

Path planning algorithm for unmanned surface vehicle formations in a practical maritime environment

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Abstract

Unmanned surface vehicles (USVs) have been deployed over the past decade. Current USV platforms are generally of small size with low payload capacity and short endurance times. To improve effectiveness there is a trend to deploy multiple USVs as a formation fleet. This paper presents a novel computer based algorithm that solves the problem of USV formation path planning. The algorithm is based upon the fast marching (FM) method and has been specifically designed for operation in dynamic environments using the novel Constrained FM method. The Constrained FM method is able to model the dynamic behaviour of moving ships with efficient computation time. The algorithm has been evaluated using a range of tests applied to a simulated area and has been proved to work effectively in a complex navigation environment.

Keywords: USV formation, Path planning, Fast marching method

1. Introduction

2 In recent years, with the benefits of reducing human casualties as well as increas-
3 ing mission efficiencies, there have been increasing deployments of USVs in both
4 military and civilian applications. However, current available USV platforms

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5 have low payload capacity and short endurance times. In order to overcome
6 these shortcomings; the current and future trend of USV operations is to de-
7 ploy multiple vehicles as a formation fleet to allow cooperative operations. The
8 benefits of using USVs formation operations include wide mission area, improved
9 system robustness and increased fault-tolerant resilience.

10 Fig. 1 describes a hierarchical structure of a USV formation system. The
11 structure consists of three layers, i.e. Task management layer, Path planning
12 layer and Task execution layer. The Task Management Layer allocates the
13 mission to individual USVs based on a general mission requirement. A mission
14 can be generally defined as a set of way-points including mission start point and
15 end point. According to the mission requirements, the second layer, i.e. the
16 Path Planning Layer, plans feasible trajectories for a USV formation. It should
17 be noted that cooperative behaviour for formation path planning is vital. Each
18 vehicle should establish good communication to ensure formation behaviour. In
19 addition, path re-planning needs to be considered if the formation is travelling
20 in a dynamic environment. Generated paths will then be passed down to the
21 Task Execution Layer, to calculate specific control for each vehicle. In order to
22 improve the robustness of system as well as to minimise system error, real-time
23 velocity and position information is fed back to the Path Planning Layer to
24 modify the path. Also, planned trajectory information is sent back to the Task
25 Management Layer in order to facilitate mission rearrangement. The whole
26 structure is acting as a closed loop system to ensure safety of a USV formation.

27 As observed from the USV hierarchical structure, the Path Planning Layer plays
28 an important role as it connects both the Task Management Layer and the Task
29 Execution Layer and navigates the formation. Path planning is a complicated
30 task and can be viewed as a multi-optimisation problem. The planned trajectory
31 should be optimised in terms of several aspects such as total distance, navigation
32 time and energy consumption. Also, collision avoidance is important for trajec-
33 tory. The formation should not collide with any static obstacles (islands, buoys)
34 and other moving vessels. To the best of the authors' knowledge, although sev-

35 eral work such as Borrelli et al. (2004), Barfoot and Clark (2004) and Cao et al.
36 (2003) studied formation path planning for unmanned aerial vehicle (UAV), un-
37 manned ground vehicle (UGV) and mobile robots, there is currently no work
38 specifically focused on developing a robust formation path planning algorithm
39 for USVs. This is possibly due to the reasons of high uncertainty and complexity
40 of obstacles in an ocean environment.

41 Therefore, this paper aims to propose a practical path planning algorithm for
42 USV formation in real navigation environments. It is the first work specifically
43 solving the USV formation problem with algorithm practicability as the main
44 feature of this research. A number of previous works have developed path
45 planning algorithms for USVs; however, nearly all of them (Tam and Bucknall
46 (2013), Naeem et al. (2012), Thakur et al. (2012)), with the notable exception of
47 Kim et al. (2014), simulated algorithms in simple self-constructed environments
48 rather than real ocean environments. The algorithm designed in this paper is
49 able to extract information from a real navigation map to construct a synthetic
50 grid map, where both static and dynamic obstacles are well represented. By
51 using such a map, a collision free path may be generated which can be directly
52 used as a guidance trajectory for practical navigation.

53 The rest of the paper is organised as follows. Section 2 reviews related work in
54 terms of formation path planning. Sections 3 and 4 describe fundamentals of
55 the method used in this paper as well as the algorithm which models static and
56 dynamic obstacles. Section 5 introduces the USV formation path planning algo-
57 rithm. Proposed algorithm and methods are verified by simulations in section
58 6. Section 7 concludes the paper and discusses the future work.

59 **2. Literature review**

60 Due to limited resources studying USV formation path planning, and also in
61 order to give a more thorough review of the current research situation; literature
62 from not only USV, but also UAV, UGV and unmanned underwater vehicle

63 (UUV) have been reviewed in this section. For simplicity, we have named all
64 kinds of autonomous vehicle as 'unmanned vehicle' in following section.

65 *2.1. Formation control structure*

66 For unmanned vehicle formations, maintenance of the formation shape is of great
67 importance. To maintain the shape, several control structures including leader-
68 follower, virtual structure and behaviour based approaches have been proposed
69 by a number of researchers. In leader-follower approach (Liu et al. (2007),Cui
70 et al. (2010),Morbidi et al. (2011),Peng et al. (2013)), one vehicle is assigned as
71 leader vehicle, which has access to overall navigation information and tracks the
72 predefined path. All the other vehicles in the formation are followers aiming to
73 maintain the desired geometric configuration. In terms of virtual structure ap-
74 proach (Ren (2008),Ghommam et al. (2010),Cong et al. (2011),Mehrjerdi et al.
75 (2011)), the formation is treated as a rigid body and maintained by making each
76 vehicle in the formation follow a reference point in the rigid body. Both of these
77 approaches adopt a centralised control topology, where all the important control
78 decisions are made within at the centre of the system. In comparison, behaviour
79 based approach allows the utilisation of decentralised control. It breaks down
80 the formation tasks into several sub tasks according to different behaviours. In
81 the work of Balch and Arkin (1998), formation maintenance is integrated with
82 other missions such as goal keeping and collision avoidance and the control of
83 each vehicle is the result of a weighted function of these missions.

84 *2.2. Multiple vehicles formation path planning*

85 The nature of unmanned vehicles formation path planning is an optimisation
86 process of multiple objectives, which is more complicated than single vehicle
87 path planning. Fig. 2 compares optimisation objectives of these two kinds of
88 path planning problems. It is noted that besides single vehicle path planning
89 optimisation criteria, more attention is paid to address formation behaviours
90 in formation path planning. The planned trajectories of the formation should,

91 to the most extent, maintain the predefined shape. Also, a certain degree of
92 flexibility such as shape variation or change is preferred to accommodate the
93 navigation environment, which is beneficial to the formation's safety.

94 To achieve formation path planning, a number of different approaches have been
95 proposed, which could be categorised based on two disciplines:

- 96 • Deterministic approach
- 97 • Heuristic approach

98 Deterministic approach is achieved by following a set of defined steps to search
99 for the solution whereas heuristic approach only searches inside a subspace of
100 the search space without following rigorous procedures (Tam et al. (2009)) .

101 Heuristic approach is designed to provide solutions when classic search methods
102 fail to find exact solutions. Its speciality is in dealing with multi-optimisation
103 problems with fast computational speed. Therefore, a number of heuristic search
104 based algorithms such as genetic algorithm (Zheng et al. (2004), Yang et al.
105 (2006), Kala (2012), Qu et al. (2013)), particle swarm optimisation (Duan et al.
106 (2008), Bai et al. (2009)) and ant colony asexual reproduction optimisation
107 (Asl et al. (2014)) have been used for formation path planning. The algorithms
108 normally use decentralised control topology, where each vehicle of the forma-
109 tion has its own path planning process and cooperates with others through a
110 co-evolution process. However, heuristic path planning algorithm is not able
111 to rigorously maintain the formation shape. Even though trajectories can be
112 coordinated by introducing certain fitness functions, the uncertainty and ran-
113 domness of a heuristic search makes the path hard to follow a predefined shape
114 and heuristic path planning suffers problems of incompleteness and inaccuracy
115 of search results.

116 In contrast, deterministic path planning approach has the features of search
117 completeness and consistency. Among them, artificial potential field (APF)
118 is becoming a key method due to its easy implementation and good collision
119 avoidance capability. The theory behind it is to construct two different potential

120 fields, i.e. attractive and repulsive fields around target point and obstacles
121 respectively. An attractive field is constructed across the space with magnitude
122 proportional to the distance to the target point; whereas, a repulsive field is
123 built within a certain area called -"influence area"- around obstacles and the
124 magnitude is inversely proportional to the distance to the obstacle. Based on the
125 potential field, the vehicle can then be guided by following total field gradient.
126 Detailed explanation of this can be referred to Khatib (1986) and Ge and Cui
127 (2002).

128 In terms of implementation of APF in formation path planning, besides potential
129 fields around target point and obstacles, new fields need to be constructed to
130 keep formation distances as well as avoid collision between vehicles within the
131 formation. Wang et al. (2008) first constructed such potential fields by referring
132 to the concepts of electric field. Each vehicle was treated as point in the electric
133 field with varying electrical polarity. If the distance between vehicles was larger
134 than the expected value, opposite charges were used to attract them to move
135 towards each other; otherwise, like polarities were used to prevent them from
136 colliding when two vehicles were moving within close proximity.

137 Paul et al. (2008) also applied APF method to solve the problem of UAV forma-
138 tion path planning. Attractive fields between leader-follower as well as follower-
139 follower were built to keep formation shape, and repulsive fields were used to
140 prevent internal collision as well as collision with obstacles. To increase control
141 accuracy as well as to better address the formation shape maintenance prob-
142 lem, attractive potential field was a function of the error value between desired
143 distance and actual leader-follower or follower-follower distance such that any
144 deflection from the desired position can be quickly modified and corrected.

145 Yang et al. (2011) published work on motion planning for UUV formation in
146 an environment with obstacles based on APF. The algorithm concentrated on
147 overall mission requirements instead of development of individual vehicle's con-
148 trol law and treated UUV formation as a multibody system with each vehicle
149 modelled as a point mass with full actuation. Potential fields for formation

150 path planning were constructed for particular mission requirement, ocean envi-
151 ronment and formation geometry.

