Dynamic analysis of mooring cables with application to floating offshore wind turbines

Crescenzo Petrone[1], Nicholas D. Oliveto[2] and Mettupalayam V. Sivaselvan[3]

Abstract

Floating offshore wind turbines are recently being considered widely for adoption in the wind power industry, attracting interest of several researchers and calling for the development of appropriate computational models and techniques. In the present work, a nonlinear finite element formulation is proposed and applied to the static and dynamic analysis of mooring cables. Numerical examples are presented, and in particular, a mooring cable typically used for floating offshore wind turbines is analyzed. Hydrodynamic effects on the cable are accounted for using the Morison approach. A key enabling development here is an algorithmic tangent stiffness operator including hydrodynamic coupling. Numerical results also suggest that previously empirical hydrodynamic coefficients could be obtained by fully coupled fluid-structure interaction. Convergence rate and energy balance calculations have been used to demonstrate the accuracy of computed solutions. The introduction of the developed cable model in a framework for the study of the global behavior of floating offshore wind turbines is subject of current work. Source code developed for this work is available as online supplemental material with the paper.

[1]Postdoctoral Research Associate. Department of Structures for Engineering and Architecture, University of Naples Federico II, via Claudio, 21, 80125, Naples, Italy. E-mail: crescenzo.petrone@unina.it.
[2]Research Scientist. Department of Civil, Structural & Environmental Engineering, University at Buffalo, 212 Ketter Hall, Buffalo, NY 14260, USA. E-mail: noliveto@buffalo.edu.
[3]Assistant Professor. Department of Civil, Structural & Environmental Engineering, University at Buffalo, 212 Ketter Hall, Buffalo, NY 14260, USA. E-mail: mvs@buffalo.edu.

Keywords

Cables; Nonlinear dynamics; Large deformations; Energy dissipation; Fluid-Structure Interaction; Floating offshore wind turbine.

Introduction

Offshore wind turbines are considered an attractive option in the solution of many issues associated with onshore turbines (Skaare et al. 2007). In addition to steadier breezes and higher annual mean wind velocity, they can also guarantee higher energy efficiency. In waters that are approximately 20 m deep, offshore wind turbines are typically installed on piled or gravity-based foundations. On the other hand, floating foundations are required to support wind turbines in waters that are 50–80 m deep. No shallow waters exist on the west coast of the US and nearly 60% of the estimated US offshore wind facilities are located in waters that are 60m deep or more (Musial and Ram 2010). Moreover, for aesthetic reasons, it is sometimes desirable to locate the turbines far off the coast where they cannot be seen. Therefore, the floating offshore wind turbine (FOWT) technology is becoming a strong candidate for the extraction of the majority of offshore wind energy in the US (Martin 2011).

Different FOWTs concepts and prototypes have been developed during the last few decades. In particular, three main concepts can be identified based on the way the wind turbine is stabilized, namely (*i*) tension leg platforms, (*ii*) spar buoy and (*iii*) barge FOWTs (Jonkman and Matha 2011). Tension leg platform turbines are stabilized by taut vertical mooring lines submerging a buoyant platform. In spar buoy systems, stability is achieved using ballasts that lower the center of gravity of the turbines below the center of buoyancy. Finally, barge turbines provide a large water plane area to stabilize the turbine through buoyancy. Some hybrid solutions have also been conceived, combining more than one of the concepts mentioned above. It should be noted that mooring systems in FOWTs are also required for station-keeping purposes.

Given their increasing popularity, different modelling tools and techniques have been recently developed and implemented for the study of FOWTs, such as NREL's fully-coupled simulator FAST (Jonkman and Buhl 2007) and the coupled simulator by Hydro Oil & Energy SIMO/RIFLEX/HAWC (Skaare et al. 2007). These software programs provide a unique platform coupling the different components of a FOWT. However, whereas quite detailed and reliable approaches are used to account for aerodynamics and hydrodynamics of the wind turbine, a simple quasi-static approximation is typically employed for the mooring systems, neglecting any influence of their dynamics and interaction with water. The need to perform additional research studies on the behavior of mooring and anchoring systems was clearly identified by The European Wind Energy Association (EWEA) (2013) and by Matha et al. (2011). Dynamic interaction between mooring cables and FOWTs can cause the loads on the turbines to increase as much as 50% (Hall et al. 2013).

Several studies have focused on the definition of models for the study of mooring systems used in FOWTs. These were mainly aimed at assessing the accuracy and approximations of different models and their influence on the response of FOWTs. An overview of available simulation codes and modeling approaches was presented by Cordle and Jonkman (2011). Matha et al. (2011) proposed a multi-body approach, whereby the mooring lines are divided into multi-body elements connected by spring-damper elements, and cable-fluid interaction is accounted for by the Morison approach (Morison et al. 1950), as detailed in the following section. Additional studies revealed the need to consider more detailed non-linear mooring system models. Kvittem and Moan (2012) investigated the behavior of a single semi-submersible wind turbine using both linear and nonlinear mooring line models for three different mooring line configurations. Masciola et al. (2013) coupled FAST with OrcaFlex, a time-domain program capable of modeling cable dynamics and hydrodynamic loads of floating offshore vessels. They concluded that the quasi-static mooring approximation can lead to underestimating peak mooring line loads. Hall et al. (2013) coupled FAST with ProteusDS (Buckham et al. 2004), a mooring line model incorporating dynamics and cable-fluid interaction, as well as cable bending and torsional stiffness. Three different floating wind turbines were analyzed using both quasi-static and dynamic mooring models for different load cases, namely free-decay tests, periodic steady-state operating conditions and stochastic operating conditions. It was concluded that quasi-static models are not adequate for evaluating mooring line loads and may lead to an inaccurate estimation of both blade and tower bending moments. In a recent study, Masciola et al. (2014) used a lumped-mass modeling approach of the mooring line for implementation in FAST. The approach was chosen due to its simplicity, low computational cost, and ability to provide physics similar to those captured by higher-order models.

Another important aspect is related to modeling of the interaction of mooring lines with surrounding water. An extensive literature survey on the topic, including work related to offshore oil platforms, (Journée and Massie 2001; Gobat and Grosenbaugh 2006; Frigaard and Burcharth 1989; Mavrakos et al. 1996; Sarkar and Taylor 2002; Webster 1995; Faltinsen 1990) shows that, due to the slenderness of mooring cables, the Morison approach is particularly suited and typically employed to evaluate the fluid-cable interaction in mooring line systems. This approach was therefore used in this study and is described in the following section.

