Curricular Orientations to Real-World Contexts in Mathematics

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A common claim about mathematics education is that it should equip students to use mathematics in the 'real world'. In this article, we examine how relationships between mathematics education and the real world are materialised in the curriculum across a sample of eleven jurisdictions. In particular, we address the orientation of the curriculum towards application of mathematics, the ways that real-world contexts are positioned within the curriculum content, the ways in which different groups of students are expected to engage with real-world contexts, and the extent to which high-stakes assessments include real-world problem solving. The analysis reveals variation across jurisdictions and some lack of coherence between official orientations towards use of mathematics in the real world and the ways that this is materialised in the organisation of the content for students.

Keywords: mathematics; real-world contexts; modelling; curriculum; International Instructional System Study; Center for International Education Benchmarking.

Introduction

Education systems around the world tend to value mathematics as a school subject that is not only studied by a large majority of students throughout the years of compulsory schooling but is also widely used as a key indicator of the success of individual students (as a necessary or highly desirable component of school leaving qualifications) and of education systems themselves (as measured by international testing regimes such as PISA and TIMSS). Various reasons may be offered for the importance attached to mathematics as a part of the school curriculum; among these is the claim that mathematics has a functional role in relation to participation in the 'real world' beyond the mathematics classroom. The question of the nature of the relationship between school mathematics and the real world arose for us during a review of curriculum documents for primary and secondary mathematics from eleven jurisdictions, carried out as part of the International Instructional Systems Study (see Creese, Gonzalez & Isaacs, this issue). As we reviewed the aims, content and assessment materials from the various jurisdictions, we began to notice differences in the ways that applications of mathematics in the real world were conceptualised, integrated into the mathematical content and materialised in assessment instruments. In this article we attempt to map out this variation across the jurisdictions studied and raise questions about the coherence of the conceptualisation and instrumentation of the mathematics – real world relationship within individual jurisdictions.

Background

The function of mathematics within the curriculum has long been a source of difference and debate. Opinions range from seeing the study of mathematics as introducing students to part of their cultural heritage, similar to the study of literature or history, to considering mathematics as mental training for the elite or the rational human, or primarily as a tool to enable students to engage successfully in study of other subjects, in their 'everyday' life and in future employment. These different orientations towards the subject have different implications for curriculum content and pedagogy (Ernest, 1991; Huckstep, 2000; Keitel 2006). They can also lead to different curricula for groups of students who are perceived to have different future needs. In spite of these debates about its function, mathematics occupies a privileged position within curricula around the world. It tends to be accorded a relatively large amount of time and is generally compulsory for all students at least to the end of lower secondary school. At upper secondary level, even where mathematics is optional, there is a high level of participation in many countries. There have been voices that question the justification for this privileged position. For example, in the context of the UK, where there is a strongly utilitarian curriculum discourse, Brammall and White have argued that most students will have learnt all the mathematics they will ever need by the age of eleven (Bramall & White, 2000). Nevertheless, the importance of mathematics is both reflected and reinforced by national policies and, in particular, by the influence of international testing regimes such as PISA and TIMSS. The results of such tests, especially the ranking of countries by test outcomes, have been used in many countries to fuel policy debates and to focus attention and investment in mathematics education (Pons, 2012).

These two major testing regimes occupy rather different positions with respect to the function of mathematics. Whereas TIMSS defines its scope according to the mathematical content of curricula, PISA is concerned with the extent to which education systems prepare young people for participation in adult life, in particular equipping them with 'mathematical literacy', defined as:

> An individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2013, p.17)

Such capacity to use mathematics is portrayed not only as a benefit to individuals, enabling them to be competent and successful in their everyday activity, but also as a benefit to society as a whole by producing citizens who are capable of contributing to the economic, technological and political activity of OECD member states. Competition among states in the PISA rankings may thus be seen as a surrogate for competition in the economic world.

Although PISA purports to address 'mathematical literacy' rather than 'mathematics', recognising that its focus on application of mathematics does not comprehensively cover the full scope of school mathematics curricula, its assessment framework as well as its results have had a strong influence on policy, curriculum and research in mathematics education (Kanes, Morgan, & Tsatsaroni, 2014). This framework, which classifies assessment tasks according to their relationship to students' real world experience (personal, societal, scientific or educational, occupational), clearly positions the mathematical knowledge and skills assessed by PISA tasks as 'useful' within a wide range of aspects of students' current and future lives.

The relationship between school mathematics and the real world is, however, more complicated than may be suggested by OECD. Indeed, the idea that school mathematics is used in the real world and that the contextualised tasks that students experience in the mathematics classroom are similar to participation in 'real' activities has been questioned. A substantial body of research into how people use mathematics in their everyday lives and in a wide variety of occupations suggests that much of this activity does not closely resemble the techniques taught in school and that some apparently mathematical situations in everyday life may be addressed without using mathematics at all (Hoyles, Noss, & Pozzi, 2002; Lave & Wenger, 1991; Nunes, Schliemann, & Carraher, 1993). From the reverse perspective, researchers in mathematics education have pointed to differences between the kind of activity and solutions expected of students engaging with contextualised school tasks and the activities and solutions that would be valued outside of school mathematics (Cooper & Dunne, 2000; Gellert & Jablonka, 2009; Kanes et al., 2014) and have even challenged the common assumption that school mathematics has any relationship to everyday practices (Dowling, 1998).