152 It should be noted that APF is prone to a local minima problem, which makes
153 the algorithm fail to 'jump out' of local minimum point and reach the target
154 point. Although methods proposed in Sheng et al. (2010) and Xue et al. (2011)
155 solved it by introducing virtual target point the impact was a sacrifice in com-
156 putation time consequently potential field with single global minimum point is
157 preferred. Garrido et al. (2011) used the fast marching (FM) method to con-
158 struct potential field with the target point as single minimum point for robot
159 formation path planning. As a method for solving the viscosity solution of the
160 eikonal function, the FM can successfully simulate the propagation of electro-
161 magnetic waves. The potential field in which electromagnetic wave transmits
162 has good properties such as absence of local minima. Besides, the gradient of
163 such a potential field is smoother than conventional one, which is more suitable
164 for a vehicle to track. Gomez et al. (2013) further improved the FM method
165 to fast marching square (FMS) method and increased the safety of planned
166 trajectories.

167 In this paper, the authors improve upon the work of Gomez et al. (2013) and
168 developing its application specifically for USV formation with emphasis on path
169 planning in a dynamic environment. A new constrained FM method is proposed
170 to model the dynamic behaviour of moving ships for collision avoidance. In
171 addition, path replanning capability is incorporated to improve the completeness
172 of the algorithm.

173 **3. Eikonal equation and fast marching method**

174 *3.1. Fast marching method*

175 The fast marching method was first proposed by J.Sethian in 1996 to track the
176 evolution of interfaces by numerically solving the viscosity solution of eikonal
177 equation :

$$|\nabla(T(\mathbf{x}))|W(\mathbf{x}) = 1 \quad (1)$$

178 where \mathbf{x} represents the point in metric space, i.e. $\mathbf{x} = (x, y)$ in 2D space and $\mathbf{x} =$
 179 (x, y, z) in 3D space. $T(\mathbf{x})$ is time matrix representing the arrival time of inter-
 180 face front at point \mathbf{x} , and $W(\mathbf{x})$ is speed matrix and describes local propagating
 181 speed at point \mathbf{x} . By using an upwind finite difference approximation scheme,
 182 the solving process of FM is similar to Dijkstra’s method but in a continuous
 183 way.

184 When applying FM method to the path planning problem, a more intuitive way
 185 to interpret it is from the potential field perspective. In Fig. 3, two round
 186 obstacles are located near the centre of the map; while the start and end points
 187 are at northwest and southeast corners respectively. The map is represented by
 188 binary grid map, where each grid in collision free space has value 1 and grids in
 189 obstacle areas have value 0.

190 FM is then applied on such a grid to simulate an interface propagation process.
 191 The interface is used to help build up a potential field, whose potential value on
 192 each grid point is the local interface arrival time. The interface begins to proceed
 193 from the start point on the grid map by taking local grid values to determine
 194 propagation speed. The evolution process of interface is shown in Fig. 4, where
 195 the brighter the colour is, the longer the arrival time. When the interface reaches
 196 the target point, the potential field (Fig. 5a) is created. The meaning of the
 197 colour in the figure is the same as Fig. 4’s. In the field, the potential value
 198 at each point represents local arrival time of the interface, which subsequently
 199 indicates local distance to the start point if a constant speed matrix is used.
 200 Since the interface begins propagating from the start point, the potential of the
 201 start point is therefore the lowest and is equal to zero. Potential values at other
 202 points increase as the interface advances and reach highest value at the end
 203 point. Because the interface is not allowed to transmit inside an obstacle area,
 204 obstacles’ potentials are infinite. Compared with the potential field generated

205 by APF, the potential field of FM has features of global minimum, which avoids
 206 local minima problems and increases the completeness of the algorithm. Based
 207 on the potential field obtained, the gradient descent method is then applied to
 208 find the shortest collision free path by following the gradient of the potential
 209 field. Such algorithm is shown in Algorithm 1. The algorithm first determines
 210 the highest potential value (max), and uses function *RescaleField* to rescale the
 211 potential field within the range 0 to max . It then computes the gradient of the
 212 rescaled potential field and finds an optimal path connecting the end point and
 213 the point with the lowest potential. The start point will be eventually added
 214 into the path if the lowest point is not the start point. Path generated by using
 215 the Algorithm 1 is shown as red line in Fig. 5b. It should be noted that the
 216 shortest path is defined in geodesic terms, which means that path has shortest
 217 Euclidean distance if the environment has constant $W(\mathbf{x})$ and is a weighted
 218 Riemannian manifold with varying $W(\mathbf{x})$ (Garrido et al. (2011)).

Algorithm 1 *Path_Gradient_Descent* Algorithm

Input: potential field (T), start point (p_{start}), end point(p_{end}), stepSize

```

1:  $max \leftarrow T.max$ 
2:  $T \leftarrow RescaleField(T, 0, max)$ 
3:  $grad \leftarrow ComputeGrad(T)$ 
4:  $path \leftarrow PathCalculator(grad, p_{end}, stepSize)$ 
5: if  $path.endpoint! = p_{start}$  then
6:    $path.Add(path, p_{start})$ 
7: end if
8: return  $path$ 
  
```

219 **4. Planning space representation**

220 In path planning problems, safety always holds priority no matter what appli-
 221 cation. To generate a safe trajectory, it is necessary to properly represent the
 222 environment in which the path planning algorithm is implemented. It is espe-

223 cially important for USV navigation environments, which include a great deal
224 of maritime uncertainties. Sufficient safe distance should always be maintained
225 between USV and obstacles (both static and dynamic). In this section, the
226 FM based map representation method for both static environment and moving
227 obstacles is described.

228 4.1. Static obstacles representation

229 One of the problems associated with path planning by directly using the FM
230 method is the generated path is too close to obstacles. Such a drawback is
231 especially impractical for USVs, because near distance areas around obstacles
232 (mainly islands and coastlines) are usually shallow water, which is not suitable
233 for marine vehicles to navigate. Hence, it is important to keep the planned path
234 a certain distance away from obstacles.

235 To tackle this problem, FMS method proposed by Gomez et al. (2013) for indoor
236 mobile robots is used in this paper. The basic concept behind FMS is to apply
237 the conventional FM algorithm twice but with different purposes:

- 238 • **step1** : FM is applied on original binary environment map (\mathbf{M}_o) to create
239 safety map (\mathbf{M}_s). Instead of calculating a single interface's propagation
240 by using a USV's mission start point; in this process, multiple interfaces
241 are emitted from all points that represent obstacles (points with value
242 0 in the binary map) and continue to advance until it reaches the map
243 boundary. Generated map (\mathbf{M}_s) is shown in Fig. 6b, where each point
244 is assigned a value, ranging from 0 to 1, representing the shortest local
245 arrival time. Since constant propagating speed is used, the local shortest
246 arrival time also determines the shortest distance to obstacles. The further
247 the distance to an obstacle is, the higher the value will be. Such values
248 can be viewed as indices to indicate the safety of local points. Low values
249 represent current locations may be too close to obstacles and consequently
250 may not be safe to proceed; hence USVs should be encouraged to keep
251 travelling in the areas with high index value.

252 • **step2** : FM is used again over the safety map (M_s) to generate the
253 potential field. USV's mission start point is now the algorithm's start
254 point. Since M_s is used as a speed matrix in this step, which gives non-
255 constant speed over the space, the interface now tends to remain in places
256 with high propagating speed. The generated potential field should follow
257 the trace of the interface, which is shown in Fig. 7b. Note the field's shape
258 is different to that of Fig. 6b, which was generated by using a constant
259 propagating speed matrix. Potential of nearby obstacles is always higher
260 than at other places', which act as a protecting layer to prevent the path
261 passing too close to obstacles. This can be proved by result paths shown
262 as red lines in Fig. 7a and Fig. 7b.

263 4.2. *Dynamic obstacles representation*

264 To prevent collision with dynamic obstacles or moving ships, most studies in
265 path planning research have adopted the concept of a 'safety area' ('ship domain'
266 in marine vessels collision avoidance) to model the area from which all other
267 vehicles are prohibited. The shape of such area is usually circular and the
268 centre of the area is located on the obstacle's instantaneous position. However,
269 in USV path planning, circular shape area is not always practical, especially
270 when a ship is travelling at high speed, which holds more risks at fore areas
271 than aft and sides. It is more realistic to assign the shape of safety area of a
272 ship according to its velocity.

273 In this paper, a new method called 'Constrained FM method' has been devel-
274 oped to model the ship domain of a dynamic vessel. In contrast to conventional
275 FM, the Constrained FM method propagates the interface within a certain space
276 rather than over the whole configuration space. Since the points explored by
277 the algorithm have been dramatically reduced, the computation time of the
278 Constrained FM is relatively low. Such a feature increases the capability of the
279 algorithm to deal with dynamic collision avoidance, which requires fast com-
280 putation speed to handle the position change of a moving obstacle. Fig. 8

281 compares these two algorithms by propagating interfaces from four start points.
 282 Configuration space is constructed as a 400*400 pixels area. It can be observed
 283 from Fig. 8b that four propagations have been restrained in four small circular
 284 areas. In terms of computation time, conventional FM spends 0.101 s to explore
 285 the space whereas it only takes 0.053 s for the Constrained FM, a near 50 %
 286 improvement.

287 To model a dynamic vessel, the Constrained FM method is implemented twice
 288 in the algorithm, the flow chart of which is show in Fig 9. It first reads in
 289 velocity (V_i) of the i^{th} ship, where i is the index of the vessel. Based on V_i , the
 290 algorithm starts to build the ship domain by adopting the shape proposed in
 291 Tam and Bucknall (2010). Ship domain alters its shape according to specific
 292 velocity; a more circular shape is constructed if vessel is travelling with low
 293 speed and half-elliptical shape is used for a high speed vessel. The dimension
 294 of the ship domain is computed by following two equations to calculate aft and
 295 fore sections respectively. For aft section, it is defined as:

$$SA_{Aft} = \begin{cases} r_{aft} & \text{if } r_{aft} \geq r_{min}, \\ r_{min} & \text{otherwise.} \end{cases} \quad (2)$$

296 where r_{min} is the minimum distance must be retained between two vessels.
 297 And r_{aft} is computed by:

$$r_{aft} = \begin{cases} velocity \times time & \text{if } velocity \times time < DisLimit, \\ 2 \times DisLimit - (velocity \times time) & \text{otherwise.} \end{cases} \quad (3)$$

298 where $time$ is the scaling factor and defined as 1.0 min in this paper which is
 299 appropriate to establish the area a vessel could potentially cover in such time
 300 period. However, it should be noted that such a parameter could be customised
 301 according to specific needs in a practical navigation situation. $DisLimit$ is a

302 predefined scalar variable to limit the maximum allowable area on the side and
 303 stern sections.