Recently, Oliveto and Sivaselvan (2014a) extended the 3D finite-deformation beam model developed by Simo (1985) to include viscous damping, and applied it to describe the dynamic behavior of flexible cables. The formulation was verified with the commercial software ABAQUS and validated with shake table tests on electrical conductor cables performed at the SEESL laboratory at the University at Buffalo (Oliveto and Sivaselvan 2014b). In the present work, the above 3D beam model is appropriately modified and applied to the static and dynamic behavior of mooring cables in water. As mentioned above, the interaction of the cable with the surrounding fluid is accounted for using the Morison approach. Using this approach, accurate evaluation of the hydrodynamic forces acting on the cable involves the correct calculation of large cable rotations. Therefore, going beyond previous formulations, the 3D finite deformation beam formulation proposed here allows for an exact representation of finite rotations.

The paper is organized as follows. In the following section, the Morison approach is summarized. Next, the governing equations of the 3D finite-deformation beam model are described. A new aspect is the introduction in the equations of motion of terms accounting for fluid-beam interaction. Then, linearization and discretization of the weak form of the equations of motion is presented, leading to the definition of a tangent operator and a system of equations solvable by means of an iterative scheme of the Newton type. The main focus in these sections is the derivation of the tangent operators associated with the hydrodynamic forces. Finally, examples are presented to investigate the performance of the numerical implementations. In particular, dynamic analyses were carried out of a cantilever beam in water and of a realistic mooring system. It is shown that, while they are generally derived from experiments, the hydrodynamic coefficients needed in the Morison approach can be also extracted from a fully coupled fluid-structure interaction (FSI) analysis. Full source code for all these developments is available as online supplemental material with this paper.

The Morison approach for mooring-to-fluid interaction

As is generally done in the literature, the interaction of the mooring cables with the surrounding water is accounted for in this work using the Morison approach. Additional drag and inertia forces are used to represent the effects of the water on the cable. Such forces, per unit length of cable, may be written as

$$\mathbf{f}_{drag} = -\lambda_{drag} \left\| \mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right\| \left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right) \tag{1}$$

$$\mathbf{f}_{inertia} = \mathbf{f}_{disturbance} + \mathbf{f}_{Froude-Krylov} = -\lambda_{inertia} \left(\mathbf{a}_{0\perp} - \mathbf{a}_{w\perp} \right) - 0.25 \rho_w \pi D^2 \mathbf{a}_{w\perp} \tag{2}$$

where $\lambda_{drag} = 0.5 \rho_w D C_D$ and $\lambda_{inertia} = 0.25 \rho_w \pi D^2 C_M$; ρ_w = density of water; D = diameter of the cable; $\mathbf{v}_{0\perp}$ and $\mathbf{v}_{w\perp}$ = cable and water velocity vectors in the plane orthogonal to the cable (Fig. 1); $\mathbf{a}_{0\perp}$ and $\mathbf{a}_{w\perp}$ = cable and water acceleration vectors in the plane orthogonal to the cable; C_M and C_D = empirical coefficients that can be determined experimentally in a variety of ways (Journée and Massie 2001). C_M and C_D are influenced by several factors, including Reynolds number, dimensions of the cable and surface roughness.

If $\mathbf{t}$ is the tangent to the cable, the drag force and the additional inertia force act in the plane $\mathcal{N}$ orthogonal to $\mathbf{t}$ (Fig. 1). Note that fluid-cable interaction in the tangential direction is not considered in this model.

The drag force takes into account the viscous terms related to skin friction drag and form drag. Such force is proportional to the square of the relative velocity between cable and fluid, and its direction is the same as that of the relative velocity vector.

The additional inertia force is composed of the Froude-Krylov force and the disturbance force. The Froude-Krylov force is related to the pressure gradient in the accelerating flow around the perimeter of the cable, and is equal to the product of the mass of water displaced by the cable and the acceleration of the undisturbed flow. While investigating the behavior of mooring lines in floating offshore wind turbines, Masciola et al. (2014) assumed that for large water depths, water acceleration is typically negligible and therefore the Froude-Krylov contribution to the inertia force can be generally omitted. On the other hand, the disturbance force is related to the change of flow pressure due to the presence of the cable, and is equal to the product of a given percentage of displaced mass of water and the relative acceleration between fluid and cable. The latter contribution vanishes if the acceleration of the fluid is equal, in direction and magnitude, to the acceleration of the cable.

Governing Equations

The governing equations of the 3D finite deformation beam model used in this paper are presented, namely kinematics, equilibrium and constitutive equations. The considered formulation is basically the one originally developed by Simo and Vu-Quoc (1986), and extended by Oliveto and Sivaselvan (2014a) to include energy dissipation. However, a new aspect is the introduction in the formulation of a model for the interaction between beam and surrounding fluid. As described above this is based on the Morison approach.

Kinematics

The motion of the beam is defined uniquely by the position of the line of centroids, $\mathbf{x}_0(S,t)$, and a rotation tensor $\mathbf{R}(S,t)$, determining the orientation of a moving (current) frame $\mathbf{t}_i(S,t)$, attached to the cross section, relative to its initial (reference) position, $\mathbf{E}_i$. In other words, $\mathbf{R}(S,t)$ represents a rigid rotation of the cross section such that

$$\mathbf{t}_i\left(S,t\right)=\mathbf{R}\left(S,t\right)\cdot\mathbf{E}_i \tag{3}$$

The reference and current configurations of the beam, and their corresponding coordinate systems, both defined with respect to a fixed global reference system $\mathbf{e}_i$, are shown in Fig. 2.

