SCHEDULING AND ROUTING FREIGHT TRAINS WITH MULTIPLE COST FUNCTIONS

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ABSTRACT

This study presents a multi-objective optimisation model for effectively scheduling and routing freight trains with minimum disruption caused to surrounding train traffic given the existing infrastructure capacity. Most existing timetabling policies around the world favour passenger train operations over the freight ones. The objective of the study is to facilitate freight operations with the proposed methodology. The optimisation model is formulated here as a mixed integer program (MIP) which captures simultaneous scheduling and routing options of trains. The optimiser is applied to a real case scenario on the Brighton Main Line (BML) in southeast England. Given the network configuration, the optimiser is shown to be able to schedule and (re-)route requested freight trains with minimised additional costs induced to the system. We also examine selected scenario with marginal cost analysis and find that the train allocation process produced by the optimisation model is somehow similar to existing practice in small scale applications, while the proposed algorithm is more systematic, generalizable to large-scale applications and multiple cost functions.

Keywords: train scheduling, mixed train operations, multi-objective optimisation, mixed integer programming, branch and bound algorithm

1. INTRODUCTION

The rail industry has experienced significant demand growth around the world. In Great Britain (GB), we have seen the total volume of railway traffic increased to 58b person-kilometres on franchised journeys in 2012 from 40b person-kilometres only a decade earlier (see Figure 1). Freight volumes have also increased by 60% over the last decade. Nevertheless, Figure 1 reveals that the infrastructure supply (in terms of 'route open for traffic' and 'total net tkm' (train-kilometre) is relatively steady over the same period. This can be understood as a consequence of the fiscal and environmental constraints that restrict the construction of new infrastructure. To maximise the efficiency of utilising existing infrastructure resources, a number of countries (including Germany, Switzerland, Japan, and Great Britain) have privatised their train operations. In a privatised environment, there is an Infrastructure Manager (IM) which is responsible for allocating and managing all infrastructure resources. The infrastructure resources include tracks, stations, signalling, and power supply. The Railway Undertakings (RUs), which include both passenger (TOCs) and freight train operators (FOCs), will then have to bid for the right of using the resources for running their train services and hence making profit. The role of the IM here is to allocate the infrastructure resources among the RUs such that the corresponding benefit brought to the entire railway system can be maximised. Solving the IM's problem on allocating infrastructure resources is regarded as a Railway Capacity Allocation Process (RCAP).

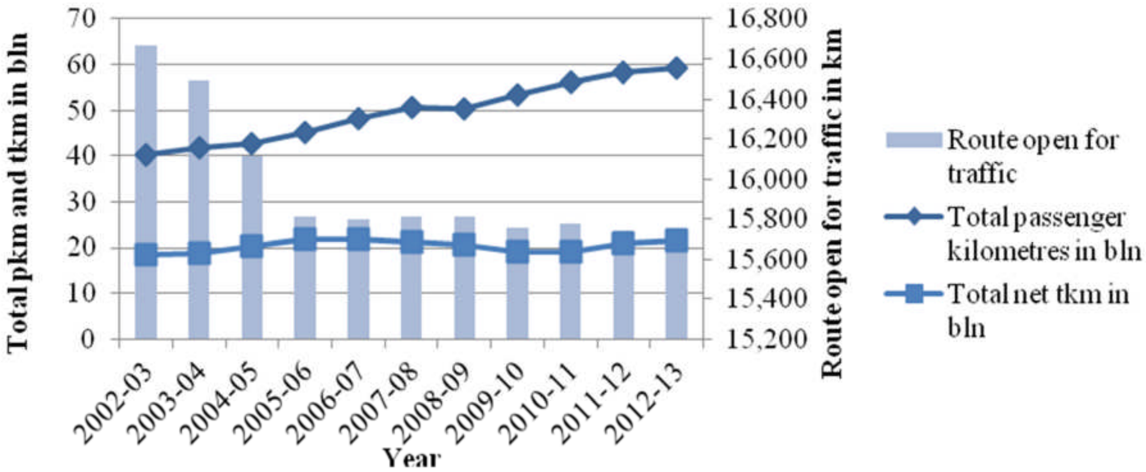

FIGURE 1 Number of person-km (pkm) and train-km (tkm) in billions (bln) against the length of tracks (in: km) open for railway traffic in GB (Source: *1*).