304 For fore section, the equation is defined as:

$$SA_{fore} = \begin{cases} velocity \times time & \text{if } velocity \times time < DisLimit, \\ r_{min} & \text{otherwise.} \end{cases} \quad (4)$$

305 After the determination of dimension of ship domain (C_{SD}), the Constrained
 306 FM method will be used to propagate the interface within C_{SD} with the source
 307 point located at the instantaneous position of the vessel to be modelled (See
 308 Fig. 10a and Fig. 10b). Since other ships are ruled out of entering into a
 309 ship domain, which makes the domain act like an obstacle; potential values
 310 obtained by running FM method in ship domain are therefore reset to be zero
 311 as $T(C_{SD}) = 0$.

312 Then, a new area called 'collision avoidance area' (CA) is constructed so that
 313 any path violating the ship domain will be re-calculated to produce an updated
 314 trajectory. CA 's dimension is controlled by scalar variable $CAScalar$ as:

$$S_{CA} = S_{SD} \times CAScalar \quad (5)$$

315 where S_{CA} and S_{SD} are the area dimension for collision avoidance area and
 316 ship domain area respectively. Equation 5 shows that CA has the same shape
 317 as ship domain but enlarged. Constrained FM method is applied again within
 318 CA by using all points in the ship domain as start points (See Fig. 10c and
 319 Fig. 10d). Generated CA will be further scaled to make potential values inside
 320 range from 0 to 1 so that it has uniform representation as the static potential
 321 map generated by FMS method.

322 Fig. 10e illustrates ship domains generated under different speeds. Low speed
 323 ships are given a circular shape ship domain so that equal collision risks are
 324 distributed around ship. When the ship is travelling at high speed, fore section

325 holds more risks than other sides. Therefore, more emphasis is placed on this
326 area and the area is increased in proportion to speed.

327 Another kind of collision avoidance of dynamic obstacles, especially for forma-
328 tion path planning, is to prevent internal USVs in the formation from colliding.
329 When two USVs are moving too close to each other from any direction, a re-
330 pulsive force is needed to maintain safety. Therefore, constrained FM method
331 is still used here but with a circular shape to model formation USVs.

332 5. USV formation path planning

333 The flow chart for USV path planning algorithm is show in Fig. 11. The
334 algorithm adopts leader-follower formation control structure along with on-line
335 path planning scheme to largely maintain formation shape. Leader USV's target
336 point is mission end point and fixed; whereas, followers' target points are re-
337 planned during each time step according to formation shape requirement. Based
338 on these target points, FM method is iteratively applied for each USV to search
339 for collision free path in real time.

340 Specific algorithm procedure is discussed here. During each time cycle t , leader
341 USV's path is searched first. The algorithm generates a static environment map
342 by using FMS method introduced previously. Since the static environment does
343 not change during the path planning period, generated map is stored as M_{static} .
344 Then, based on instantaneous positions and velocities of moving obstacles as
345 well as other USVs in formation, dynamic obstacles representation algorithm is
346 used to model the behaviours of vessels. Synthetic map combining static and
347 dynamic obstacles is finally compounded such that FM method can be used to
348 calculate path for leader vehicle.

349 Once the leader's path is determined, the algorithm starts to iterate to compute
350 paths for followers. Similar procedures are followed; however, since follower's
351 target points are re-planned during each time step, it is possible that the target
352 point is located within the obstacle (see Fig. 12a) such that the algorithm

353 fails to find the path. Hence, a sub target re-planning algorithm is used to
 354 'remove' the target point to a new feasible place with minimum impact on
 355 overall performance. It is computed based on distance reduction scheme as well
 356 as dynamic characteristics of the USV and summarised as Algorithm 2:

Algorithm 2 *Sub-Target-Re-planning* Algorithm

Input: sub target point (p_{sub}), USV's current point (p_{usv}), distance reduction scalar ($RdScalar$)

```

1: while  $p_{sub} = obstacle$  do
2:    $p_{sub} \leftarrow (p_{sub} + p_{usv}) \times RdScalar$ 
3: end while
4: return  $p_{sub}$ 

```

357 In the Algorithm 2, the parameter $RdScalar$ varies based on the dynamics of
 358 USV, i.e. if the USV has high manoeuvrability, it is able to reduce the distance
 359 travelled by a large amount thereby setting $RdScalar$ with a small value such
 360 as 0.1. Sub target re-planning procedure is shown in Fig. 12b. Based on sub
 361 target points, the algorithm computes the trajectory for follower vehicles until
 362 all of them have been updated, which is the end of time cycle t . Then it will
 363 continue the path planning process until leader vehicle arrives at the final target
 364 point.

365 6. Simulations

366 To validate the algorithm, simulations have been carried out using two differ-
 367 ent tests in the dynamic environment with one moving obstacle and dynamic
 368 environment with multiple moving obstacles. We use practical simulation areas
 369 to further test the algorithm's capability dealing with real navigation require-
 370 ment. The algorithm has been coded in Matlab and simulations are run on the
 371 computer with a Pentium i7 3.4 Ghz processor and 4Gb of RAM.

372 In the simulations, we assume that identical USVs are used in formation. Speed

373 of leader USV is set as constant such that it is easier for other USVs to fol-
374 low. Followers, however, can vary their speeds according to their positions in
375 formation. For example, follower USV needs to remain at the same velocity as
376 leader's when it is moving at desired formation position. If current position of
377 follower deviates from the desired position, it is required for follower to speed
378 up to catch up or slow down to wait for the leader.

379 *6.1. Simulation in dynamic environment with one moving obstacle*

380 In the first test, simulation area is selected near Portsmouth harbour (Fig.
381 13a), which is a large natural water area and one of the busiest harbours in
382 the UK. The dimension of the area is 2500 m×2500 m, which is transferred
383 to a 500 pixels×500 pixels grid map (Fig. 13b). The start and end points for
384 USV formation are marked as red and purple markers in Fig. 13a. To test
385 the capability of the algorithm dealing with dynamic obstacle, a moving vessel
386 with a constant speed of 6 knot and a constant course of 284° is added into the
387 simulation area.

388 Simulation results recording the movement sequences of the formation are rep-
389 resented in Fig. 14. Each representative sequence is depicted in both a binary
390 map and the corresponding potential map. In binary maps, the leader USV is
391 drawn in red, and follower1 and follower2 USVs are in magenta and blue. The
392 track of the target ship (TS) is represented as red circles. The binary map is
393 generated based on leader USV's view with its instantaneous position drawn as
394 black square marker.

395 Since the harbour has a narrow channel, the line formation shape is selected as
396 the desired formation shape with a formation distance of 15 pixels (75 m). How-
397 ever, to validate the algorithm's capability of formation generation, a triangle
398 formation shape is assigned as the initial shape shown in Fig. 14a. In Fig. 14b,
399 safety potential map of the simulation area along with TS is shown. It is clear
400 that both static obstacle area (in dark blue) and safe area (in red) have been
401 identified. In addition, the TS has also been well represented with a circular

402 ship domain and collision avoidance area. After time step 5, the formation forms
403 the line shape and keeps such shape entering into the channel area (Fig. 14c -
404 Fig. 14f). Fig. 14g - Fig. 14l illustrate how the formation avoids the TS. When
405 the formation approaches close to the TS, port side turning is adopted by the
406 leader, and two followers will follow this behaviour. In the corresponding safety
407 potential maps (Fig. 14h, Fig. 14j and Fig. 14l), it can be observed that each
408 USV can stay well outside the ship domain and inside the collision avoidance
409 area of TS to generate a collision avoiding trajectory. After the collision risk is
410 avoided, the formation moves towards target point and reaches it at time step
411 113.

412 Evaluations of the algorithm performance and USV formation behaviour are
413 given in Fig. 15. Fig. 15a shows the overall trajectories for the formation,
414 and all of them remain a safe distance away from static obstacles, which proves
415 that the algorithm is able to generate acceptable safe paths in a complex envi-
416 ronment. Furthermore, in Fig. 15b, distances between TS and each USV are
417 recorded. It is noted that the closest distances for leader and two followers are
418 approximately 21 pixels, 17 pixels and 25 pixels, which demonstrates that for-
419 mation can effectively avoid moving obstacle. In terms of formation behaviour,
420 distance errors between actual positions and desired positions for follower1 and
421 follower2 are shown in Fig. 15c. It may be concluded that during initial time
422 steps, large errors occur since two followers are not located at their desired po-
423 sitions. However, both of them can fast navigate to their formation positions by
424 following generated trajectories, and once the formation is formed the formation
425 shape can be well maintained as the error values remain relatively small.

426 *6.2. Simulation in dynamic environment with multiple moving obstacles*

427 A more complex simulation is done in a dynamic environment with multiple
428 moving vessels. Ocean area near Plymouth harbour shown in Fig. 16a is selected
429 as the testing area. In Fig. 16b, planning space has been transformed into a
430 square area with 500×500 pixels dimension representing 2.5×2.5 km area. Now,

431 three virtual target ships are added into the environment travelling at 20 knot
432 (TS1), 6 knot (TS2) and 12 knot (TS3) respectively.

433 The formation now starts with line shape and the desired formation shape is
434 triangular with formation distance as 15 pixels (75m). Movement sequences of
435 the formation are represented in Fig. 17, which includes both the original binary
436 maps as well as the potential maps. In the potential maps, it is shown that the
437 algorithm can well define the ship domain and collision avoidance areas of three
438 target ships based on their velocities. TS1 has the highest velocity thereby
439 forming an half-elliptical shape. In contrast, the other two ships are relatively
440 slow, so more circular shapes are assigned. Between them, because TS3 has
441 larger speed than TS2, generated area of TS3 has a longer radius than TS2's.
442 In addition, to prevent internal collision, internal USV is viewed as a circle with
443 radius representing safe distance in potential map.

444 To assess the algorithm, first of all, trajectories generated by the algorithm are
445 shown in Fig. 18a. It is clear that each path maintains a good position to
446 the others and does not collide with any static obstacles. Fig. 18b shows the
447 distances between target ships and each USV for whole simulation time period.
448 Smallest distance occurs at time step 61 with the value of 11 pixels (55 m)
449 between TS2 and follower1, which means that the formation does not collide
450 with any target ships. In terms of formation behaviour, Fig. 18c records the
451 distance error values. Except the initial formation generation stages, the values
452 remain close to zero for most of simulation time, which means that the formation
453 shape is well maintained.