Nevertheless, in textbooks, classrooms and assessments across the world we find mathematical tasks that involve a non-mathematical context. Such tasks differ in the extent to which the context relates to a 'real' application of mathematics, that is, in how closely the activity demanded of the student resembles that of a participant in the non-mathematical context that is represented. European curricula have traditionally included the genre of ‘word problem’ that tends to be a thinly disguised exercise in translation from words into symbols, followed by implementation of a familiar routine (e.g. John has three more sweets than Jane. Jane has six sweets. How many sweets does John have?) (Sutherland, 2002). Such problems are not found outside the mathematics classroom; indeed they refer primarily and ritually to educational traditions rather than to other realities (Gerofsky, 1996). They demand little or no engagement with features of the context and students often approach them by homing in on key words (such as ‘more’) in order to decide what operation to use (Nesher & Teubal, 1975). Less routine tasks may involve students in making complex decisions about what mathematics to use and which aspects of the context to take into account. At the other extreme, there is some interest among mathematics educators in the use of mathematical modelling in schools (Galbraith, Henn, & Niss, 2007; Stillman, Blum, & Salett Biembengut, 2015). Modelling is an activity engaged in by ‘professional’ mathematicians, constructing a mathematical description of a real situation in order to help address a real world problem (e.g. population change, stresses on buildings, traffic flow, spread of diseases, setting insurance premiums). It involves making assumptions and approximations in order to simplify extremely complex situations as well as, crucially, testing solutions against reality in order to decide whether they are adequate. These characteristics are rarely found in school mathematics tasks, which are generally well defined and self-contained. Research into school applications of mathematical modelling has often explored tensions between student expectations, system requirements and teaching aims (e.g. Lowrie, 2011). Contextualised tasks also differ in their role within the didactic context: as an application of mathematical knowledge and skills that have already been learnt; as a means of supporting the development of problem solving skills; or as a vehicle for enabling students to learn new mathematics by relating it to their other experience. The last role, using non-mathematical experience as a means of making sense of mathematical concepts, lies at the heart of Realistic Mathematics Education, developed in the Netherlands and based on the theoretical principles of Hans Freudenthal (Van den Heuvel-Panhuizen & Drijvers, 2014). Contextualising mathematics is also widely seen as a means of motivating students, by reducing its abstractness, by relating to student interests, and by showing them that mathematics is useful in the world outside the classroom and may have relevance for their current and future lives.

The term ‘problem solving’ is widely found in mathematics curriculum documents, often in association with ‘real world’. It is important to note, however, that the scope of mathematical problem solving is wider, including working with purely mathematical problems as well as those that involve extra-mathematical scenarios. The meaning of the term is also contentious: ‘problem’ is commonly used as a synonym for ‘task’, but researchers in the field of mathematical problem solving tend to reserve the term for tasks for which the solution path is unknown to the solver (Schoenfeld, 1985). The PISA 2012 framework relates problem solving to real-world contexts through the processes of formulating situations mathematically and interpreting mathematical outcomes, distinguishing these from employing mathematical techniques.

Previous research studies have used the relationship between mathematics and the real world as a means of analysing curricula. Wu and Zhang (2006) highlight a new consensus that “mathematics is now presented in the same light as other sciences that seek to understand the real world” (p.192) but treat problem solving and modelling together. Li and Ginsburg (2006) compared instances of mathematical and non-mathematical contexts in US and Asian mathematics textbooks and found strong classification within all Asian textbooks and the US algebra textbooks but weaker classification, with more non-mathematical examples, in US non-algebra texts; and they relate this to differences in cultural traditions of authority. In this article we take this issue further by mapping variation not only in the extent of use of non-mathematical contexts but also in the ways in which these feature in the discursive construction of school mathematics.

Methods

This paper emerges from the research project International Instructional Systems Study which sought to delineate the curricula of eleven jurisdictions selected as attracting international attention through high PISA or TIMMS scores (in mathematics, science and or literacy): Australia (New South Wales and Queensland), Canada (Alberta and Ontario), Finland, Japan, Singapore, Hong Kong, Shanghai and the United States (Florida and Massachusetts). While some of these jurisdictions rank very highly across the subjects, others are less exceptional in mathematics. In particular, Florida was chosen in order to include a US state with performance contrasting to that of Massachusetts. The overarching project produced subject reports in each of nine curriculum areas for each jurisdiction, organising the reports under the headings: orientation, coherence and clarity, scope, levels of demand, progression, assessment, and key competencies (see Creese, Gonzalez and Isaacs, this issue, for the definition of these terms and for discussion of the aims and achievements of the wider project).

Our data selection and analytical aims were framed by the wider project aims and structures but as subject experts we also had the remit to focus on features of the curricula that struck us as significant or unusual from a mathematics education perspective. The issue of how the eleven jurisdictions related to mathematics in real-life contexts was one of these foci, chosen both for its importance within international mathematics education discourse and because it was immediately apparent as a feature of curriculum discourse in many of the eleven jurisdictions. It has been noted (e.g. Schmidt, 2013) that there is a trend, or at least an aspiration, in the PISA countries' curricula towards more integration of mathematical modelling with content, and towards a greater variety of contexts. In addressing our focus on real-world contexts in mathematics, we sought to characterise differences in how they are brought into curriculum texts. We have therefore examined: how these jurisdictions articulate and operationalise their aims; how variations between and within jurisdictions reflect debates in the research literature about the purpose of engaging students with real-world examples; what counts as engagement with mathematics in the real world; the curricular emphasis to be given to applications of mathematics; and the pedagogic language used to frame these as mathematical experiences.

Data collection was carried out in a similar way to previous comparative studies in mathematics (e.g. Hodgen, Marks, & Pepper, 2013; Norris, 2012; Sutherland, 2002), though on a larger scale. A core team amassed a database of policy and curriculum documents and sample assessment materials through web-based research. We cannot claim that the documents collected in this way provide a complete picture of the curriculum in each of the jurisdictions. The extent of the information available to us also varies between jurisdictions. Nevertheless, the database of materials can be said to represent the public face of the curriculum in each case.

One team of 'country' researchers used this data to understand the education system and the curriculum offer of each jurisdiction and to produce descriptive summaries of educational context. Another team, including ourselves, focused on subject curricula and assessment materials; our understanding of these materials was informed by the descriptive summaries for each jurisdiction. Two distinct phases of subject analysis were planned to allow consultation between the teams and with external country experts, thus strengthening internal coherence and surface validity.

