



A heuristic model of bounded route choice in urban areas



E.J. Manley^{a,b,*}, S.W. Orr^b, T. Cheng^b

^a Centre for Advanced Spatial Analysis, University College London, Gower Street, London WC1E 6BT, United Kingdom

^b Department of Civil, Environment and Geomatic Engineering, University College London, Gower Street, London WC1E 6BT, United Kingdom

ARTICLE INFO

Article history:

Received 11 July 2014

Received in revised form 28 January 2015

Accepted 12 March 2015

Available online 19 April 2015

Keywords:

Route choice

Heuristics

Road transportation

Bounded rationality

Spatial cognition

ABSTRACT

There is substantial evidence to indicate that route choice in urban areas is complex cognitive process, conducted under uncertainty and formed on partial perspectives. Yet, conventional route choice models continue make simplistic assumptions around the nature of human cognitive ability, memory and preference. In this paper, a novel framework for route choice in urban areas is introduced, aiming to more accurately reflect the uncertain, bounded nature of route choice decision making. Two main advances are introduced. The first involves the definition of a hierarchical model of space representing the relationship between urban features and human cognition, combining findings from both the extensive previous literature on spatial cognition and a large route choice dataset. The second advance involves the development of heuristic rules for route choice decisions, building upon the hierarchical model of urban space. The heuristics describe the process by which quick, 'good enough' decisions are made when individuals are faced with uncertainty. This element of the model is once more constructed and parameterised according to findings from prior research and the trends identified within a large routing dataset. The paper outlines the implementation of the framework within a real-world context, validating the results against observed behaviours. Conclusions are offered as to the extension and improvement of this approach, outlining its potential as an alternative to other route choice modelling frameworks.

© 2015 The Authors. Published by Elsevier Ltd. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Over recent years there has been a growing recognition of the fundamental limitations of traditional route choice modelling. The increasing availability of fine-grained route choice analyses (Nakayama et al., 2001; Papinski et al., 2009), combined with improved analytical capabilities afforded by enhanced computational power, have prompted a reconsideration of the conventions ascribed within Wardrop's Equilibrium (Wardrop, 1952). Wardrop's behavioural assumptions of pure rationality, homogeneity and unlimited knowledge and foresight, inherent within many route choice models, have been questioned by numerous authors (Garling, 1998; Avineri, 2012).

In response, a number of new models have emerged that aim to tackle existing limitations. One strong focus has rested upon the bounded nature of route choice decision making. Bounded rationality has been an important research stream in behavioural economics for some time (Simon, 1957; Sen, 1977), yet the methodologies developed within this field have only recently filtered into route choice modelling. By far the greatest application of behavioural economics within transportation

* Corresponding author at: Centre for Advanced Spatial Analysis, University College London, Gower Street, London WC1E 6BT, United Kingdom. Tel.: +44 (0) 20 3108 3884.

E-mail addresses: ed.manley@ucl.ac.uk (E.J. Manley), s.orr@ucl.ac.uk (S.W. Orr), tao.cheng@ucl.ac.uk (T. Cheng).

behaviour has involved the incorporation of Prospect Theory and Cumulative Prospect Theory (Avineri and Bovy, 2008; Gao et al., 2010; Ben-Elia and Shiftan, 2010; Xu et al., 2011), building on the research of psychologists Daniel Kahneman and Amos Tversky (Kahneman and Tversky, 1979). The Prospect Theory model aims to reflect the non-linear relationship between risk – such as the risk of delay involved in choosing a route – and choice by augmenting the utility of potential alternatives. While these models have brought an enhanced sophistication to the modelling of travel behaviour, an extension into the wider potential offered by behavioural economics has yet to emerge (Avineri, 2012).

An alternative approach to modelling bounded decisions, not previously incorporated within route choice modelling, has been offered in the form of *heuristic decision-making*, a theory put forward by Gerd Gigerenzer and Daniel Goldstein (Gigerenzer and Goldstein, 1996). This theory states that when individuals are required to make decisions under uncertainty, they use simple rules and inferences to determine the relative value of each alternative. The heuristic approach is presented as an alternative to other econometric models of decision-making (including Prospect Theory). As Gigerenzer states: 'Logic and probability are mathematically beautiful and elegant systems. But they do not always describe how actual people ... solve problems' (Gigerenzer, 2008).

Central to the heuristic decision-making framework is the concept that the human cognition contains a 'toolbox' of rule-sets that enable the completion of both simple and complex tasks in a quick and efficient manner (Gigerenzer, 2004). These rule-sets, known as heuristics, impose low cognitive computational load and require minimal volumes of information in order to reach a decision (Gigerenzer and Todd, 1999). The heuristics reflect how individuals use cognitive shortcuts to reach intuitively correct decisions.

A number of heuristic frameworks have been outlined and validated (Snook and Cullen, 2006; Pohl, 2006; Goldstein and Gigerenzer, 2002; Brandstatter et al., 2006), each modelling decision making processes through simple rule-based structures. The most widely applied, the Take-The-Best (TTB) heuristic, deals in differentiating between multifaceted alternatives, basing judgement on ranked *cues*, where each cue describes a separate attribute of each alternative. In traversing the set of cues in order, judging between two alternatives, a selection is made at the point of the first significant deviation between the cue values, stopping the decision process and selecting the alternative with the higher cue value. This process is known as 'one reason' decision making, replicates how individuals use cues as indicators of broader utility, when faced with scarce information. This simple inferencing process has been demonstrated to work effectively across a range of scenarios, including investment decisions (Bröder, 2000), court judgements (Dhimi, 2004), political election strategies (Graefe and Armstrong, 2012) and medical decisions (Dhimi and Harries, 2001). The TTB heuristic has also been demonstrated to match or outperform multiple regression (Czerlinski et al., 1999) and machine learning algorithms (Brighton et al., 2006).

While heuristic decision-making represents an alternative method for choice modelling, research into spatial cognition offers an alternative approach for describing how individuals make choices in urban space. Many conventional route choice models typically represent urban space through the road network and its attributes alone. However, research from neuroscience, cognitive science and behavioural geography indicates that the relationship between individuals and space is much more complex. Instead it has been shown that urban space is mentally encoded and recalled by individuals within a hierarchical structure (Montello, 1998; Tversky, 1993; Golledge et al., 1985; Hirtle and Jonides, 1985), where certain salient features are more strongly recalled by individuals, and so feature centrally within the route choice decision process. In a highly influential work, urban planner Kevin Lynch identified five types of urban feature – paths, nodes, districts, edges and landmarks – as forming the basis of spatial knowledge (Lynch, 1960). The process of the brain recording space as a series of point (O'Keefe and Dostrovsky, 1971; O'Keefe and Nadel, 1978) and region-based (Hafting et al., 2005; Solstad et al., 2008) objects has been furthermore observed in neurological studies, while others have outlined the process by which the brain uses these features during navigation (Wiener and Mallot, 2003; Wiener et al., 2004). A more recent study, using large-scale GPS routing data, has identified non-linear attraction to particular locations on the road network during navigation, resulting in widespread deviation from more optimal alternative routes (Manley et al., 2015). Despite the evidence supporting the importance of the spatial hierarchy in navigation decision-making, application of this model has not been widely applied within transportation studies. While a few route choice models have incorporated landmarks and anchor points as a potential influence on navigation (Prato et al., 2012; Kaplan and Prato, 2012; Arentze et al. 2014; Chown et al., 1995), the modelling of route choice within a complete hierarchical representation of space has not yet been undertaken.

This paper draws together research into heuristic decision making and the cognition of space in introducing a novel approach to modelling route choice decision making. The route choice framework encodes choices as sets of simple heuristics, based on a hierarchical spatial structure corresponding with the construction of cognitive spatial knowledge. The heuristic modelling approach represents an alternative methodology to the utility maximising paradigm currently dominant in route choice modelling. The use of a hierarchical representation of space recognises that navigation decisions in urban areas may be based on an array of spatial features, rather than purely road connectivity and route conditions. While this paper will introduce one configuration and implementation of this new framework for route choice modelling, it is hoped that the framework will serve as an alternative methodology for future route choice modelling, within wider urban transportation modelling stacks where individual-level behaviours are represented (e.g. activity-based, agent-based models).

To achieve these objectives, this paper integrates previous findings within the literature with observations drawn from a large dataset of route traces, leading to the development of a new route choice modelling framework. The paper presents the implementation of the model within a real-world environment. The paper is structured as follows. The next section describes the route dataset that will support the development of the model framework. The following third section outlines the development of the hierarchical model of urban space across which route choices will be made. The model is based on the

principles of spatial cognition and memory established within the literature. The fourth section details the development of route choice heuristics, describing the decision process by which a route is chosen through urban space. A description of the implementation and validation of the complete modelling approach for London, United Kingdom is presented in Section 5. The final section concludes in discussing the main advances presented by this approach in addition to the potential for improvement and elaboration.