454 7. Conclusions and future work

455 This paper introduced and discussed a path planning algorithm for the USV
456 formation navigation. Fahimi (2007) and Antonelli et al. (2006) have previously
457 investigated the problem of USV formation, the emphasis of these works is on
458 robust control (Level 3 in Fig. 1) instead of path planning (Level 2 in Fig. 1).

459 The algorithm we introduced in this paper is the first work specifically dealing
460 with the USV formation path planning problem. The algorithm developed is
461 based on the FM method, which has features of fast computation speed and
462 low computation complexity. To particularly address the dynamic problem in
463 path planning, a Constrained FM method has been proposed and developed
464 to construct two areas, i.e. ship domain area and collision avoidance area, to
465 ensure the planned trajectory to not violate any forbidden area. In addition, the
466 output from the algorithm shows that collision free paths can be generated for
467 formations for complex, practical and for both static and dynamic environments.
468 More importantly, since all of the simulations are taken in real navigation areas,
469 it is worth mentioning that the algorithm is practical and can potentially be
470 developed to advance navigation in manned ships.

471 For future work, the algorithms proposed will be improved in several ways. First,
472 the practicability of planned paths can be further increased. COLREGS, which
473 is the international maritime collision avoidance regulation, is largely obeyed
474 by most navigators when taking collision avoidance manoeuvres and should
475 also be integrated into current algorithms. Second, the trajectory could be
476 optimised in terms of aspects such as energy consumption, and environment
477 influences such as current and wind. Thirdly, a mission planning module can
478 be included into the algorithm. The module is a self-decision making system,
479 which can accordingly assign different missions based on specific requests. This
480 will enormously improve the autonomy of USVs, which is the ultimate goal of
481 this research.

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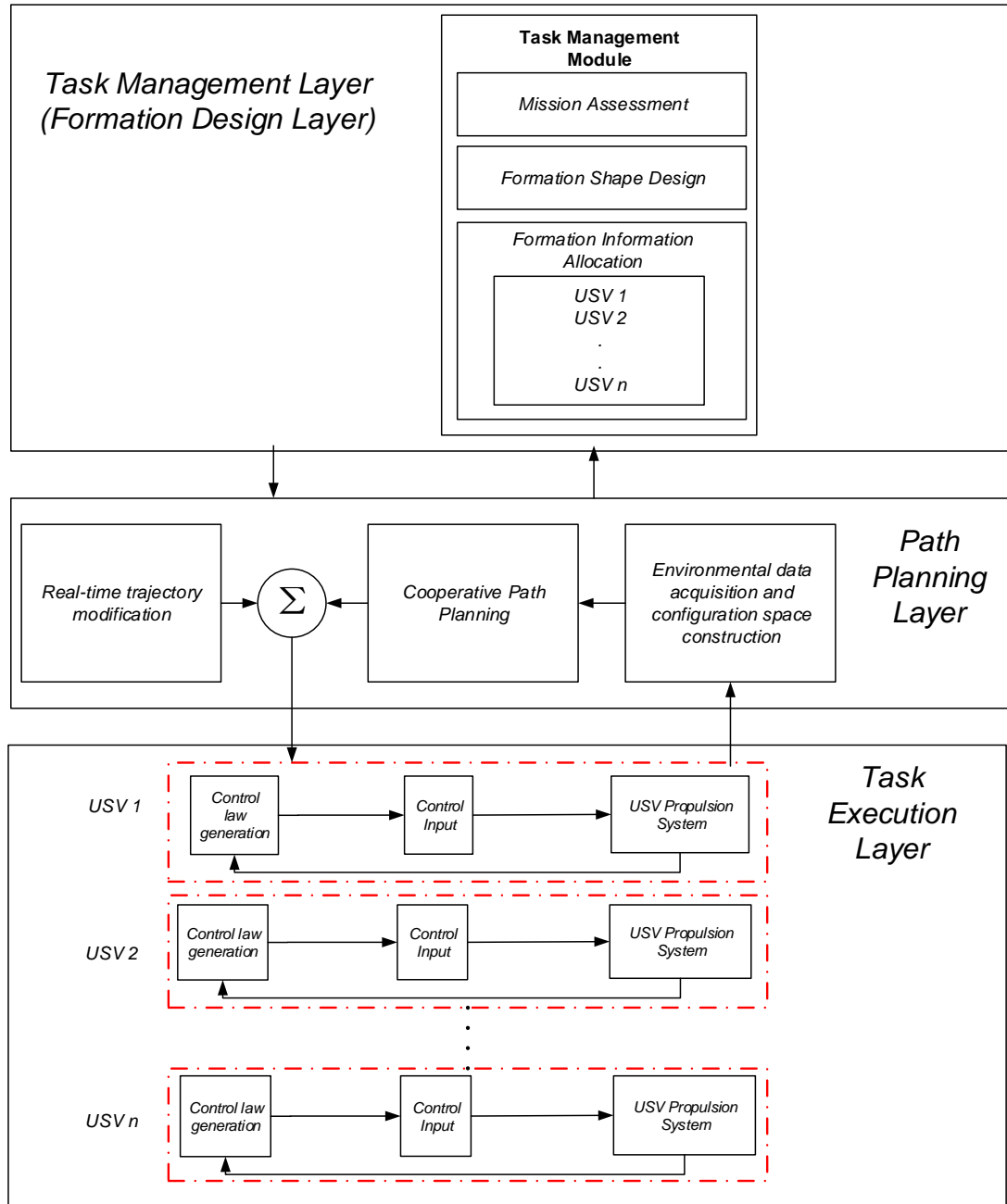


Figure 1: Hierarchy of multiple USVs system.

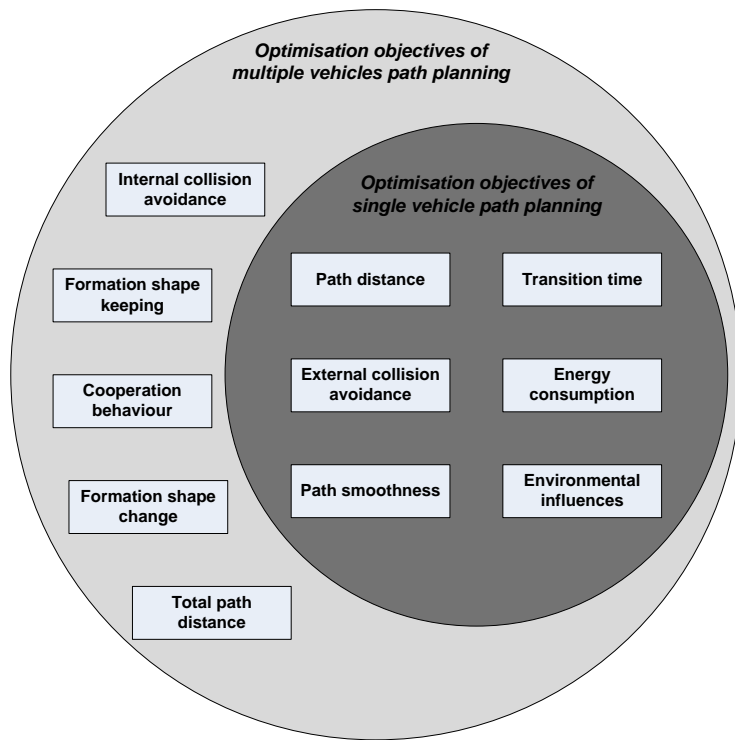


Figure 2: Comparison of formation and single vehicle path planning.

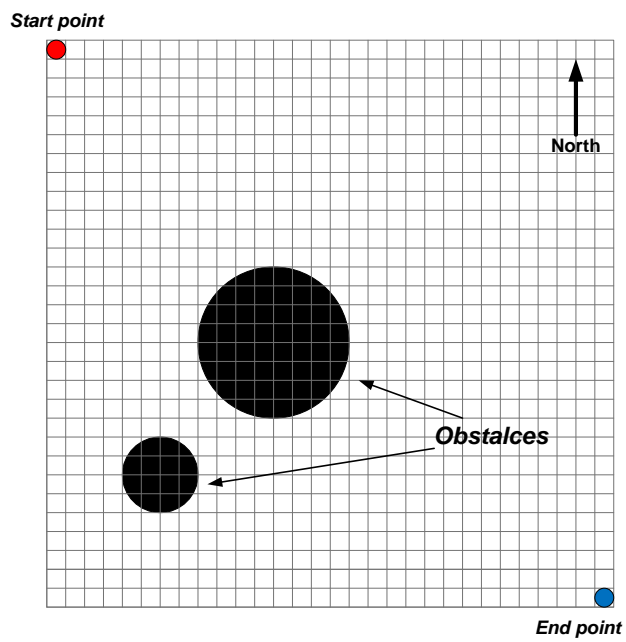


Figure 3: Grid map.