Phase 1: Production of jurisdiction reports summarising the features of each mathematics curriculum for the primary and secondary grades.

The jurisdiction reports were mainly descriptive of the curricula; the selection of descriptive content and its organisation was influenced by the seven project framework categories listed above. The analytic content of these reports was a commentary framed by questions based on our curricular knowledge and experience as mathematics educators: were the scope, key skills and assessment coherent with the orientation? Were they coherent with each other? Did the detail of the curriculum appear usable by teachers? Was any curriculum notably unusual in approach? In this paper we draw mainly on four of the project categories: orientation, scope, key competence skills, and assessment.

One particular issue in compiling the jurisdiction reports was the availability and reliability of English-language texts. For both Shanghai and Japan, where no translation was available for parts of the curriculum, we consulted with native speaker consultants in order to interrogate the documents. These consultants were PhD students studying at our institution. It proved important that they had both education in mathematics at upper secondary level and recent educational experience (as students or teachers) within the jurisdiction, both to deal with the mathematical vocabulary in the documents and to help us to understand the context. For example, our Shanghai consultant pointed out that the commitment throughout the mathematics curriculum to ‘Research and Practice’ is delivered through a weekly, afternoon combined studies lesson rather than in morning mathematics lessons. Consultants made initial translations of document headings, and then took part in repeated and extended face-to-face discussions with one of us, focussing on the detailed wording of the curriculum documents. The outcomes of these discussions were cross-referenced with available English-language sources and literature related to the mathematics curriculum (e.g., for Shanghai: Lim, 2007; National College for Teaching and Leadership, 2014; Wang et al., 2012).

Phase 2: Production of a cross-jurisdiction analysis comparing the two US curricula to the nine non-US curricula within the project framework.

For mathematics, the move to compare curricula required a new analytic approach. The jurisdiction reports had captured and evidenced characteristics of each curriculum but had not attempted a holistic mapping or categorisation. For this second phase, a new set of questions was developed and applied to the jurisdiction reports, referring back to the original documents only if necessary. These questions focused on specific aspects of the curriculum mathematics content or organisation chosen by us either as a known indicator of curricular difference or as an unusual feature observed during the first phase. An example of a familiar indicator is the structure of differentiated pathways in the curriculum that have different mathematical content. Hodgen, Marks, and Pepper (2013) surveyed 24 countries and reported on the proportions of upper secondary students taking ‘basic’ or ‘advanced’ mathematics courses. Participation in ‘any’ mathematics varied from under 20% (e.g. in England, not one of our target jurisdictions) to over 95% (including Japan and Finland from our project). Participation in ‘advanced’ mathematics, defined as courses aiming to prepare students for mathematically demanding tertiary study, varied from under 15% to over 30% (including Japan and Singapore). In this case, our new question developed at phase 2 was ‘what different pathways are there and how do they operationalise the curriculum aims?’

An example of an unusual feature that we chose to investigate further was the way that historical or cultural aspects of mathematics were featured in the curriculum documents. Our interest in this arose from the specification in Shanghai that students from grades 1-2 onwards should meet topics from the history of mathematics. This requirement continues to grades 10-12 where all students aiming to qualify for university must study at least one credit on the theme of ‘historical and modern mathematics’.

Phase 3: Analysis across jurisdictions of the relationship between mathematics and real-life contexts.

A third phase of analysis has drawn together strands from the jurisdiction descriptions and the cross-jurisdiction analysis, focusing explicitly on how the curricula present the relationship between mathematics and real-life contexts. We selected sections of the reports of the previous two phases that appeared related to our theme. These were then used to address the following key questions:

> How do these jurisdictions conceptualise relationships between mathematics and the ‘real world’? (Orientation)
>
> How is student experience of applications of mathematics organised within the curriculum? (Curricular Organisation)
>
> Do orientation to the real world and the organisation of student experience vary for different groups of students? (Curricular Differentiation)
>
> How do applications of mathematics feature in assessment materials? (Assessment)

We address each of these questions separately, then conclude with reflections on coherence within jurisdictions and variation between them.

Orientation

Throughout this study we conceptualised curricula as 'artifacts of intention' (Schmidt et al, 1996, p. 25), articulating the vision, aims, goals and rationale for studying mathematics for different groups of pupils and proposing a structure to guide the complexity of classroom practice . We have considered these articulations as presenting the orientation of each curriculum. The trend towards using international comparisons as part of curriculum reform (Lee, 1998) means that these orientations have shared elements, one of which is the aim that students should be able to connect mathematics to real-world contexts. Despite the almost universal agreement that such connections are desirable, we see different approaches between the jurisdictions in how they emphasise them and what purpose they give them. Here we outline three main approaches and exemplify them. In subsequent sections we will consider – still drawing on the curriculum texts – how these orientations are operationalised within guiding structures.

Pupils should use Mathematics as a tool for everyday life

The curricula differ in how explicitly they state that schools should teach pupils to use mathematics in real-world contexts and, given the broad consensus noted above, this surprised us. No curriculum had a total absence, but in some cases references to the real world were left implicit. One way this could happen was in the wider intention that students should apply mathematics in 'a range of contexts': as we will see, this intention may have a limited interpretation of solving problems expressed in words or in new mathematical contexts. Similarly, the skills involved in modelling a real-world problem could be subsumed in a commitment to 'problem solving' that does not specify the source or referents of the problem. The 2006 Singapore curriculum has been an example of this: although the ‘Mathematical Problem Solving’ pentagon was prominent in the text and understood as important by teachers (Wu and Zhang, 2006), there are few references to developing real-world applications within the list of mathematical content. Singapore’s revised curriculum, now being implemented across lower secondary from 2014, has much more explicit treatment of connections and modelling both in the introduction and the listed learning experiences.