Equations of motion

The equations of motion of the 3D finite deformation beam model considered in this work are given by:

$$\frac{\partial \mathbf{n}}{\partial S} + \tilde{\mathbf{n}} + \mathbf{f}_{drag} + \mathbf{f}_{inertia} = A_\rho \cdot \mathbf{a}_0 \tag{4}$$

$$\frac{\partial \mathbf{m}}{\partial S} + \frac{\partial \mathbf{x}_0}{\partial S} \times \mathbf{n} + \tilde{\mathbf{m}} = \mathbf{I}_\rho \cdot \dot{\mathbf{w}} + \mathbf{w} \times \left(\mathbf{I}_\rho \cdot \mathbf{w} \right) \tag{5}$$

where $\mathbf{n}$ and $\mathbf{m}$ = force and moment resultants in the current configuration; $\tilde{\mathbf{n}}$ and $\tilde{\mathbf{m}}$ = distributed applied forces and moments per unit undeformed length of the beam; A_ρ and $\mathbf{I}_\rho$ = mass and mass moment of inertia per unit undeformed length of the cable; $\mathbf{w}$ and $\dot{\mathbf{w}}$ = rotational velocity and acceleration vectors, all represented in the current configuration. Moreover the notation $\mathbf{a}_0 = \ddot{\mathbf{x}}_0$ is used for cable acceleration.

Note that Eq. (4) and (5) were obtained by adding two terms, namely $\mathbf{f}_{drag}$ and $\mathbf{f}_{inertia}$, to the equations of motion of the geometrically nonlinear beam model (Simo 1985, 1986; Simo and Vu-Quoc 1988; Oliveto and Sivaselvan 2014a). By omitting the Froude-Krylov term in Eq. (2) , the hydrodynamic forces $\mathbf{f}_{drag}$ and $\mathbf{f}_{inertia}$ will be taken as

$$\mathbf{f}_{drag} = -\lambda_{drag} \left\| \mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right\| \left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right) \tag{6}$$

$$\mathbf{f}_{inertia} = -\lambda_{inertia} \left(\mathbf{a}_{0\perp} - \mathbf{a}_{w\perp} \right) \tag{7}$$

Constitutive equations

By assuming large deformations but small strains, as is generally done in the literature, the stress resultants in the reference configuration, $\mathbf{N}^e$ and $\mathbf{M}^e$, are linearly proportional to the corresponding strains $\mathbf{\Gamma}$ and curvatures $\mathbf{\Omega}$ through a constant and diagonal elasticity tensor $\mathbf{C}$ defined as

$$\mathbf{C} = \mathrm{diag}\left[\mathbf{C}^N, \mathbf{C}^M\right] \tag{8}$$

where

$$\mathbf{C}^N = diag[GA_1, GA_2, EA], \quad \mathbf{C}^M = diag\left[EI_1, EI_2, GJ_t\right] \tag{9}$$

where E = Young's modulus; G = shear modulus; A = area of the rigid cross section; A_1 and A_2 = effective cross-sectional areas for shearing; I_1 and I_2 = area moments of inertia of the cross section; and J_t = torsion constant.

A Kelvin-Voigt damping model was introduced in the beam formulation by Oliveto and Sivaselvan (2014a) in order to account for viscous forms of energy dissipation. The internal dissipative forces and moments in the reference configuration, $\mathbf{N}^d$ and $\mathbf{M}^d$, are taken as linearly proportional to the corresponding strains $\dot{\mathbf{\Gamma}}$ and curvatures $\dot{\mathbf{\Omega}}$ through a constant and diagonal tensor $\mathbf{C}_d$ defined as

$$\mathbf{C}_d = \mathrm{diag}\left[\mathbf{C}_d^N, \mathbf{C}_d^M\right] \tag{10}$$

where

$$\mathbf{C}_d^N = diag[\mu GA_1, \mu GA_2, \eta EA], \quad \mathbf{C}_d^M = diag\left[\eta EI_1, \eta EI_2, \mu GJ_t\right] \tag{11}$$

where μ and η = retardation time constants transforming the elastic moduli E and G into viscous constants, akin to stiffness proportional damping coefficients.

The constitutive equations, relating the total internal forces and moments to the corresponding strains, strain rates, curvatures and curvature rates, are given by

$$\mathbf{N} = \mathbf{N}^e + \mathbf{N}^d = \mathbf{C}^N \cdot \mathbf{\Gamma} + \mathbf{C}_d^N \cdot \dot{\mathbf{\Gamma}} \tag{12}$$

$$\mathbf{M} = \mathbf{M}^e + \mathbf{M}^d = \mathbf{C}^M \cdot \mathbf{\Omega} + \mathbf{C}_d^M \cdot \dot{\mathbf{\Omega}} \tag{13}$$

Expressions for strains $\boldsymbol{\Gamma}$, curvatures $\boldsymbol{\Omega}$, and their corresponding rates $\dot{\boldsymbol{\Gamma}}$ and $\dot{\boldsymbol{\Omega}}$, can be found in (Oliveto and Sivaselvan 2014a).

Weak form of the equations of motion

The weak form of the equations of motion is obtained by multiplying the equations of motions (4) and (5) by an admissible variation $\boldsymbol{\eta} = \left[\boldsymbol{\eta}_u, \boldsymbol{\eta}_\theta\right]$ and integrating by parts. This gives:

$$\begin{aligned} G(\boldsymbol{\varphi}, \boldsymbol{\eta}) &= \int_0^L \left[\left(\frac{\partial \boldsymbol{\eta}_u}{\partial S} - \boldsymbol{\eta}_\theta \times \frac{\partial \mathbf{x}_0}{\partial S}\right) \cdot \mathbf{R} \cdot \mathbf{N} + \frac{\partial \boldsymbol{\eta}_\theta}{\partial S} \cdot \mathbf{R} \cdot \mathbf{M}\right] dS \\ &- \int_0^L \left(\tilde{\mathbf{n}} \cdot \boldsymbol{\eta}_u + \tilde{\mathbf{m}} \cdot \boldsymbol{\eta}_\theta\right) dS + \int_0^L \left\{A_\rho \mathbf{a}_0 \cdot \boldsymbol{\eta}_u + \mathbf{R} \cdot \left[\mathbf{J}_p \cdot \dot{\mathbf{W}} + \mathbf{W} \times \left(\mathbf{J}_p \cdot \mathbf{W}\right)\right] \cdot \boldsymbol{\eta}_\theta\right\} dS \\ &- \int_0^L \boldsymbol{\eta}_u \cdot \mathbf{f}_{drag} dS - \int_0^L \boldsymbol{\eta}_u \cdot \mathbf{f}_{inertia} dS = 0 \end{aligned} \qquad (14)$$

where $\mathbf{N}=\mathbf{R}^T \cdot \mathbf{n}$ and $\mathbf{M}= \mathbf{R}^T \cdot \mathbf{m}$ = reference force and moment resultants; $\mathbf{J}_\rho = \mathbf{R}^T \cdot \mathbf{I}_\rho \cdot \mathbf{R}$ = time independent reference mass moment of inertia per unit undeformed length of the beam; $\mathbf{W} = \mathbf{R}^T \cdot \mathbf{w}$ = reference angular velocity vector.