An effective allocation and utilisation of infrastructure resources can reduce unnecessary congestion significantly (see *2*). Nevertheless, it is found that current policies of a number of IMs around the world tend to favour passenger train operations over freight ones due to the demand for passenger trains (*3*). As an illustration of this imbalanced demand, there are respectively 1,000 freight and 19,000 passenger trains traversing the GB network per day in 2012 (*4*). Such pure demand-based allocation scheme undoubtedly is hurting the freight train industry. The demand for freight trains is expected to grow significantly over the next decade due to the improvements in rail infrastructure and technology combined with the increasing surface road haulage costs (*5*). With the projected increase in freight train demand, both freight TOCs and the IM are actively looking for new solutions to accommodate this future need. Consequently, there is a need to revisit the current RCAP and explore ways of improving freight train operations.

This paper presents the use of optimisation model for determining the timetable of freight trains with the least disruptions caused to existing train traffic with consideration of multiple objectives. In the optimisation framework, the timetable of each train is represented by a series of arrival and departure times over a set of control points (which can be a station, junction, etc.) along its service route (see example in *6, 7, 8, 9, 10*). An example is shown in Figure 2 in which the horizontal and vertical axes represent the time and position along the train route respectively. Each line on the diagram represents a train run which is specified by a series of departure $\sigma_{n,s}$ and arrival times $\tau_{n,s}$ at station *s* for each train *n* as specified by the timetable. Each track section is further disaggregated into a series of 'blocks'. Under the current railway signalling system, each block only accommodates up to one train at a time to ensure safe operations. As a consequence, congestion can occur when the traffic volume is high especially during the peak period. The capacity allocation problem is about determining how we should route and schedule the trains such that we can maximise the utilisation of limited capacity within a given time period. In addition, we also need to determine what kinds of trains (e.g. passenger, freight, fast, slow, etc.) we should give priority to at different times and locations when allocating the limited infrastructure capacity.

Considering the current practice of scheduling freight and passenger trains, the objective of the optimisation is to minimise the delays incurred by freight trains on passenger trains (*7, 8, 9*) and a number of other objectives including minimising costs due to extra running times and re-routing. Given the nature of train timetable and operational constraints, the optimisation is formulated as a mixed integer program (MIP, see *6* and *8*) and solved by branch and bound method (*10)* implemented in the IBM CPLEX solver. The proposed optimisation model can be used to determine the operational capacity of train runs along the service line. The model can also derive the maximum number of additional trains (e.g. ad-hoc freight trains, *9*) that can be inserted into a pre-scheduled timetable given a network configuration. The optimisation model is applied to a case study of Brighton Main Line in south-east England. Given the network configuration, the optimiser is able to schedule and route requested freight trains with minimised additional costs induced.

The paper is organised as follows: the next section starts with presenting the formulation of the optimisation model. It is followed by the case study of BML and discussion of results. Finally, the paper ends with some concluding remarks and suggestion for future studies.

METHODOLOGY

This section presents the specification of timetable, constraints, and formulation of the optimisation model for deriving freight train timetable with multiple objectives.

