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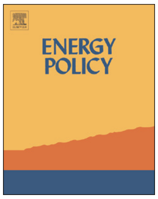

Determining the impact of regulatory policy on UK gas use using Bayesian analysis on publicly available data

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HIGHLIGHTS

- We investigate the impact of a UK policy to require new boilers to be high efficiency.
- Theoretically informed models are developed and applied to national data.
- Bayesian analysis is used to find best fit parameters and compare model performance.
- The policy is prescriptive and simple to enforce; it improves stock boiler efficiency.
- Significant energy and carbon savings may be associated with this policy.

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ABSTRACT

This paper presents a novel method to analyse policy performance, using the example of legislation in the UK to require domestic boilers fitted since 1 April 2005 to be condensing. A technological uptake model based on the logistic equation is combined with four physical and economic models; Bayesian techniques are used for data analysis. Projections of energy savings are presented and the impact of different policy implementation dates investigated.

Boiler efficiency is estimated to improve by a factor of 1.25 ± 0.15 on replacing a conventional with a condensing boiler. Estimated savings of the policy are $176,000^{+86,000}_{-127,000}$ GW h (or 32^{+16}_{-23} MTons of CO_{2e}) between introduction in 2005 and 2013. Total estimated savings by 2050 of introducing the legislation in 2005 are $2,000,000^{+1,000,000}_{-1,500,000}$ GW h (or 368^{+184}_{-276} MTons of CO_{2e}), approximately 5.6 times the average annual domestic UK emissions from domestic gas use of approximately 66 ± 5 MTons of CO_{2e}.

Nomenclature

Symbol	Units	Meaning
n	day	Model variable: day index number counting from 1st January 1990.
$N_D{}^n$	Millions	Predicted number of dwellings in England on day n
$N_B{}^n$	Millions	Predicted number of gas boilers in England on day n
$N_C{}^n$	Millions	Predicted number of condensing gas boilers in England on day n
$N_{NC}{}^n$	Millions	Predicted number of new condensing gas boilers in England on day n after the switch date
$N_{RC}{}^n$	Millions	Predicted number of replacement condensing gas boilers in England on day n after the switch date
$N_D{}^0$	Millions	Parameter: number of dwellings in England at the start of the analysis period 1st January 1990, when $n=0$
$N_B{}^0$	Millions	Parameter: number of dwellings in England with a boiler at the start of the analysis period 1st January 1990, when $n=0$
$N_C{}^0$	Millions	Parameter: number of dwellings in England with a condensing boiler at the start of the analysis period 1st January 1990, when $n=0$
r_D	Millions per day	Parameter: intrinsic linear rate of dwelling creation
r_B	Millions per day	Parameter: intrinsic rate of growth

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		of boilers in the stock
r_{C1}	Millions per day	Parameter: intrinsic rate of growth of condensing boilers in the stock before 1st April 2005
r_{C2}	Millions per day	Parameter: intrinsic rate of growth of condensing boilers in the stock after 1st April 2005
γ	Fraction	Parameter: fractional change in efficiency upon replacement of a conventional with a condensing boiler
F^{EUK}	Ratio	Set parameter: ratio of dwellings in UK to England
η_{NC}	Fraction	Model variable: fractional efficiency of non-condensing boilers
η_{C}	Fraction	Model variable: fractional efficiency of condensing boilers
η	Fraction	Model variable: fractional efficiency of the stock
E	GW h/Quarter	Predicted quarterly gas consumption in the UK
T_{ext}	°C	Data: measured quarterly average external temperature
T_{BAL}	°C	Parameter/model variable: balance temperature
T_{k}	°C	Parameter: internal temperature constant
P	Index	Data: measured quarterly average gas consumer price index
S	MW h/quarter/dwelling	Model variable: gas needed/demanded for space heating
E	MW h/quarter	Model variable: UK quarterly gas demand
W	MW h/quarter/dwelling	Model variable: gas needed/demanded for water heating
W_{k}	MW h/quarter/dwelling	Parameter: water heating constant
G_{TP}	°C/price	Parameter: change in internal temperature due to price changes
G_{P}	MW h/quarter/dwelling/price	Parameter: change in gas use for space and water heating due to price changes
G_{WP}	MW h/quarter/dwelling/price	Parameter: change in gas use for water heating due to price changes
G_{WT}	MW h/quarter/dwelling/°C	Parameter: change in gas use for water heating due to external temperature changes
G_{SP}	MW h/quarter/dwelling/price	Parameter: change in gas use for space heating due to price changes
G_{ST}	MW h/quarter/dwelling/°C	Parameter: change in gas use for space heating due to changes in the external-internal temperature difference
A, B		Parameters: dummy parameters
$Prob$		Probability distribution
H		Hypothesis
D		Data: either gas consumption or number of houses
$\mathcal{E}$		Long-run price elasticity of domestic gas use
Ω		Parameter set

1. Introduction

The potential consequences of climate change have induced many countries to commit to reduce their carbon emissions (IPCC, 2007; UNFCCC, 2012). For example, the UK government has committed to an 80% reduction in its carbon account from 1990 levels by 2050 (Climate Change Act, 2008). A raft of policies, spanning energy supply and demand in the industrial, transport, domestic and commercial sectors, aims to bring about this transformation (DECC, 2011); however, assessing the efficacy of policies remains challenging due to the complex interplay of economic, social and physical factors (Foxon, 2011). The full impact of a policy may take many years to become apparent.

Domestic water and space heating were responsible for approximately 26% of UK energy consumption in 2012, primarily supplied by the local combustion of natural gas, which is thought to account for approximately 81% of domestic consumption for heat (DECC, 2014c). A range of mitigation policies have been proposed to decrease carbon emissions associated with domestic heating (DECC, 2011). Building regulations are projected to deliver 44% of residential sector energy savings in the fourth Carbon Budget (DECC, 2012). This paper presents an analysis to determine the efficacy of energy consumption policies, using the example of legislation to mandate the installation of high efficiency boilers for new and replacement systems via the Building Regulations (ODPM, 2005).

1.1. Condensing boiler legislation

On 1 April 2005 an amendment to the Building Regulations came into force in England and Wales requiring that, apart from exceptional circumstances, all domestic gas boilers for new and replacement systems should be rated SEDBUK (Seasonal Efficiency of Domestic Boilers in the UK) A or B (ODPM, 2005). The minimum required efficiency (defined in terms of gross calorific value of natural gas) is 86%, necessitating the use of a condensing boiler (ODPM, 2005). The proportion of condensing boilers in the domestic stock has subsequently risen from ~5.7% of all boilers in 2004 to 42.8% in 2011 (DECC, 2014c). In this paper we treat the 2005 regulations as triggering a step change in condensing boiler uptake and therefore stock efficiency; however, significant improvements in boiler efficiency will have occurred prior to, and after, this date due to a range of factors including legislation, maturing technology and market competition. In this analysis all non-condensing boilers have one fixed efficiency regardless of age, condensing boilers have a different fixed efficiency, changes in the fixed efficiencies for each boiler type are incorporated into uncertainty estimates.

The introduction of condensing boilers, of higher efficiency than traditional boilers, is expected to decrease the carbon intensity of heating. However, reductions in gas usage may be partially offset by consumer comfort taking and rebound (Sorrell, 2007; Sorrell et al., 2009). Additionally, upgrading a dwelling's boiler may result in a physical rebound whereby the system is capable of achieving thermostat set-point over a wider range of external conditions, or simply achieve set-point faster than previously, thus raising the mean internal temperature and increasing the heat losses from the dwelling (Deurinck et al., 2011). Further, the in situ performance of installed boilers may be below their designed efficiency for a range of complex reasons, including return water temperatures being too high for condensing operation; a setting that may be adjusted at installation, servicing or during operation by professionals and occupiers (Orr et al., 2009). Field trials of condensing boilers in the UK reported efficiencies approximately 5% lower than SEDBUK ratings, unfortunately the efficiency of non-condensing boilers was not investigated (Orr et al., 2009). Any resulting efficiency change of boilers in the stock associated with the Condensing Boiler Legislation is therefore part of a complex socio-technical problem.

1.2. Evaluation of the impact of the condensing boiler legislation

In this paper, the impact of the Condensing Boiler Legislation on domestic gas use is presented as a case study of the impact of policy on energy demand. National energy consumption statistics have been analysed utilising models of gas demand. Models of varying complexity have been investigated, incorporating technological uptake based upon the logistic equation for population dynamics, simple thermodynamics and economics. Bayesian techniques were used to provide parameter probability distributions, co-variant error analysis and select the model with the highest probability of describing the available data, accounting for our prior knowledge of the parameters that were investigated. After the model with the highest probability of describing the data was identified, it was used to estimate a range of characteristics of the stock, such as the average balance temperature, specific heat loss and improvement in efficiency associated with fitting a replacement boiler. The energy savings attributable to the legislation that requires newly fitted boilers to be condensing were estimated and projected savings to 2050 compared to those estimated assuming later introduction of the policy. Our approach of summing annual energy and CO_2 savings to 2050, and assuming a constant efficiency of the building stock may result in overestimates of cumulative savings since it is likely buildings will become better insulated and that one or more technologies will supersede condensing boilers over this period (DECC, 2011). This paper provides enough information to allow the reader to estimate cumulative savings over shorter periods, if required.

