally accepted that we live in a world that is increasingly shaped by mathematics, with mathematical models underpinning so much of the pervasive and ubiquitous technological infrastructure of our society, This change has considerable implications for skills in workplaces and careers of all sorts. (see, for example, Hoyles et al, 2002).

In 2012, the EPSRC together with the Council of Mathematical Sciences (CMS) commissioned a report from Deloitte into the Economic Benefits of Mathematical Sciences in UK (Deloitte, 2013). This report quantified the ways in which Mathematical Sciences Research (MSR) in 2010 influenced economic performance in the UK and its economic value in terms of direct employment and Gross Value Added generated. The introduction of the report set the scene with a brief survey of some of the areas in which MSR effects everybody's daily lives and how the "fruits of MSR" can be discerned in to quote :

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- smart phones which use mathematical techniques such as linear algebra to maximise the amount of information that can be transmitted across a limited spectrum;
- mathematical models predicting the movement of weather systems to allow airplanes to quickly and safely return to the skies after major meteorological events such as the 2010 Icelandic volcanic ash cloud;
- healthcare that applies the insights from fluid mechanics to better understand blood-related diseases in order to save lives;
- the latest Hollywood blockbusters that take advantage of the mathematics behind software for 3D modelling to showcase cutting- edge special effects; and

- the performance of elite athletes at the 2012 Olympic Games who have maximised their performance using tools that harness mathematical tools and techniques such as inverse dynamics."

A major question for me is *who* actually "appreciates this mathematical infrastructure"? Rather few in my view, which is something we in the mathematics community need to work on together. But returning to the Deloitte report, I note its methods: "first identify those jobs that can be classified as mathematical science occupations and then to use publicly available data to examine how these occupations are distributed across different sectors of the UK economy….." . These analyses came up with what to many were somewhat surprising figures: "that the quantified contribution of mathematical science research to the UK economy in 2010 is estimated to be approximately 2.8 million in employment terms (around 10 per cent of all jobs in UK), and £208 billion in terms of Gross Value-added contribution (around 16 percent of total UK GVA)". These outcomes were calculated using what is a bespoke and I understand invisible Deloitte model, which I find somewhat ironic given my thesis of the need to combat the invisibility of underlying models.
Nonetheless findings such as these, along with reports that document skills shortages (see for example the report from the Confederation of British Industry, CBI, 2015) have fuelled demands for enhanced quality and quantity of mathematics education provision at every phase of education: put simply, we need more people at every level to be competent and confident in mathematics. This turns the spotlight on to education. We are among just a few countries that allow students to drop mathematics after compulsory schooling; in England after GCSE, (see, for example, Hodgen et al 2013). However there was a major announcement in 2011 by the then Secretary of State for Education, Michael Gove (June 29th, 2011) stating the intention that: *"….within a decade the vast majority of pupils are studying maths right through to the age of 18…"* (Gove, 2011*)* **.**
This announcement led to a flurry of activity to increase and enhance post-16 mathematics provision in England over and above seeking to increase recruitment of students to Advanced-level mathematics and Advanced-level Further mathematics. It is noteworthy that A-level mathematics was the most popular choice for students in summer 2014, a remarkable and 'first-time' result. In addition, there has been a steady annual increase in the number of students entering Further mathematics A-Level, a major success story since the

nadir in recruitment in 2001. As for non-A-level post-16 provision in mathematics, a new Level 3 course, Core maths has been devised and piloted and will available for teaching from September 2015. Core Maths is aimed at students who have passed GCSE mathematics at grade C or above but who are not taking A-Level mathematics. It forms a major part of the government's plan to increase participation in mathematics education with the aim that by 2020, the vast majority of all students in post-16 education will continue to study some form of mathematics, GCSE, A Level or Core Maths.

But for these welcome initiatives to be successful, I claim there must be more explicit attention to motivation, and by that I mean genuine engagement with the subject.