2. Route dataset

The development of the model is supported by a large dataset of minicab route choices. This dataset contains the routes of 690,045 journeys undertaken by 2970 Addison Lee minicab drivers within London, United Kingdom, between December 2010 and February 2011. The routes are constructed from GPS point data, and aligned with job records, providing complete origin–destination routes.

It should be noted that the Addison Lee drivers are not expected to have held professional driving positions in London previously, nor are they required to have passed ‘The Knowledge’ test, required of Black Cab drivers in London. While Addison Lee do provide their new drivers with some training, it is expected that there is considerable variance in the knowledge of London amongst the drivers. This has been confirmed through analyses carried out during earlier research, where drivers were found to not follow optimal paths and the widespread use of satellite navigation routing was ruled out (Manley et al., 2015).

The dataset is split into calibration and validation partitions according to a 80:20 percentage split of the randomly sorted routes. This divides the dataset into 552,036 routes for the construction and calibration of the models, reserving the remaining 138,009 routes for the final validation stage.

3. Modelling hierarchical urban space

As described earlier, cognition and memory of space is influenced by an array of spatial features, used distinctly during the course of route choice. One widespread interpretation within the literature is that urban space is mentally encoded within a hierarchy, where less prominent objects are recalled within the context of more salient features. In defining urban space for heuristic decision-making, a spatial hierarchy is developed, with each level of the hierarchy taking on a different role during route choice.

Spatial hierarchies have been shown to exist at a number of scales. Stevens and Coupe identified a hierarchy of cities (Stevens and Coupe, 1978), while Hirtle and Jonides presented a hierarchical organisation of landmarks (Hirtle and Jonides, 1985). Other models incorporate different types of urban feature – Golledge presented a hierarchical model that linked landmarks to paths to within areas (Golledge, 1978); Montello outlined a model of spatial memory that moved from landmarks to routes and then to survey (or layout) knowledge (Montello, 1998); Tomko and Winter present a framework structured around Lynch’s urban elements (Tomko and Winter, 2013; Lynch, 1960); while Richter and colleagues introduce a hierarchical network approach for modelling indoor spaces (Richter et al., 2011).

Like those proposed by Golledge and Montello, the model developed for this work represents the movement from coarse to granular spatial knowledge. In this case, however, the model aims to reflect how spatial objects are generalised and recalled for the purposes of spatial decisions. As such, the urban environment is broken down into *regions*, *nodes*, and *roads*, where each feature type is encapsulated within the former as part of a hierarchy. A conceptual representation of the arrangement of these layers is shown in Fig. 1.

The implication of this schema is that each level of the hierarchy is involved in a different type of route choice decision. This model replicates the cognitive of spatial features during route finding described elsewhere (Wiener and Mallot, 2003; Wiener et al., 2004). Regions are positioned at the top of the hierarchy, used for the formation of a coarse *route plan* that only broadly directs subsequent route choices. Nodes fall below regions, referring to decision points on the road network, used in the construction of more specific paths made within the constraints of the previously defined regional route plan. While roads, in being at the base of the hierarchy, and following the previous decisions made at regional and nodal level, are involved only in the execution of the route between nodes.

The remainder of this section will outline the implementation of this hierarchical representation of space. Each level of the hierarchy is built from the spatial representation below it, as such, the models are presented here in reverse order, beginning with the road segments, ending with the region definitions. The model has been implemented in London, United Kingdom.

3.1. Roads

Roads sit at the base of the spatial hierarchy. They are involved in the execution of route plans defined at higher levels of the hierarchy, based on regionalised interpretations of the urban area. As such only a simple road network dataset is required. In the case of London, the Ordnance Survey Integration Transport Network (ITN) dataset, a GIS dataset covering all road segments within the United Kingdom, is used. All directional, turn and speed regulations are included within the road network model.

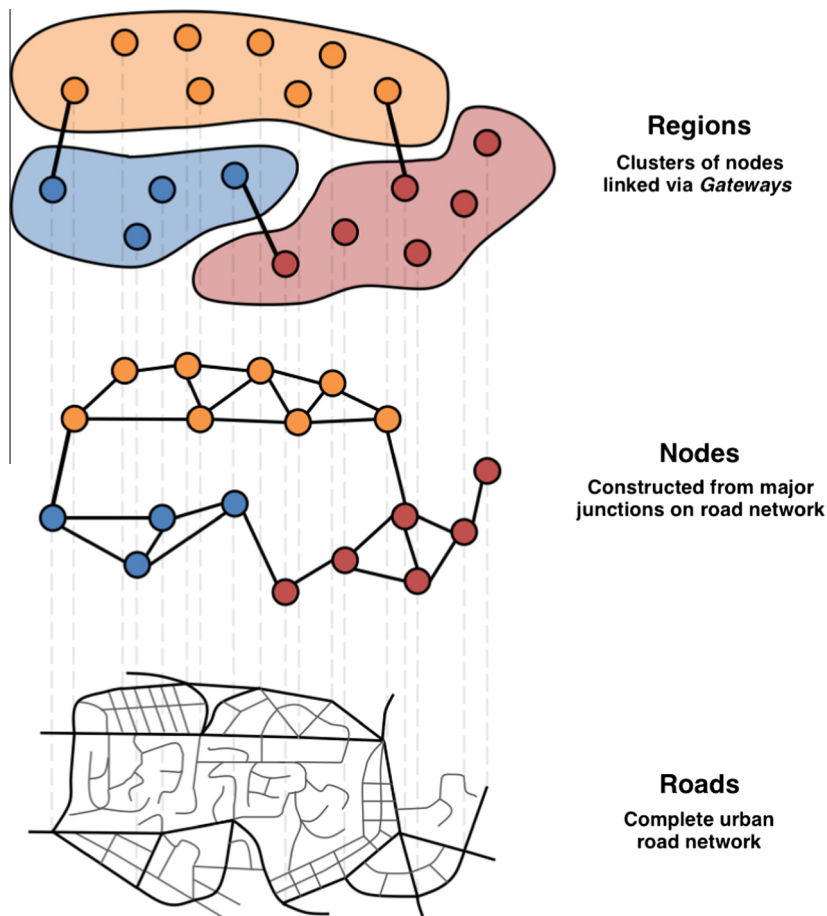


Fig. 1. Conceptual representation of hierarchical model of space, indicating the connections between each hierarchy of the spatial representation.

3.2. Nodes

Nodes represent locations in urban space that prompt a route choice decision, points where a number of alternative choices are available to the decision-maker. The locations chosen to represent these points are road junctions (or intersections), linking with the road network and enclosed within a regional model above it. At these locations, individuals are unable to proceed as they may have been previously, and are faced with an intermediate choice about their overall route plan. At this point they make a choice to move to another node, based on the region-based route plan, and this choice is executed on the road network.

The use of road junctions as an indicator of a decision point is well supported in the literature. Lynch noted that junctions as have ‘compelling importance for the city observer’, being places where ‘decisions must be made’, and where ‘people heighten their attention’ (Lynch, 1960). Passini (Passini, 1984) reported how individuals use particular locations during route planning, associating salient features with actions (e.g. turn left). The notion that point-like features are highly important in spatial memory and navigation has been widely recognised, both through neuroscience and behavioural studies (O’Keefe and Dostrovsky, 1971; Siegel and White, 1975; Couclelis et al., 1987; Tversky, 1993; Winter et al., 2008). While these features refer to any point-like location, including salient buildings and landmarks, the process of driving ensures greater engagement with road junctions, rather than buildings.

The construction of nodes builds on the intersections formed in road network. However, clearly not all junctions are important navigation points, nor are all junctions recalled in the same way. As such, junctions will be ranked according to their importance. Many road network datasets incorporate a categorisation system providing a hierarchical indication of a road’s importance. In defining nodes, the classifications of two intersecting roads is used to rank the importance of a particular junction. In the case of the Ordnance Survey ITN datasets, the classification includes six road classes. Taking these classes, and removing the most minor classifications – Local Streets and Alleys – a four-point node ranking is constructed as follows:

1. Junctions between only Motorways and A-Roads.
2. Junctions linking B-Roads to Motorways or A-Roads.
3. Junctions between B-Roads.
4. Junctions between Minor Roads and Motorways, A-Roads or B-Roads.

With the nodes defined, a directed network is constructed based on the recorded flow between each pair of junctions. By extracting the number of routes passing between each pair of junctions, the weighted network encodes the connectivity of junctions, reflecting the likelihood of an individual travelling to any connecting node based on their current location. To preserve the node hierarchy, separate weighted networks are constructed for each level of the node hierarchy – at level 1, only Motorway and A-Road junctions are included, whereas at level 4, all junctions (beyond Local Streets and Alleys) are incorporated. Each network is constructed using the calibration dataset.