The impact of policies on their intended outcomes have been widely researched, and to a certain extent standardised through guidance provided by policy-makers (e.g. Treasury, 2011; EC, 2011). The approach adopted in this paper is based on simple theoretical models to analyse a top down and exhaustive data set (all domestic gas use); the potential impact of the Condensing Boiler Legislation is determined by analysing the performance of the stock before and after its introduction, then modelling the stock performance in the absence of the legislation in a counterfactual manner. The clear start date associated with this policy has resulted in different rates of population growth of condensing boilers in the stock being recorded before and after the introduction of this legislation. This enables the use of the full national dataset rather than a sub-set of the population or results of technology trials.

1.2.1. Bayesian analysis

This paper presents a Bayesian statistical analysis, the use of which is widespread in many disciplines, particularly medicine (e.g. Ashby, 2006; Lee and Chu, 2012). However, Bayesian data analysis has not yet been widely adopted in energy demand research; applications of Bayesian techniques include investigating building energy performance with regression models (Hsu, 2014), estimation of missing data for energy use intensities (EUIs) for the non-domestic stock incorporating prior information through regression modelling (Choudhary, 2012), investigating the spatial characteristics of EUI distributions (Choudhary and Tian, 2014), forecasting energy demand (Crompton and Wu, 2005) and energy efficiency (Chen et al., 2015). We have been unable to find any publication detailing the use of Bayesian analysis combined with Bayesian model comparison to investigate the impact of demand and efficiency policies, and consequently no research which applies this approach to the effect of the Condensing Boiler Legislation.[1]

The use of Bayesian methods enables the comparison of different hypotheses (models) through the relative probability associated with each model providing an accurate representation of the data. This model comparison incorporates prior information about the potential distribution of parameter values, thus incorporating known (or anticipated) physical and economic characteristics. Bayesian model comparison also effectively penalises more complicated models, since they expand the prior probability space, and has been termed a numerical implementation of Occam's razor (Jeffreys, 1939; Good, 1968; Jefferys and Berger, 1992). The principle of Occam's razor has been developed by many authors, but in probabilistic terms may be expressed as "*if two hypotheses H and* H_1 *explain the facts equally*, *then the simpler of the two is to be preferred*" (Good, 1968); Good suggests a development of this principle for the purpose of the selection of hypotheses when probabilities are not equal, the Sharpened Razor: "*choose the hypothesis whose strong explanatory power is the greater*, *or the greatest*", this is applied in the analysis presented here.

1.2.2. Theoretically based models

This analysis uses simple models incorporating population dynamics, building physics and economics, providing insight not only into the magnitude of the policy impact, but also on the physical parameters that characterise the building stock and their relationship to the key drivers of external temperature and energy price. Typically, energy demand analyses utilise standard statistical relationships, rather than models and relationships informed by theory, as illustrated by the examples in Section 1.2.1.

The analyses of Summerfield et al. (2010) and Hamilton et al. (2013) on energy demand from the UK housing stock provide contrast to that presented here. Summerfield et al analyse the same national datasets as used here, as described in Section 2.1, naturally excluding additions to the data in subsequent years (Summerfield et al., 2010). They apply multiple linear regression to determine a relationship between temperature, price and domestic gas demand; however, while providing some useful insights, the models applied are not physically informed, and include third order polynomials with no physical justification. Increasingly complicated models incorporating higher order polynomials will always reduce the residuals, unless a fit is already exact, as illustrated by Jefferys and Berger (1992).

Hamilton et al. (2013) analyse the UK Homes Energy Efficiency Database (HEED), applying a counterfactual analysis of pre- and post-energy efficiency intervention (such as fitting a condensing boiler) and utilising control group of dwellings. They found an 8% reduction in gas use associated with fitting a condensing boiler, but were unable to provide a detailed error analysis. This paper also highlights the challenges of interpreting results derived from datasets that are a sub-set of the population, at national scale.

2. Methods

The method used to investigate the impact of a policy to increase the efficiency of domestic boilers on gas consumption is outlined below. Firstly, the data sources and associated issues are discussed, followed by the models used to both represent the number of boilers in the stock of different types, and for gas use in homes. Finally, the Bayesian methods employed to analyse the

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[1] In addition to a broad literature review, a systematic search of bibliographic databases was undertaken, including Web of Science and Google Scholar, to identify papers investigating energy demand and Bayesian model comparison using the search terms "energy demand" OR "energy efficiency" AND "model comparison" AND Bayes*; no publication date or other limits were set.

data are reviewed.

2.1. Data sources

The data used for this analysis were UK government published data for gas consumption, consumer price index for gas, external temperature, dwelling and boiler numbers. While extensive data is available in the UK, the varying geographical scales, differences in aggregation, available time-scales and lack of error estimates present challenges for analysis, as outlined below.

Statistics, published by the Department of Energy and Climate Change (DECC), were used to identify domestic gas consumption (DECC, 2014a) and average quarterly temperature (DECC, 2014e). The available UK gas use figures include space heating, water heating and cooking, estimated currently to comprise approximately 77%, 21% and 2% of total consumption respectively (DECC, 2014c). Gas use due to cooking is therefore included in this analysis but is not explicitly discussed.

The available quarterly data for domestic gas use begins in 1998, limiting the time span of the analysis (DECC, 2014a). Average quarterly external temperatures in the UK, T_{ext} are also provided, broadly weighted according to population density (DECC, 2014e). The average temperature has been taken from 17 stations in the UK, with four stations in England double weighted. This is a crude reflection of the distribution of homes in the UK, but introduces additional errors and uncertainty into the analysis. UK consumer gas price index, P, is also provided in the DECC statistics; it is the component of the retail price index associated with gas prices measured from a base of 2005 prices and compiled by the Office for National Statistics (DECC, 2014b).

The number of dwellings (N_D), gas boilers (N_B) and condensing boilers (N_C) are available for England for complete years (DCLG, 2014b). The different geographical extent of gas consumption, temperature and price data, representing the whole UK (DECC, 2014a,b,e), necessitated scaling of the England-only data, as described by Eq. (7) and discussed in Section 2.3.1. We assume the ratio of dwellings in England to those in the whole UK to be constant. Similarly, we assume that the uptake rate of condensing boilers is the same in England and the whole UK. Approximately 83% of all UK dwellings are in England, which is therefore likely to dominate energy consumption trends.

2.2. Models of boiler uptake

A series of models has been developed to investigate the impact of the 2005 UK policy to require all newly fitted boilers to be condensing (ODPM, 2005). The models are physically or theoretically informed wherever possible, incorporating building physics and economics, as discussed in Section 2.3. A key component of these models is the number of boilers and condensing boilers; the technological uptake component of the four models, which also addresses differences in the time-steps of the data, is discussed in this section.

Gas consumption for space heating is strongly dependent upon external temperature, so it is desirable to analyse gas consumption on a seasonal basis, in line with the quarterly gas, price and temperature data (DECC, 2014a,b,e). However, the data for N_D, N_B and N_C is provided on an annual basis (DCLG, 2014b). Quarterly values of N_D, N_B and N_C could be estimated from simple interpolation, but such a model does not account for the non-linear characteristics of technology diffusion (Lund, 2006) and would not be suitable to estimate future energy savings associated with the introduction of condensing boilers.

A number of different models of technological diffusion (Rogers, 2003; Stoneman, 1995) could be employed to investigate the increase in the number of condensing boilers in the UK stock. The model employed here is based on a simple physically bounded growth curve, in the manner adopted in studies of population dynamics, and described by the logistic equation Verhulst, 1845; Case, 1999) (Eq. (2) gives its differential form). Partly as a result of its simplicity, this model requires the input of only three sets of data, all of which were available from UK national statistics (Section 2.1): the number of boilers, condensing boilers and dwellings in the UK. In this model, the number of dwellings with boilers in England, N_B, is limited by the total number of dwellings in the stock.[2] Similarly, the number of condensing boilers in England is limited by the number of dwellings with boilers. Such technological diffusion models have been studied widely, including analysis of the uptake of energy technologies, showing reasonable agreement to data, and for low exponential growth rates[3] generate the S-shaped curves typical of technology uptake (Rogers, 2003; Lund, 2006). An alternative model accounting for the potential drivers of boiler replacement could provide further insight, such as a model assuming that the likelihood of boiler replacement is related to the age of the installed boiler; however, no data is available to support the investigation of such effects.

The number of condensing boilers in the stock depends on both the rate of replacement of existing boilers with condensing boilers, and the rate at which condensing boilers are installed in properties that have previously not been centrally heated. Additionally, the rate at which gas central heating is installed for the first time in dwellings comprises two components: that for existing dwellings and the rate for new build properties. Models of the increase in the number of condensing boilers in the stock pre- and post-regulation are outlined below.

Before regulation requiring condensing boiler installation: The rate of first time installation of boilers in dwellings may be modelled as a population growth, with increasing carrying capacity (total number of possible boilers). This increase in the total number of dwellings is modelled first, followed by repeated application of the logistic equation.