Objective functions and constraints

Given the arrival and departures of a train *n* as specified by a timetable (see Figure 2), we can first derive the corresponding dwell time

$$d_{n,s} = \sigma_{n,s} - \tau_{n,s}, \tag{1}$$

for each train n at each station s as the difference between the associated the arrival and departure times. We can also compute the running time

$$T_{n,s} = \tau_{n,s+1} - \sigma_{n,s}, \tag{2}$$

for each train n between each station pair (s, s+1). Following this, we can set up and compute the cost C_n associated with each train n in the optimisation problem as:

$$C_n = \alpha_n E_n + \beta_n D_n \tag{3}$$

where $E_n = \sum_{s=1}^{S_n} (\tau_{n,s} - \hat{\tau}_{n,s})^2$ is the total squared difference between the ideal arrival time $\hat{\tau}_{n,s}$ and scheduled arrival time $\tau_{n,s}$ of train n over all stations s = 1, 2,…, S_n along its service path. This cost component E_n represents the lost in revenue for serving passengers or delivering goods on time due to disruptions. The cost component $D_n = \sum_{s=1}^{S_n} T_{n,s}$ is the total running time that train n is associated with along the path, which can be regarded as a reflection of the effectiveness and fuel consumption of the service path (e.g. a service path takes longer distance and/or time can be regarded as consuming more energy and hence money). The parameters α_n and β_n are the coefficients (e.g. their monetary values) associated with schedule delays E_n and running times D_n. For example, UK Department for Transport (*11*) suggests the values for α_n and β_n as £5.76 (per person-hour) and £14.4 (per person-hour) respectively following the empirical study by (*12*). Finally, it should be noted that we adopt the cost components E_n and D_n simply as an illustration while the construction of the cost function (3) is generic and users can incorporate other cost components as they wish. Further review and discussions on cost functions can be found in (*13*) and (*14*).

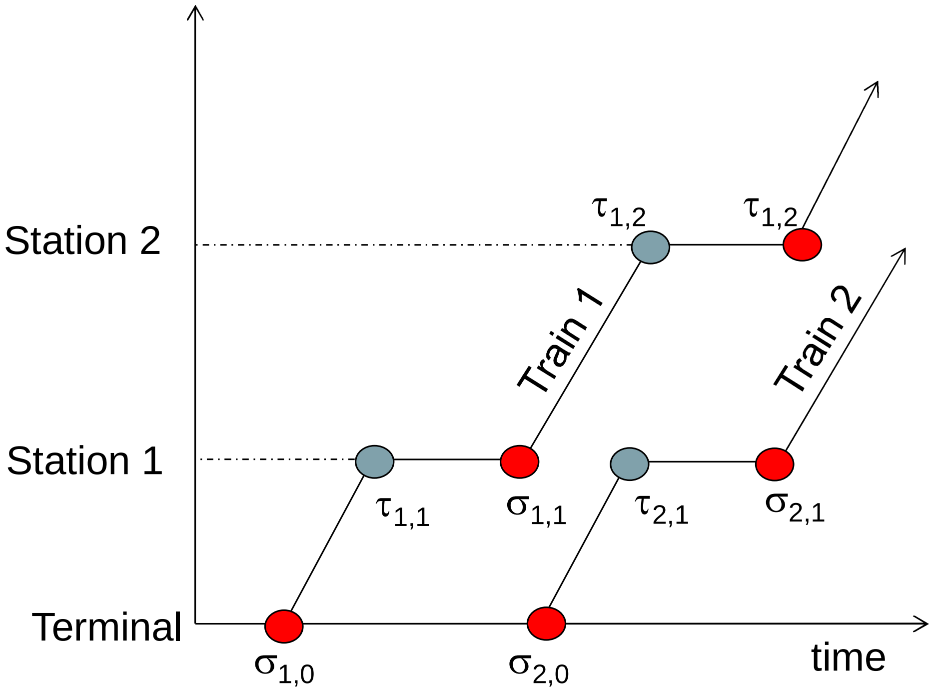

FIGURE 2 A realisation of timetable with train diagram

Following the formulation of the cost functions, a multi-objective optimisation is then formulated to determine an optimal timetable, i.e. the optimal set of arrival times $\boldsymbol{\tau} = [\tau_{n,s}]$ and departure times $\boldsymbol{\sigma} = [\sigma_{n,s}]$ for all trains *n* over all stations *s*, that minimises the total cost:

$$\min_{(\boldsymbol{\sigma},\boldsymbol{\tau})} C = \sum_{n=1}^{N} C_n \qquad (4)$$

where *N* is the total number of trains to be scheduled. The objective (4) is subject to a set of operational constraints including minimum separation between trains, minimum section running times, and minimum dwell times (*13, 14, 15*).