Towards an engaging mathematical culture: some general issues

I now turn to what I see as a widespread endemic challenge to all these efforts to widen engagement with mathematics in England, namely to open up the 'black box' of mathematics to more people (for an early paper concerning this issue see, Resnick et al, 2000. As argued above the status of mathematics is generally high in England: it is regarded as important. Yet it is still socially acceptable to profess inability in the subject and even to boast about incompetence: 'I never could do maths'. The ground-breaking Williams review referred to this phenomenon and its implications: "A parent expressing such sentiments can hardly be conducive to a learning environment at home in which mathematics is seen by children as an essential and **rewarding** part of their everyday lives." (Williams, 2008, emphasis added). That mathematics should be to some extent intrinsically 'rewarding' (as well as extrinsically) is for me critical for engagement. This in turn raises a challenge to the prevailing mathematical culture as well as to mathematics education, where a major part of this culture still tends to be about speed and 'winning the race', as nicely captured in the quote below from Alex Bellos:

'maths is plagued by the genius cult, which tells students that it is only worth pursuing if you are going to be the best. Athletes don't quit their sport just because one the their teammates outshines them. And yet I see promising young mathematicians quit every year, even thought they love mathematics, because someone in their range of vision was 'ahead' of them'. (Bellos 2014).

It is reasonable to conjecture that such a cultural view of mathematics has a particularly negative influence on some groups, not least females: attitudes of girls towards maths is significantly less positive than attitudes of boys. There is an extensive literature on this topic, so here I simply refer to the OECD report, PISA 2012. It might be that women are less inclined to join a community where 'winning' is the dominant aspiration. This again is a huge topic, and I merely point to an emerging theme argued to underpin enhancing participation, and that is to support females' sense of belonging to the community, as captured in the following quotation from Good et al, (2012):

" communicating an incremental view of math intelligence in educational environments may begin to address pipeline issues for women in science, math, engineering, and technology. Doing so may help eliminate the culture of "talent" and the mentality of the "weed-out system" that pervades many of these classrooms and that can send fixed-ability messages to women. Learning environments that foster a culture of potentiality in which anyone can develop their skills may create room for many more females to feel that they belong in these fields and, thus, to encourage many more females to pursue math and science degrees" (Good et al, 2012).

But how can this objective be taken forward? I address this question by highlighting the importance of supporting more 'authentic' engagement with what it is to be mathematical, a challenge in several respects. The subject is highly regarded by policy makers and parents and generally conceived as 'hard' with high stakes tests. All these factors tend to work together to reinforce the view of mathematics from outside the mathematics community as simply *procedural*: a set of rules to follow in order to perform calculations. In fact when I presented the popular TV programme Fun and Games in the late 80s, as the 'resident mathematician', I was asked on several occasions to describe what mathematicians actually *did* (Hoyles, 1990). It was clear to me that the pervasive view among the general public was that mathematicians as they become more advanced simply perform longer and longer calculations of say long multiplication or division. Thus mathematics procedures and calculations tend to be the **visible** face of mathematics for most people. There was – and I suggest still is - little idea of what the subject is all about, its structure, logical framework and network of interrelated concepts - its very essence. This leads almost inevitably to ignorance of why on earth mathematics might be important. Thus I argue that to increase engagement with mathematics ways to tackle this invisibility must be developed.

To illustrate this dilemma, I give just one example. Simon Jenkins a leading Guardian journalist wrote in 2014:

"I learned maths. I found it tough and enjoyable. Algebra, trigonometry, differential calculus, logarithms and primes held no mystery, but they were even more pointless than Latin and Greek. Only a handful of my contemporaries went on to use maths afterwards. (Jenkins, Guardian, 18 February 2014). This and similar statements indicate all too clearly how little is known about the nature of the subject. Mathematics is not merely a collection of 'fragments of content', which individually may indeed be irrelevant. Rather mathematics can be thought of as a 'way of thinking' that prioritises patterns and structures expressed in formal relationships, an appreciation of which can add a new dimension to the intellectual armoury of any citizen.

The IMA has done much to combat the invisibility of mathematics through its Research Case Studies**,** *Mathematics Matters***.** These are two-sided articles that summarise some of the modern applications of contemporary maths research (http://www.ima.org.uk/i_love_maths/mathematics_matters.cfm.html)

Each case study provides a glimpse of the underlying mathematics and I hope are widely read and promoted. To mention a few here:

- Building the Digital Society: from data to information,
- Smarter Phones for all: improving signal transmission,
- Networking for the future: balancing individual and society needs,
- Advancing the Digital Arts with its associated video

One more caught my attention recently : Finding new planets (IMA, Mathematics Matters http://www.ima.org.uk/viewItem.cfm-cit_id=384244.html). It seems even more topical now with the apparent 'discovery' of a new planet rather like Earth.