The number of households in England may be used as a proxy for the number of dwellings, depending both on the rate of new-build and the rate of conversion of current properties into different dwellings. The model used here follows the government projection of a linear increase in the number of households in England over the next 10–20 years, based on demographic trends, but neglecting economic issues, government policies and other factors (DCLG, 2010, 2013)

$$N_D^n = N_D^{n-1} + r_D \tag{1}$$

where $N_D{}^n$ is the number of dwellings in England at the start of the time step, n the number of days since the start and r_D the number of new dwellings per day (or the rate of dwelling building). The number of dwellings at the start of the analysis period, $N_D{}^0$, is a model parameter. The number of conventional and condensing boilers in England, N_B, is assumed to be limited by the number of dwellings in the stock and is modelled by the logistic equation

$$\frac{dN_B}{dt} = r_B N_B \left(1 - \frac{N_B}{N_D}\right) \tag{2}$$

where r_B is the intrinsic rate of growth of boilers in the stock (per

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[2] To first order, the impact of large dwellings with more than one boiler and flats where it is impractical to fit a gas boiler is not considered.

[3] At high growth rates, solutions to the logistic equation become chaotic (May, 1976).

day) and N_D (the number of dwellings) is the carrying capacity or the population limit. For the nth day this becomes

$$N_B^n = N_B^{n-1} + r_B N_B^{n-1}\left(1 - \frac{N_B^{n-1}}{N_D^n}\right) \tag{3}$$

where ${N_B}^n$ is the number of boilers in England at the start of the time step. The number of boilers at the start of the analysis period, ${N_B}^0$, is a model parameter. A small time step of one day ensures that the model is effectively continuous when used to predict quarterly values. The number of condensing boilers in the stock before introduction of the new regulation may be similarly modelled, limited by the number of boilers

$$N_C^n = N_C^{n-1} + r_{C1} N_C^{n-1}\left(1 - \frac{N_C^{n-1}}{N_B^n}\right) \tag{4}$$

where ${N_C}^n$ is the number of condensing boilers at the start of the time step and r_{C1} the intrinsic rate of growth of condensing boilers in the stock before introduction of legislation to require their uptake. The number of condensing boilers at the start of the analysis period, ${N_C}^0$, is a model parameter.

Boiler uptake after introduction of condensing boiler regulation: Following introduction of the legislation to require new boiler installations to be condensing, the number of newly installed non-condensing gas boilers in England and Wales is assumed negligible in this analysis. The exceptional circumstances in which they may be fitted are detailed in the Amendments to Approved Document L1 (2002), Appendix G (ODPM, 2005), and require a clear justification, independent of home-owner preference of the room in which the boiler may be fitted. Under this regulation, difficulties in fitting a condensate drain or the need for an unusually long balanced flue, neither of which is required for a conventionally flued non-condensing boiler, are acceptable reasons for not fitting a condensing boiler. No national statistics are available detailing the number of non-condensing boilers fitted since 1 April 2005; however, we expect that few dwellings would qualify under these stringent criteria. The UK boiler market has subsequently become dominated by condensing boilers.

The number of condensing boilers from replacement of systems existing at the date of introduction of the legislation, $N_{RC}^{n=1/4/2005}$, increases at a new intrinsic rate of growth, r_{C2}

$$\begin{aligned} N_{RC}^{n=1/4/2005} &= N_C^{n=31/3/2005} \\ N_{RC}^n &= N_{RC}^{n-1} + r_{C2} N_{RC}^{n-1}\left(1 - \frac{N_{RC}^{n-1}}{N_B^{n=1/4/2005}}\right) \\ N_{NC}^n &= N_B^n - N_B^{n=31/3/2005} \\ N_C^n &= N_{RC}^n + N_{NC}^n \end{aligned} \tag{5}$$

where ${N_{RC}}^n$ is the number of replacement condensing boilers and ${N_{NC}}^n$ the number of new condensing boilers at time step n.

Solution of the models of boiler uptake requires the determination of seven parameters: ${N_D}^0$, r_D , ${N_B}^0$, r_B, ${N_C}^0$, r_{C1} and r_{C2}.

2.3. Models of the relationship between gas use, external temperature, price and the efficiency of boilers

Four simple models of domestic gas consumption are developed in this section utilising the strongest external physical driver, external temperature, the consumer price index for gas as a driver of heating behaviour, boiler efficiency and the relationships describing boiler uptake derived above. We have assumed that the average efficiency of non-condensing boilers in the stock, and therefore of those being replaced at each timestep, is constant. This may not be the case, for example, if the oldest and least efficient boilers are replaced first.[4] Similarly, we have assumed that the improvement in efficiency associated with fitting condensing boilers is constant. As noted in Section 2.1, UK domestic gas use is dominated by space and water heating; combining gas consumed for water heating and cooking into one parameter, W (for convenience referred to henceforth as “water”)

$$E = \frac{N_B^{UK}}{\eta}(S + W) \tag{6}$$

where E is the UK quarterly gas demand, ${N_B}^{UK}$ the number of boilers in the UK during that quarter as predicted by Eq. (3) (the value for the middle of the quarter was used), η the stock heating efficiency and S the energy used for gas space heating per domestic property.[5] Eq. (6) is expanded below, linking available data to simple models. Firstly, a simple relationship between the number of dwellings in the UK and those in England is presented, followed by a discussion of stock efficiency. A series of models are then developed incorporating the impact of external temperature and price on gas use.

2.3.1. Addressing geographical coverage issues with the data

A technology uptake model for the number of condensing boilers in England was developed in Section 2.2. However, quarterly gas consumption is available for the whole UK (DECC, 2014a). Assuming a constant ratio of boilers in England to the whole UK, a scaling factor, F^{EUK}, may be employed to represent the number of boilers in the UK, ${N_B}^{UK}$, as a function of the number of boilers in England, N_B

$$N_B^{UK} = F^{EUK} N_B \tag{7}$$

As the parameter F^{EUK} is a scaling factor in an equation with multiple unknowns (Eq. (6)), it cannot be determined in this analysis and was explicitly entered into the model. An approximation from the ratio of domestic gas use in Great Britain to England in 2011, of 1.15 is used, with an assumed error of 5% (DECC, 2013).

2.3.2. Stock efficiency

The efficiency of condensing boilers, η_C, can be related to the efficiency of conventional boilers, η_{NC}

$$\eta_C = \eta_{NC}(1 + \gamma) \tag{8}$$

where γ is the fractional change in boiler efficiency between a conventional and condensing boiler and is a parameter in the models to be determined.

The average gas use efficiency of the stock, η, may then be represented by

$$\eta = \eta_{NC}\left(1 + \frac{\gamma N_C}{N_B}\right) \tag{9}$$

where N_C is the number of dwellings with condensing boilers in England and N_B is the number of dwellings with condensing and conventional boilers in England. Eq. (6) for the UK domestic energy demand from natural gas becomes

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[4] The efficiency of both non-condensing and condensing boilers is likely to have improved over time, so errors due to our modelling of replacement as a random process will tend to offset.

[5] The adopted categories “space heating” and “water heating” may not reflect the full complexity of the temperature and price dependence of gas use for water and space heating, which cannot be distinguished in this analysis. However, the authors have introduced these terms to help the reader navigate his or her way through the paper; it is likely that these interpretations and ascriptions would be substantially supported by future research.

$$E = \frac{F^{EUK}}{\eta_{NC}} \frac{N_B}{\left(1 + \frac{\gamma N_C}{N_B}\right)} (S + W) \tag{10}$$

It is not possible to directly determine the parameter η_{NC} from the data available in this analysis, so an explicit estimate was entered into the model. A range of estimates for the efficiency of boilers in the building stock are available, but, the in situ performance is not well characterised. For example, government supplied boiler efficiencies for the stock are estimated by matching responses to the English Housing Survey to SEDBUK quoted efficiencies where possible or simply by matching the boiler type and fuel to typical figures (DECC, 2014d); values therefore do not account for in situ performance factors such as the state of maintenance, quality of installation and losses due to un-insulated pipework. We have used 72 ± 2% in accordance with the range suggested for the UK's Standard Assessment Procedure for post 1998 conventional boilers (Building Research Establishment, 2014). Typical quoted efficiencies of condensing boilers are 80–84%[6] (Building Research Establishment, 2014).

2.3.3. Models of space and water heating demand

Specific models were developed to explore the relationship between domestic gas use, external temperature, consumer gas price index and the number of condensing boilers. While the physical relationship between a temperature differential and heat loss is well established, the relationship in occupied dwellings is less clear. Physical factors which may correlate to temperatures or time such as the incident solar radiation, wind speed and change in heat gains from devices within the property, combine with behavioural issues such as a price dependency of heat demand, ventilation requirements and seasonal variation in occupant clothing choices. Improvements to the building fabric (e.g. insulation or double glazing), the water heating system (e.g. better insulated hot water tanks or insulated pipes) and cooking (e.g. a better insulated oven) were not addressed to simplify the analysis; their impact is incorporated into the error estimates outlined in Section 3.