The minimum train separation constraint is to ensure there is a safety margin between each pair of successive trains. Most mainline systems adopt a fixed block signalling system in which rail tracks are discretised into a series of blocks. The basic principle of the fixed block system is that a train is only allowed to proceed into a block when the previous train has left it (see Figure 3). Referring to Figure 3, denote the arrival and departure time of train *n* at block '*j*' between station pair (*s*, *s*+1) as $\sigma_{n,s,j}$ and $\tau_{n,s,j}$ respectively. The shaded region in the figure represents the location and time period (during times t_{in} and t_{out}) that is occupied by the train of interest during which other trains are prohibited from entering. Following the specification in the current UIC (International Union of Railways) operational code (*16*),

$$t_{in} = \tau_{n,s,j} - \frac{\delta_{n,j}}{v_{n,j}}, \quad (5)$$

where $\delta_{n,j}$ is the visual distance of train *n* to the entrance of block *j*; $v_{n,j}$ is the nominal speed of train *n* travelling through block *j*. The time t_{in} represents the time when the driver of train *n* observes the signal aspect at block *j* and starts to take according action(s). Moreover,

$$t_{out} = \sigma_{n,s,j} + \frac{L_n}{v_{n,j}}, \quad (6)$$

where L_n is the length of train *n*. The time t_{out} represents the time when the tail of the train *n* clears from the block section. Following (5) and (6), the signal blocking constraint can then be written mathematically for all station pairs (*s*, *s*+1) and signal blocks *j* as

$$\tau_{n_f,s,j} \geq \sigma_{n,s,j} + \frac{L_n}{v_{n,s,j}}, \quad (7)$$

in which n_f denotes the train following immediately after train *n* arriving at block *j*. Here we introduce the notation n_f instead of simply using ‘*n*+1’ is due to the consideration of ‘re-routing’ as to be discussed in latter section.