It should be noted that the creation of a hierarchy is not a strict condition of this methodology. While the volume of junctions considered during this stage of route choice will be considerably lower than that of the number of road segments, a threshold may be used in place of a hierarchy (e.g. using only junctions associated with Motorways and A-Roads). As will be shown later, however, the hierarchy enables the modeller to reflect variation in a decision-makers awareness of a node, reflecting a transition from detailed (all nodes used) to coarse (only higher level nodes used) knowledge of space.

3.3. Regions

Regions represent the least granular representation of the urban environment. Through a regional definition, individuals generalise about an entire area of the city, easing the requirements upon cognition during decision-making. Using the region definitions, a route plan is constructed which informs which nodes will be selected. Regional boundaries are formed again using lower spatial representations, in this case node definitions. Regions represent clusters of nodes, where nodes share a particular characteristic causing them to be considered part of a structurally homogenous entity.

There have been few attempts made to quantify how neighbourhoods are perceived and recalled by city residents. While many conventional boundary definitions exist, these usually relate to administrative or electoral boundaries, and tend not to reflect how individuals perceive regions. Instead individuals, as Lynch argues, identify regions through shared 'texture, space, form, detail, symbol, building type, use, activity, inhabitants, degree of maintenance, topography' (Lynch, 1960). Where regionalisation in urban space has been studied, participants have engaged with controlled, virtual environments (Wiener and Mallot, 2003, 2006; Kuipers et al., 2003; O'Sullivan, 2009), without a direct relationship with real-world spatial features.

3.3.1. Region definition: method

In view of the absence of an established methodology, regions are constructed for London from the observed behaviours of the minicab drivers. For this purpose, the node network outlined in SubSection 3.2 is used. This weighted network is segmented using *community detection methods*, breaking the network into regions that are internally strongly interconnected, while less well connected with external nodes. Strongly interconnected regions reflect common usage – if one node is used, it is more likely that another within the same region will be used subsequently – and thus is indicative of a perceived homogeneity. In this sense, regions do not follow those definitions suggested by Lynch, but instead follow the functional nature of route choice.

The community detection algorithm used to identify regions within the weighted network was the *Louvain Method* (Blondel et al., 2008). The Louvain Method approach builds communities through the agglomeration of nodes into groups. As nodes are added to a group, their impact on a modularity metric is calculated. Modularity (known as Q) is measured as the degree of connectivity within a region in relation to random network arrangement, with a higher values indicative of tightly-grouped regions. The method will continue to change the configuration of communities until no further improvements can be made to the modularity score.

Aside from optimising for modularity, the construction of communities using this approach is influenced by two additional factors. First, the method incorporates stochasticity – and so variation in regional definitions – that must be accounted for during region production. Second, it incorporates a resolution parameter (R) that influences community size. This parameter is incorporated within the measurement of modularity, influencing the strength of attraction between nodes within groups. Where values set to below 1, a stronger inter-community modularity is required than necessary through the standard modularity function, resulting in smaller, more tightly-clustered groups.

3.3.2. Region definition: results

In view of the influence of stochasticity and the potential role of varied resolution, the production of regional definitions is carried out over a number of trials. Resolution settings were furthermore tested five times each to test for variance in cluster size and specification.

As the results of the trials shown in Table 1 demonstrates, little variation in modularity score was found both between trials and as the resolution parameter is adjusted. The indication is that, no matter what resolution parameter is selected, a reasonable distinction of regions can be made by the algorithm. However, greater insight is found in visually inspecting the generated regions, allowing an assessment of their alignment with known regional definitions and the identification of variation in boundary definitions between trials.

Table 1
Resolution parameter evaluation.

| R | Trial 1 | | Trial 2 | | Trial 3 | | Trial 4 | | Trial 5 | |
|-----|---------|----------|---------|----------|---------|----------|---------|----------|---------|----------|
| | Q | Clusters | Q | Clusters | Q | Clusters | Q | Clusters | Q | Clusters |
| 1 | 0.785 | 80 | 0.788 | 77 | 0.786 | 77 | 0.785 | 79 | 0.786 | 79 |
| 0.8 | 0.787 | 81 | 0.781 | 80 | 0.786 | 80 | 0.779 | 81 | 0.789 | 81 |
| 0.6 | 0.783 | 85 | 0.778 | 86 | 0.778 | 86 | 0.789 | 82 | 0.779 | 84 |
| 0.4 | 0.768 | 94 | 0.766 | 93 | 0.77 | 92 | 0.765 | 91 | 0.767 | 92 |
| 0.2 | 0.732 | 113 | 0.733 | 112 | 0.733 | 110 | 0.735 | 109 | 0.736 | 110 |

Through visual inspection it is found that very little variation in region definition is found between trials, despite the stochasticity inherent to the Louvain Method. As such, any ‘averaging’ of regional definitions across multiple trials was considered unnecessary. On inspecting the regions generated at each resolution, there are indications that the best results are achieved with a resolution setting of 0.2. At this level,¹ as is shown in Fig. 2, some well-known areas of Central London are extracted, including the City of London, Soho, Camden and Knightsbridge. Regions are also defined along with some of the important routes in and around Central London. Interestingly, in many cases, major roads are divided into multiple regions, suggesting that these roads are used in sections rather than in their entirety.

The resulting regional definitions describe which nodes are used together and, as such, may reflect the way in which drivers recall regions during vehicular navigation. Using these definitions, each node is assigned to a region. Regions are then linked where node-to-node connections between regions exist. These points will be referred to as *gateways*, and reflect the locations at which an individual choose to move between one region and another (indicated in Fig. 1 as the links between regions).

4. Heuristic modelling of route choice

The structure of the heuristic route choice model aligns with the model of space defined above. In travelling between origin and destination, the decision-maker moves from a coarse to a finely-grained route plan. In the first level of the hierarchy, individuals make region-based choices, forming their rough plan within which subsequent finer-grained choices are constrained. In the second tier, individuals refine their route based on the nodes within the previously chosen regions. In the lowest level, individuals select the roads that link together the chosen nodes, arriving at the final chosen route.

An example of this process is outlined in Fig. 3, showing how an individual's choice of regions (defined by origin and destination points) limits the subsequent selection of nodes, which in turn limits the selection of road segments. Fig. 3 highlights the choice rulesets involved at each level of the hierarchy, more detail on each will be provided within this section.

This section will describe the specification and calibration of each the three choice models. At each level, heuristic choice models are utilised, reflecting how quick, ‘good enough’ decisions are taken under uncertainty (Gigerenzer, 2008). Following the definition of these models, a final subsection will describe how behavioural heterogeneity is introduced through estimation error and bounded spatial knowledge.

4.1. Region-based choice

The selection of regions towards a destination represents the most strategic level of the route planning model. At this level, an approximate path is specified, made up of regions within which more specific elements of the route will be selected. A set of heuristics are developed to represent this process of selecting a region-based route to a destination.

The region selection process is conducted in a step-by-step fashion, moving from region-to-region until the destination is found. This selection process consists of two stages – first, an identification of regions adjacent to the current region that satisfy one set of heuristics, based on an Elimination By Aspects heuristic; then following that, identification of the most favourable region from the subset of accepted alternatives, based on a Take-The-Best heuristic.

4.1.1. Stage 1 – Region Elimination By Aspects

The first stage in the decision-making process prunes the number of alternative regions to only those satisfying distinct criteria. This heuristic follows the *Elimination by Aspects* (EBA) theory of decision making, which states that alternatives will be ignored if they do not fit within specific definitions of acceptability (Tversky, 1972).

This decision process is based on *gateways*, pairs of nodes linking one region to a potential next region. The characteristics of these gateways provide the attributes on which a quick, initial judgement is made as to the suitability of a potential next region. Should a gateway be deemed attractive according to the heuristics in place, the entire region will be considered within a second, more comprehensive assessment.

The EBA ruleset eliminates all potential gateways that do not result in the decision-maker being closer to their destination. These rules specify the maximum distance and angular deviation away from the final destination that would be deemed

¹ For completeness, lower resolutions ($R = 0.1, 0.05$) were checked too, but found to reduce modularity score significantly, leading to high fracturing of central areas.