Four simple models were developed encapsulating different characteristics of gas demand. Each model assumes a system in thermal equilibrium and all models satisfy the condition that space heating may never be negative: dwellings may not produce gas and export it. Where models otherwise predict negative space heating gas demand in any quarter (in Summer months with warm external temperatures), it is forced to zero:

- Model I attempts to minimise the number of parameters used to explain gas use, while accounting for both external temperature, T_{EXT}, and gas price, P. It assumes that gas use for both water heating and space heating is equally sensitive to the price

  $$S + W = G_{ST}(T_{BAL} - T_{EXT}) + W_K + G_P P \tag{11}$$

  $$G_{ST}(T_{BAL} - T_{EXT}) = 0 \text{ if } G_{ST}(T_{BAL} - T_{EXT}) < 0$$

  where G_{ST} is the quarterly heat loss per dwelling per degree of interior-exterior temperature differential, T_{BAL} is the balance temperature, a parameter representing the lowest external temperature at which free internal gains meet heating demand, and W_K is a constant water usage. G_P is the linear change in gas demand due to changes in the consumer price index for gas. A linear relationship between gas price and use was chosen for its simplicity; a more complicated model could be adopted in the future. Model I utilises 12 unknown parameters: T_{BAL}, G_{ST}, G_P, W_K and the fractional change in efficiency, γ, in addition to the 7 unknown parameters from the technological uptake relationships developed in Section 2.2.
- Model II attempts to model price sensitivity of thermal comfort, while gas use for water heating is assumed to be independent of price. In this model the balance temperature (T_{BAL}) of dwellings is not a parameter but linearly dependent on the consumer gas price index. This model is described by

  $$\begin{aligned} T_{BAL} &= G_{TP}P + T_K \\ S &= G_{ST}(T_{BAL} - T_{EXT}) \\ W &= W_K \\ S &= 0 \quad \text{if } S < 0 \end{aligned} \tag{12}$$

  where G_{TP} is the rate of change of internal temperature as gas price increases, T_K is a constant temperature, W_K is the constant water use and all other symbols are as above. This model has the same number of parameters as model I, 12, but incorporates a slightly more complicated model of gas use.
- Model III is similar to model II as it incorporates a price dependency of the gas use for space heating, but it also includes a separate price dependency for water heating

  $$\begin{aligned} T_{BAL} &= G_{TP}P + T_K \\ S &= G_{ST}(T_{BAL} - T_{EXT}) \\ W &= G_{WP}P + W_K \end{aligned} \tag{13}$$

  where G_{WP} is the rate of change of gas use with gas price for water heating and all other symbols are as above. This model has a total of 13 parameters, one more than models I and II.
- Model IV is the most complicated model analysed, with 14 parameters. It builds on model III by incorporating a temperature dependence of the gas used for water heating, in addition to its price dependence, to account for differences in the cold water supply temperature and seasonal differences in hot water demand

  $$\begin{aligned} T_{BAL} &= G_{TP}P + T_K \\ S &= G_{ST}(T_{BAL} - T_{EXT}) \\ W &= G_{WP}P + G_{WT}T_{EXT} + W_K \end{aligned} \tag{14}$$

  where G_{WT} is the rate of change of gas use for water heating with external temperature and all other symbols are as above.

Models I–IV represent simplified models of domestic gas use in the UK based on available data, to investigate the effect of implementing a policy to increase the efficiency of boilers in the housing stock. The models represent incrementally greater complexity; the best model to represent available data has been determined using Bayesian techniques (Section 2.4.2). The following section describes the derivation of error estimates for model inputs.

2.3.4. Error analysis for data analysis

The absolute uncertainty or error in the number of dwellings (N_D), dwellings with boilers (N_B), dwellings with condensing boilers (N_C) and quarterly gas use (E) is not provided directly by the metadata. It would be extremely difficult and time consuming to

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[6] The true *in situ* performance of boilers in the UK is an important characteristic of the energy performance of the housing stock, being required to calculate the carbon savings and cost implications of alternative heating systems. Recent estimates of *in situ* performance, including those listed here as a baseline, derive from modelled performance or laboratory tests (Palmer and Cooper, 2013; Building Research Establishment, 2014). Results from the Energy Saving Trust field trial of 43 condensing boilers indicated an average efficiency of regular and combination boilers of 85.3 ± 2.5% and 82.5 ± 4% respectively (Orr et al., 2009); we have been unable to identify similar test results for non-condensing boilers.

extract this uncertainty from the information available. Nevertheless the analysis combines these data elements and an estimate is required to ensure that the data is properly weighted in the evaluation of the likelihood function (Section 2.4).

In order to provide estimates, two simplified, but plausible, models of the relationships between time and N_D, N_B, and N_C and temperature, price and E were used to determine the best fit deviation and therefore the uncertainty of these data. While this method does provide meaningful error values, which are an amalgamation of all the random uncertainties, they are model dependant. The quality and validity of the models therefore contributes to the errors associated with the parameters values derived in Section 3. However, as they are common to all models described in Section 2.3.3 they will not alter the model comparison.

Systematic errors in external temperature, price of gas and date on which measurements were taken were not estimated in this analysis. Finally, it is likely that condensing boilers consume more electricity than non-condensing boilers, due primarily to the longer gas flow path through the heat exchanger. The effect is not included in the SEDBUK boiler rating system, and no systematic study of it has been reported in the literature. The impact on electricity use in dwellings is small and confounded with many other sources of change. It has therefore not been possible to provide an empirical estimate of its magnitude.

2.3.5. Condensing boiler uptake error

Eqs. (1)–(5) were used as the basis for a model of boiler uptake dynamics. Uniform non-informative prior distributions including all reasonable values were selected for the seven parameters ($N_D{}^0$; r_D; $N_B{}^0$; r_B; $N_C{}^0$; r_{C0}; r_{C1}) (Table 1). The error in values was initially arbitrarily set to 1 for all parameters, enabling estimation of the true error from residuals. The one sigma error for N_D, N_B and N_C is model dependent and was estimated as ±0.12 million houses.

2.3.6. Gas use error

A very simple model of gas use was utilised to estimate the error in domestic natural gas energy demand

$$E = A\left(T_{BAL} - T_{EXT}\right) + B$$
$$A\left(T_{BAL} - T_{EXT}\right) = 0 \quad \text{if } A\left(T_{BAL} - T_{EXT}\right) < 0 \tag{15}$$

The unknown parameters in the model are a heat loss dependent variable, A, the balance temperature, T_{BAL}, and water heating parameter, B. As for models described in Section 2.3, if T_{EXT} is greater than T_{BAL}, the difference is set to zero. The one sigma error derived from the residuals between this model and the data is ±9500 GW h per quarter.

Table 1
Parameter estimates, uncertainties and prior ranges for models I, II, III and IV.

Symbol	Model I ($\pm 1\sigma$)	Model II ($\pm 1\sigma$)	Model III ($\pm 1\sigma$)	Model IV ($\pm 1\sigma$)	Prior Range
N_D^0	19.4 ± 0.1	19.4 ± 0.1	19.4 ± 0.1	19.4 ± 0.1	5–25
r_D	0.00041 ± 0.00002	0.00041 ± 0.00002	0.00041 ± 0.00002	0.00041 ± 0.00002	0–1
N_B^0	13.6 ± 0.2	13.6 ± 0.2	13.6 ± 0.2	13.6 ± 0.2	5–20
r_B	0.000276 ± 0.000009	0.000276 ± 0.000009	0.000276 ± 0.000009	0.000276 ± 0.000009	0–1
N_C^0	0.11 ± 0.02	0.12 ± 0.03	0.13 ± 0.02	0.13 ± 0.02	0–1
r_{C1}	0.00033 ± 0.00004	0.00031 ± 0.00004	0.00030 ± 0.00004	0.00030 ± 0.00003	0–1
r_{C2}	0.00122 ± 0.00004	0.00123 ± 0.00004	0.00123 ± 0.00004	0.00123 ± 0.00004	0–1
γ	0.46 ± 0.14	0.33 ± 0.15	0.25 ± 0.15	0.26 ± 0.15	−10–10
G_{ST}	0.49 ± 0.02	0.48 ± 0.02	0.47 ± 0.02	0.37 ± 0.03	−15–15
G_P	−0.006 ± 0.002				−15–15
T_{BAL}	14.2 ± 0.3				0–30
G_{TP}		−0.020 ± 0.006	−0.015 ± 0.006	−0.018 ± 0.008	−15–15
T_K		15.7 ± 0.5	15.3 ± 0.5	15.1 ± 0.7	0–30
G_{WT}				−0.11 ± 0.02	−15–15
G_{WP}			−0.004 ± 0.002	−0.004 ± 0.002	−15–15
W_K	1.6 ± 0.2	1.1 ± 0.1	1.4 ± 0.2	3.1 ± 0.4	−50–50

2.4. Data analysis

The relationship between UK domestic gas consumption and temperature, consumer gas price index and the number of condensing boilers in the stock was investigated using Bayesian analysis. The four models constructed in Section 2.2 contain components for the uptake of boilers and domestic gas consumption; each model was analysed independently to estimate its best fit parameters and their confidence intervals.[7] The probability of each model accurately describing the data was then compared using these best fit parameters. Parameter estimation is described in Section 2.4.1. Model comparison is discussed in Section 2.4.2.