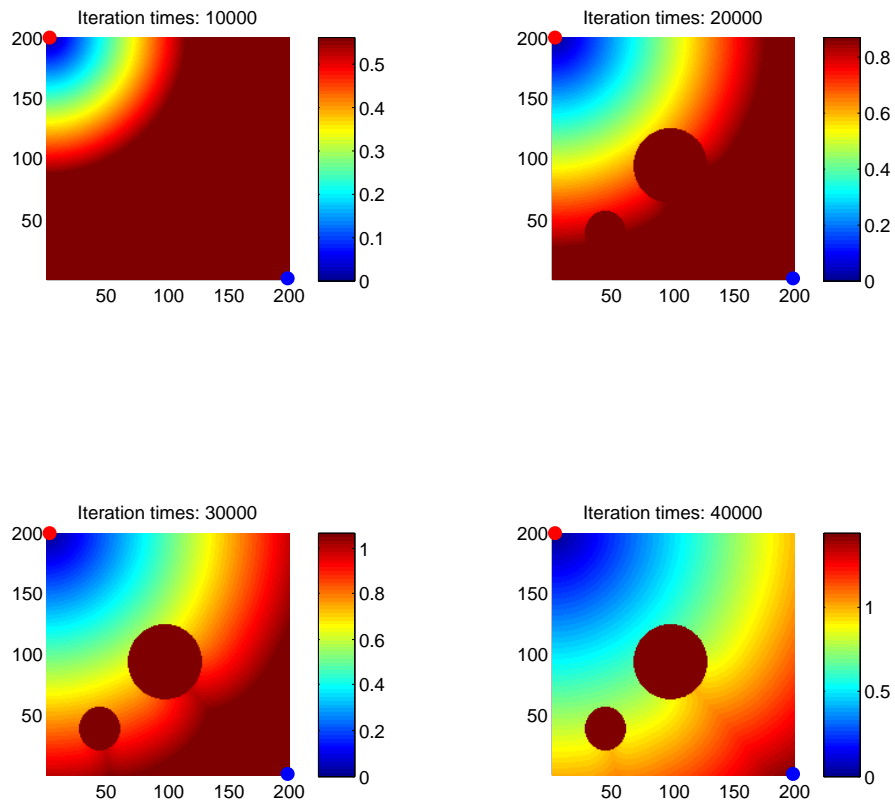
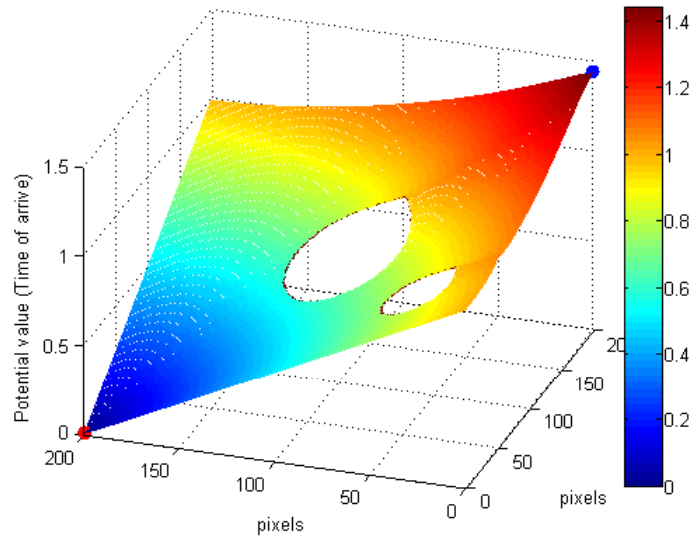
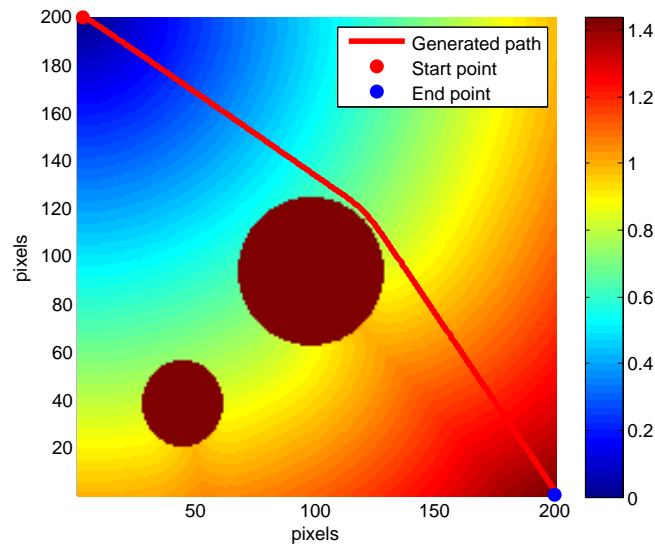


Figure 4: Simulating interface propagation process by using FM method. Interface starts to emit from $(0, 200)$ and ends at $(200, 0)$. Processes are recorded at iteration times 10000, 20000, 30000, 40000 respectively. Colour in the figure represents the local interface arrival time. The brighter the colour is, the longer the arrival time will be.

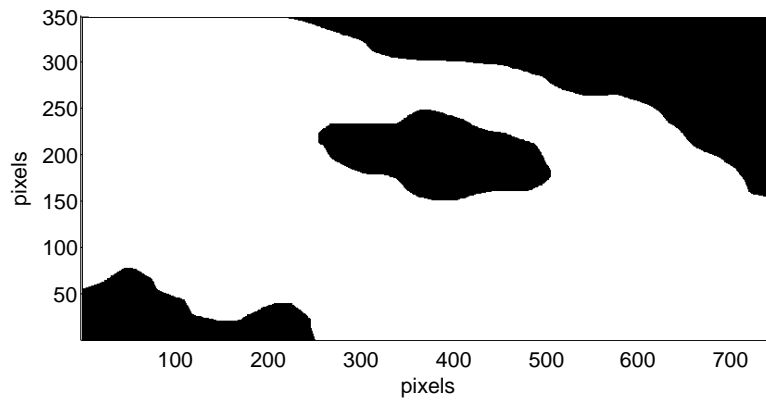


(a)

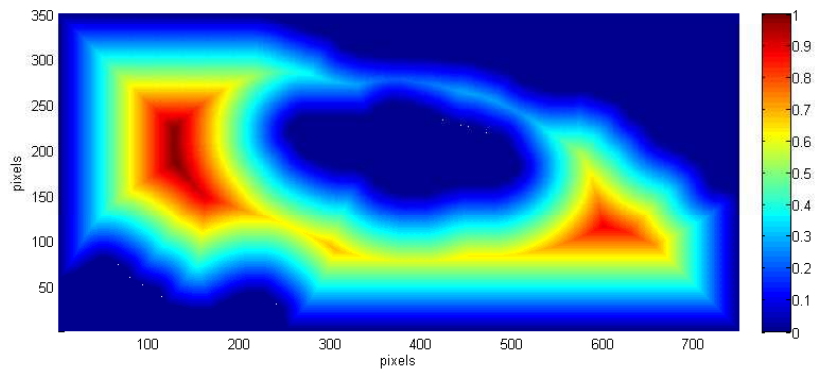


(b)

Figure 5: (a) Potential field generated by running FM method. Local potential value represents local interface arrival time. (b) Path generated by following gradient of potential field.



(a)



(b)

Figure 6: (a) Original environment map (M_o) in binary format. (b) Safety map (M_s) generated by FM method.

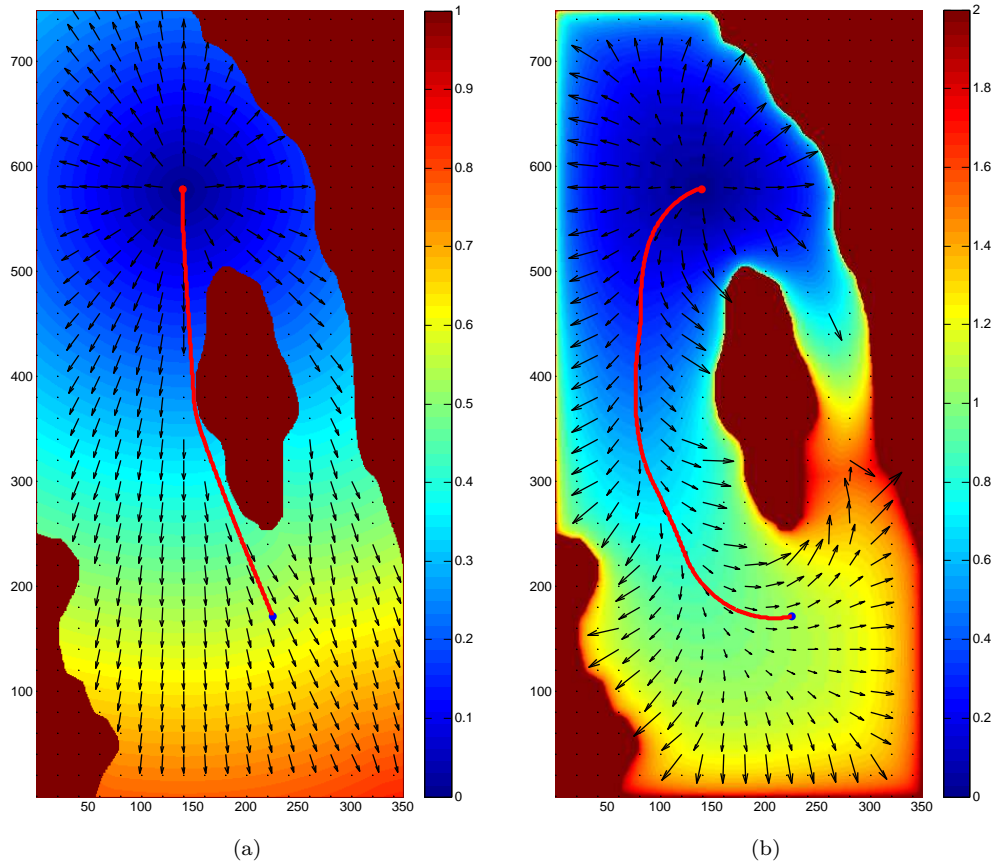


Figure 7: (a) Potential field and corresponding path generated by FM method. (b) Potential field and corresponding path generated by FMS method.

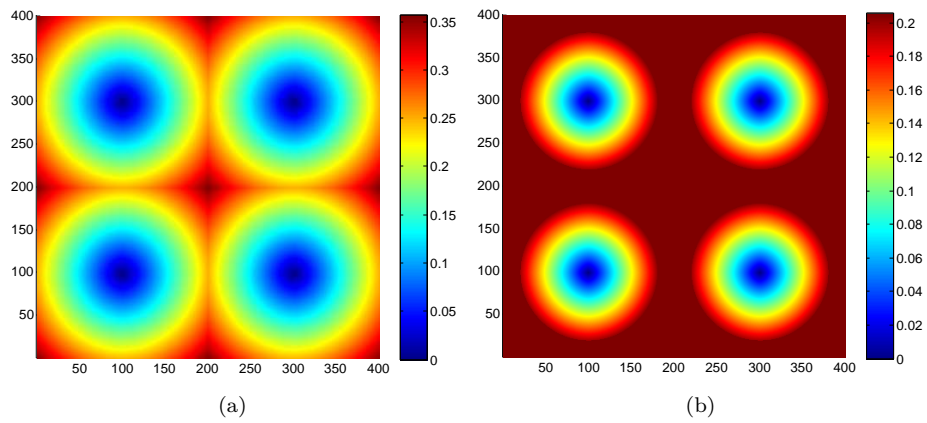


Figure 8: Comparison between conventional FM and constrained FM method. (a)Interface propagation from four start points by using conventional FM method. (b) Interface propagation from four start points by using constrained FM method. Constrained area is built as circle with radius of 20 pixels.