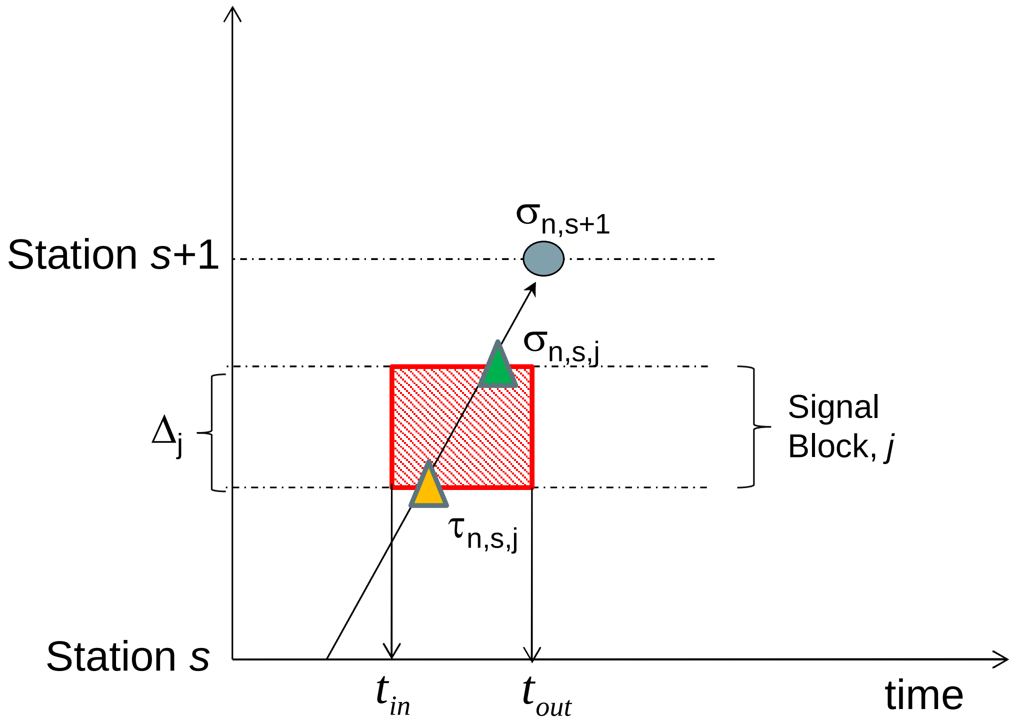

FIGURE 3 Representation of fixed block system

In addition to (7), we also have a set of minimum section running time constraints to reflect the speed limit imposed on each track section (s, s+1). It is presented mathematically as:

$$\tau_{n,s+1} \geq \sigma_{n,s} + \frac{\Delta_{s,s+1}}{v^*_{n,s}}, \tag{8}$$

where $\Delta_{s,s+1}$ is the distance between stations s and s + 1, $v^*_{n,s}$ is the maximum speed limit for train n travelling from station s toward s + 1.

Finally, we have a set of minimum dwell time ($d^*_{n,s}$) constraints which define the minimum time have to be spent by each train n at a station s.

$$d_{n,s} = \sigma_{n,s} - \tau_{n,s} \geq d^*_{n,s} \tag{9}$$

The minimum dwell time $d^*_{n,s}$ on each train n at each station s will typically be determined by a number of factors on the demand side such as demand level of passengers or freight for that specific train at that specific station, and/or the consideration of connectivity where it is necessary to ensure a long enough dwell time for passengers or goods to transfer from one train to another at the station or interchange (*13, 14*).

Extension to incorporate re-routing option

In addition to scheduling of trains, we also consider the possibility of re-routing trains in the optimisation framework. Re-routing is a strategy used in railway operation where a train service can be assigned to an alternative route or path due to specific circumstances such as line closure or line occupied by another train services. Despite the extra cost due to the additional distance travelled and energy consumed, re-routing can indeed be a cost-effective strategy when excessive congestion occurs on the nominal train route.

Given a predefined configuration of network and train paths, the optimisation problem (3) – (9) presented above can be solved as a standard quadratic programming problem (due to the quadratic function in E_n) for a timetable, in terms of $\boldsymbol{\tau} = [\tau_{n,s}]$ and $\boldsymbol{\sigma} = [\sigma_{n,s}]$, that minimises the multi-objective cost function (3). It is noted however the previous formulation can only deal with scheduling of train services but not (re-)routing which involves structural changes in network and service path configuration, and hence the constraint set (7) – (9). Consequently, we need to revise the optimisation formulation in order to incorporate the feature of re-routing.

Following (*15*), we adopt a mixed integer programming (MIP) formation to capture the structural change in constraints for (re-)routing options. We start with introducing a binary integer variable I_n which equals to zero if train n follows its nominal route; I_n will be set to be one if train n is assigned to an alternative route.

With this binary variable I_n for each n, the signal blocking constraint set (7) is replaced by the following paired constraint set for all trains n, stations s and s', and blocks j and j' as:

$$\tau_{n_f,s,j} + I_n M \geq \sigma_{n,s,j} + \frac{L_n}{v_{n,s,j}},$$

$$\tau_{n'_f,s',j} + (1 - I_n) M \geq \sigma_{n,s',j} + \frac{L_n}{v_{n,s',j'}}, \tag{10}$$

in which M denotes an arbitrarily large number where we set it to be '99,999'. The first set of constraints in (10) is essentially the same as constraints (7) when $I_n = 0$, i.e. train n is assigned to its nominal service route. When train n is assigned to an alternative route, I_n will be set to one and hence the first set of constraints in (10) will be disabled with the introduction of the large quantity M which implies the associated set of constraints will be satisfied no matter what values of $\tau_{n+1,s,j}$ are. On the other hand, the second set of constraints in (10) will be effective with the removal of the 'M' term. This second set of constraints is structurally identical to the first set with exception of the alternative set of n'_f, s' and j' in place of the original set of n_f, s and j to represent the new operational circumstance along the alternative service path.