Note the presence in Eq. (14) of $\mathbf{f}_{drag}$ and $\mathbf{f}_{inertia}$. Previous formulations of the 3D finite deformation beam model do not account for these terms. Therefore the derivations that follow are significantly different.

Linearization of the weak form

The weak form of the equations of motions is linearized and discretized, in time and space, leading to the definition of a tangent operator and a system of equations to be solved by an iterative procedure of the Newton's type. In this process, extensions of Newmark's time integration scheme and Newton's method to large rotations are used. Details of these can be found in Simo and Vu-Quoc (1988), and Oliveto and Sivaselvan (2014a). If the Newton iteration counter is denoted by i, and the time step counter by n, the weak form of the equations of motion at configuration $\boldsymbol{\varphi}_{n+1}^{(i)} = \left[\mathbf{x}_{0,n+1}^{(i)}(S,t), \mathbf{R}_{n+1}^{(i)}(S,t)\right]$ is given by:

$$
\begin{aligned}
&G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) = \int_0^L \left[\left(\frac{\partial \boldsymbol{\eta}_u}{\partial S} - \boldsymbol{\eta}_\theta \times \frac{\partial \mathbf{x}_{0,n+1}^{(i)}}{\partial S}\right) \cdot \mathbf{R}_{n+1}^{(i)} \cdot \mathbf{N}_{n+1}^{(i)} + \frac{\partial \boldsymbol{\eta}_\theta}{\partial S} \cdot \mathbf{R}_{n+1}^{(i)} \cdot \mathbf{M}_{n+1}^{(i)}\right] dS \\
&-\int_0^L \left(\tilde{\mathbf{n}} \cdot \boldsymbol{\eta}_u + \tilde{\mathbf{m}} \cdot \boldsymbol{\eta}_\theta\right) dS + \int_0^L \left\{A_\rho \mathbf{a}_{0,n+1}^{(i)} \cdot \boldsymbol{\eta}_u + \mathbf{R}_{n+1}^{(i)} \cdot \left[\mathbf{J}_p \cdot \dot{\mathbf{W}}_{n+1}^{(i)} + \mathbf{W}_{n+1}^{(i)} \times \left(\mathbf{J}_p \cdot \mathbf{W}_{n+1}^{(i)}\right)\right] \cdot \boldsymbol{\eta}_\theta\right\} dS \\
&-\int_0^L \boldsymbol{\eta}_u \cdot \mathbf{f}_{drag,n+1}^{(i)} dS - \int_0^L \boldsymbol{\eta}_u \cdot \mathbf{f}_{inertia,n+1}^{(i)} dS = 0
\end{aligned}
\tag{15}
$$

The linear part of equation (15) is then given by

$$
L\left[G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right)\right] = G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) + \delta G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) \tag{16}
$$

where $G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right)$ = unbalanced force at configuration $\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right)$ and $\delta G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right)$= linear in the incremental displacement field $\Delta \boldsymbol{\varphi}_{n+1}^{(i)}\left(\delta \mathbf{u}_{n+1}^{(i)}, \delta \boldsymbol{\theta}_{n+1}^{(i)}\right)$, leads to the definition of a tangent operator. This can be decomposed into the geometric and material stiffness terms, the inertia term, the damping term, and two terms related to the addition of the hydrodynamic forces

$$
\begin{aligned}
\delta G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) = {}& \delta G_G\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) + \delta G_M\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) + \delta G_I\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) + \delta G_D\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) \\
&+ \delta G_{FB,D}\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right) + \delta G_{FB,I}\left(\boldsymbol{\varphi}_{n+1}^{(i)}, \boldsymbol{\eta}\right)
\end{aligned}
\tag{17}
$$

For the derivation of the first three terms, the reader is referred to Simo and Vu-Quoc (1986; 1988), and for the fourth term to Oliveto and Sivaselvan (2014a). The following section describes the derivation of the terms related to the fluid-beam interaction. The subscripts n, denoting that a quantity is evaluated at time t_{n+1}, and the superscript i, denoting the Newton iteration counter are dropped to alleviate the notation.

Fluid-beam interaction tangent operators

The tangent operators related to the fluid-beam interaction are obtained by differentiating the hydrodynamic forces, $\mathbf{f}_{drag}$ and $\mathbf{f}_{inertia}$, as follows:

$$\delta G_{FB,D}\left(\boldsymbol{\varphi},\boldsymbol{\eta}\right) = -\int_0^L \boldsymbol{\eta}_u \cdot \delta\mathbf{f}_{drag}\, dS \tag{18}$$

$$\delta G_{FB,I}\left(\boldsymbol{\varphi},\boldsymbol{\eta}\right) = -\int_0^L \boldsymbol{\eta}_u \cdot \delta\mathbf{f}_{inertia}\, dS \tag{19}$$

Differentiating Eq. (6) gives

$$\begin{aligned}\delta\mathbf{f}_{drag} &= -\lambda_{drag}\,\delta\left(\left\|\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right\|\left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right)\right) = \\ &-\lambda_{drag}\frac{\left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right)\otimes\left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right)}{\left\|\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right\|}\cdot\left(\delta\mathbf{v}_{0\perp} - \delta\mathbf{v}_{w\perp}\right)\end{aligned} \tag{20}$$

Considering that shear deformations are small, the velocity of the cable in plane $\mathcal{N}$ may be written as $\mathbf{v}_{0\perp} = \mathbf{v}_0 - \left[\mathbf{v}_0 \cdot \left(\mathbf{R}\cdot\mathbf{E}_3\right)\right]\left(\mathbf{R}\cdot\mathbf{E}_3\right)$, where the notation $\mathbf{v}_0 = \dot{\mathbf{x}}_0$ is used. Recalling from Simo and Vu-Quoc (1988) that

$$\delta\mathbf{v}_0 = \frac{\gamma}{\beta\cdot h}\delta\mathbf{u} \tag{21}$$

$$\delta\mathbf{R} = \delta\hat{\boldsymbol{\theta}}\cdot\mathbf{R} \tag{22}$$

where γ and β = parameters of Newmark's time-integration scheme and h = time step, then it follows that

$$\begin{aligned}\delta\mathbf{v}_{0\perp} &= \frac{\gamma}{\beta\cdot h}\left[\mathbf{I} - \left(\mathbf{R}\cdot\mathbf{E}_3\right)\otimes\left(\mathbf{R}\cdot\mathbf{E}_3\right)\right]\cdot\delta\mathbf{u} \\ &+\left[\left(\mathbf{R}\cdot\mathbf{E}_3\right)\otimes\mathbf{v}_0 + \mathbf{v}_0\cdot\left(\mathbf{R}\cdot\mathbf{E}_3\right)\mathbf{I}\right]\cdot\left(\mathbf{R}\cdot\mathbf{E}_3\right)^{\wedge}\cdot\delta\boldsymbol{\theta}\end{aligned} \tag{23}$$

Note that the hat notation denotes the skew symmetric tensor associated with a given vector.