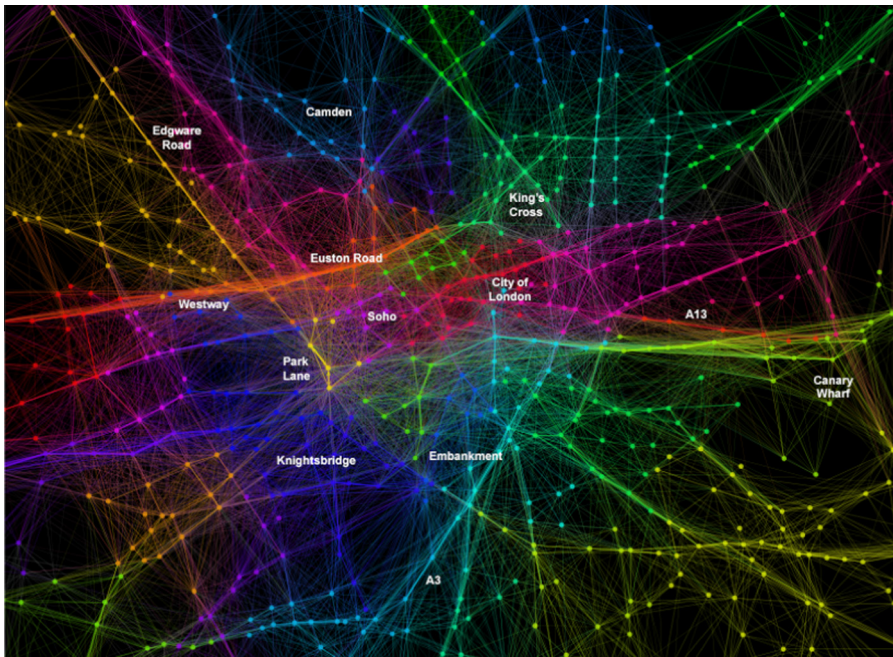


Fig. 2. Region definitions in Central London ($R = 0.2$), where each region is distinguished by a different colour. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

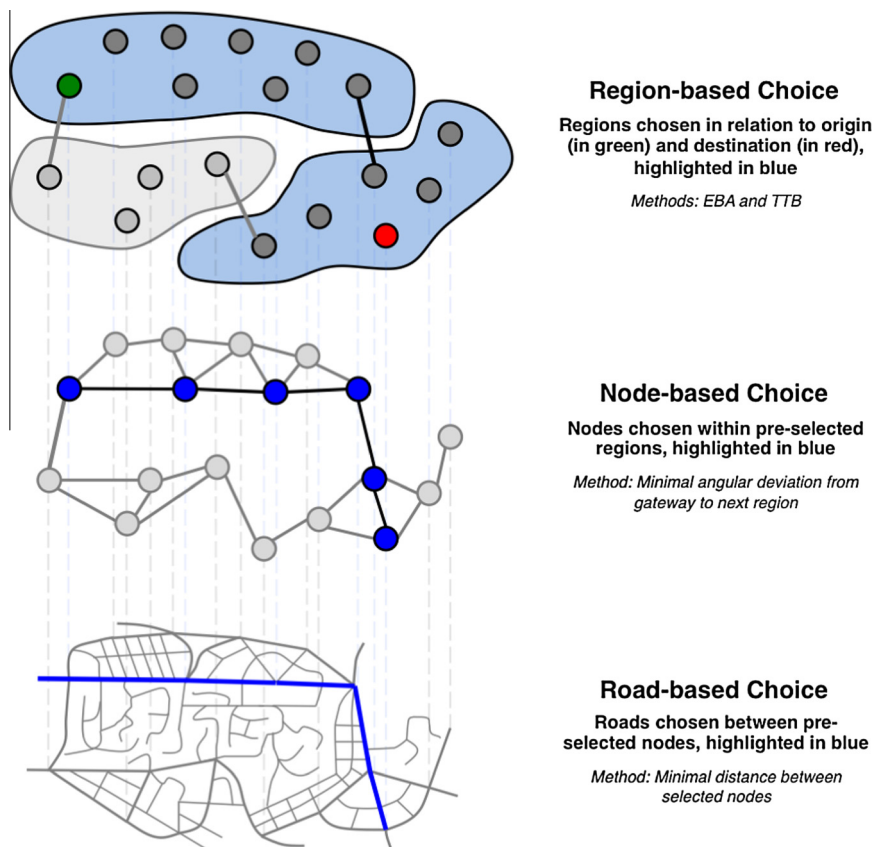


Fig. 3. Hypothetical example of the heuristic route choice process executed across the hierarchical representation of space, indicating how higher level decisions influence subsequent choices at lower levels of the spatial hierarchy. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

acceptable to the decision-maker. The rules defined here are *non-compensatory*, with each rule assessed independently with no trading off between attributes. These rules state that gateways will only be deemed attractive if they fulfill all of the stated criteria, failing to do so will mean the gateway is eliminated and not considered during the second stage of decision-making.

For a gateway to be deemed acceptable, the following criteria must be met. These rules are executed at the entry to each selected region (i.e. the end of the previously selected gateway), relative to the location of the destination. Regions may be considered for each potential gateway.

1. **Deviation from target rule:** (current location, potential gateway exit node) < (90° + angle (current location, destination)) AND angle (current location, potential gateway exit node) > (angle (current location, destination) – 90°).
2. **Distance rule:** distance (potential gateway exit node, destination) < distance (current location, destination).
3. **Gateway direction rule:** angle (potential gateway entry node, potential gateway exit node) < (90° + angle (current location, destination)) AND angle (potential gateway entry node, potential gateway exit node) > (angle (current location, destination) – 90°).

The satisfaction of each rule results in its inclusion in the set of alternatives considered in the second stage of decision-making.

4.1.2. Stage 2 - Take-The-Best Region

The second stage of decision-making aims to identify a single stand out candidate region from those preselected by the EBA heuristic. This process follows the *Take-The-Best* (TTB) heuristic, whereby the attributes of an array of alternatives are compared until the first significant distinction is identified (Gigerenzer, 2008; Bröder, 2000).

The TTB heuristic operates according to a very simple iterative process of comparisons of alternatives through single attributes, known as *cues*. The comparison of cues is carried out in order of a predefined importance of each attribute. The TTB process proceeds by running through the list of attributes, where a distinction is identified between two alternatives, the one scoring more favourably is carried forward and the other rejected from further consideration. Where no distinction is found between two or more nodes via the first cue, a second comparison is made using the next cue in the list. The process continues while no distinction is found through all specified attributes. Should there remain no distinction between two alternatives following a comparison between all attributes then an alternative is chosen at random.

The specification of the TTB model involves two stages – the specification of the cues on which decisions are made, and the order in which they will be compared. This specification of the TTB heuristic is described below. Following these descriptions, the process by which they are involved in region-based selection will be outlined.

4.1.2.1. Cue specification. The cues specified for the TTB model must reflect those preferences that individuals consider important and meaningful at a regional level. This refers to strategic elements of preference, that may be difficult to monitor at a more granular spatial resolution. The cues chosen for inclusion within this model, outlined in Table 2, incorporate factors relating to the minimisation of delay, distance travelled, and intra-regional deviation.

The cue values for each region are calculated along the least angular node path from the current location through the current region to the potential gateway. For each node-to-node pair the five metrics are calculated, with the sum providing the cue value for each region.

4.1.2.2. Cue ordering. The cue order is established by observing the choices undertaken during the trips within the calibration dataset. In order to calculate cue values at each level of the node hierarchy (see Section 3.2), the calibration dataset was split four ways, assigning 138,009 trips to each level in the hierarchy. The cue ordering observed across each trip within the calibration dataset is undertaken according to the following process:

1. Identify the order of regions traversed en route to the destination.
2. Iterating through each region, identify a range of potential next regions according to the Stage 1 decision process detailed above. Limit number of regions under consideration to two, but allowing multiple gateways into region.
3. Construct a node-to-node path to each potential next region based on a principle of minimising the angular deviation from the target gateway. For each path extract the full range of attributes according to the methodology above.
4. Rank all regions according to their utility with respect to each assessed attribute.
5. Check rankings to see whether the observed selected region is ranked highest according to any of the attributes, and where it is, the instance is recorded. A region can be ranked first according to more than one attribute, in all instances these are counted.

Through an assessment of all regional selections, a ranking is derived as to the most popular reasons for the selection of a region. The results from this process are shown in Table 3. These results indicate that individuals view the minimisation of angular deviation within the region as the strongest cue for a favourable region selection. The minimal distance to target cue is shown to be chosen with the lowest frequency, indicating that decisions are more likely made at the regional level, rather than in sole reference to the location of the target. This order of cues is applied during the execution of the TTB process.

Table 2
Implementation of cues used in selecting between regions within TTB model.

| Cue | Method of implementation |
|------------------------------------|--|
| Least total deviation from gateway | For each node-to-node traversal the angular deviation between the target node and region gateway is calculated. These deviations are summed for the complete route to provide this cue value |
| Least total distance | Given the absence of certainty with respect to the exact travelled distance between nodes – with a naturally wide variation of routes available between pairs of nodes – inter-nodal distances are calculated as the Euclidean distance between nodes. Inter-nodal distances are summed for the region path to yield the cue value |
| Least historic travel time | Calculated using travel times observed within the calibration dataset. For each <i>observed</i> node-to-node traversal the travel time is extracted. Travel times between all nodes are summed and the mean calculated for all trips. Mean values are summed for the entire region path to provide the cue value |
| Fastest historic mean speed | Calculated for the complete region path as the mean travel time over the total Euclidean distance between node-to-node pairs |
| Least distance from target | Calculated as the Euclidean distance between the gateway exit node and the destination target location |

Table 3
Cue order derived from region selections during 552,020 journeys.

| Rank | Cue | Observed selections |
|------|------------------------------------|---------------------|
| 1 | Least total deviation from gateway | 259,470 |
| 2 | Least total distance | 212,101 |
| 3 | Least time | 203,944 |
| 4 | Fastest mean speed | 141,843 |
| 5 | Least distance from target | 103,194 |
| 6 | Random | – |

4.1.2.3. *Selection process.* With the ordering of cues defined, the selection of the next region proceeds according to the TTB methodology. Alternative regions are compared using each cue in order, until only one alternative remains. These alternatives include only those regions that have passed through the initial EBA selection phase. The process is similar to that adopted during the cue ordering – potential next regions are first preselected, and then ranked according to each cue – the difference lies in step 5, where differentiating between alternatives. At this stage, instead one option that is clearly better than the alternatives is sought.