Likewise, with the introduction of an alternative path route, the minimum running time constraints (8) are revised as

$$\tau_{n,s+1} + I_n M \geq \sigma_{n,s} + \frac{\Delta_{s,s+1}}{v^*_{n,s}},$$

$$\tau_{n,s'+1} + (1 - I_n)M \geq \sigma_{n,s'} + \frac{\Delta_{s',s'+1}}{v^*_{n,s'}},$$

(11)

and the minimum dwell time constraints (9) as

$$\sigma_{n,s} - \tau_{n,s} + I_n M \geq d^*_{n,s},$$

$$\sigma_{n,s'} - \tau_{n,s'} + (1 - I_n)M \geq d^*_{n,s'}$$

(12)

With the routing option, we can also refine the cost function (3) as

$$C_n = \alpha_n E_n + \beta_n D_n + \gamma_n R_n \tag{13}$$

where R_n is total number of re-routing that train n has to perform along its designated path. The coefficient γ_n represents the inconvenience and monetary cost caused by re-routing. This parameter is estimated to be £35 per extra train-kilometre to represent the extra distance travelled due to re-routing (*11*).

With the introduction of the binary variable, the new optimisation problem (13), subject to (10)-(12), now becomes a mixed integer program which can be solved by a branch and bound technique (*10, 17*). The branch and bound algorithm is available in a number of existing packages such as IBM CPLEX (*17*) which is used in the present study.

Finally, it is noted that the formulation (10)-(12) considers only two possible routing options, while it can be extended to capture multiple routing options by introducing additional binary variables (see *15, 17*). Li et al. (*17*) further present an analysis on the complexity of the resultant MIP with additional binary variables and show that complexity grows exponentially with the number of binary variables. It is hence suggested advanced solution techniques (e.g. *18, 19*) may need to be explored for applications involving a large number of possible routing options, while research into algorithmic design is beyond the scope of the present paper and we leave it for further studies.

CASE STUDY – Brighton Main Line, England

The optimisation framework is now applied to Brighton Main Line (BML) in southeast England (see Figure 4). The Brighton Main Line is approximately 80-km long electrified connection linking London Victoria and London Bridge with Brighton via East Croydon and Gatwick Airport. The line itself has a complex structure with a variable number of tracks (four tracks from London down to Balcombe Tunnel Junction and two tracks thereafter), different speed limits along the line, multiple branch lines (e.g. at Junctions Horsham, Lewes), and sidings (e.g.

along Ardingly, Lovers Depot). Passenger operators that operate on the BML include Southern and First Capital Connect. We select the section between Gatwick Airport and Brighton which is highlighted in Figure 4. This is one of the busiest sections along BML. We consider a 24-hr period over a nominal weekday with both directions: from Brighton toward Gatwick and hence Central London (the 'Up' direction) and from Gatwick toward Brighton (the 'Down' direction). Both directions consist of two available tracks between Brighton and Haywards Heath where re-routing of trains can take place. For passenger trains, there are two different train classes running through the section during the study period: Classes 375 and 442 with Class 375 used for the express connection. The signal block (7) and running time (8) constraints for different trains over different sections are constructed based on the network configuration and speed limits provided by UK Network Rail. The minimum dwell times (constraint (9)) are set to be 50-sec and 3-min for passenger and freight trains respectively following local regulation and field observations.