Another consequence of the invisibility of mathematics is the widespread ignorance of the diverse careers that are accessible to people with mathematical expertise. I elaborate one further example, that of computer animation. The UK is at the forefront in this sector with many major films to its credit. But rather few appreciate that *there are mathematical models embedded in the computer programs that produce the visual effects*. I quote as illustration some extracts from the video produced as part of the NextGen report (Nesta 2010): "Learning about Word, Excel and PowerPoint is not going to get you a career in the high-tech creative industries. … The sooner that parents, teachers, pupils realize that the better chance that this country can build on the firm foundations we have established. What we want is for people to learn **computer programming**…………….. The digital world is changing in front of our eyes and with it new careers are opening up requiring new combinations of skills.” I will return to the theme of computer programming later. Here I dwell on the following statement from this NESTA video: *“It’s that combination of art, which allows you to go beyond what is there and maths which helps you understand what is there that is absolutely crucial…. In our work, we’re simulating how buildings collapse – that is physics, we understand how rivers flow, computational fluid dynamics. They all work together. We wrap science and art together at every turn, at every minute of our working day….”* And finally the presenter states: *“My dream graduate would be somebody with Maths, Physics and Art. Sadly there are very few of them.”*

This is just one interdisciplinary context where there is a an urgent need for more people who are confident to use mathematical skills to solve problems creatively. There are many more and not all in high skills sectors. Throughout UK industry, commerce and public sector organisations, mathematical modelling and mathematical systems are becoming more pervasive. And given technological advances and intensified competition, it is imperative that the modelling systems work better, *and* more open to inspection and monitoring. In knowledge-based sectors, individuals are increasingly required to understand more of the workings of the programs they employ, thus challenging the idea that the computer is ideally conceived as a 'black box'. The trend to devolve some responsibilities to employees at all levels inevitably involves them in a degree of explicit interaction with the mathematical models of the system, instantiated as abstractions within computer programs. So one must ask what might be effective interactive synergies between employees and computer models of workplace practices and products. This question raises epistemological and cognitive issues concerning the knowledge and understandings (respectively) involved. This is a matter of *design* and *education*.

Towards an engaging mathematical culture at work

We[1] addressed this challenge in research on characterising and then promoting what we termed the Techno-mathematical Literacies (TmL) used in highly automated workplaces (Hoyles, Noss et al 2010). The kinds of basic mathematical literacies required at work had

[1] The research referenced here, along with most of my work, has been undertaken collaboratively in teams based at the London Knowledge Lab, and almost all in collaboration with Prof Richard Noss.

previously been identified and schematically characterised as knowing how to calculate and estimate, and having a feel for numbers, percentages and proportions. With the advent of pervasive computer modelling, the core of what is required I would argue has changed and involves not only interactions with specific calculated results, but with a model acting on an entire range of general data (a simple example is a spreadsheet "rule" applied to a whole column of numbers). These interactions can be schematically located along a dimension of computer-model-use that ranges from entirely opaque (e.g. checkout operators) to transparent (e.g. engineering analysts). Thus TmL may include gathering and validating data for computer-based models (which itself entails systematic precise measurement and data entry techniques); manipulating the models (which requires flexibility with designed representations and tools); modifying variables to improve a model's predictive power in simulating workplace practices or outcomes; interpreting numerical, symbolic and graphical information derived from models; forming judgements on the basis of data derived from models; and, last but by no means least, communicating decisions 'vertically' within the firm to justify a plan or to explain a trend, 'horizontally' within a project team or 'outwards' to clients. (for a summary of the findings of our TmL research, see Hoyles, Noss et al , 2013). What is clear from our work in the TmL research that it is often very hard for workplace employees, and indeed even their managers, to appreciate to their work , let alone its key characteristics.

I give as an example the use of statistical process control used in many manufacturing industries where process capability indices are calculated with the aim to give employees an idea of the stability and efficiency of the process for which they are responsible. These indices are derived from the data produced by measuring 'key' variables and calculating their variability. Random variation is of course inevitable and expected, and such variation needs to be distinguished from 'special courses' that amplify variation and need investigation. However this mathematical basis of the indices is all too frequently 'lost' on the workers. We found that employees across many levels of the workplace interpreted both charts and these indices 'pseudo-mathematically', with little if any connection to data or underlying mathematical relationships. In the case of charts, employees, including trainers and managers, often failed to understand control limits as artifacts of the distribution, and in the case of process capability indices, did not link higher values with reduced variation. There was a tendency to conceive both as arbitrary targets imposed by management, as one worker told us “you can be beaten up for low Cp’s (one of the process indices) ”. The mathematical models were presented in a language (mainly algebra) which was just inaccessible to most: just too baffling.