This assessment is initially made using the top ranked cue – Least Total Deviation from Gateway – and if a *clear difference* is not found between two or more alternatives, the next ranked cue is used to differentiate between those remaining alternatives. A clear difference is determined according to whether a considered alternative represents a specified percentage improvement on the current best alternative. For this attribute, three specifications will be tested during the validation phase, with 10%, 20% and 30% improvement thresholds chosen for consideration. An improvement is classed as a reduction where travel time, angular deviation and distance are concerned, or an increase in the case of the speed cue. For example, if one alternative holds an expected travel time of 200 s, and another of 178 s, the second option would be narrowly accepted as it falls outside of the 10% threshold of the alternative (e.g. 180 s).

The assessment passes through until only one alternative remains. If this is not achievable using the five separate cues then an alternative is selected at random from those that remain. The region selection process then restarts, taking that last selected as the first region in the next iteration. This process continues until the destination region is found.

4.2. Node-based choice

The choice of nodes represents an increase in the granularity of the route plan, as initially laid out through the choice of regions. This stage represents a less strategic and less cognitively intensive choice process than the selection of regions, concerned only with the traversal of small, preselected regions rather than navigation across an entire city. As such, the ruleset by which the node-based route is constructed is somewhat simpler.

The objective of this stage is to select a string of nodes that take the driver from the entrance to a region – the origin gateway – to the exit of that region – the target gateway. In view of the fine spatial scale at which these choices are made, it is assumed that the individual will be well placed to make near optimal choices. As such, the optimal node route is chosen that minimises angular deviation from the target gateway. The usage of this metric aligns with findings elsewhere within the literature that individuals aim to minimise angular deviation from their target during route choice (Dalton, 2003; Turner, 2009). Once a node route has been specified between region gateways, a route is constructed for the subsequent regions to the point at which the destination node is reached.

4.3. Road-based choice

The final stage of the route choice process involves the selection of roads. Within this model, the selection of roads is considered the least cognitively intensive process, heavily influenced by the more strategic choices made during node and region selection. Considering the fine spatial scale involved in this stage – be it navigating between subsequent nearby

nodes – there are unlikely to be many alternative routes available to an individual at this stage. As such, it was decided that roads would be selected according to a simple minimal distance measure. Road-based routes are constructed between each preselected node, and to the journey origin and destination points. This stage finalises the generation of the complete route.

4.4. Behavioural heterogeneity

The model described thus far is deterministic, not accounting for any potential variation in individual behaviour. Behavioural heterogeneity in this model is generated by introducing two additional factors into the modelling framework – one, referring to an individual's ability to reason accurately over space, and second, bounding the extent of a decision-maker's spatial knowledge.

4.4.1. Estimation error

Within the model, decisions are made in relation to unseen objects, and thus naturally based on subjective interpretation. It has been widely noted that the estimation of the attributes of unseen alternatives is prone to error (Golledge et al., 2001). Thus in prescribing a deterministic representation of behaviour, one must reconsider the role of error in estimation across a population of decision-makers.

Estimation, in the case of this route choice framework, is incorporated in the estimation of angular deviation, distance, speed and travel time between locations. Previous literature provides some context on which bounds in error estimation can be established.

Numerous studies have been previously carried out into individual ability to estimate the direction of unseen locations. Such studies usually explore these errors by asking individuals to point towards unseen locations, within a small-scale learned environment. In one such study, Barber and Lederman identified an unsigned mean error estimation in sighted individuals of 18.8° with a 12.5° standard deviation (Barber and Lederman, 1988). Klatzky furthermore demonstrated how errors increase with the size of the angle, producing the results shown in Table 4 (Klatzky et al., 1990).

In view of the richness of the dataset available, the Klatzky definitions are implemented. These errors are implemented within the calculation of nodal path angularity within a region, adding error specifically into the calculation of the angular deviation between two nodes. Errors are drawn from a normal distribution, where the mean is extracted from the error estimations in Table 4 according to the angular bound within which the angle lies (e.g. all angles below 60° are designated a mean error of 11°, below 90° drawn with 12° mean error). The standard deviation of the distribution is specified as $\sigma/2$ (in view of this data being unavailable). The error value is drawn from the distribution then signed either positive or negative (according to a random uniform draw), and added to the actual angle specification. The value is utilised as any other calculation of angular deviation would be, within the region selection process.

Estimates for error within the remaining attributes are less forthcoming within existing literature, with little research conducted into these processes, and nothing identified that is set within the urban context. As such, it was decided that a Normal distribution would be utilised to represent errors in estimation for each of the remaining attributes. Normal distributions have been noted to widely correspond to a range of psychological and biological processes (Haslam et al., 2003), thus would appear to represent a reasonable expectation for distribution of error around a value.

Errors relating to distance, speed and travel time are all drawn from a Gaussian distribution with a mean of 0 and standard deviation defined as a percentage, indicative of a percentage variation around the actual value. This standard deviation value will be tested at both 10% and 20%, indicative of the anticipated error variation around the real value. An error term is calculated separately for each of the attributes, then added to the total value in each case.

These error terms are applied only during the regional decision process, and not during node or road traversal. This is because these actions are considered to be based upon local cues, rather than cognitive recollections of spatial relations. In being close to the locations being traversed, an individual able to better understand the relationship between locations, so reducing the potential for estimation error.

4.4.2. Bounded spatial knowledge

Spatial knowledge refers to an individual's bounded memory of the spatial configuration of the environment around them. This knowledge is driven by prior experiences, with regular interaction with areas of space leading to more comprehensive mental representations of that area. These experiences thus lead to a variation in the accuracy of an individual's knowledge across space, where some areas are very well known and others hardly known at all. This knowledge will influence the decision-making abilities of the individual.

During the construction of the nodal representation of space, a hierarchy in node definitions was identified (see Section 3.2). At one end of the hierarchy are major junctions, at the other only very minor junctions. In modelling heterogeneity in spatial knowledge, it is assumed that there exists variation in spatial knowledge across the population, ranging from only very basic knowledge to very detailed knowledge of the network. In line with the node hierarchy, four equal classes of driver are introduced, ascribing 25% of drivers with knowledge of only level 1 nodes (only major junctions), 25% with level 1 and 2 nodes, another 25% with level 1, 2 and 3 nodes, and a final 25% with knowledge of all nodes across each level. The drivers will only be able to use the nodes that they have knowledge of in constructing their route. This restriction applies throughout the decision-making process, including the calculation of minimal angular paths during the TT

Table 4
Error in estimation of angular direction of target location.

| Angle (°) | Approximate average unsigned error (°) |
|-----------|--|
| 60 | 11 |
| 90 | 12 |
| 135 | 18 |
| 180 | 15 |
| 225 | 23 |
| 270 | 28 |
| 300 | 29 |

stage. While the equal split of spatial knowledge in this way is limiting, and ignores interpersonal heterogeneity, no datasets were identified to enable a more accurate specification.

The introduction of heterogeneity in spatial knowledge completes the specification of the model, providing an opportunity to observe the role of this particular facet of the model. Fig. 4 shows the construction of two routes both running from a location in north-west London to east London. The only factor altered during the construction of these two routes is the level of spatial knowledge – the purple route constructed where the driver is granted only knowledge of level 1 nodes, the orange route reflecting a journey that considers all nodes from level 1 through to level 4. The routes demonstrate that the more experienced driver is able to select a route that avoids the straighter, yet more congested routes chosen by the less experienced driver, leading to a lower expected travel time.

5. Model implementation and validation

For the purposes of model assessment, the route selections observed within the validation dataset – the remaining 20% of routes not included during model definition (encompassing 138,009 routes) – are predicted using the modelling approach. Any samples within this dataset falling below 500 metres in route length are excluded, as to not artificially increment the accuracy of the modelling approach. Connections between nodes are assessed on the basis of whether the original journey was taken during the daytime or evening.

In implementing the model, its sensitivity to the specification of a number of attributes must be explored. This includes the influence of variation in the difference thresholds required between alternatives where comparing cues (detailed in SubSection 4.1.2), and the role of standard deviation in estimation error (described in SubSection 4.4.1). The influence of these attributes will be assessed during the first stage of the validation, and the best performing model configuration established for complete route validation.