Similarly, water velocity in plane $\mathcal{N}$ is given by $\mathbf{v}_{w\perp} = \mathbf{v}_w - \left[\mathbf{v}_w \cdot (\mathbf{R}\cdot\mathbf{E}_3)\right](\mathbf{R}\cdot\mathbf{E}_3)$ and, since $\mathbf{v}_w$ is considered constant within a time step,

$$\delta\mathbf{v}_{w\perp} = \left[(\mathbf{R}\cdot\mathbf{E}_3)\otimes\mathbf{v}_w + \mathbf{v}_w\cdot(\mathbf{R}\cdot\mathbf{E}_3)\mathbf{I}\right]\cdot(\mathbf{R}\cdot\mathbf{E}_3)^{\wedge}\cdot\delta\boldsymbol{\theta} \tag{24}$$

Substituting Eqs. (23) and (24) into Eq. (20), this becomes

$$\begin{aligned}\delta\mathbf{f}_{drag} = &-\frac{\gamma\cdot\lambda_{drag}}{\beta\cdot h}\boldsymbol{\eta}_u\cdot\left[\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|\mathbf{I}+\frac{(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp})\otimes(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp})}{\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|}\right]\cdot\\ &\left[\mathbf{I}-(\mathbf{R}\cdot\mathbf{E}_3)\otimes(\mathbf{R}\cdot\mathbf{E}_3)\right]\cdot\delta\mathbf{u}\\ &-\lambda_{drag}\left[\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|\mathbf{I}+\frac{(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp})\otimes(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp})}{\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|}\right]\cdot\\ &\left[(\mathbf{R}\cdot\mathbf{E}_3)\otimes(\mathbf{v}_0-\mathbf{v}_w)+(\mathbf{v}_0-\mathbf{v}_w)\cdot(\mathbf{R}\cdot\mathbf{E}_3)\mathbf{I}\right]\cdot(\mathbf{R}\cdot\mathbf{E}_3)^{\wedge}\cdot\delta\boldsymbol{\theta}\end{aligned} \tag{25}$$

Furthermore, differentiating Eq. (7) gives

$$\delta\mathbf{f}_{inertia} = -\lambda_{inertia}\delta\left(\mathbf{a}_{0\perp}-\mathbf{a}_{w\perp}\right) \tag{26}$$

Recalling from Simo and Vu-Quoc (1988) that

$$\delta\mathbf{a}_0 = \frac{1}{\beta\cdot h^2}\delta\mathbf{u} \tag{27}$$

and following the same procedure as for differentiation of velocities, Eq. (26) becomes

$$\begin{aligned}\delta\mathbf{f}_{inertia} = &-\frac{\lambda_{inertia}}{\beta\cdot h^2}\left[\mathbf{I}-(\mathbf{R}\cdot\mathbf{E}_3)\otimes(\mathbf{R}\cdot\mathbf{E}_3)\right]\cdot\delta\mathbf{u}\\ &-\lambda_{inertia}\left[(\mathbf{R}\cdot\mathbf{E}_3)\otimes(\mathbf{a}_0-\mathbf{a}_w)+(\mathbf{a}_0-\mathbf{a}_w)\cdot(\mathbf{R}\cdot\mathbf{E}_3)\mathbf{I}\right]\cdot(\mathbf{R}\cdot\mathbf{E}_3)^{\wedge}\cdot\delta\boldsymbol{\theta}\end{aligned} \tag{28}$$

Substituting Eqs. (25) and (28) into Eqs. (18) and (19), finally leads to

$$\delta G_{FB,D}(\boldsymbol{\varphi},\boldsymbol{\eta}) = \frac{\gamma\cdot\lambda_{drag}}{\beta\cdot h}\int_0^L \boldsymbol{\eta}_u\cdot\left[\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|\mathbf{I}+\frac{\left(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right)\otimes\left(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right)}{\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|}\right]\cdot \left[\mathbf{I}-\left(\mathbf{R}\cdot\mathbf{E}_3\right)\otimes\left(\mathbf{R}\cdot\mathbf{E}_3\right)\right]\cdot\delta\mathbf{u}dS + \lambda_{drag}\int_0^L \boldsymbol{\eta}_u\cdot\left[\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|\mathbf{I}+\frac{\left(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right)\otimes\left(\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right)}{\left\|\mathbf{v}_{0\perp}-\mathbf{v}_{w\perp}\right\|}\right]\cdot \left[\left(\mathbf{R}\cdot\mathbf{E}_3\right)\otimes\left(\mathbf{v}_0-\mathbf{v}_w\right)+\left(\mathbf{v}_0-\mathbf{v}_w\right)\cdot\left(\mathbf{R}\cdot\mathbf{E}_3\right)\mathbf{I}\right]\cdot\left(\mathbf{R}\cdot\mathbf{E}_3\right)^{\wedge}\cdot\delta\boldsymbol{\theta}dS \tag{29}$$

$$\delta G_{FB,I}(\boldsymbol{\varphi},\boldsymbol{\eta}) = \frac{\lambda_{inertia}}{\beta\cdot h^2}\int_0^L \boldsymbol{\eta}_u\cdot\left[\mathbf{I}-\left(\mathbf{R}\cdot\mathbf{E}_3\right)\otimes\left(\mathbf{R}\cdot\mathbf{E}_3\right)\right]\cdot\delta\mathbf{u}dS + \lambda_{inertia}\int_0^L \boldsymbol{\eta}_u\cdot\left[\left(\mathbf{R}\cdot\mathbf{E}_3\right)\otimes\left(\mathbf{a}_0-\mathbf{a}_w\right)+\left(\mathbf{a}_0-\mathbf{a}_w\right)\cdot\left(\mathbf{R}\cdot\mathbf{E}_3\right)\mathbf{I}\right]\cdot\left(\mathbf{R}\cdot\mathbf{E}_3\right)^{\wedge}\cdot\delta\boldsymbol{\theta}dS \tag{30}$$