2.4.1. Parameter estimation

The parameters, Ω, within each model describe a posterior probability space, with considerable co-variance across the 12 parameters in models I and II, 13 in model III and 14 in model IV. Following Bayes' theorem, the joint probability distribution of the parameters, $Prob(\Omega|D, M_i)$, given the data, D, is

$$Prob\left(\Omega|D, M_i\right) = \frac{Prob\left(D|\Omega, M_i\right) \times Prob\left(\Omega|M_i\right)}{Prob\left(D|M_i\right)} \tag{16}$$

where M_i represents a model, $Prob(D|\Omega, M_i)$ is the likelihood function, the probability of measuring the recorded data given the estimated parameters and model, and $Prob(\Omega|M_i)$ is the prior distribution, the estimated initial probability distribution of the parameters. $Prob(D|M_i)$, the evidence, is the probability of observing the recorded data given the model; it normalises across all possible models and is not required to determine the most likely values of the parameters. Model parameters may be estimated from Eq. (16) by optimising across the probability space, using the prior distributions of parameters and likelihood function, as described below.

In the absence of detailed prior knowledge of the probability distributions of the parameters, flat, uninformative, distributions were employed. Wide physically informed limits to the distributions were selected to ensure all reasonable parameter values were included

$$Prob\left(\Omega|M_i\right) = \prod_{n=1}^{N}\left(\frac{1}{\Omega_n^{max} - \Omega_n^{min}}\right) \quad \text{for } \Omega_n^{min} \le \Omega_n \le \Omega_n^{max}$$
$$Prob\left(\Omega|M_i\right) = 0 \quad \text{otherwise} \tag{17}$$

where $\Omega_n{}^{max}$ is the maximum value of the parameter, Ω_n, and $\Omega_n{}^{min}$ its minimum value; the range of prior for each parameter is shown in Table 1. Likelihood functions were estimated by assuming data points conform to a Gaussian (normal) distribution, in the absence of error estimates for data (Section 2.1), with uncertainty in the kth data point estimated as described in Section 2.3.4

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[7] 67% confidence intervals are used throughout. Confidence intervals are not assumed symmetric, but are the smallest 67% interval containing the peak of the probability distribution.

Table 2
Number of parameters, prior density and evidence for models I, II, III and IV.

Property	Model I ($\pm 1\sigma$)	Model II ($\pm 1\sigma$)	Model III ($\pm 1\sigma$)	Model IV ($\pm 1\sigma$)
Number of parameters	12	12	13	14
Prior density	6.17e – 11	6.17e – 11	2.06e – 12	6.86e – 14
Evidence	3.31e – 66	4.16e – 66	1.19e – 64	9.13e – 65

$$Prob\left(D|\Omega,\, M_i\right) = \prod_{k=1}^{N} \frac{1}{\sigma_k\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{E_{ik}(\Omega) - D_k}{\sigma_k}\right)^2 \right) \tag{18}$$

where $E_{ik}(\Omega)$ is the estimate of the data point k under model M_i and using parameters Ω. Substituting into Eq. (16), parameters and their confidence intervals were estimated numerically from the peak in the probability distribution, using the optimisation software MINUIT (James and Roos, 1975). Table 1 summarises the model parameter, prior range, most probable value and uncertainty for each model. See the nomenclature table for the physical description of each parameter and its units.

2.4.2. Model comparison

The evidence, the probability of obtaining the recorded data given the model, may be used to compare the relative probabilities of different models describing the observed data

$$Prob\left(D|M_i\right) \approx Prob\left(D|\Omega_{MP},\, M_i\right) \times Prob\left(\Omega_{MP}|M_i\right) \times \sigma_{\Omega|D} \tag{19}$$

where Ω_{MP} are the best-fit parameters and $\sigma_{\Omega|D}$ is their uncertainty (MacKay, 2007, p. 349). The final two terms in Eq. (19) are the Occam factor. The Occam factor accounts for the fact that for a given level of evidence, the fit to the data must increase as the number of parameters increases (Section 1.2.1). The evidence for models I, II, III and IV are shown in Table 2; model III is the most probable, having the highest evidence, despite utilising 13 parameters (MacKay, 2007, p. 343). The most probable of the models to describe the data therefore includes a separate price dependency for space and water heating, a temperature dependence of space heating, but does not include an additional temperature dependence of water heating (Section 2.3.3). Model III is therefore used in the rest of this paper to investigate the impact of the Condensing Boiler Legislation on UK gas use, as discussed in Section 3.

3. Results and discussion

Tables 1 and 2 provide the results of the parameter estimation and model comparison described above, indicating that model III, illustrated by Eq. (13), best describes the numbers of dwellings, dwellings with boilers and boilers that are condensing and the UK gas consumption data. This model utilises different price parameters for temperature dependent (primarily space heating) and temperature independent (assumed to be primarily water heating) energy use. The model parameters shown in Table 1 represent the best estimates in this study and provide some insight into the performance of the stock and uptake of condensing boilers.

The fixed ratio of the England to UK scaling factor to the efficiency of the stock of non-condensing boilers (F^{EUK}/η_{NC}), required to solve Eq. (13) in the absence of more data, introduces significant uncertainty into estimates of certain parameters. Three parameters are inversely proportional to this fixed ratio: the quarterly heat loss per dwelling per degree of interior-exterior temperature differential (G_{ST}), temperature independent gas use dependence on energy price (G_{WP}) and the constant representing other contributions to temperature independent gas consumption, W_K; all other parameters are independent of it. The ratio of the number of boilers in England to the whole UK (F^{EUK}) is well characterised (Section 2.3.1), although it may change over time. However, the in situ performance of boilers in the UK is not well known as discussed in Section 2.3.2; our results may be scaled accordingly if a different estimate is utilised.

3.1. Uptake of condensing boilers

Table 1 and Fig. 1 highlight the clear impact of the legislation to

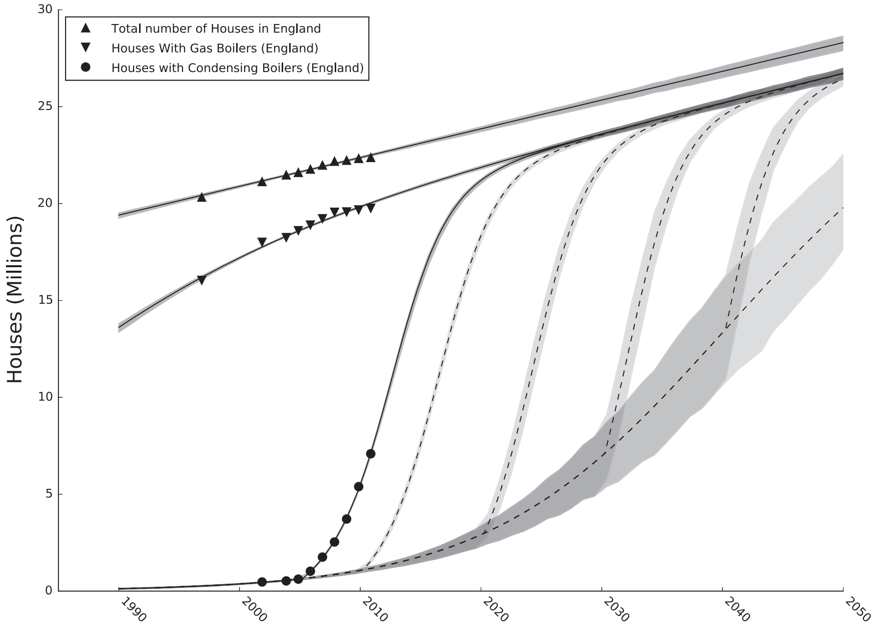

Fig. 1. The total number of house in England (upward pointing triangles), number of those houses with boilers (downward pointing triangles) and the number of those that are condensing boilers (circles). The solid lines with shaded uncertainty bands shown through each set of points represent the predictions of model III. The dashed lines with shaded uncertainty bands represent the predictions of the number of condensing boilers from model III, with different dates for the introduction of the Condensing Boiler Legislation. Predictions using the dates of 1 of April 2010, 2020, 2030, 2040 and 2050 were used.