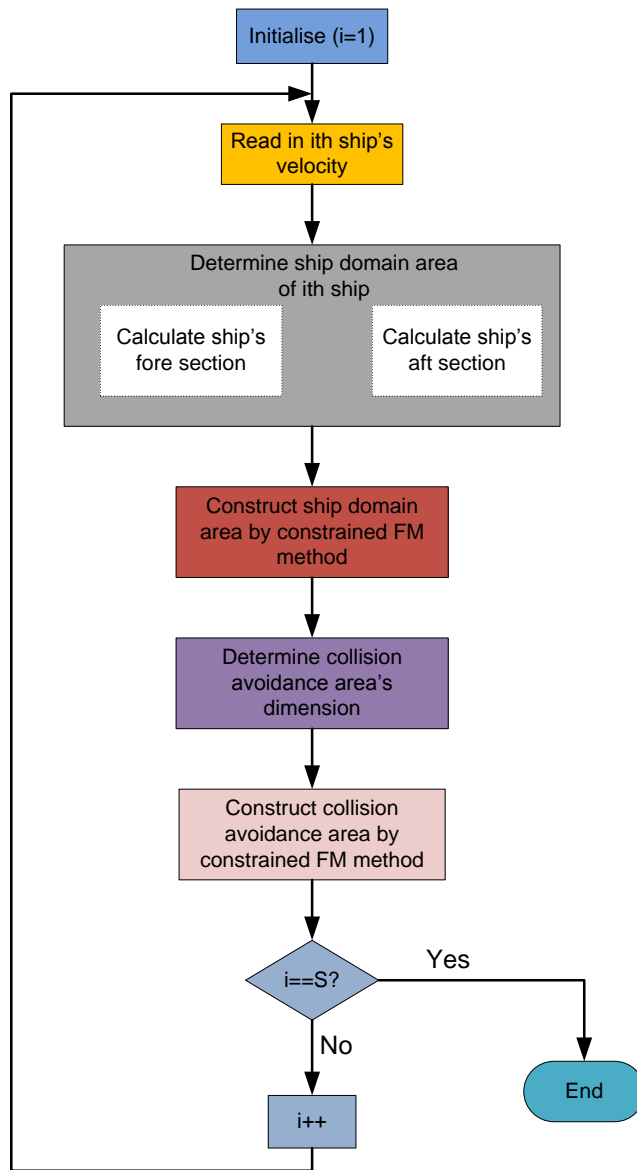


Figure 9: Algorithm flow chart of moving ships modelling by using constrained FM method

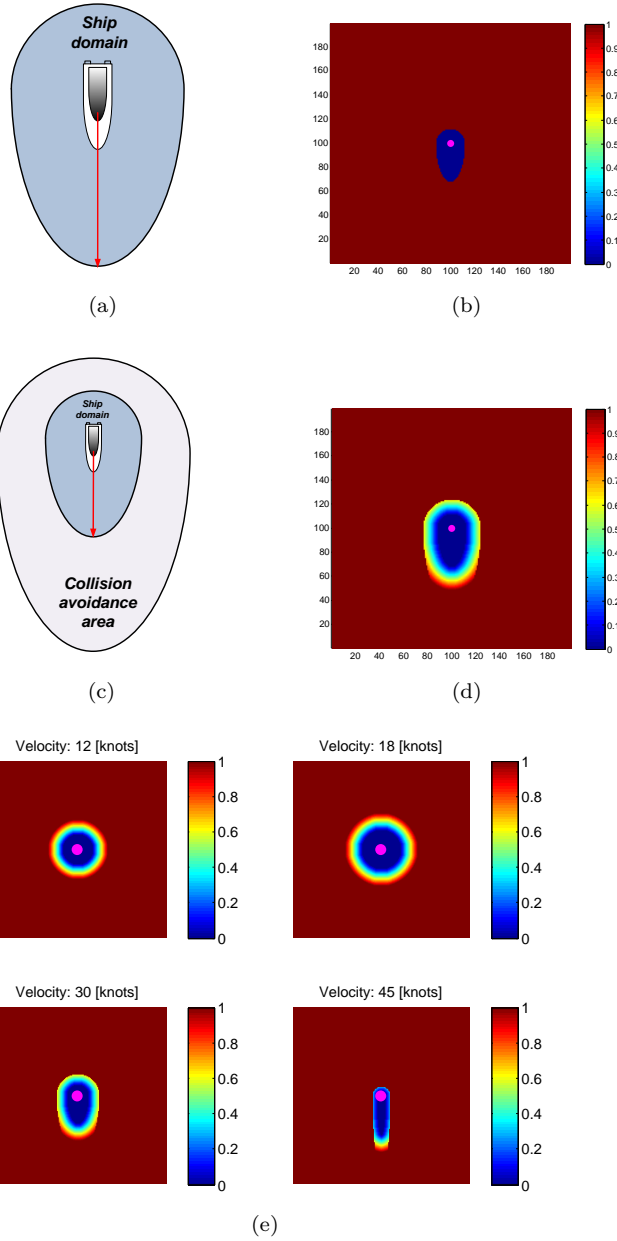


Figure 10: (a) Ship domain area. (b) Ship domain constructed using constrained FM method by using ship's position as start point. (c) Ship domain and collision avoidance area. (d) Collision avoidance area constructed using constrained FM method by using points in ship domain as start points. (e) Different ship domains under different speeds.