Space Discretization of the Weak Form

The finite-element discretization in space of the linearized weak form is performed, as in (Simo and Vu-Quoc 1986), using the standard Galerkin method. The incremental displacement field, rotation field and admissible variation are interpolated, on an element basis, using the same interpolation functions, that is

$$\delta\mathbf{u}(S)=\sum_{i=1}^{N}N_i(S)\delta\mathbf{u}_i,\qquad \delta\boldsymbol{\theta}(S)=\sum_{i=1}^{N}N_i(S)\delta\boldsymbol{\theta}_i,\qquad \boldsymbol{\eta}(S)=\sum_{i=1}^{N}N_i(S)\boldsymbol{\eta}_i \tag{31}$$

where N = number of nodes of the element; $N_i(S)$ = element shape function associated with node i; $\delta\mathbf{u}_i$ and $\delta\boldsymbol{\theta}_i$ = incremental displacement and rotation fields at node i; $\boldsymbol{\eta}_i$ = admissible variation at node i. Furthermore, the rotation tensor $\mathbf{R}$ is interpolated as follows:

$$\mathbf{R}(S)=\exp\left[\hat{\boldsymbol{\chi}}(S)\right];\boldsymbol{\chi}(\xi)=\sum_{i=1}^{N}N_i(\xi)\boldsymbol{\chi}_i \tag{32}$$

where $\hat{\boldsymbol{\chi}}$ = skew-symmetric tensor associated with the total rotation vector $\boldsymbol{\chi}$.

Substituting these interpolations into the linearized weak form, leads to the following discrete approximation of the linearized weak form:

$$\sum_{i,j=1}^{N} \boldsymbol{\eta}_i \cdot \left[\mathbf{P}_i\left(\boldsymbol{\varphi}\right) + \mathbf{K}_{ij}\left(\mathbf{R}_n, \hat{\boldsymbol{\Omega}}_n, \boldsymbol{\varphi}\right) \cdot \Delta\boldsymbol{\varphi}_j \right] = 0 \quad \forall \boldsymbol{\eta}_i \tag{33}$$

where $\mathbf{P}_i$ = residual or out-of-balance force; $\Delta\boldsymbol{\varphi}_j$ = incremental displacement and rotational field; the discrete tangent operator $\mathbf{K}_{ij}$ = sum of the material stiffness operator, $\mathbf{S}_{ij}$; the geometric stiffness operator, $\mathbf{G}_{ij}$; the inertia operator, $\mathbf{M}_{ij}$; the damping operator, $\mathbf{D}_{ij}$; the operators associated to the hydrodynamic forces, $\mathbf{FD}_{ij}$ and $\mathbf{FI}_{ij}$; that is

$$\mathbf{K}_{ij} = \mathbf{S}_{ij} + \mathbf{G}_{ij} + \mathbf{M}_{ij} + \mathbf{D}_{ij} + \mathbf{FD}_{ij} + \mathbf{FI}_{ij} \tag{34}$$

Expressions for $\mathbf{S}_{ij}$, $\mathbf{G}_{ij}$, $\mathbf{M}_{ij}$ can be found in (Simo and Vu-Quoc 1988), while the expression for $\mathbf{D}_{ij}$ was derived in (Oliveto and Sivaselvan 2014a; Oliveto 2013). From Eq. (29), the discrete drag force operator takes the form

$$\mathbf{FD}_{ij} = \begin{bmatrix} \mathbf{a}_{ij} & \mathbf{b}_{ij} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{35}$$

with

$$\mathbf{a}_{ij} = \frac{\gamma \cdot \lambda_{drag}}{\beta \cdot h} \int_{I_e} \left[\left\| \mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right\| \mathbf{I} + \frac{\left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right) \otimes \left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right)}{\left\| \mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right\|} \right] \cdot \left[\mathbf{I} - \left(\mathbf{R}\cdot\mathbf{E}_3\right) \otimes \left(\mathbf{R}\cdot\mathbf{E}_3\right) \right] N_i N_j dS \tag{36}$$

$$\mathbf{b}_{ij} = \lambda_{drag} \int_{I_e} \left[\left\| \mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right\| \mathbf{I} + \frac{\left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right) \otimes \left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp}\right)}{\left\| \mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right\|} \right] \cdot \left[\left(\mathbf{R}\cdot\mathbf{E_3}\right) \otimes \left(\mathbf{v}_0 - \mathbf{v}_w\right) + \left(\mathbf{v}_0 - \mathbf{v}_w\right) \cdot \left(\mathbf{R}\cdot\mathbf{E}_3\right) \mathbf{I} \right] \cdot \left(\mathbf{R}\cdot\mathbf{E}_3\right)^{\wedge} N_i N_j dS \tag{37}$$

Moreover, from Eq. (30), the discrete added mass operator may be written as

$$\mathbf{FI}_{ij} = \begin{bmatrix} \mathbf{c}_{ij} & \mathbf{d}_{ij} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{38}$$

with

$$\mathbf{c}_{ij} = \frac{\lambda_{inertia}}{\beta \cdot h^2} \int_{I_e} \left[\mathbf{I} - \left(\mathbf{R} \cdot \mathbf{E}_3 \right) \otimes \left(\mathbf{R} \cdot \mathbf{E}_3 \right) \right] N_i N_j dS \tag{39}$$

$$\mathbf{d}_{ij} = \lambda_{inertia} \int_{I_e} \left[\left(\mathbf{R} \cdot \mathbf{E}_3 \right) \otimes \left(\mathbf{a}_0 - \mathbf{a}_w \right) + \left(\mathbf{a}_0 - \mathbf{a}_w \right) \cdot \left(\mathbf{R} \cdot \mathbf{E}_3 \right) \mathbf{I} \right] \cdot \left(\mathbf{R} \cdot \mathbf{E}_3 \right)^{\wedge} N_i N_j dS \tag{40}$$