Some curricula subscribe closely to the PISA view that students should be able to use mathematics to solve problems drawn from their personal lives and possible future employment, and problems drawn from technological and scientific contexts. Ontario is an example where this is at the heart of the curriculum, starting from its introduction: ‘Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations.’ As in the other Canadian jurisdiction, Alberta, the curriculum identifies seven process expectations amongst which it distinguishes between problem solving - ‘the primary focus and goal of mathematics in the real world’ – and connecting – important for grasping general principles and appreciating ‘the role of mathematics in human affairs’. Connecting appears again in titles of curriculum strands, and the majority of content expectations are accompanied by a sample problem in a real-life context. This extends to topics such as quadratic relations. Elsewhere these are treated as algebraic and graphic, but in Ontario’s grade 10 Academic course the topic strand is headed by ‘collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada)’. Assessment follows the same emphasis, with teachers reporting on four equal categories of which the last involves application of knowledge to familiar and new contexts. Canadian students perform very well on PISA tests, and Ontario is close to their average. In particular, they are strong on interpreting mathematics in context (Brochu et al, 2014).

The real world as a vehicle for learning Mathematics

A second orientation within these curricula treats the real world as the origin of mathematical concepts, and interactions with context as essential to learning. Both Canadian jurisdictions refer to a constructivist pedagogy in which the environment provokes curiosity and frames learning. Finland has the strongest emphasis on this orientation. The importance accorded to mathematics in the Finnish curriculum is related to the individual development of the student: their intellectual growth and their participation in problem solving and social interaction. The development of mathematical thinking is highlighted and seen as a means of developing creativity and precision. The use of mathematics as a tool in solving everyday problems is mentioned and, in grades 6-9, developing capability in modelling everyday problems is identified as a core task of mathematics instruction. However, application of mathematics to real world problems is presented throughout secondary education primarily as a vehicle for developing mathematical thinking and concepts and as a means of inspiring and encouraging independent experimentation and investigation rather than as an aim in itself.

The real world as a motivator for learning Mathematics.

In some jurisdictions, real life applications are not presented as a goal of learning mathematics in school, nor as a necessary part of learning. Instead they provide a motivation for learning school mathematics, which may otherwise be entirely symbolic. One example is Japan, where the curriculum centres on developing a ‘zest for life’ and there has been a recent concern that insufficient students pursue STEM subjects past compulsory education. Similarly, Shanghai's 'National Curriculum Standard' aims to provide students with a good foundation of mathematical skills that they can apply to everyday life, to stimulate their interests and enlighten their thought and at the same time develop positive attitudes towards mathematics. Wang et al. (2012) report that this last goal was given a new emphasis in the 2004 reforms that aimed to present students as active learners. Enjoyment is treated as stemming from students' appreciation that mathematics is a part of human culture, relating it to everyday life, social and cultural development.

In a different approach to motivation, several jurisdictions warn that mathematics is a necessity for economic competition in a newly technological world. In Singapore, Florida, Massachusetts and Hong Kong, the motivation for students to succeed in mathematics is to take part in an increasingly competitive and technological workforce. Leaton Gray (2004) notes the prevalence of a 'youth as future' rhetoric in education policy documents. Here it is combined with a discourse of mathematics as controlling the risks that accelerated economic progress opens up in social relations and personal lives. The curricular tension between re-expressing established national cultural norms and forward-planning has been highlighted by Unesco: 'there is the need not to be sidelined by the changes gathering speed worldwide and nationally that have an effect on the economy, the labour market, business, finance, social relations, and communications' (Amadio, Tedesco & Opertti, 2015, p.6). Mathematics is discursively harnessed by such documents, combining traditional, grounded, back-to-basics common sense with the abstract, flexible, strategic skills essential for 21-st century living.

Curricular Organisation

Curriculum frameworks are multi-layered, including cross-cutting key competencies that can be instantiated across traditional subject areas and transversal themes that integrate disciplines. There is variety in how national education systems organise these layers (Tedesco, Opertti & Amadio, 2013). Pepper (2011), reporting on a European Commission curriculum survey, notes that problem solving appears within the definition of each of the key competencies identified, cutting across all subject disciplines, yet in several jurisdictions it is articulated and assessed primarily within mathematics. Hence the question of how problem solving is organised within mathematics becomes significant. The core of all the mathematics curricula in our study is a specification of learning experiences or activities within traditional mathematical content domains such as number, algebra and geometry. However, focussing on the aim of relating mathematics to real-life contexts, we note a variety of modes of organisation. We characterise these modes of organisation as an expectation that permeates all mathematics content areas, as one that cross-references contexts with content, or as local containment within a specialised unit, or one or two content areas.

Permeation

As discussed above, the mathematical content of the Canadian jurisdictions is permeated by an expectation that mathematics will be used in a real-world context. The majority of content expectations in the grades 1-8 Ontario documents are accompanied by a sample problem that outlines a word problem, investigation of a scientific or consumer context or problem related to concrete objects. Investigation, problem solving and connection between representations are also recurrent themes throughout grades 9-12. In addition some content expectations are in themselves stated as real world problems. For example, investigating optimal values of measurements is an important topic across grade 9, and solving problems involving financial applications in the topic 'Functions' in grade 11. These features of the Ontario curriculum make it particularly coherent in relating content to process skills and to overall curricular goals. This coherence extends beyond mathematics, as one of the four categories generic to assessing achievement in all subjects is 'Application: use of knowledge and skills to make connections within/between different contexts'.

Cross-referencing

Cross-referencing is the most frequent organisational strategy in another group of jurisdictions. The use of real-world contexts is set out within one or more of the process skills, usually under a title such as 'connecting', 'modelling' or 'problem solving'. These skills are intended to cross-cut the curriculum as interacting dimensions, so that students might meet examples of modelling travel with triangles, or making connections between steady growth and linear equations. There is, however, potential variation in how these dimensions interact, for example whether it is envisaged that *any* process can be experienced while teaching *any* content, or that some interactions are recommended for mathematical or pedagogic reasons.