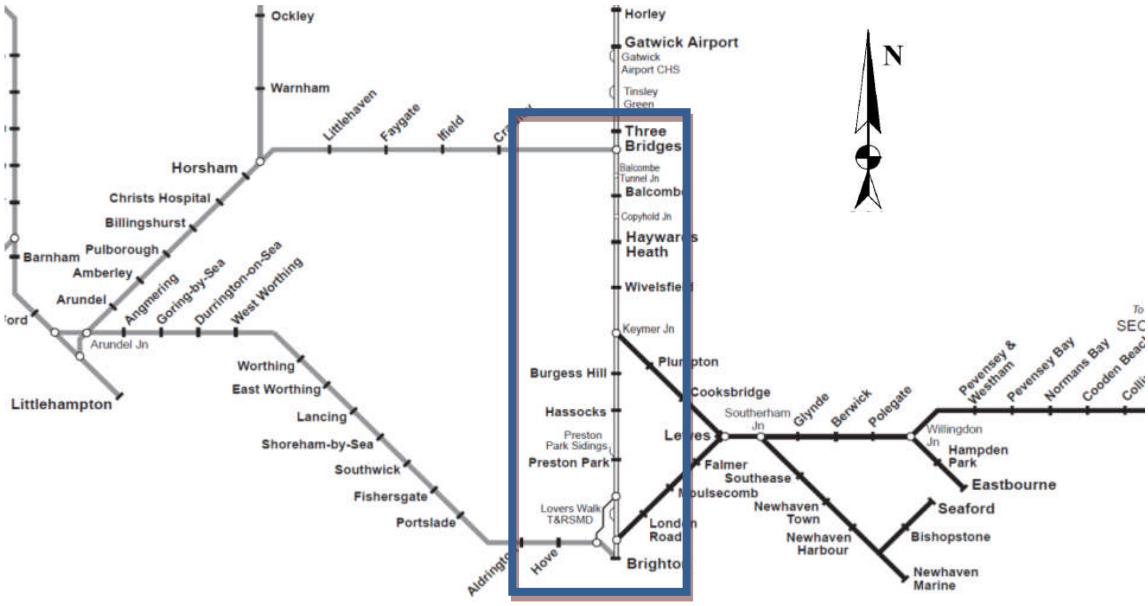

FIGURE 4 Brighton Main Line, Southeast England
(The case study section is highlighted by the rectangle)

In the current scenario, there are a total of four freight trains scheduled to run (two on each direction), all scheduled before 07:00 every weekday. The aim of this study is to explore the potential number of extra freight trains that we can actually schedule and the corresponding impact on existing train traffic through the optimisation framework. The optimisation model is implemented on a standard Windows 7 (64-bit) desktop computer and computed by IBM CPLEX solver. The cost coefficients are set to be the values following the standard in the UK as specified in the previous section. The computer takes less than a minute to solve the optimisation problem without re-routing option while it takes 6 minutes when re-routing option is allowed due to the complexity induced by the binary constraint (see also, *17*).

With the optimiser, we explore the relationship between system costs with respect to number of freight trains scheduled into the existing timetable. Table 1 summarises the results, fpr both with and without re-routing options, of optimal scheduling of extra freight trains between the BML Gatwick and Brighton section. The results show that the additional cost induced by additional freight trains increases non-linearly with the number of additional freight trains to schedule in all cases. The non-linearity is due to the extra delays and running times due to the congestion caused. It is revealed that the re-routing option will be used, in addition to re-scheduling, when the number of additional freight trains reaches five due to high train running time and passenger waiting time costs because of congestion. It is found that with the re-routing option the optimiser can reduce the total additional operational cost by up to 20% (from £9.4k to £7.3k) with number of additional freight trains is 10. The benefit associated with re-routing increases with the number of freight trains increases, and this highlight the value of effective infrastructure planning in handling heavy train traffic. The analysis herein provides insight on using re-routing strategy and the benefit of building extra track(s) from infrastructure planning perspective.