Some then draw the conclusion that the mathematics is ‘too hard’ and should be ‘black-boxed’. However many of us in education, committed to enhance engagement with the subject, seek to use the dynamic and visual functionality of computer software to help users glimpse and interact with the underlying mathematical structures. In the TmL research project, we built what we called technologically enhanced boundary objects, TEBOs, designed to be used with team leaders and shopfloor workers in order to make the calculations of the process more visible: that is where manipulations could easily be made by users that exploit visual output so that they could ‘see’ the basis for the process calculations.

An example of one TEBO we co-designed (with workplace employees) and implemented was for the process capability index, Cp, that summarises the spread of a distribution in relation to the required specification for the process and is calculated using the formula below:.

$$Cp = \frac{USL - LSL}{6\sigma}$$

where: USL = upper specification limit

LSL = lower specification limit

σ = standard deviation

The TEBO we built is shown in Fig1: by manipulating the mean and spread of the normal curve of the measurement data, the user can come to see what the formal definition for Cp ‘*means*’ , that is the number of times the range of the data fits into the allowed limits of the specification. So a low Cp means that the variation is large. Thus a data point might be close to being outside specification, possibly leading to serious repercussions, legally or for the process as a whole.

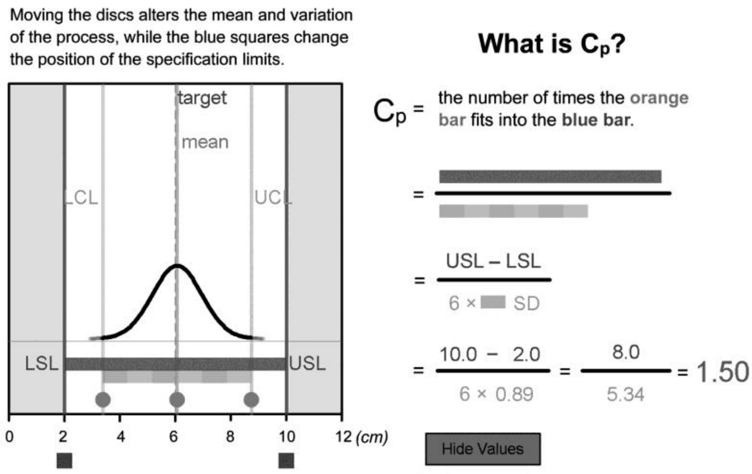

Fig ???1 : Screen shot of the TEBO for Cp

To give some evaluations of the use of this and other TEBOs, I quote from the words of the process engineers and SPC tutors from one workplace:

"We tried many ways to make it [the process index] visual for the people on the shop floor. The web-based tool [our TEBO] helped us get across this concept where the process was sitting between the specification and control limits. The problem is visualisation versus calculation. Response to the formulae is laughter. Eyes glaze over. They really lose it. We want them to REALLY understand what capability is and the use of the tool helps".

But what about designing for enhancing engagement of school students, both primary and secondary, with mathematics?

Towards an engaging mathematical culture in school

School is of course rather different from the workplace: the goal of any activity is to learn – mathematics in this case. Any project in school must, in addition to supporting and assessing students' interactions with technology, address the curriculum and the teacher's role in using and deploying the technology (see Hoyles and Lagrange, 2009). Crucial for the success of any intervention is that teachers develop pedagogical and technological confidence through professional development and collaborative ongoing support.