The approaches taken towards validation differ according to the level of hierarchy being assessed. At the node and region levels, model predictions are compared directly against observed behaviours. This is because these incorporate more strategic route planning choices, not so susceptible to variations in individual behaviours. Complete route accuracy is only assessed across all routes, rather than on a route-by-route basis. This alternative method assesses whether the full variation in fine-scale routing choices are accounted for within the complete modelling framework. Modelled and observed routes are not compared directly due to the inherent heterogeneity in the production of both routes.

The final stage of validation tests the similarity between modelled and optimal routing. Given the widespread use of cost minimisation within the model framework (albeit at a range of spatial scales), these tests will establish how closely route sets generated through the heuristic modelling framework match those generated through simple shortest distance path calculation.

5.1. Region and node choice accuracy

In assessing how well the model predicts observed selections requires an evaluation of both the region and node selection processes. Region selections are assessed in three ways – firstly, correct identification during the EBA pre-selection phase; second, correct selection of region during TTB phase; and third, correct prediction of the gateway selected between two regions. During the second phase of region selections a range of models are tested, exploring the model's sensitivity to variation in attribute assignment.

5.1.1. Pre-selection of regions

At the pre-selection phase, two regions are carried forward for further assessment, incorporating all valid connections between the regions that are found. Validation journeys are compared region-by-region, with the presence or otherwise of the next region following pre-selection is recorded. For all trips, it was found that the actual next selected region was identified within the top two options on 86.5% of occasions. This indicates that the pre-selection model functions effectively in identifying potential next regions.

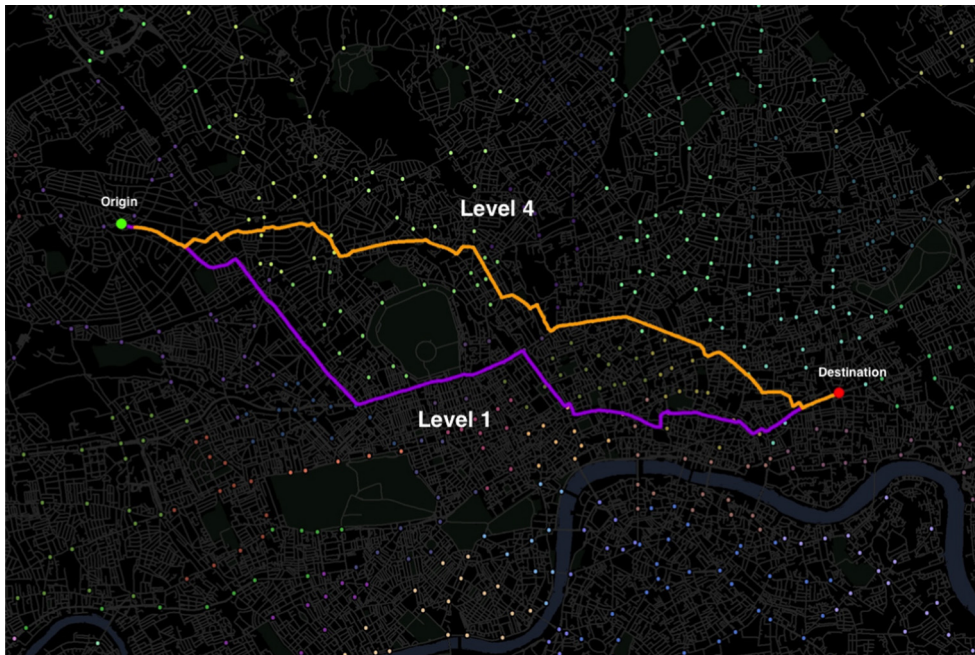


Fig. 4. Route choice variation by level of spatial knowledge.

5.1.2. Selection of region and gateway from alternatives

Following the pre-selection of two potential next regions, the TTB heuristic sorts between all possible gateways into the potential next regions. The next validation step therefore identifies whether the next region is correctly estimated. The model is tested using different implementations of the similarity sensitivity measure (e.g. how different regions must be to prompt a selection) and the error standard deviation (e.g. how much error is introduced at each selection). A range of values for these attributes are tested to provide an insight into the sensitivity of the model. The scores associated with each implementation are shown in Table 5.

As one may observe from Table 5, each model configuration exhibits a reasonable ability to predict the next selected region. Each configuration, furthermore, demonstrates an adequate performance in relation to its selection of the correct gateway towards the chosen region, particularly where one considers that two regions may be connected by multiple potential gateways (particularly where more nodes are considered). However, it is further apparent that variations in both the similarity and error scatter metrics impact very little on model performance, with each model performing similarly across both indicators. This trend is indicative of there often existing a single alternative that clearly stands out above alternatives during the selection process. As such, whether a 10%, 20% or 30% improvement threshold is utilised, one particular alternative will usually be selected. By the same token, variation in error overall makes little impact upon the selection of the alternative, only playing a role where a small difference exists between two alternatives.

In view of the results presented in Table 5, it was decided that Model C would be adopted for further testing. This selection was made due to performing better than any of the alternative models, albeit with only minimal improvement. This model will be used in reporting all further test results.

Cue utilisation. Exploring the result for Model C in more detail, during the process the cue used to determine the selection of a region and gateway was recorded. These results are shown in Table 6, demonstrating a high utilisation of the least total deviation cue, with most of the additional cues barely utilised. This does suggest a strong reliance upon the least deviation cue, relative to its selections identified during the ranking process, representing a possible limitation to this approach.

5.1.3. Selection of nodes through region

Beyond regional selections, node-to-node selection processes are of significant importance too, and provide a route into comparing this modelling approach against those presented earlier. In this instance, node selections are modelled within the context of the region they are within. Given an individual traversing a particular region, en route to a gateway into another region, a path is constructed that minimises the deviation from that gateway. Next node selections are made on the basis of the success rate of predicting a subsequent node, based on the observed current node, minimising deviation from the observed gateway node. The model indicates a prediction success rate for predicting node-to-node selections of 81.5%.

Table 5

Model performance according to region and gateway selection by model type.

| Model | Model attributes | Correct prediction (%) | |
|-------|---------------------------|------------------------|----------------------------------|
| | | Next region selected | Next region and gateway selected |
| A | Similarity 10%; error 10% | 60.29 | 38.65 |
| B | Similarity 20%; error 10% | 60.31 | 39.02 |
| C | Similarity 30%; error 10% | 60.37 | 39.25 |
| D | Similarity 10%; error 20% | 60.25 | 38.60 |
| E | Similarity 20%; error 20% | 60.31 | 38.90 |
| F | Similarity 30%; error 20% | 60.35 | 38.98 |

Table 6

Cues used for differentiation during 138,009 modelled journeys.

| Rank | Cue | Taken selections | Percentage selected |
|------|--|------------------|---------------------|
| 1 | Least total deviation from destination | 210,580 | 68.78 |
| 2 | Least total distance | 48,527 | 15.85 |
| 3 | Least time | 25,921 | 8.46 |
| 4 | Fastest mean speed | 15,459 | 5.05 |
| 5 | Least distance from target | 5650 | 1.85 |
| 6 | Random | 30,098 | 9.83 |

5.2. Complete route accuracy

The second validation stage tests the accuracy of the complete route choice model. At this stage, equivalent routes are modelled for each of the 138,009 observed routes (using the same origin and destination), incorporating the complete selection process from regional choice, to the node and road segment selection. The distribution of modelled routes is compared against the observed set of routes using a range of tests.

In the first instance, statistical variation between the observed and modelled traffic flow on each road link is calculated. On a summary level, 128,442 road segments were covered by the modelled routes with a mean flow of 151.19 (s.d. = 454.55). This is comparable with the validation dataset that covers 146,596 road segments, with a mean flow of 125.15 (s.d. = 387.21). Taking in the total distances of each route set, the modelled routes total a distance of 1.12 billion metres, averaging 8028.16 metres per journey, slightly longer than the observed mean distance of 7869.81 metres.

Comparing the model and observed flow on each road segment, the Mean Error (ME) in flow is found to be 8.465 (s.d. = 211.07). Taking absolute values, the Mean Absolute Error (MAE) is 60.37 (s.d. = 202.43).

In order to gain insight into the spatial variation in model fit, linear regression is calculated between the modelled and observed traffic flow on a road-by-road basis. The line of regression passes along $y = 4.543 + 0.784x$, where y represents the real flows on each road and x the modelled flow. The model indicates a reasonable fit with the observed distribution, with the R^2 at 0.76. The strength of similarity is likely to in part associated with the matching distribution of origin and destination locations. Inspecting the outliers, the regression indicates that the model underperforms along routes with very high traffic flow, with an over-prediction of routes along minor roads.