We noted considerable variation in the detail of guidance given in the cross-referencing curricula. In Japan, applying mathematics in everyday situations is stated as an intended mathematical activity across each of the grades 1-9, and in Hong Kong from grades 4-6, but in each case with no further penetration into the text. Queensland's document specifies two dimensions of *content* strands that describe 'what' is to be taught and learnt and *proficiency* strands that describe 'how' content is explored or developed. For primary and lower secondary, there is little detail within the content listing of how the interactions are conceived; in contrast, the upper-secondary courses elaborate their content lists with a comprehensive, cross-referenced set of suggested learning experiences. There is also a wealth of official exemplification online[1], including a suggested grade 8 unit on Mathematical Design in which students produce optimal and efficient designs for various real-world situations, including problems involving several content areas and assessment of modelling skills. The relationship is formalised in the 2-way table format for recording students' achievement. Singapore's new curriculum also uses the heading of 'learning experiences' in order to specify scientific and consumer applications of mathematical content, described as 'integral parts of learning that topic'. Finally, Massachusetts' upper secondary curriculum lists modelling as a conceptual category alongside content categories. Rather than list modelling expectations as such, it then identifies with a * content standards that have a modelling component. Nearly a third of the standards are connected to modelling at grades 9-12, across the main strands of number, functions and data. For example, 'Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* *'. Although this is a simple system it communicates the prominence of modelling: it is the only one of the eight US standards of mathematical practice to be featured in this way. Massachusetts' state assessments up to grade 10 also emphasise modelling with mathematics, including skills of interpreting representations of data, making sense of worded problems that model functional and geometric contexts, and estimating numerical answers from inequalities, calculations and diagrams.

Units of Work

Finally there are jurisdictions that base their approach to real-world contexts within specified units of work, either devoted to process skills or within a particular content area. Of course, some of the jurisdictions where the approach is permeating or cross-referenced also have units that focus on real-world applications, and this is particularly true of the Australian jurisdictions. We classify Shanghai as using units of work. Throughout grades 1-9, there is a strand of extended (non-assessed) material that involves researching and exploring real-life problems across the content learnt. However, this strand has no hours explicitly allocated in contrast to the 200 hours allocated for content, thus giving it a minor emphasis. For university–track students in grades 11 and 12, there is a more substantial approach through compulsory units of work. Arts students must study applications of mathematics in the arts, decision making and statistics, while Science/Engineering students study geometry, statistics and vectors – topics that are potentially applicable although not actually seen in context in the *GaoKao* examinations. In addition both streams must also choose at least one of the courses 'Modelling with mathematics' and 'Historical and modern mathematics'. Japan has a similar approach with its recommended 'Use of Mathematics' elective for grades 11-12.

Finland's approach to real-life contexts is also framed most explicitly via units of work. Implicitly, it starts in primary and lower secondary with 'Thinking skills and methods' listed as a strand amongst other content strands, and in the early grades this is given only a little more detail within measurement and data handling strands. We see this is as reflecting the orientation that the real world is a vehicle through which to learn mathematics, thus it does not need to be specified separately. In upper secondary, the emphasis shifts. The basic syllabus includes two (out of six) compulsory courses on

Mathematical Models and two ‘specialisation’ courses, Commercial Mathematics and Mathematical Models III. The contents of the three Mathematical Models courses each specify the study of particular mathematical concepts to be used in modelling real-world phenomena as well as the process skills of perceiving regularities in and dependencies between real-world phenomena. We note that the specified units in these jurisdictions are all aimed at older students, and they are introduced after the curriculum has been differentiated. In the next section we look more closely at how the real world is used within different curricular pathways.

Concentration of real-world applications within certain content areas is not unusual, for example in sections on measurement, statistics and data handling. However there appears to be a special role for the mathematical topic of functions and relations as models of real world situations. In lower secondary school this topic can include linear and quadratic relations, even exponential relations in Singapore and the US. The standard approach is algebraic, graphic and numeric, with connections being drawn between all these representations. In some curricula, it also includes discussion of real-world situations that could be modelled with these relations. For example, in Florida a suggested word-problem for grades 6-8 is ‘The height of a tree was 7 inches in the year 2000. Each year the same tree grew an additional 10 inches. Write an equation to show the height h of the tree in y years. Let y be the number of years after the year 2000. Graph the height of the tree for the first 20 years.’ Although we might question the likelihood of constant growth, it is a real context for a linear relation and has potential to extend to comparison with non-linear growth. In Japan, (where such examples do not appear in the curriculum document) a grade 9 national test problem asks students to extrapolate the temperature of a cooling object from linear data shown on a graph and consider which of four other detailed real-world scenarios could use a linear model.

In upper secondary school the range of functions widens and some students study calculus in preparation for mathematics-rich university courses. Calculus originates in the study of motion and has applications in studying any situation where there is predictable change. Western researchers argue that real-world experiences of rate, growth and covariation underpin a sophisticated understanding of calculus (Thompson, 1994, Carlson et al, 2002) and so it is interesting to observe the specification of such pre-calculus experiences in the curriculum for lower age groups. There is no explicit pre-calculus in the Shanghai or Hong Kong mathematics curricula, although related topics may have been covered in science. In Japan the grade 8 Functions text specifies the study of linear functions including how they represent concrete phenomena; however only the inclusion of 'slope' and 'rate of change' in the note vocabulary suggests that teachers should draw students' attention to variation. It appears that, in practice, Japanese teachers draw on professional knowledge that supplements the published curriculum, transforming such marginal indicators into credible guidance on curricular intentions (Schmidt & Prawat, 2006). At the other extreme, in Alberta, there is an explicit and prominent emphasis on understanding different language and representations of rate of change and slope of graphs. This occurs notably when quadratic relations are introduced in grades 10. Although this is relatively late, it follows a slow progression through earlier grades in the algebraic complexity of the linear equations that students are expected to meet, represent and model situations with. In grade 7, these are of the form $ax = b$, $ax + b = c$, with integer coefficients a, b, c. In grade 8 this extends only to $\frac{x}{a} = b, \frac{x}{a} + b = c$ with $a \neq 0$, and $a(x + b) = c$, still with integer coefficients a, b, c, before the full range in grade 9. This makes some sense from the perspective of modelling contextual situations, because the change from multiplication to division changes the type of operation implied in a verbal presentation of the problem. It may suggest why Canadian students from all provinces performed best in the Change and Relationships aspect of PISA content, exceeding the OECD average by the greatest margin (Brochu et al., 2014).