I will briefly present a glimpse of one research-based intervention that follows these principles with which I have been involved for some years, Cornerstone Mathematics (CM). (I add that there are many others). CM consists of three units, each focused on key mathematical topics in in Key Stage 3, (students aged 11-14 years) and designed and tested over several years. Each unit explicitly exploit the dynamic and multi-representational potential of digital technology to enhance learners' engagement and understanding of the mathematical ideas at stake. I illustrate the flavour of CM by reference to its first unit on linear functions, as described in Hoyles, Noss, Vahey et al (2013, and elaborated further in Clarke-Wilson et al (2015) At the heart of the software environment is “a simulation, or a ‘journey’, of an object that can be tracked in a graph and a table, as well as captured in algebraic or narrative form. Students receive feedback on any journey they have constructed by visually ‘seeing it happen’; the mathematics plays out in terms of motion and vice versa. Students can control their object’s journey by manipulating the position-time graph or its algebraic representation”. Thus the system links algebraic expressions, graphs, tables, and narrative through the phenomenon of motion, and through the activities offered in the CM unit, students explore and exploit these links to construct journeys and explore their own ‘journeys’.

I go on to illustrate briefly another way to exploit the functionality of digital technology to make the processes of mathematical calculations more visible.

Programming to ‘see how things work’

Over many years, I have used a variety of computer software to design environments and activities to help students achieve a better grasp of mathematical structure: so they see beyond the procedural side of mathematics to its conceptual basis and logical nature [2] . One example is CM above. However another constant thread has been concerned with programming environments. My early research was stimulated by the inspirational ideas of Seymour Papert, and engagement with the programming language, Logo (see for example, Hoyles, & Sutherland, R. (1992) , Hoyles & Noss, R. (1992), Coincidentally I had joined the MIT turtle workshop at the ICME Congress in 1974 mentioned earlier. It left a lasting impression on me. Since that time the popularity of programming in schools has waxed and waned. But recently there has been a remarkable and massive change triggered in part by a lecture by Eric Schmidt: CEO Google , in 2011 who stated:

“*I was flabbergasted to learn that today Computer Science isn't even taught as standard in UK schools*. *…Your IT curriculum focuses on teaching how to use software,* ***but gives no insight into how it's made”.*** MacTaggart Lecture, 2011

[2] Much of this work has been undertaken in collaboration with colleagues at the Institute of Education, in particular Richard Noss. I wish to acknowledge their collective and indivudal contributions with thanks.

This along with reports (including the NESTA report above) has culminated in a national movement to bring back programming into schools in England, leading to the introduction of a new statutory computing curriculum in Sep 2014. (Computing at School http://www.computingatschool.org.uk). Alongside this new curriculum, there is a large and international 'coding' movement.

There are some incredibly powerful programming environments used across the age ranges and curricula, with some following the Logo tradition: *NetLogo* a multi-agent programmable modeling environment used by tens of thousands of students, teachers and researchers worldwide (Wilensky, 1999; and *Scratch* a project born in the Lifelong Kindergarten Group at the MIT Media Lab and designed especially for ages 8 to 16, although used by people of all ages. (https://scratch.mit.edu/)

But what about programming and mathematics? Again there is a long history. Limitations of space allow me only to remark that programming can certainly be exploited so students ask *why* did that happen, and can answer with reference to the program- and the mathematical structures - that they had built (or helped to build) themselves., So students have some ownership of the process and in so doing glimpse the need for mathematics. Seymour Papert wanted children through programming to "control a computer to make whether music, animation, graphics **that they care about** using a mathematical language, a program" https://www.youtube.com/watch?v=xMzojQFyMo0 , emphasis added). And as he said all those years ago in 1972: "let the students learn mathematics as applied mathematics … in the sense that mathematical knowledge is an instrument of power, making it possible to do things of independent worth that one could not otherwise do … (Papert, ICME 1972).

Learning school mathematics as applied mathematics takes me a paper by a previous President of IMA Professor Tim Pedley, who wrote in Mathematics Today: 'what I mean by a good applied mathematician: it is someone who knows a lot of mathematics at the appropriate level, but who in addition has experience and facility at translating problems from outside mathematics into mathematical form … Part of the experience and facility will involve judgement, based on a deep understanding of the underlying physics (or biology or chemistry or whatever."

But what if the field of application is education, which is hugely complex, and still more so when embedding computer use? Additionally ,platforms and software inevitably change or become obsolete. But this is not a matter for 'despair' among designers and educators, but rather for renewed effort inspired by the evident *mathematical* engagement of students, along with the commitment and collaborative input of school leaders, researchers, and of course teachers. I end by returning to the quote at the beginning of this paper, to underline its importance to me: over and over again we are made aware of the complexity of the task of teaching and how it is in teaching that 'the real genius' lies.

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