5.3. Against shortest distance routing

The final stage of validation tests the similarity between heuristic modelled routes and those generated by optimal distance paths. Once again, the flows generated by each approach are compared on a road segment basis, providing an indicator of overall similarity in generated traffic flow.

Where the heuristic model arrived at a mean flow of 151.19 (s.d. = 454.55) across 128,442 road segments, the equivalent shortest distance path route set results in a mean flow of 128.19 (s.d. = 393.55) across 120,239 road segments. On a segment basis, the Mean Error is found 31.43 (s.d. = 201.09), with the MAE falling roughly around the same point at 64.26 (s.d. = 193.12). The cause of these discrepancy is indicated by a difference in mean journey lengths, with the heuristic approach averaging 8028.16 m per journey, where shortest distance routes are found to exhibit a considerably lower mean distance of 6721.98 m per journey.

With the comparison of route set distributions through linear regression an R^2 of 0.81 is found. It is clear that despite the deviations in route flow indicated above, the distribution of traffic is broadly similar. However, like above, the strength of similarity is likely to be strongly influenced by the matching distribution of origin and destination locations across the network.

6. Discussion and conclusions

The model of route choice introduced in this paper has sought to advance the representation of both bounded decision-making and spatial cognition during the route choice process. The approach built on literature from diverse domains, incorporating elements of quick and strategic decision-making, where rough, high-level plans are made initially and then later refined, and variable spatial representations, reflecting the multifaceted nature of human cognition of space. The framework outlined reflects a novel approach towards representing route choice, offering potential for incorporation within other transportation models (particularly those that model individual behaviours explicitly, e.g. activity-based or agent-based models). The results generated through the implementation of the framework are promising, however there are plenty of areas for potential improvement and extension of the approach.

One area for potential elaboration surrounds the definition of the hierarchical representations of space. While the process via which individuals utilise different spatial objects during route choice is well understood, the computational definition of these objects is a novel research area. The definitions outlined in Section 3 offer one possible approach, yet these models remain untested, and would require development for their application within other cities. While the usage of community detection methods to establish functional spatial regions is novel, there are numerous ways by which regions may be perceived or defined. Likewise, there is potential within this model of space for more fully incorporating heterogeneity in spatial knowledge, something only crudely adapted during the implementation of the model (during Sections 4.1.2 and 4.4.2). Future research should be directed towards closing the gap between those urban features used by drivers during route choice and the computational representation of these features used within modelling this process.

The heuristic rule sets outlined in Section 4 aim to reflect the quick, bounded nature of route choice decision making. Yet the implementation of these rules should be investigated further, particularly within the contexts of alternative cities. The implementation of the framework indicated an over reliance upon the deviation cue and an under prediction along routes with high traffic flow, further work may seek to address these limitations. Like the past work carried out in defining discrete choice models with a wide range of different attributes, this decision-making framework may be manipulated in a multitude of ways.

The modelling framework introduced during this paper is partly intended to act as a springboard for further research. The model outlined aims to better incorporate aspects of bounded rationality, strategic decision-making and spatial cognition that have been previously poorly incorporated within route choice models. While the implementation of this framework is open to alteration, the model structure aims to serve as alternative approach to conventional discrete choice and optimal route modelling approaches.

Acknowledgements

This work is part of the STANDARD project – Spatio-Temporal Analysis of Network Data and Road Developments, supported by the UK Engineering and Physical Sciences Research Council (EP/G023212/1) and Transport for London (TfL). The authors are grateful to Addison Lee Limited for their provision of minicab GPS traces for the purposes of this research.

References

- Arentze, T.A., Dellaert, B.G., Chorus, C.G., 2014. Incorporating mental representations in discrete choice models of travel behavior: modeling approach and empirical application. *Transport. Sci.* <http://dx.doi.org/10.1287/trsc.2013.0513>.
- Avineri, E., 2012. On the use and potential of behavioural economics from the perspective of transport and climate change. *J. Transport Geogr.* 24, 512–521.
- Avineri, E., Bovy, P.H., 2008. Identification of parameters for a prospect theory model for travel choice analysis. *Transport. Res. Rec.: J. Transport. Res. Board* 2082 (1), 141–147.
- Barber, P.O., Lederman, S.J., 1988. Encoding direction in manipulatory space and the role of visual experience. *J. Visual Impair. Blindness* 82 (3), 99–106.
- Ben-Elia, E., Shifitan, Y., 2010. Which road do I take? A learning-based model of route-choice behavior with real-time information. *Transport. Res. Part A: Policy Pract.* 44 (4), 249–264. <http://dx.doi.org/10.1016/j.tra.2010.01.007>.
- Blondel, V.D., Guillaume, J.L., Lambiotte, R., Lefebvre, E., 2008. Fast unfolding of communities in large networks. *J. Stat. Mech.: Theory Exp.* 2008 (10), P10008, ISSN: 1742-5468.
- Brandstatter, E., Gigerenzer, G., Hertwig, R., 2006. The priority heuristic: making choices without trade-offs. *Psychol. Rev.* 113 (2), 409.
- Brighton, H., 2006. Robust inference with simple cognitive models. In: *Papers from the AAAI Spring Symposium*, pp. 17–22.
- Bröder, A., 2000. Assessing the empirical validity of the take-the-best heuristic as a model of human probabilistic inference. *J. Exp. Psychol. Learn. Memory Cognit.* 26 (5), 1332–1346, ISSN: 0278-7393. <<http://www.ncbi.nlm.nih.gov/pubmed/11009261>>.
- Chown, E., Kaplan, S., Kortenkamp, D., 1995. Prototypes, location, and associative networks (PLAN): towards a unified theory of cognitive mapping. *Cognit. Sci.* 19 (1), 1–51. http://dx.doi.org/10.1207/s15516709cog1901_1, ISSN: 03640213.
- Couclelis, H., Golledge, R., Gale, N., Tobler, W., 1987. Exploring the anchor-point hypothesis of spatial cognition. *J. Environ. Psychol.* 7 (2), 99–122. [http://dx.doi.org/10.1016/S0272-4944\(87\)80020-8](http://dx.doi.org/10.1016/S0272-4944(87)80020-8), ISSN: 02724944.
- Czerlinski, J., Gigerenzer, G., Goldstein, D., 1999. How good are simple heuristics? In: *Simple Heuristics that Make us Smart*. Oxford University Press, USA, New York, gerd giger edn.
- Dalton, R.C., 2003. The secret is to follow your nose route path selection and angularity. *Environ. Behav.* 35 (1), 107–131.
- Dhami, M., 2004. Conditional bail decision making in the magistrates' court. *Howard J. Crim. Justice* 43 (1), 27–46. <http://dx.doi.org/10.1111/j.1468-2311.2004.00309.x>, ISSN: 0265-5527.
- Dhami, M., Harries, C., 2001. Fast and frugal versus regression models of human judgement. *Thinking Reason.* 7 (1), 5–27. <http://dx.doi.org/10.1080/13546780042000019>, ISSN: 1354-6783.
- Gao, S., Frejinger, E., Ben-Akiva, M., 2010. Adaptive route choices in risky traffic networks: a prospect theory approach. *Transport. Res. Part C: Emerg. Technol.* 18 (5), 727–740.
- Garling, T., 1998. Behavioural assumptions overlooked in travel choice modelling. *Travel Behaviour Research: Updating the State of Play*, pp. 3–18.