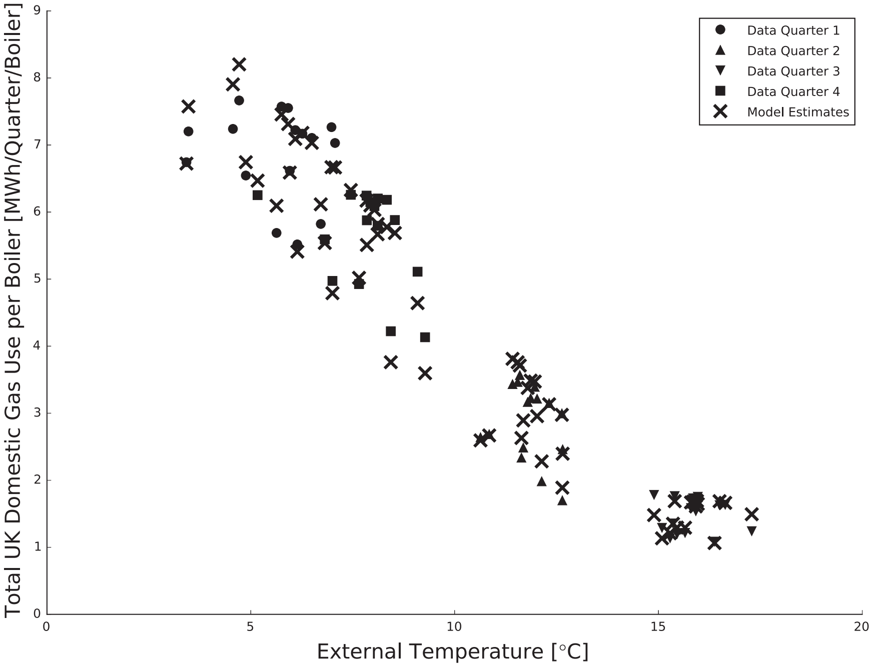

Fig. 2. The UK domestic gas use per boiler as a function of external temperature. The points are the measurements taken during quarters 1 (circles), 2 (upward pointing triangles), 3 (downward pointing triangles) and 4 (squares). The crosses represent the estimate of model III using the external temperature, consumer gas price index, the number of houses, houses with boilers and houses with condensing boilers during the quarter.

require boilers fitted from 1 April 2005 to be condensing. The rate of installation of condensing boilers in properties for the first time increases by roughly an order of magnitude from 170^{+18}_{-25} per day one year before the introduction of the Condensing Boiler Legislation (1 April 2004) to 1850^{+12}_{-47} per day one year after the introduction of this legislation (1 April 2006). The modelled peak in the number of properties being fitted with condensing boilers for the first time, on the basis of the data outlined in Section 2.1, is $6,322^{+156}_{-204}$ per day, achieved on 20 August 2012; on this date 582^{+13}_{-15} properties had boilers installed for the first time.

3.2. Heating characteristics

The heat energy loss per UK dwelling per unit temperature per quarter (G_{ST}) was converted to a specific heat loss coefficient per property, 215 ± 10 W/°C, which is the average value over the period studied.[8] The specific heat loss per property is expected to decrease over the duration of the analysis due to improvements to new building efficiency driven by regulation (DCLG, 2014a), and increasing uptake of retrofit measures such as insulation and windows (DCLG, 2014b). The specific heat loss per dwelling estimated here is lower than that estimated by the Building Research Establishment Housing Model for Energy Studies (BREHOMES) of 247 W/°C (Utley and Shurrock, 2008) and Summerfield et al. of 240–320 W/ °C (Summerfield et al., 2010). However, Summerfield's technique of fitting a third order polynomial to the data, followed by selecting the approximately linear central section of this fit to estimate specific heat loss creates a stronger perceived relationship between temperature and heating energy use, accounting for the latter difference (Summerfield et al., 2010). Similarly, differences in methodology account for the difference between our estimate of specific heat loss and that from BREHOMES, a theoretical bottom up model informed by the physical survey of several thousand properties and containing many assumptions about the heat loss from windows, walls, floors, roofs and ventilation through occupant behaviour and fabric air tightness (Shorrock and Dunster, 1997). BREHOMES also assumes whole house heating where the only barrier to heat loss is the external fabric, whereas the UK Energy Follow-Up Survey (Building Research Establishment, 2013) found that 65% of houses had one or more habitable room that was unheated by the main heating system, effectively reducing the heat loss area (though demand for heat will reduce by less than the difference in actual floor area).

The balance temperature in UK dwellings, the external temperature at which the desired internal temperature is just reached through free heat gains (solar, metabolic and appliances), is a function of the price of gas in this model. The effect of dwelling thermal efficiency is incorporated into the balance temperature estimated by the model through the constant T_K, (Eq. (13)); however, this parameter is estimated over the duration of the data, providing an average value. The impact of improvements in building fabric on the balance temperature therefore cannot be estimated. In model III the balance temperature falls as a result of rising gas price index, from 14.7 ± 0.3 °C in 1998 to 13.7 ± 0.5 °C in 2013, suggesting consumers decreased their thermal comfort (indoor temperature) as the price of gas rose.

3.3. Boiler efficiency

The estimated fractional change in the boiler efficiency, γ, between a conventional and a condensing boiler is given by Eq. (8). For model III, $\gamma = 0.25 \pm 0.15$; while the uncertainty in this value is high, it indicates a reduction in gas use on boiler replacement. Utilising $\eta_{NC} = 0.72$ the estimated efficiency of condensing boilers η_C is 0.9 ± 0.1, as compared to the 0.8–0.84 range provided in the Standard Assessment Procedure (Building Research Establishment, 2014).

3.4. Gas use dependence on external temperature

Fig. 2 shows the dependence of UK domestic gas use on

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[8] There are 92.5 × 24 = 2190 h per quarter, G_{ST} is 0.47 ± 0.02 MW h/quarter/dwelling/°C. The specific heat power loss per property is therefore 0.47 × 1,000,000/2190 = 215 W/°C.

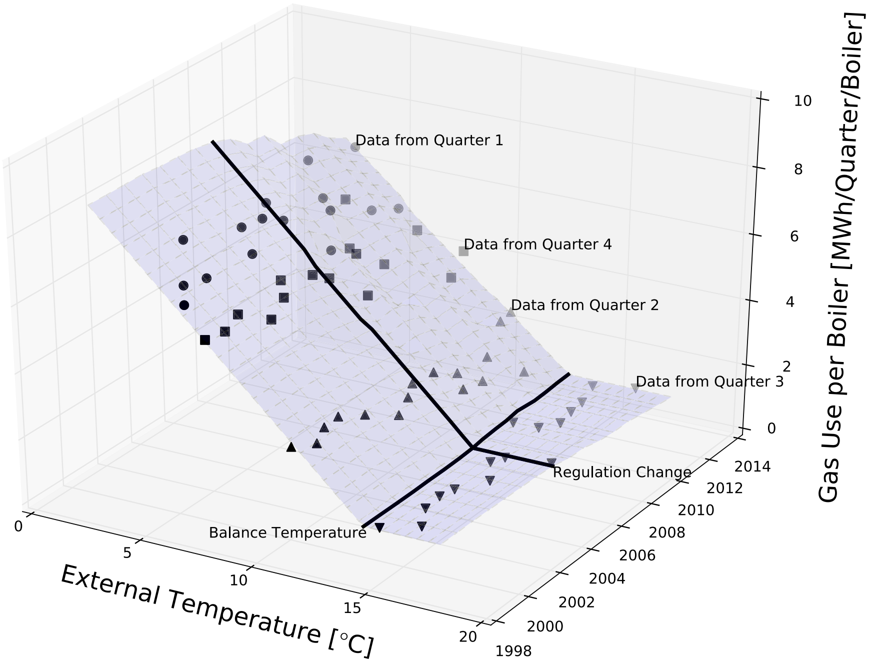

Fig. 3. The total UK domestic gas use per boiler as a function of the external temperature and year. The points are the measurements taken during quarters 1 (circles), 2 (upward pointing triangles), 3 (downward pointing triangles) and 4 (squares). The wire surface represents the estimate of model III using the external temperature, consumer gas price index, the number of houses, houses with boilers and houses with condensing boilers during the quarter. The solid line moving across the plot indicates the balance temperature, which is the temperature at which the heat demand to reach the desired internal temperature is just met by free heat gains. The date of the regulation change is also shown.

quarterly external temperature, indicating a very good agreement between observed and modelled data. Total gas use in model III is linearly dependent on the number of boilers in the stock and their average efficiency; Fig. 2 shows the gas use per boiler to eliminate the impact of the number of boilers. The relationship between external temperature and gas use is further explored in Fig. 3, which also shows the year. The figure indicates very good agreement between the model and data. The impact of price on thermal comfort is illustrated by the evolution of the balance temperature, which only varies within the analysis as a function of gas price; it generally decreases over time.