Finally, from equation (14), the discrete unbalanced force is given by

$$\begin{aligned} \mathbf{P}_i = \int_{I_e} \Bigg\{ & \mathbf{\Xi}_i \cdot \begin{bmatrix} \mathbf{n} \\ \mathbf{m} \end{bmatrix} - N_i \begin{bmatrix} \tilde{\mathbf{n}} \\ \tilde{\mathbf{m}} \end{bmatrix} + N_i \begin{bmatrix} A_\rho \mathbf{a}_0 \\ \mathbf{R} \cdot \left[\mathbf{J}_p \cdot \dot{\mathbf{W}} + \mathbf{W} \times \left(\mathbf{J}_p \cdot \mathbf{W} \right) \right] \end{bmatrix} + \\ & \lambda_{drag} N_i \begin{bmatrix} \left\| \mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right\| \left(\mathbf{v}_{0\perp} - \mathbf{v}_{w\perp} \right) \\ 0 \end{bmatrix} + \lambda_{inertia} N_i \begin{bmatrix} \mathbf{a}_{0\perp} - \mathbf{a}_{w\perp} \\ 0 \end{bmatrix} \Bigg\} dS \end{aligned} \tag{41}$$

where

$$\mathbf{\Xi}_i = \begin{bmatrix} N_i^{'} \mathbf{I} & \mathbf{0} \\ -N_i \left[\dfrac{\partial \mathbf{x}_0}{\partial S} \times \right] & N_i^{'} \mathbf{I} \end{bmatrix}$$

Numerical examples

A series of numerical simulations are carried out to assess the performance of the formulation described above. A first example involves the free vibration of a cantilever beam in water. The goal is to compare the Morison approach with fully coupled FSI analysis carried out using COMSOL (COMSOL Inc. 2013b). Having verified our formulation, in a second example, we analyze the behavior of a realistic mooring cable subjected to typical wind turbine loads. Convergence rates and energy balance calculations are presented for each example to illustrate the performance of the computations.

Free vibration of cantilever in water

The first numerical example consists of statically applying and then instantaneously releasing a 5 cm vertical displacement at the free end of a cantilever beam immersed in water, which is initially at rest. The beam considered is cylindrical and characterized by the following parameters: length, L = 30 cm; diameter, D = 2 cm; Young's modulus, E = 1 MPa; Poisson's ratio υ = 0.3; and mass density, ρ = 1000 kg/m^3.

Analysis using proposed formulation

The beam was discretized in space using 60 two-noded (linear) elements. Reduced (one-point) Gaussian integration was used for the evaluation of the internal force vector, the fluid-beam force vectors, the material and geometric stiffness matrices, and the fluid-beam matrices, while two-point Gaussian integration was used for the inertial force vector and the inertia matrix. The parameters used in the time integration scheme were β=0.25 and γ=0.5.

Two analyses were performed, one with no fluid-beam interaction ($C_D = C_M = 0$) and the other using $C_D = 3.0$ and $C_M = 1.5$. The choice of these parameters is based on recommendations in literature and so as to obtain the best match with the results of a fully coupled fluid-structure interaction analysis presented in the following section. Furthermore, we demonstrate that these are consistent with values we can extract from fully coupled FSI analyses. The time step used was h = 0.002 s. Note that no viscous damping was considered in the analyses so that damping is entirely due to fluid-beam interaction.

The vertical displacement history of the free end of the beam is plotted in Fig. 3 for the two considered cases. The figure clearly shows the decay of motion due to the drag force and, as expected, a period elongation due to the added mass.

The rate of convergence of Newton's method is given for several time increments in Table 1, where the norm of the unbalanced force vector P_i at each iteration is listed. The reliability of calculations was also assessed by verifying the energy balance. The sum of strain energy, kinetic energy, drag energy and added mass energy should be constant and equal to the initial strain energy prior to release. Fig. 4(a) and (b) show the energy components for the two analyses considered. The energy error (Fig. 5) was in both cases smaller than 2.5×10^{-4}.

Fully coupled fluid-structure interaction analysis

In order to verify that the effects of the fluid on the motion of the beam are captured correctly by the proposed formulation based on the Morison approach, the free vibration problem was also solved using COMSOL (COMSOL Inc. 2013b). The Fluid-Structure Interaction interface in COMSOL combines fluid flow with solid mechanics to capture the interaction between the fluid and the solid structure. The fluid flow is described by the Navier-Stokes equations (COMSOL Inc. 2013c).

The 3D model of the beam in water defined in COMSOL is shown in Fig. 6. The properties of the fluid, modeled by a 0.5 m x 0.5 m x 0.7 m square box surrounding the beam, were density=1000 kg/m^3 and dynamic viscosity=0.001 Pa s. The beam was characterized by no additional damping. An *open boundary* condition was selected for the fluid walls, meaning that fluid can both enter and leave the boundaries of the domain shown in Fig. 6.

A Backward Differentiation Formula (BDF) (COMSOL Inc. 2013a) time integration scheme was used in the analysis, with the same time step used for the proposed formulation, namely 0.002 s. The vertical displacement of the free end of the beam is shown in Fig. 7, where it is compared to the displacement history obtained using the proposed formulation with $C_D = 3.0$ and $C_M = 1.5$. The results are in good agreement, considering that they are based on different models.

Evaluation of drag and added mass coefficients from fully coupled FSI analysis

The values of C_M and C_D were selected based on the following calculations carried out using the results from COMSOL. Assuming small displacements, the total force at the fluid-beam interface, acting orthogonally to the undeformed cable, can be evaluated as:

$$F = -\int_0^L 0.5\rho D C_D \operatorname{sgn}\left(v\left(S\right)\right) v^2\left(S\right) dS - \int_0^L 0.25 C_M \rho_w \pi D^2 a\left(S\right) dS \quad (42)$$

where $v\left(S\right)$ and $a\left(S\right)$ = velocity and acceleration of the beam in the direction orthogonal to the undeformed beam.

The values of C_M and C_D can be then evaluated from the following relationships:

$$C_D = -\frac{F}{0.5\rho D \int_0^L \operatorname{sgn}\left(v\left(S\right)\right) v^2\left(S\right) dS} \quad (43)$$

$$C_M = -\frac{F}{0.25\rho_w \pi D^2 \int_0^L a\left(S\right) dS} \quad (44)$$

Note that Eq. (43) is obtained by neglecting the second term in Eq. (42) is neglected, whereas Eq. (44) is obtained when the first term in Eq. (42) is set to zero (Frigaard and Burcharth 1989).