Curricular differentiation

General curriculum statements about the importance of preparing students to use mathematics as they participate in adult life do not on the whole recognise explicitly that 'adult life' and its mathematical demands will be very different for different groups of students. We were therefore interested to see how this function of school mathematics is operationalised through the provision of differentiated curriculum pathways and whether the relationship between mathematics and real life is construed differently for different groups.

Few jurisdictions divide students into different pathways for mathematics until the later years of lower secondary school and, where a differentiated curriculum exists at primary or lower secondary level, its rationale is generally expressed in terms of the learning needs of the students rather than in terms of their future use of mathematics. For example, Queensland and Massachusetts suggest that adjustments to the curriculum may be made for students with special educational needs and those learning in an additional language or dialect. Singapore shows the earliest division of students: initially into two pathways in primary years 5 and 6 and then into three pathways from the beginning of lower secondary school. The pathway at lower secondary for the lowest attaining students includes a unit called 'Integrative Contexts', where students should have opportunities to learn through 'meaningful real-world contexts', including money, time scheduling and packaging problems, and 'practical hands-on experience'. Similarly, in Ontario the secondary pathway (grades 9 and 10) for lower attaining students stresses practical applications, concrete examples and 'hands-on learning'. In these jurisdictions, it seems that for younger, lower attaining students the use of real-world contexts is seen as a vehicle for making sense of mathematics, while higher attainers are expected to be able to learn mathematics in more abstract ways. At the same level in Singapore, the pathways for higher attaining students include a unit ‘Applications of Mathematics in Practical Situations’, involving mainly applications of number, rather than new content, again suggesting that these students are seen to learn mathematics first before applying it.

At upper secondary level (typically from age 16), there is some form of curricular differentiation in all the jurisdictions studied. This takes a variety of forms ranging from optional courses for those intending to study mathematically rich programmes at university level to provision of up to five different pathways for students intending different kinds of university programmes, other forms of further education, apprenticeships or direct entry into the workplace. At one extreme, the two US jurisdictions do not define alternative pathways, though higher attaining students may experience an accelerated route through the curriculum and an optional calculus course is offered in Florida. Similarly, Hong Kong defines an optional ‘Extended’ curriculum, though this may not be offered by all schools as it does not count towards university entrance. Upper secondary students in Japan are offered a single sequence of three mathematics courses. The curricula in these jurisdictions do not distinguish between the uses that various groups of students may make of mathematics in their lives but differentiate primarily by the quantity of mathematics studied.

Other curricula have a binary split that maps to real-world relevance: Finland distinguishes a Basic curriculum that includes a substantial emphasis on modelling, while the Advanced curriculum is defined in terms of abstract mathematical topics. The extended curriculum in Shanghai for university bound students in the Arts/Social

Sciences stream includes applications in the arts, decision making and statistics, whereas the Science/Engineering stream students only encounter additional pure mathematics topics.

New South Wales and Queensland each offer three different pathways that provide credit for university entrance and two further pre-vocational oriented pathways. The content of the university-oriented pathways are defined primarily in terms of abstract mathematical topics, although the stated curriculum objectives claim that this content is aimed towards the requirements of particular disciplines, defining the pathways in terms of the intended trajectories of the students (for example, in New South Wales the university oriented pathways with the least advanced mathematical content is recommended for those intending to study life sciences or commerce, while those intending to study physical sciences or engineering are recommended to take one of the more advanced pathways). In contrast, the pre-vocational pathways include strong orientation towards real world applications. In New South Wales students study units such as ‘Mathematics and Driving’ and ‘Mathematics and the Human Body’ that bring mathematics to bear on real world activities and experiences. This is similar to the Canadian jurisdictions, which also offer pre-vocational pathways that emphasise applying mathematics to everyday and work contexts. It is interesting to note that the pre-vocational pathway in Queensland focuses on applications of mathematics not only because of their potential usefulness in everyday life, work or further learning but also in order to build student confidence and to overcome negative attitudes towards mathematics. Thus connections to the real world are presented for this group of non-university bound students as a vehicle for supporting motivation to learn.

Differentiating pathways at upper secondary level in terms of student trajectories does not necessarily involve any substantial attention to real world applications even for those students who are construed as needing to learn mathematics for its applicability rather than for its own sake. For example, in Singapore, the lowest level upper secondary curriculum expresses an aim to equip students with skills for data analysis and interpretation and informed decision-making rather than to prepare them for further study of mathematical topics. This aim is realised by focusing on statistical topics and the pure mathematics required to support the statistics but the approach to the subject matter remains generally abstract, with the concession to applicability being that the most theoretical aspects (such as the derivation of statistical formulae) are explicitly excluded.

There is a general tendency across jurisdictions to focus more on real-world use of mathematics in upper secondary pathways for lower attaining students and for those who are not intending higher study of mathematically rich subjects. This is not the case, however, in Shanghai where all students must choose to study at least one unit addressing the nature, history and uses of mathematics. This is consistent with the emphasis given in this jurisdiction to student enjoyment and appreciation of mathematics as part of human culture.