3.5. Gas use dependence on price

Fig. 4 shows the dependence of UK domestic gas use on the annual consumer gas price index, indicating good agreement between the data and model. Data points are not adjusted for annual or seasonal variations in temperature and model predictions

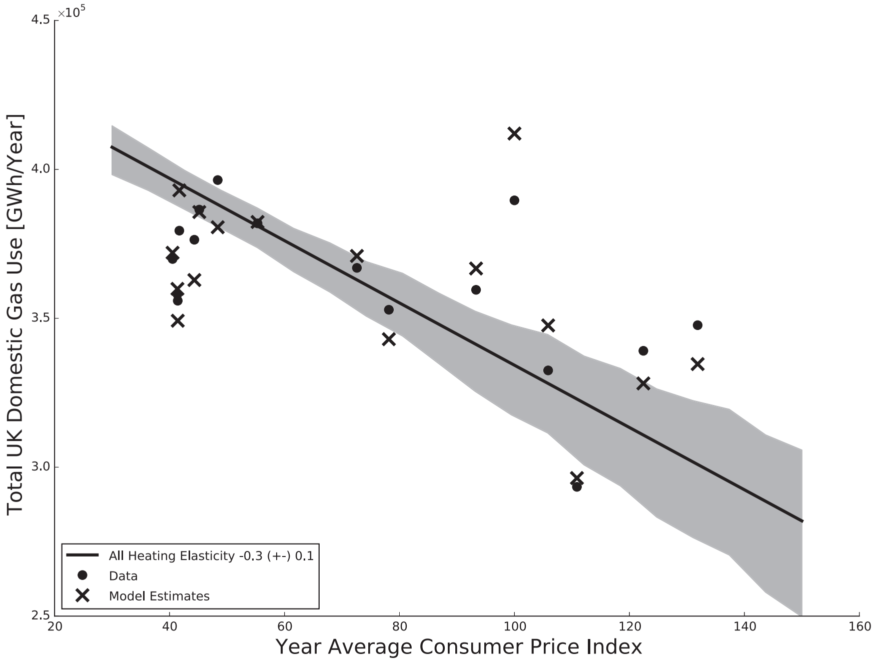

Fig. 4. UK gas use per quarter vs. year average consumer price index for gas. Circles represent data, crosses model estimates and the price elasticity of gas use, estimated from Eq. (20), is indicated by the line.

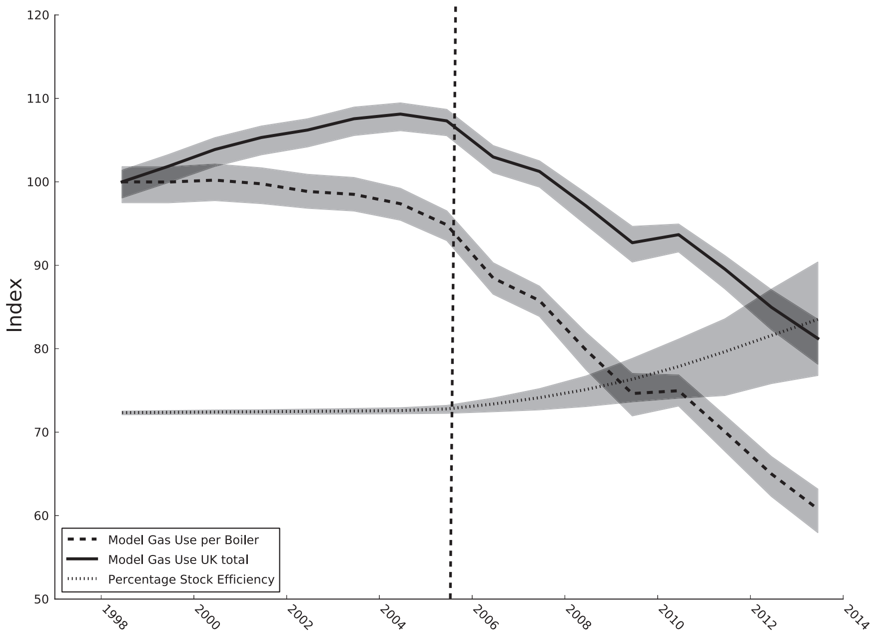

Fig. 5. The modelled underlying trend in the total UK gas use (solid line), gas use per boiler (dashed line) shown as an index with the 1998 value set to 100 and estimated domestic boiler stock efficiency (dotted line). To remove the effects of the temperature variations, the average temperature between 1998 and 2013 is used by model III. The effects of price and improved efficiency determine the reduction in gas use, whereas the increasing number of boilers increases gas use. The dashed vertical line indicates the date of the introduction of the regulation to promote the uptake of condensing boilers.

incorporate this factor. The long-run price elasticity of domestic gas use, $\mathcal{E}$, is indicated by the line on the graph and was estimated over the full period of data availability, according to

$$\mathcal{E} = \frac{\mathrm{d}E}{\mathrm{d}P}\frac{P}{E} \tag{20}$$

Average values were used for the external temperature, number of conventional and condensing boilers to provide a modelled best fit to the total period. The uncertainties account for variations in T_{EXT}, N_C and N_B in addition to the error analysis described above. Estimated total UK domestic gas use price elasticity is -0.3 ± 0.1, consistent with the value estimated by Summerfield et al. (2010) and that estimated in the Netherlands by Koopmans and te Velde (2001) of -0.28. Deeming gas use with no

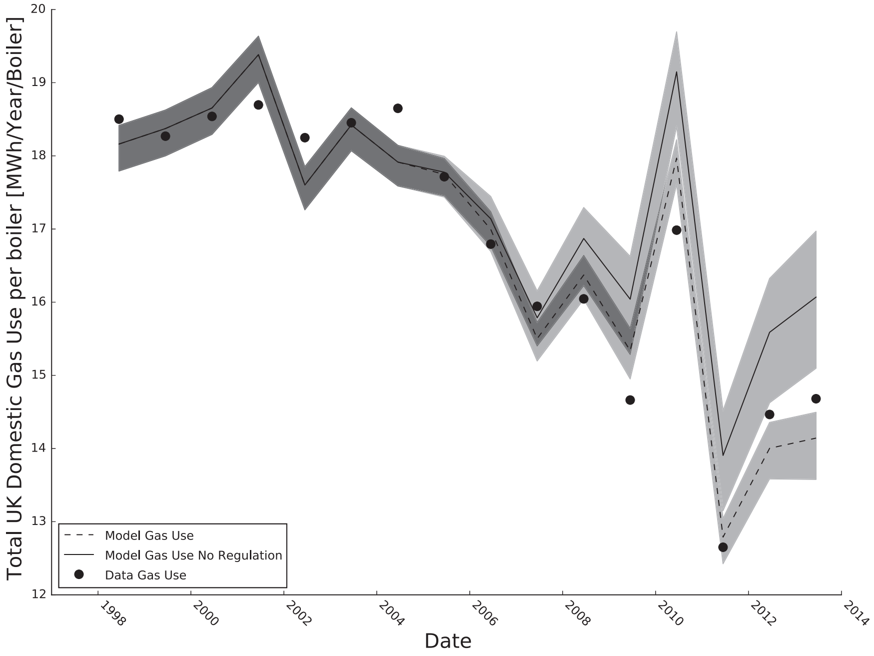

Fig. 6. The yearly gas use per boiler. Data is represented by points. The dashed line with shaded uncertainty bands is the prediction from model III using the best fit parameters. The solid line with shaded uncertainty bands is the prediction from model III using the best fit parameters, but assuming the Condensing Boiler Legislation was not introduced in 2005.

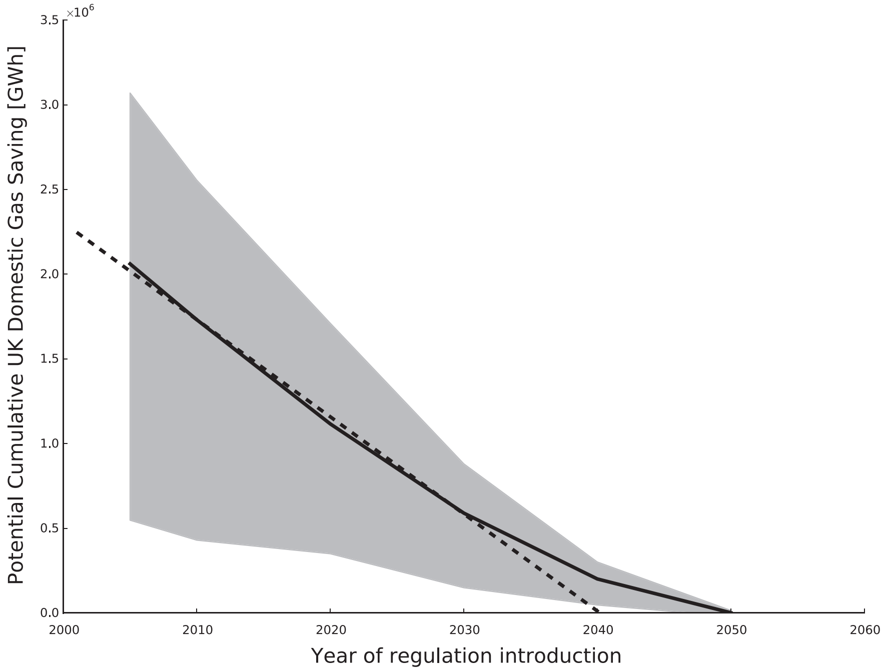

Fig. 7. The solid line shows the potential cumulative domestic gas use savings with shaded error bars over time, until 2050. The dashed line shows a simple fit to the predictions between 2005 and 2030.

temperature dependency, summer use, to be water heating, as in Section 2.3, the price elasticities of space and water heating were estimated as -0.2 ± 0.1 and -0.3 ± 0.2 respectively.

3.6. *The impact of Condensing Boiler Legislation on UK gas use*

The impact of the Condensing Boiler Legislation is explored in this section, using the model and best fit parameters developed above. The modelled trend in total UK domestic gas use, removing the effects of changes in external temperature, and domestic gas use per boiler is shown in Fig. 5. Increases in total UK gas use are driven by increases in the number of boilers in the stock while decreases in gas use are driven by increases in gas price and improving stock efficiency; estimated average boiler efficiency is also shown in Fig. 5.