- Gigerenzer, G., 2004. Fast and frugal heuristics: the tools of bounded rationality. *Blackwell handbook of judgment and decision making*, p. 82.
- Gigerenzer, G., 2008. *Rationality for Mortals: How People Cope with Uncertainty*. Oxford University Press, USA, ISBN: 0199890129.
- Gigerenzer, G., Goldstein, D.G., 1996. Reasoning the fast and frugal way: models of bounded rationality. *Psychol. Rev.* 103 (4), 650.
- Gigerenzer, G., Todd, P.M., 1999. *Simple Heuristics that Make us Smart*. Oxford University Press.
- Goldstein, D.G., Gigerenzer, G., 2002. Models of ecological rationality: the recognition heuristic. *Psychol. Rev.* 109 (1), 75.
- Golledge, R.G., 1978. Learning about urban environments. *Timing Space Spacing Time* 1, 76–98.
- Golledge, R., Smith, T., Pellegrino, J., Doherty, S., Marshall, S., 1985. A conceptual model and empirical analysis of children's acquisition of spatial knowledge. *J. Environ. Psychol.* 5 (2), 125–152. [http://dx.doi.org/10.1016/S0272-4944\(85\)80014-1](http://dx.doi.org/10.1016/S0272-4944(85)80014-1), ISSN: 02724944.
- Golledge, R.G., Garling, T., 2001. Spatial behavior in transportation modeling and planning. Tech. Rep., University of California Transportation Center. <http://econpapers.repec.org/RePEc:cdl:uctcwp:qt68z571sc>.
- Graefe, A., Armstrong, J.S., 2012. Predicting elections from the most important issue: a test of the take-the-best heuristic. *J. Behav. Decis. Making* 25 (1), 41–48.
- Hafting, T., Fyhn, M., Molden, S., Moser, M.-B., Moser, E.I., 2005. Microstructure of a spatial map in the entorhinal cortex. *Nature* 436 (7052), 801–806. <http://dx.doi.org/10.1038/nature03721>, ISSN: 1476-4687. <http://www.ncbi.nlm.nih.gov/pubmed/15965463>.
- Haslam, S.A., McGarty, C., 2003. *Research Methods and Statistics in Psychology*. SAGE, ISBN: 0761942939. <http://books.google.com/books?id=BeOcPM7OtfMC&pgis=1>.
- Hirtle, S., Jonides, J., 1985. Evidence of hierarchies in cognitive maps. *Memory Cognit.* 13 (3), 208–217, ISSN: 0090-502X.
- Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. *Econometr.: J. Econometr. Soc.* 47 (2), 263–291, ISSN: 0012-9682.
- Kaplan, S., Prato, C.G., 2012. Closing the gap between behavior and models in route choice: the role of spatiotemporal constraints and latent traits in choice set formation. *Transport. Res. Part F: Traffic Psychol. Behav.* 15 (1), 9–24.
- Klatzky, R.L., Loomis, J.M., Golledge, R.G., Cicinelli, J.G., Doherty, S., Pellegrino, J.W., 1990. Acquisition of route and survey knowledge in the absence of vision. *J. Motor Behav.* 22 (1), 19–43.
- Kuipers, B., Tecuci, D., Stankiewicz, B., 2003. The skeleton in the cognitive map: a computational and empirical exploration. *Environ. Behav.* 35 (1), 81–106. <http://dx.doi.org/10.1177/0013916502238866>, ISSN: 00000000. <http://eab.sagepub.com/content/35/1/81.abstract>.
- Lynch, K., 1960. *The Image of the City*, vol. 1. MIT Press, ISBN: 0262620014.
- Manley, E.J., Addison, J.D., Cheng, T., 2015. Shortest path or anchor-based route choice: a large-scale empirical analysis of minicab routing in London. *Transport Geogr* 43, 123–139.
- Montello, D.R., 1998. A new framework for understanding the acquisition of spatial knowledge in large-scale environments. *Spatial and temporal reasoning in geographic information systems*, 143–154.
- Nakayama, S., Kitamura, R., Fujii, S., 2001. Drivers' route choice rules and network behavior: do drivers become rational and homogeneous through learning? *Transport. Res. Rec.: J. Transport. Res. Board* 1752 (2001), 62–68. <http://dx.doi.org/10.3141/1752-09>, <http://trb.metapress.com/content/241u8267h12722k5> <<http://trb.metapress.com/index/241U8267H12722K5.pdf>>.
- O'Keefe, J., Dostrovsky, J., 1971. The hippocampus as a spatial map: preliminary evidence from unit activity in the freely-moving rat. *Brain Res.* 34 (1), 171–175. [http://dx.doi.org/10.1016/0006-8993\(71\)90358-1](http://dx.doi.org/10.1016/0006-8993(71)90358-1), ISSN: 00068993.
- O'Keefe, J., Nadel, L., 1978. *The Hippocampus as a Cognitive Map*. Clarendon Press, Oxford, <http://www.getcited.org/pub/101782291>.
- O'Sullivan, D., 2009. Changing neighborhoods - neighborhoods changing a framework for spatially explicit agent-based models of social systems. *Sociol. Methods Res.* 37 (4), 498–530.
- Papinski, D., Scott, D.M., Doherty, S.T., 2009. Exploring the route choice decision-making process: a comparison of planned and observed routes obtained using person-based GPS. *Transport. Res. Part F: Traffic Psychol. Behav.* 12 (4), 347–358. <http://dx.doi.org/10.1016/j.trf.2009.04.001>, ISSN: 13698478.
- Passini, R., 1984. Spatial representations, a wayfinding perspective. *J. Environ. Psychol.* 4 (2), 153–164. [http://dx.doi.org/10.1016/S0272-4944\(84\)80031-6](http://dx.doi.org/10.1016/S0272-4944(84)80031-6), ISSN: 02724944. <http://linkinghub.elsevier.com/retrieve/pii/S0272494484800316>.
- Pohl, R.F., 2006. Empirical tests of the recognition heuristic. *J. Behav. Decis. Making* 19 (3), 251–271. <http://dx.doi.org/10.1002/bdm.522>, ISSN: 0894-3257.
- Prato, C.G., Bekhor, S., Pronello, C., 2012. Latent variables and route choice behavior. *Transportation* 39 (2), 299–319.
- Richter, K.-F., Winter, S., Santosa, S., 2011. Hierarchical representations of indoor spaces. *Environ. Planning - Part B* 38 (6), 1052.
- Sen, A., 1977. Rational fools: a critique of the behavioral foundations of economic theory. *Philos. Public Affairs* 73 (4), 317–344, <http://www.jstor.org/stable/2264946>.
- Siegel, A.W., White, S.H., 1975. The development of spatial representations of large-scale environments. *Adv Child Dev Behav.* 10, 9–55. [http://dx.doi.org/10.1016/S0065-2407\(08\)60007-5](http://dx.doi.org/10.1016/S0065-2407(08)60007-5).
- Simon, H.A., 1957. *Models of Man: Social and Rational; Mathematical Essays on Rational Human Behavior in a Social Setting*. Wiley.
- Snook, B., Cullen, R.M., 2006. Recognizing National Hockey League greatness with an ignorance-based heuristic. *Canadian J. Exp. Psychol. = Revue canadienne de psychologie expérimentale* 60 (1), 33–43, ISSN: 1196-1961. <http://www.ncbi.nlm.nih.gov/pubmed/16615716>.
- Solstad, T., Boccaro, C.N., Kropff, E., Moser, M.-B., Moser, E.I., 2008. Representation of geometric borders in the entorhinal cortex. *Science* 322 (5909), 1865–1868. <http://dx.doi.org/10.1126/science.1166466>, ISSN: 0036-8075. <http://www.sciencemag.org/content/322/5909/1865.abstract>.
- Stevens, A., Coupe, P., 1978. Distortions in judged spatial relations. *Cognit. Psychol.* 10 (4), 422–437.
- Tomko, M., Winter, S., 2013. Describing the functional spatial structure of urban environments. *Comput. Environ. Urban Syst.* 41, 177–187.
- Turner, A., 2009. The role of angularity in route choice. In: Hornsby, K.S., Claramunt, C., Denis, M., Ligozat, G. (Eds.), *Spatial Information Theory, Lecture Notes in Computer Science*, vol. 5756. Springer, Berlin, Heidelberg, pp. 489–504, ISBN: 978-3-642-03831-0, 978-3-642-03832-7. http://link.springer.com/chapter/10.1007/978-3-642-03832-7_30.
- Tversky, A., 1972. Elimination by aspects: a theory of choice. *Psychol. Rev.* 79 (4), 281.
- Tversky, B., 1993. Cognitive maps, cognitive collages, and spatial mental models. *Lecture Notes Comput. Sci.* 716, 14–24. <http://dx.doi.org/10.1007/3-540-57207-42>, <http://www.springerlink.com/content/ym02092783774232>.
- Wardrop, J.G., 1952. Some theoretical aspects of road traffic research. *ICE Proc.: Eng. Div.* 1 (3), 325–362. <http://dx.doi.org/10.1680/ipeds.1952.11259>, ISSN: 0534-2767. <http://www.icevirtualibrary.com/content/article/10.1680/ipeds.1952.11259>.
- Wiener, J.M., Mallot, H.A., 2003. 'Fine-to-Coarse' route planning and navigation in regionalized environments. *Spatial Cognit. Comput.* 3 (4), 331–358. http://dx.doi.org/10.1207/s15427633scc0304_5, ISSN: 1387-5868.
- Wiener, J.M., Mallot, H.A., 2006. Path complexity does not impair visual path integration. *Spatial Cognit. Comput.* 6 (4), 333–346. http://dx.doi.org/10.1207/s15427633scc0604_3, http://www.informaworld.com/10.1207/s15427633scc0604_3.
- Wiener, J.M., Schnee, A., Mallot, H.A., 2004. Use and interaction of navigation strategies in regionalized environments. *J. Environ. Psychol.* 24 (4), 475–493. <http://dx.doi.org/10.1016/j.jenvp.2004.09.006>, ISSN: 02724944.
- Winter, S., Tomko, M., Elias, B., Sester, M., 2008. Landmark hierarchies in context. *Environ. Plann. B: Planning Des.* 35, 381–398. <http://dx.doi.org/10.1068/b33106>.
- Xu, H., Zhou, J., Xu, W., 2011. A decision-making rule for modeling travelers' route choice behavior based on cumulative prospect theory. *Transport. Res. Part C: Emerg. Technol.* 19 (2), 218–228.