TABLE 1 Results from optimising freight scheduling on BML

Number of extra freight trains	Additional cost without re-routing (£ thousands)	Additional cost with re-routing (£ thousands)	Number of re-routings
1	0.1	0.1	0
2	0.3	0.3	0
3	0.6	0.6	0
4	0.9	0.9	0
5	1.3	1.2	1
6	2.4	2.1	1
7	3.8	3.3	2
8	5.1	4.4	2
9	7.4	5.9	2
10	9.4	7.3	3

The above analysis is an overview of an operation over a 24-hr period while it is known that the train traffic is varying over time of the day and it is particularly congested during the peak hours. To gain further insight on the scheduling strategy, Table 2 shows the scheduling of freight trains over different time periods of the day. We take the scenario of scheduling 10 freight trains with re-routing option as an example. In Table 2, the 24-hr period is disaggregated into four 6-hr sub-periods starting from 00:00, where the second column shows the corresponding number of freight trains scheduled into each period. It can be seen that most of the freight trains (8 out of 10) are scheduled into the period of 00:00 - 06:00, during which there are very few train traffic on the line. It is noted that we do not consider the maintenance work during mid-night here which would reduce the network capacity and hence affect the scheduling decisions. However, the impact of maintenance work will be easy to incorporate with relevant information (e.g. schedule and details of the work) provided.

TABLE 2 Scheduling and routing of 10 freight trains over time of day and their associated marginal costs

Period	Number of freight trains scheduled	Marginal cost of freight train (£ thousands)
00:00 - 06:00	8	0.65
06:00 - 12:00	0	1.81
12:00 - 18:00	1	1.63
18:00 - 24:00	1	1.33

The allocation of trains derived by the optimiser can also be explained by observing the marginal costs induced by adding one additional train as shown in the third column in the table. The third column shows that the average marginal cost of scheduling one train into the period 00:00 – 06:00 is £0.65k, starting from £0.2k for scheduling the first train. When we attempt to schedule the 9^{th} freight train to 00:00 – 06:00, the associated marginal cost will have increased to £1.4k from the original £0.2k due to the building up of track congestion. By then this marginal cost will be higher than the one associated with the period 18:00 – 24:00 which is £1.33k. Consequently, the optimiser will choose to schedule this 9^{th} freight train into the slot 18:00 – 24:00 instead. The last train will have to be scheduled into 12:00 – 18:00 despite the afternoon traffic due to the already high operating cost in 00:00 – 06:00 and 18:00 – 24:00.

Such allocation process aligns with the current practice while the proposed algorithm makes the scheduling process systematic and generalizable to large-scale applications and multiple cost functions.

CONCLUDING REMARKS

This paper presents a multi-objective optimisation framework for scheduling and re-routing freight trains into main line service with consideration of multiple objectives including running times, delays, and re-routing costs to both passenger and freight train operators. The contributions of this paper include specification of timetable and its associated operational constraints, the mixed integer formulation capturing both scheduling and re-routing of trains, and analysis of train allocation scheme under different circumstances. The optimisation model is applied to the Brighton Main Line in southeast England. Given the network configuration, the optimiser is shown to be able to schedule and (re-)route requested freight trains with minimised additional costs induced to the existing system. We examine selected scenario with marginal cost analysis and find that the train allocation process produced by the optimisation model is similar to existing practice, while the proposed algorithm is more systematic, generalizable to large-scale applications and multiple cost functions.

It is found that current policies of many Infrastructure Managers around the world tend to favour passenger train operations over freight trains. The work presented herein can support freight train industry in the long run through incorporating more equity of train services. It is noted that the focus of the present paper lies on the formulation of timetabling optimisation instead of the optimization algorithm. We agree that it will be worthy of conducting further research on alternative algorithms (e.g. *10, 20, 21*) for improving the quality of the optimal solutions. Future work also includes investigating the impact of freight trains on the reliability and resilience of overall train service with uncertainties in running times taken into account.

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