Curricular differentiation also affects the type of real-world context that is used within mathematics. In jurisdictions where lower attainers follow a specialised pathway before upper secondary school, the contexts that are used for them are those that fall within personal or societal uses of mathematics, including money, scheduling time and travel, interpreting media reports of quantitative data, or health. These students are constructed as consumers rather than changers of society with mathematics. Occupational uses of mathematics are presented in New South Wales where there is an emphasis on mathematics in decision-making and design, and in Ontario with an emphasis on spatial mathematics and measurement. However there are notably few scientific contexts offered for mathematics in the lower grades or in the pathways for non-university students even though some science is valued for all. In the upper secondary pathways, Ontario, Alberta, and Japan are jurisdictions where scientific contexts are used within national mathematics tests, for example substituting into complex chemical formulae, predicting future motion and interpreting the real-world meaning of numerical outcomes. Such a use relies on the curriculum providing shared scientific experiences amongst the students on a mathematics pathway, and a commitment to weak disciplinary boundaries within assessment.

Assessment

The amounts and types of assessment prescribed within each of the jurisdictions studied vary considerably at primary and lower secondary levels, ranging from annual computer-gradable tests (Florida), through an on-going programme of moderated school-based portfolio assessment (Queensland), national testing of a sample of students in order to monitor standards (Finland), and end of phase examinations used to select students for different kinds of schools (Singapore, Japan). At upper secondary level, all jurisdictions have some form of terminal assessment used for certification and/or for university entrance. In most cases this assessment is in the form of externally prescribed examinations; exceptions include Ontario, where the only state-wide assessment is at the end of lower secondary while credit for graduation at upper secondary level is determined by in-school assessments, and Queensland, where school-based portfolio assessment continues through upper secondary level.

Terminal assessments and those used for determining future pathways, whether externally prescribed or determined in-school, generally have high stakes for students and in many cases also for their teachers and schools. Given the influence that high-stakes assessments have on the implementation of the curriculum in schools (Broadfoot, 1996), the degree to which such assessments are aligned with the intentions of the curriculum is an important concern for any jurisdiction. Studies of alignment in some of the jurisdictions included in this study have raised concerns about the cognitive level of assessments and the assessment of higher-level thinking and problem solving (Leung, Leung, & Zuo, 2014; Polikoff, Porter, & Smithson, 2011). As our focus in this article is on the ways in which curricula orient themselves towards the use of mathematics in the real world, it is relevant to consider to what extent the assessment instruments reflect the orientation of the curriculum. It is important to note that the quality of the data available to us in investigating this issue was very limited as high-stakes assessment materials in many jurisdictions are not publically available. In many cases we have access only to sample assessment materials, which may not be fully representative of the instruments actually experienced by students.

The extent to which the assessment materials address real world contexts varies considerably across jurisdictions, between assessments of different age groups and, where they exist, of different pathways. At one extreme, the examinations used in Japan and in Shanghai for selection to upper secondary school and university contain no contextualised items. In Japan this is a change of emphasis from the lower secondary national test B where scientific and everyday contexts do feature. In contrast, the lower secondary assessments in Massachusetts include a strong emphasis on modelling with mathematics, including skills of interpreting representations of data, making sense of worded problems that model functional and geometric contexts, and estimating numerical answers from inequalities, calculations and diagrams. Similarly, in the Finnish lower secondary national assessments, almost all multiple-choice and problem-solving items are presented as word problems involving some form of everyday context.

Where different pathways are identified at upper secondary level and examined separately, it is common to find that assessment of those students not intending to continue studying mathematically rich subjects and those expected to enter directly into the workplace includes a substantial component of contextualised items. For example, in New South Wales the High School Certificate examination for the lowest attaining students contains a high proportion of questions involving some real world context. The assessment of the most basic mathematics course for university-bound students includes a small number of items involving modelling (examinations from 2014 included one item about growth of an investment and one about the breakdown of a drug in a patient's body) but such items are not found in the examinations for the 'extension' courses for those intending to study more mathematically oriented subjects.

Of course, reference to a real-world context within an assessment item does not necessarily entail that students need to engage in any significant way with that context. For example, in Singapore, in those questions that involve a non-mathematical context, the choice of what mathematics to use is either routine application to very familiar situations or is signalled very clearly. This means that, in spite of the value accorded to modelling in the programmes of study, the end of course examinations do not assess important parts of the modelling process. In Finland, the matriculation examinations for the Basic mathematics syllabus contains both 'word problems' in which a simple situation is presented to be solved by applying standard mathematical methods and 'modelling' questions in which a more complex real-world situation needs to be described mathematically in order to solve a problem. However, contextualised items in the Advanced syllabus examination demand less engagement with the context. For example, a sample item (addressing the loci of vertices of rotating polygons) is introduced by a real-world story:

> According to one story, humanity experimented with movement with the help of regular polygons before the wheel was invented.

but proceeds to pose purely mathematical questions without further reference to humans or wheels.

An important factor influencing the extent to which assessment instruments support curriculum orientation towards use of mathematics is the format of the assessment items. In particular, multiple choice and short answer questions, especially those demanding only numerical answers such as those found in the Florida computer-gradable tests, are less likely to involve substantial engagement with the reality of the context or with the processes of mathematical modelling. Although mathematics educators have argued (and illustrated) that it is possible to design multiple-choice and short answer questions that assess high level thinking skills, including those associated with modelling (see, for example, de Lange, 1995), such items were not evident in the materials we were able to analyse. Even longer questions are sometimes structured in ways that do not demand any more engagement. Having said this, some assessment instruments in several of the jurisdictions do include more substantial items that demand that students engage in aspects of modelling, including: interpretation of a description of an everyday or scientific context and recognising when a model is appropriate (Japan grade 9); exploring the meaning of mathematical features in models of real-life linear situations (Ontario grade 9); making and justifying judgements, for example whether a loan paid off at a given rate will be completely paid at the end of a certain period of time (New South Wales upper secondary ‘General’ pathway). Moreover, a small number of jurisdictions use alternative assessment methods that expect students to carry out more extended tasks that provide opportunities to engage more fully with modelling processes. Hong Kong has recently introduced school-based assessments at upper secondary level that contribute 15% to the overall grade. These include extended mathematical investigation or problem solving and data handling tasks. Queensland's use of portfolio assessment at all levels has considerable potential for engaging in problem solving in real world contexts and in modelling. The guidance for teachers and suggested assessment materials include a strong expectation that extended modelling and problem-solving tasks should be a component of the assessed portfolio of student work.