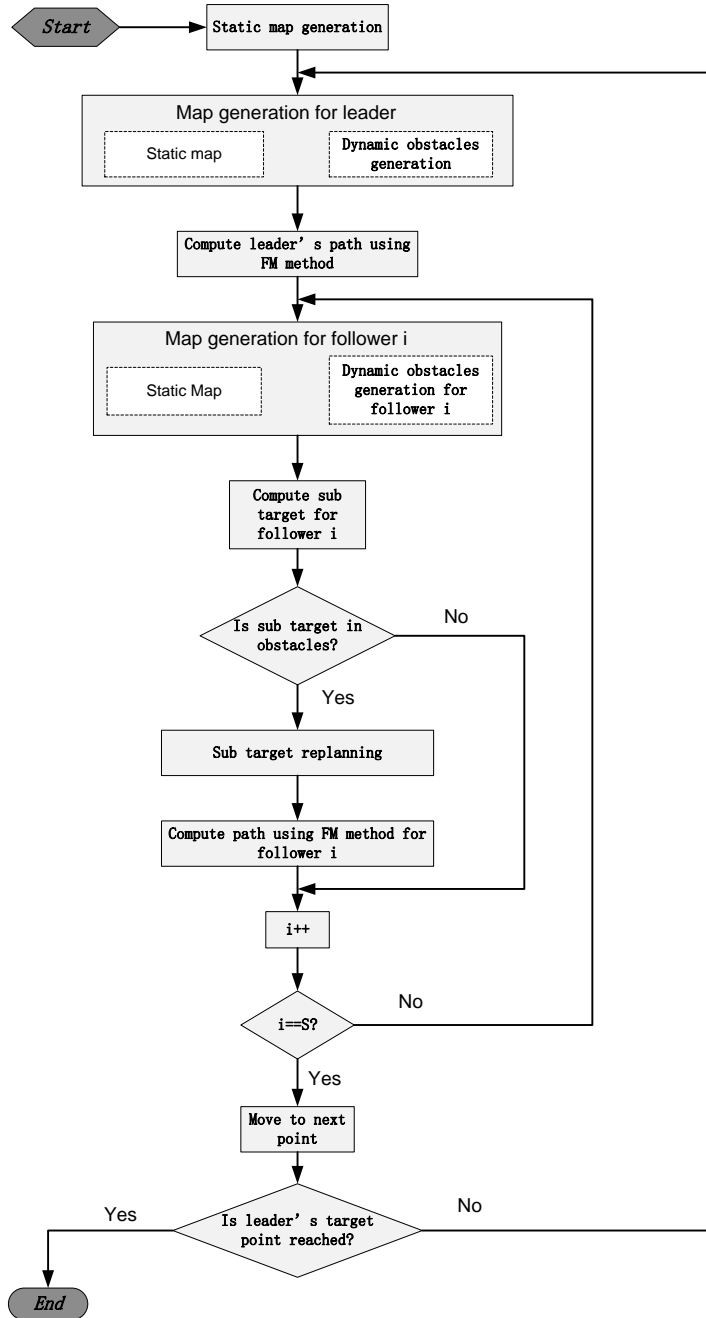


Figure 11: Algorithm flow chart of USV formation path planning

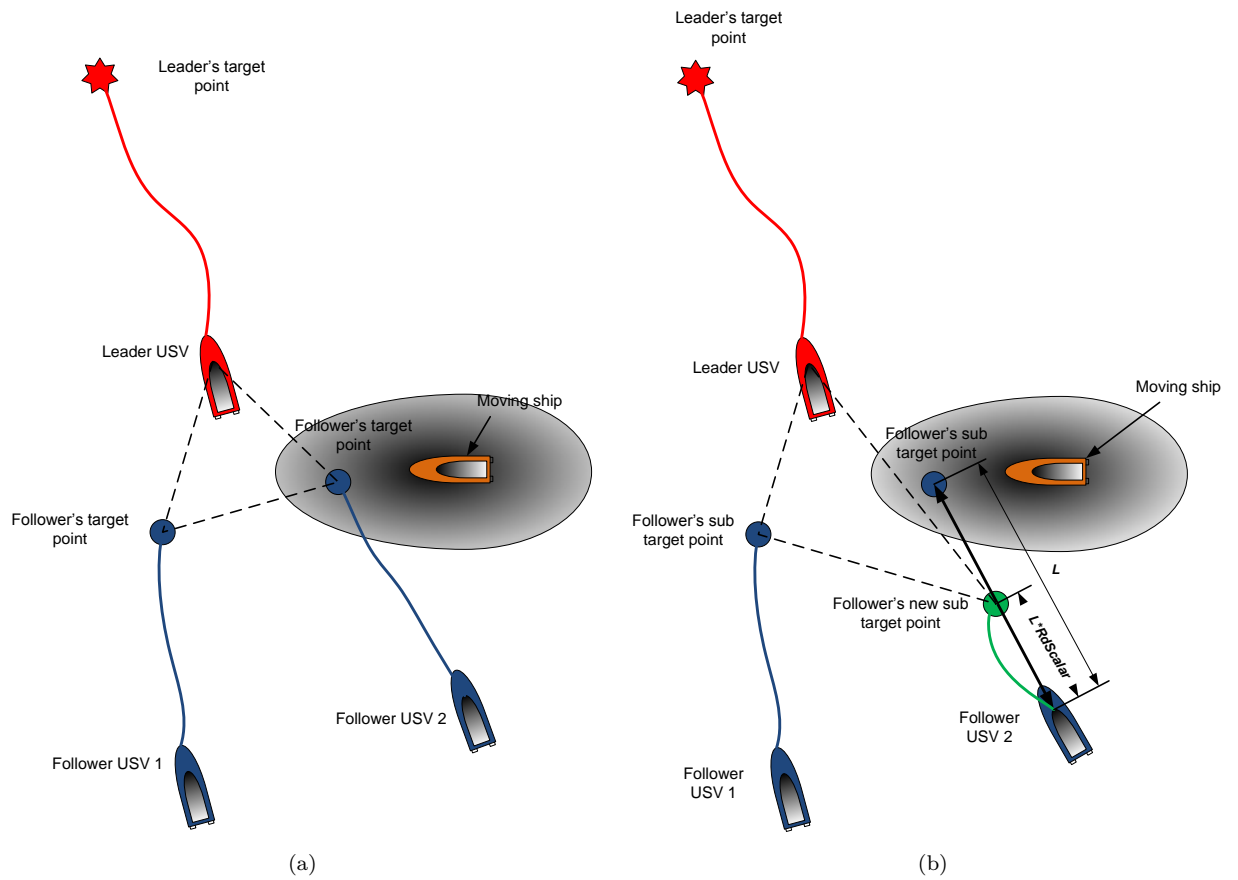
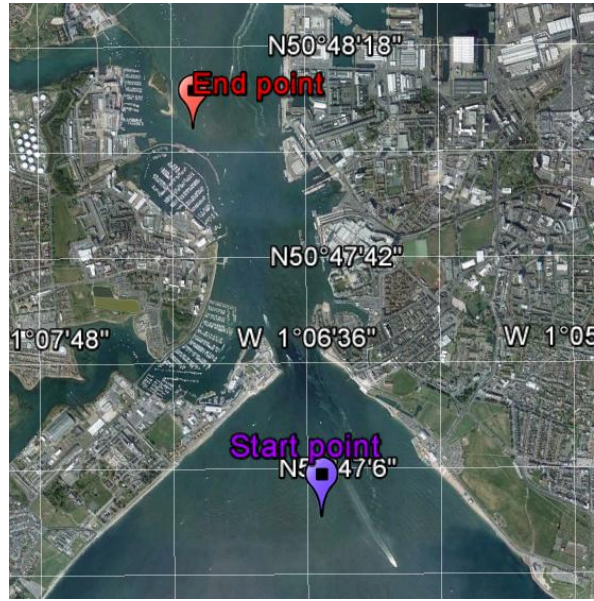
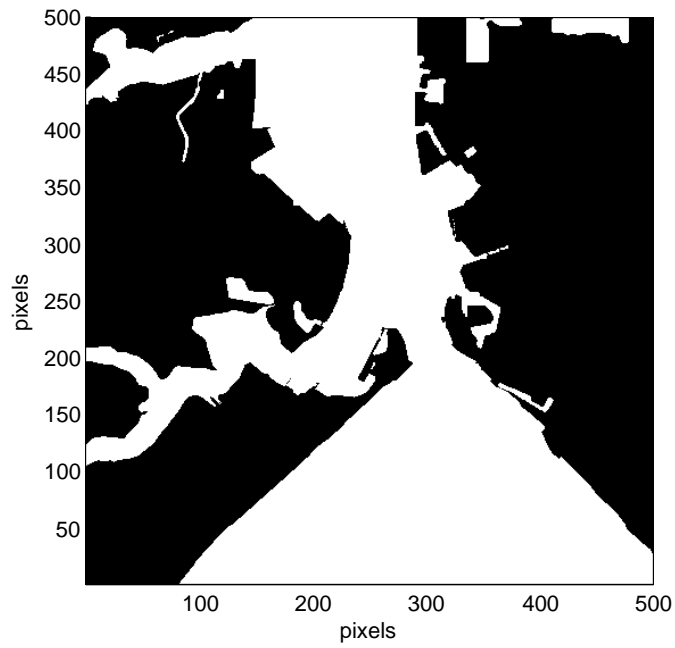


Figure 12: (a) Sub targets planned for two followers with follower2's located in moving ship's ship domain area. (b) Sub target re-planned for follower2, new target point (plotted in green colour) is outside ship domain.



(a)



(b)

Figure 13: (a) Simulation area (Portsmouth harbour). (b) Binary map of simulation area.

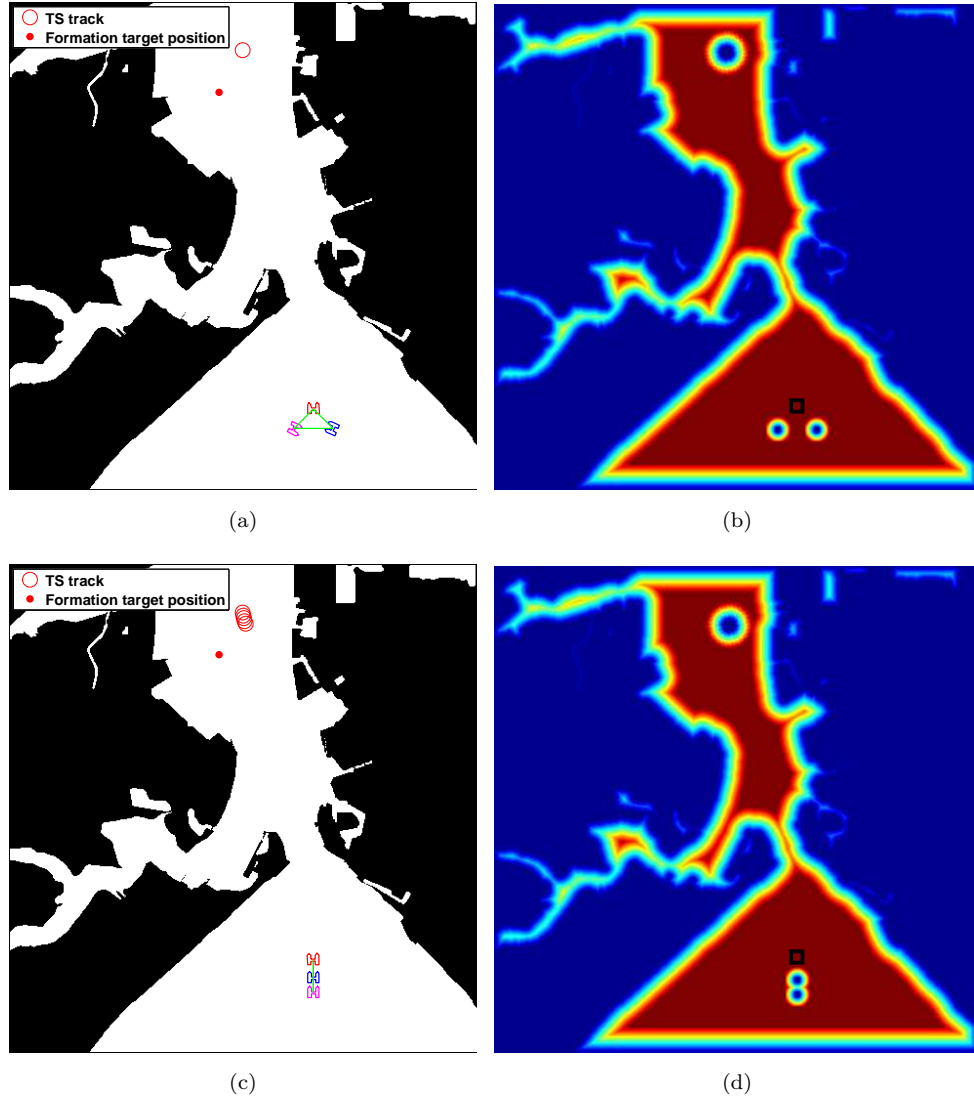
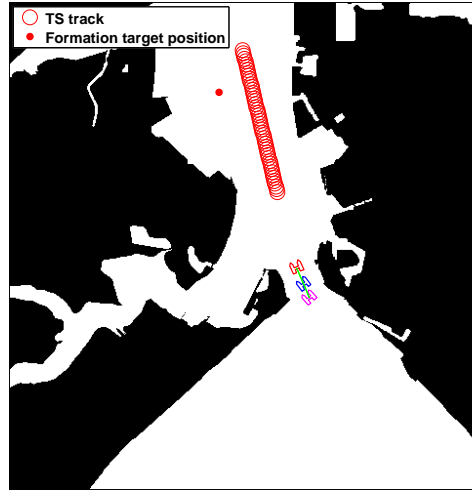
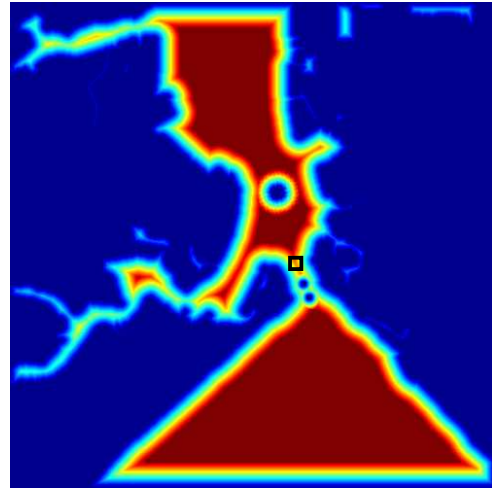


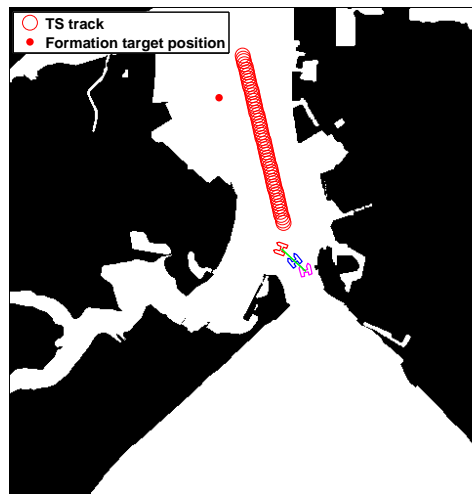
Figure 14: Formation movement sequences and corresponding potential maps. (a)-(b) Time step = 1. (c)-(d) Time step = 5.