The drag (C_D) and added mass (C_M) coefficients, obtained by Eqs. (43) and (44), are plotted as a function of time in Fig. 8. The values of interest are indicated in the figures by black dots. The added mass coefficient C_M is seen to be in the range 1.3-1.7, justifying the use of a constant value of 1.5 throughout the analysis. The drag coefficient C_D appears to be in the range 1.3-2.1 in the first half cycle of the response ($t < 0.5\,\text{sec}$), and in the range 2.6-4.5 for the remaining part of the analysis, thus confirming dependence of the drag coefficient on the Reynolds number and, consequently, on the amplitude of the velocity of motion. This variability of the drag coefficient explains the differences in Fig. 7 between the response obtained by COMSOL and that of the proposed formulation, where a constant value of 3.0 was used for the drag coefficient C_D.

In Fig. 9, the force at the fluid-beam interface given by COMSOL is compared to that obtained with the proposed formulation and the agreement is satisfactory.

Dynamic behavior of a mooring cable

The following example deals with the analysis of a mooring cable of a typical floating offshore wind turbine. The material and geometric properties of the cable were taken from Jonkman (2010) as follows: length, L=902.2 m; diameter, d=0.09 m. The mass per unit length was 77.71 kg/m, the weight in water was 690 N/m, and the equivalent extensional rigidity was EA=384243 kN. The initial configuration of the cable, shown in Fig. 10, was obtained by first imposing horizontal and vertical displacements at the right end of an initially straight and unstrained cable, and then subjecting it to its own weight. The imposed horizontal distance between the two supports of the cable was 848.67 m, whereas the vertical distance was 250 m.

Starting from this configuration, the right end of the cable, ideally connected to a floating offshore wind turbine, was subjected to an in-plane horizontal excitation (Fig. 11) representative of the motion of the platform of the NREL 5 MW - OC3 Hywind reference turbine (Jonkman et al. 2009), evaluated through the use of the software FAST (Jonkman and Buhl 2007).

Two analyses were performed, one with no fluid-cable interaction ($C_D = C_M = 0$) and the other using $C_D = 1.5$ and $C_M = 0.5$. The latter coefficients were selected based on typical values assumed in other studies (Yang et al. 2013; Hall et al. 2013). The cable was discretized in space with 40 two-noded (linear) elements. As in the previous example, reduced (one-point)

Gaussian integration was used for the evaluation of the internal force vector, the fluid-beam force vectors, the material and geometric stiffness matrices, and the fluid-beam matrices, while two-point Gaussian integration was used for the inertial force vector and the inertia matrix. The time step used in the analyses was h=0.0125 s. Again, no viscous damping was considered in the analyses to isolate the influence of fluid-cable interaction on the response of the cable.

The response in terms of displacements and axial force at midspan ("investigated point" in Fig. 10) are shown in Fig. 12 and Fig. 13. The damping effect of the drag force is clearly visible both in the displacement and axial force time-histories.

The rate of convergence of Newton's method in each analysis is given, for several time steps, in Table 2. The reliability of computations was again assessed in terms of energy balance. The energy components for the two analyses considered are shown in Fig. 14, while the energy error, defined as the difference between the input energy and the sum of the different energy components, is plotted in Fig. 15.

Concluding remarks

A nonlinear finite element formulation has been developed and applied to the dynamic analysis of mooring cables used in floating offshore wind turbines. Fluid-cable interaction was introduced in the formulation using the Morison approach. Two numerical examples have been presented. In a first example, the Morison approach is compared with fully coupled fluid-structure interaction analysis carried out in COMSOL. While generally based on empirical data, it is demonstrated in the present work that the hydrodynamic coefficients can be obtained from fully coupled FSI analysis. In the second example, the dynamic behavior of a mooring cable typically used for floating offshore wind turbines is analyzed. Energy balance plots, as well as convergence rates of Newton's method, illustrate the reliability of computations. It should be noted that a key and non-trivial aspect in the proposed formulation is the development of an algorithmic tangent operator including hydrodynamic coupling. Current and future work involve the inclusion of the cable model in a platform for the full analysis of floating offshore wind turbines, and subsequent model validation efforts. Source code for all developments in the present paper is provided as online supplemental material.

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Figure captions list

Fig. 1. Orthogonal plane $\mathcal{N}$, and normal components of water and cable velocities.

Fig. 2. Fixed and moving coordinate systems of beam in reference and current configuration.

Fig. 3. Tip vertical displacement with and without fluid-beam interaction.

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Fig. 6. Cantilever beam model in COMSOL.

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Fig. 15. Energy error for the analyzed cable (a) without and (b) with fluid-structure interaction.

Table 1. Convergence rate of Newton's method. Norm of residual (out-of-balance force) throughout iteration process.

	fluid-beam interaction			no fluid-beam interaction		
Iteration	t=1.000 sec	t=2.500 sec	t=4.000 sec	t=1.000 sec	t=2.500 sec	t=4.000 sec
1	7.00×10^{-1}	1.69×10^{-1}	3.33×10^{-1}	3.63×10^{-1}	6.77×10^{-1}	7.97×10^{-1}
2	6.02×10^{-4}	2.15×10^{-5}	1.36×10^{-4}	3.85×10^{-4}	1.31×10^{-3}	1.81×10^{-3}
3	9.28×10^{-10}	1.07×10^{-11}	4.71×10^{-11}	6.79×10^{-10}	3.24×10^{-9}	3.68×10^{-9}

Table 2. Convergence rate of Newton's method. Norm of residual (out-of-balance force) throughout iteration process.

	fluid-cable interaction			no fluid-cable interaction		
Iteration	t=10.00 sec	t=30.00 sec	t=50.00 sec	t=10.00 sec	t=30.00 sec	t=50.00 sec
1	1.63×10^{6}	3.92×10^{6}	2.38×10^{6}	1.57×10^{6}	4.39×10^{6}	2.54×10^{6}
2	3.87×10^{3}	2.27×10^{4}	8.72×10^{3}	5.00×10^{1}	1.97×10^{2}	4.80×10^{1}
3	5.95×10^{-2}	1.97×10^{0}	2.87×10^{-1}	1.03×10^{-5}	9.38×10^{-6}	7.94×10^{-6}
4	8.11×10^{-6}	7.54×10^{-6}	7.36×10^{-6}	5.15×10^{-6}		