Conclusions

Our analysis has highlighted the variety that exists within these curriculum documents, all of which espouse (at least implicitly) the inclusion of real-world contexts in school mathematics. This variety starts with the purposes of relating to the real world and its perceived benefits, continues through the organisation of the document in how teaching real-world examples is spliced into teaching mathematical content, enters the rationale for curricular differentiation in secondary education, and appears in the assessment instruments.

In discussing the orientation of the curricula we identified three main rationales for including real-world examples: mathematics as a tool for everyday life; the real world as a vehicle for learning mathematics; and engagement with the real world as a motivation to learn mathematics. Within the motivation purpose we trace a distinction between claims made on behalf of children that engagement is intrinsically pleasurable for them and arguments directed at parents and children that future employment opportunities are risky and can only be accessed with mathematical knowledge. We have shown that different curricula are orientated towards different rationales, with Ontario, Finland, Japan and Singapore given as typical examples of each of those characterised above. We do not argue that these rationales are distinct. Indeed there are often arguments for each within the exposition of intentions, and it is only when our analysis has followed into the organisation and instrumentation that the main orientation becomes clear. Instead we consider them as competing discursive currents in the international education conversation that are taken up with more or less conviction and follow-through at different times in the curricular reform cycles and in different parts of the curriculum. Taking the example of Shanghai, the emphasis on real-world examples as personally pleasurable is widely explained as a reaction to public concern that students learn too passively and do not enjoy mathematics. Thus it expresses what the curriculum *ought in future to be* superimposed on what it already is.

We have traced similarities in some of these discursive currents. Where curricular differentiation exists, the use of real-world contexts is seen as a vehicle for making sense of basic mathematics for younger students, but this seems not to be considered relevant for older students or for more advanced mathematics. In systems where the high-stakes examinations are not written by the same agencies who write the curriculum, the use of real-world contexts tends to disappear or becomes formulaic. There are other currents we cannot trace with our data because they need close attention to the relationships between curriculum, classroom practices and social culture. In considering the role of the real world as a motivation to study mathematics we might ask why and how that motivation is perceived to be necessary. We know that social and family beliefs are significant to variation across cultures. For example, studies of Chinese and Indian students in Britain suggest that these students feel that mathematics is accessible and amenable to hard work while other groups of students construe it as inaccessible save to a few (Archer and Francis, 2005; Bradbury, 2013).

We have also found evidence of variation between different parts of the curriculum in a single jurisdiction. This is worth discussing in terms of curricular coherence. There is some agreement that curricular coherence matters in relation to student outcomes, as shown by analysis of educational system variables during US curricular reforms (Newmann, Smyth, Allenworth & Bryk, 2001) and through international tests such as TIMMS (Schmidt & Prawat, 2006). Curricular coherence is argued to be pedagogically and politically demanding, requiring ‘real choices about what to teach and how to teach it and articulating those choices in a consistent manner in key policy instruments’ (Schmidt & Prawat, 2006, p.641). Yet we have shown that there are inconsistencies in how the real-world approach is operationalised within these curriculum texts, even in successful jurisdictions. Recall the case of Japan where the application to everyday life is an intended ‘mathematical activity’ but is not evidenced at all within the content listings up to grade 9, while being assessed in a complex way in year 9 national test; it then does not appear in the high school entry tests we have seen. On the other hand, Florida sets out a commitment to the US *Common Core State Standards*[2] that value modelling and mathematics as a tool. The curriculum content includes cross-referenced exemplification of modelling and problem solving, but the assessment does not allow students to show sophisticated reasoning in formulating or interpreting problems.

One issue in studying curriculum is to identify what makes up the set of key policy instruments and how these relate to practice. This is an issue not just for researchers but also crucially for the teachers, planners and parents who use them. Our study is necessarily limited both in the scope of the collection of data and in our ability to interpret what are contextually- and culturally-specific texts. A more significant limitation is that our study has not considered either the professional knowledge and values of teachers or the range of guidance and instantiation that is and has been available to them. Schmidt and Prawat (2006) argue that coherence should not be considered so much in the authoritative voice of the policy documents as in their consistency with concrete examples of what should be taught. Credibility among practitioners is more important than the authoritativeness and status of the curriculum writers in influencing how teachers take on reforms. Thus, we could consider the work that the Japanese ministry of education does with mathematics teachers to be a more significant key to good PISA outcomes than the curriculum artefacts themselves.

Finally, we return to the competing orientations to real-world relevance and how these underpin curricular differentiation. The two US states, Hong Kong and Japan have a single main pathway in the mathematics curriculum, differentiated by progress along it. Students in different teaching groups or different schools may study more mathematics and make faster progress through the content but the definition of the content is not varied. The other seven jurisdictions included in our study introduce pathways, mainly at upper secondary level, with different sub-sets of content, framed as more or less advanced mathematically, with the less advanced pathways having a stronger emphasis on real-world contexts. This differentiation is then carried through into assessment tasks. A rationale for this three-way association of older students, less advanced mathematics and real-world application is not made explicit in the curriculum documents we have studied. Positioned as the culmination of education, the discourse of policy suggests that mathematics is a tool for use in the real world, yet the differentiated pathways mean that students on the more advanced mathematics courses do not get opportunities to learn about such applications or about the practical power of mathematics as a tool. If older students on the less-advanced mathematics courses are seen to need real-world contexts to offer a vehicle for learning or to provide motivation or build confidence, then what is the rationale for not including such a focus earlier in their schooling? Such tensions have been explored in mathematics education research on the nature of mathematics and pedagogy for different groups of learners, yet our analysis suggests that the organisation and content of curriculum is often based on assumptions and traditions that are not made explicit and that may be in conflict with its stated aims.

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Notes

[1] http://www.qcaa.qld.edu.au/13656.htm

[2] http://www.corestandards.org

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