Fig. 6 shows the modelled impact of the Condensing Boiler Legislation on UK gas use since 1998. Using the pre-regulation rate as described in Section 3.7, model outputs suggest a significant reduction in UK gas use associated with the Condensing Boiler Legislation, $\approx 176,000^{+86,000}_{-127,000}$ GW h, or 32^{+16}_{-23} MTons of CO_2e,[9] between 2005 and 2013, compared to an average annual gas demand in that period of $360,000 \pm 25,000$ GW h/year or 66 ± 5 MTons of CO_2e.

3.7. *Future impacts of the Condensing Boiler Legislation*

The potential long term impacts of the 2005 Condensing Boiler Legislation on UK gas use and the effect of delaying its introduction were investigated using the predictions of model III.

Fig. 6 shows the yearly gas use with and without the Condensing Boiler Legislation for the years 2005–2013. Fig. 5 shows the change in stock efficiency with the regulation. The stock efficiency without the regulation is not shown, but can also be calculated assuming that the post 2005 condensing boiler uptake rate (r_{C1}) continues at its pre 2005 value ($r_{C1} = r_{C2} = 0.0003$ million boilers per day). There is a linear relationship between the difference in the yearly gas use and the average difference in stock efficiency for each year, which is, to first order, independent of temperature or price. Variations in the difference in gas use are obviously influenced by price and temperature, but in the years 2005–2013 their influence is much smaller than the difference due to stock efficiency improvements. The predicted gas use drops by 5000 ± 1000 GW h/year for every 1% increase in the average stock efficiency.

The impact of different dates of introduction of the Condensing Boiler Legislation was studied by changing the date at which the rate r_{C1} changes to r_{C2} in the model. Six scenarios were investigated: introduced in 2005, 2010, 2020, 2030, 2040 and not before 2050 (Fig. 1). The different scenarios for the evolution of the boilers in the stock, with associated errors, were then used to create projections of the efficiency of the stock for each scenario up to the year 2050. Assuming that the relationship between the difference in the yearly gas use to the average difference in stock efficiency continues into the future[10] it can used, together with predictions of the differences in efficiency, to project the longer term impact of the Condensing Boiler Legislation.

Fig. 7 highlights the decreasing impact of the legislation the later it is introduced. The dashed line in Fig. 7 shows a linear fit to the predictions between 2005 and 2030 to provide an estimate of the domestic gas savings per year associated with delaying introduction of the policy of approximately 60,000 GW h/year.

While the 68% credibility limits are large due to the long time span of the projections, Fig. 7 highlights the significant gas use and

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[9] Assuming a conversion factor of 0.18404 kg CO_2e per kW h, that for natural gas (Carbon Trust, 2013). This factor ignores the energy costs and emissions associated with supplying gas to the individual consumer's gas meter, and is therefore likely to understate the actual impact of domestic gas consumption. It is easy to scale the results presented here for different values of this factor.

[10] This implicitly assumes that other physical features of the stock affecting heat loss, the climate, price and behaviour are similar to the 1998 to 2013 period. It is not possible to account for the effects of unknown future contextual changes, such as the introduction of new legislation, the impact of gas prices on boiler installations and the effect of emerging and potentially disruptive technologies (such as heat pumps). Contextual and counter-factual indeterminacy is likely to become progressively more important, and thus the predictions of this analysis less reliable, as time goes on.

carbon savings that may be associated with the introduction of legislation, in 2005, to require new boilers to be condensing. Energy savings by 2050 total $2,000,000^{+1,000,000}_{-1,500,000}$ GW h (or 368^{+184}_{-276} MTons of CO_2e), approximately 5.6 times the average UK gas demand of approximately 360,000 ± 25,000 GW h (or 66 ± 5 MTons of CO_2e).

4. Conclusions and policy implications

Analysis of the efficacy of policies to reduce energy use and carbon emissions is challenging due to the complicated interactions of economy, behaviour, weather, building and technology performance. The core purpose of this paper has been to provide quantitative estimates, with a comprehensive analysis of errors, of the reduction in UK domestic gas consumption, and therefore of CO_2e emissions, through an analysis based on published and widely available statistics, brought about by the introduction of the 2005 Condensing Boiler Legislation. This paper presents the first such investigation; as far as we are aware. The paper also presents the first application of Bayesian methods incorporating model comparison, combined with physical models, to investigate the efficacy of energy demand policy. The Condensing Boiler Legislation was introduced as an addendum to the 2002 Edition of the Building Regulations for England & Wales, prior to the 2006 revision of Part L (Conservation of fuel and power) (ODPM, 2005, 2006a,b) but these two editions represent very different approaches to regulation. Whereas the 2006 revision of Part L is complicated, allowing designers a range of options for compliance and supporting design freedom within a carbon target, the Condensing Boiler Legislation, still in effect, is simple and prescriptive, requiring condensing instead of non-condensing boilers to be used in most circumstances relating to new housing or boiler replacement.

It is likely that in the absence of the Condensing Boiler Legislation, the 2006 revision of Part L would have triggered a shift in the market for condensing boilers. However, this may not have occurred immediately; this analysis suggests that the replacement boiler market is, at peak in August 2012, ~2.1 million per year, compared with just ~210,000 installations per year of boilers for the first time, and ~150,000 new dwellings per year in the stock (Section 3.1). Among the reasons for this is the fact that anecdotal evidence (we are unaware of any formal analysis) suggests that the application of the 2006 and subsequent revisions to Part L to existing dwellings has been haphazard and unenforced.

Simple models have been developed and their probability of representing the observed data compared, to explore the relationship between quarterly gas use and external temperature, consumer gas price index, the number of boilers and condensing boilers. A technological uptake model based on the logistic equation was employed to predict the number of boilers and condensing boilers in England. It was not possible to account for market disruptions due to unknown future technologies, policies and economic factors. Bayesian analysis was utilised to estimate best fit parameters, error and to compare the probability of each model representing the observed data, incorporating an Occam's factor to account for model complexity.

All models incorporated a linear dependence of gas use on the number of boilers and their efficiency. Gas use was divided into two components: that with a higher temperature-dependence, assumed to be dominated by space heating, and with low or zero temperature dependence, assumed to be dominated by water heating. The model best able to describe the observed data incorporates a linear dependence of space heating requirements on external temperature and consumer gas price index. It also includes a separate price dependency for water heating, which was not temperature dependent.

The model best able to describe the data provides a good fit to observed gas use and highlights a strong dependence on external temperature, the number of boilers and their efficiency and the consumer gas price index. A boiler efficiency improvement of 1.25 ± 0.15 upon the replacement of a conventional with a condensing boiler has been derived; the specific heat loss per dwelling is 215 ± 10 W/°C. The balance temperature is a function of price only in the selected model and reduces from 14.7 ± 0.3°C in 1998 to 13.7 ± 0.5°C in 2013, suggesting the observed increase in gas price over this period has significantly lowered demand.

The model was used to estimate the total energy savings associated with the Condensing Boiler Legislation; between 2005 and 2013 this is $176,000^{+86,000}_{-127,000}$ GW h (or 32^{+16}_{-23} MTons of CO_2e). Models of future technology uptake were then combined with the models of energy demand to project energy savings, approximately 60,000 GW h per year may be saved between 2005 and 2030 by the Condensing Boiler Legislation. This analysis suggests that carbon savings in 2020 associated with the Condensing Boiler Legislation may be as high as 11 MT CO_2e, significantly higher than the conservative estimate of 2 MT CO_2e for boiler replacements only by the Committee on Climate Change (2008). Our estimate of total energy savings by 2050 associated with the 2005 legislation is $2,000,000^{+1,000,000}_{-1,500,000}$ GW h (or 368^{+184}_{-276} MTons of CO_2e), approximately 5.6 times the average UK gas demand of approximately 360,000 ± 25,000 GW h (or 66 ± 5 MTons of CO_2e).

The above carbon and energy savings highlight the efficacy of the Condensing Boiler Legislation, which meets simple criteria for effective policy deployment (Lowe, 2000): it is unambiguous, clear to installers and homeowners, and simple to enforce through gas safety and accreditation mechanisms, in contrast to the complexity of the Building Regulations Part L. The cost effectiveness of replacing existing boilers with new condensing boilers is difficult to estimate due to the changing and non-linear nature of market prices for condensing and non-condensing boilers and the present authors have not set out to provide an estimate. Published marginal abatement cost curves, and associated analysis suggest a cost of CO_2e saved per £45 per tonne (Committee on Climate Change, 2008). It therefore appears that the Condensing Boiler Legislation has led to significant carbon and energy savings at a cost of CO_2e that is close to the UK Government's historic shadow price of CO_2e (Clarkson and Deyes, 2002), although significantly above the current market price defined by the European Union emissions trading system (EUETS). Since carbon emissions have a long life time in the atmosphere, the analysis presented here supports the early introduction of such polices to maximise the impact on climate change.

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