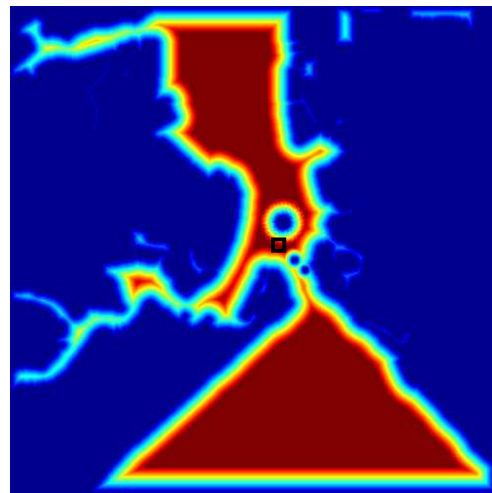
(e)



(f)



(g)



(h)

Figure 14: Formation movement sequences and corresponding potential maps. (e)-(f) Time step = 51. (g)-(h) Time step = 60.

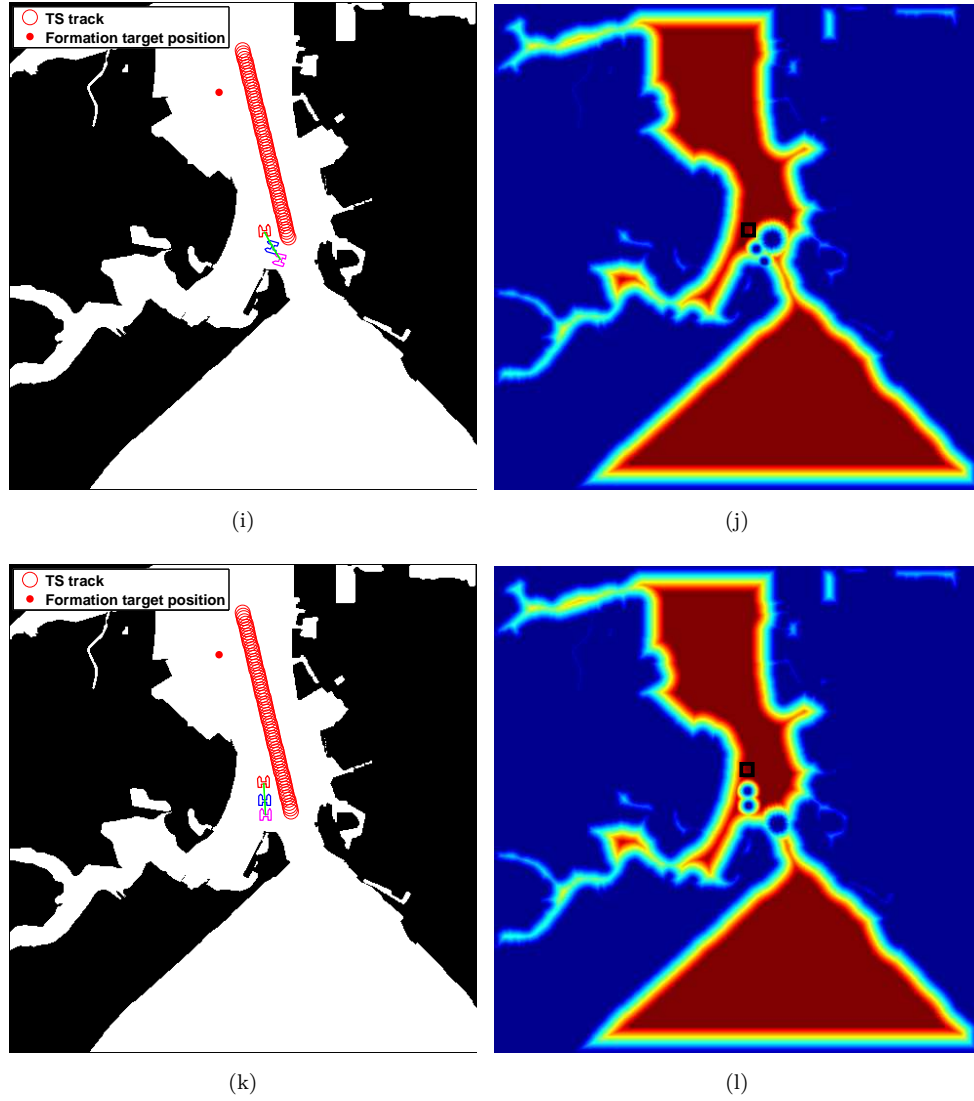


Figure 14: Formation movement sequences and corresponding potential maps. (i)-(j) Time step = 67. (k)-(l) Time step = 75.

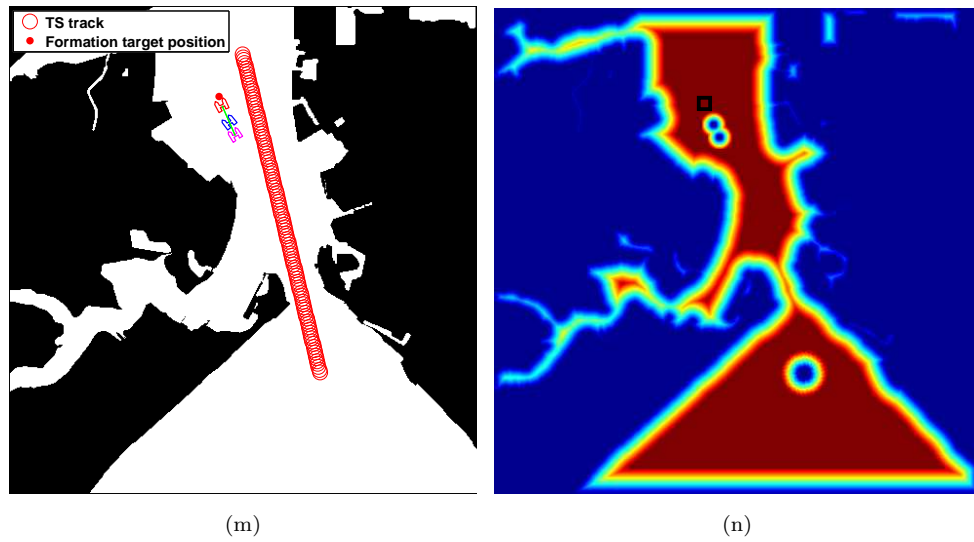
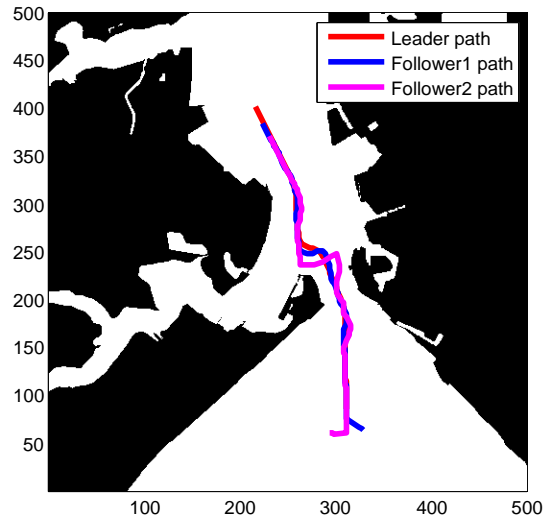
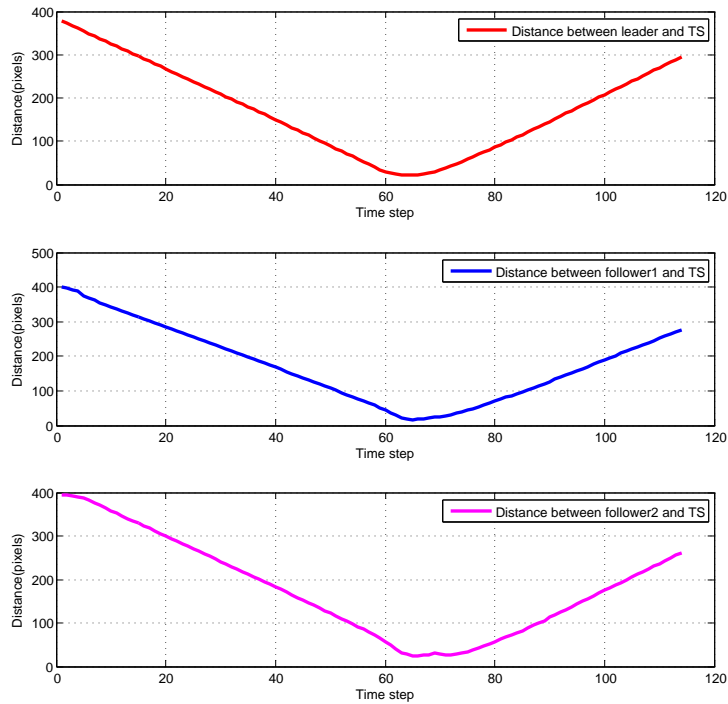


Figure 14: Formation movement sequences and corresponding potential maps. (m)-(n) Time step = 113.

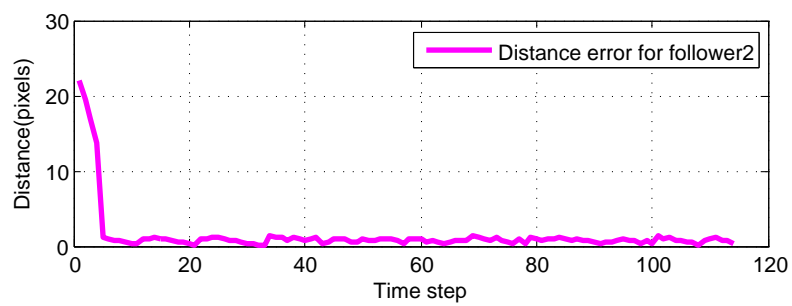
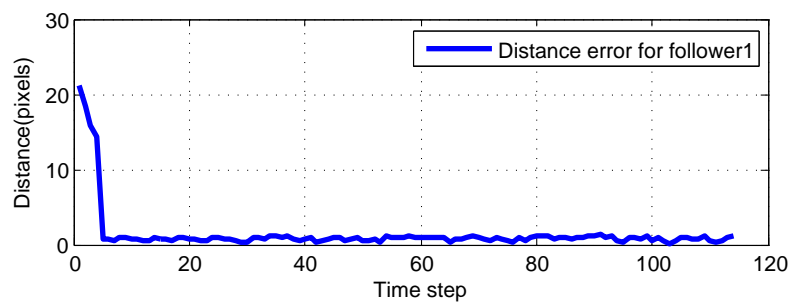


(a)



(b)

Figure 15: Evaluation results. (a) Trajectories for formation. (b) Distance between TS and each USV in formation.

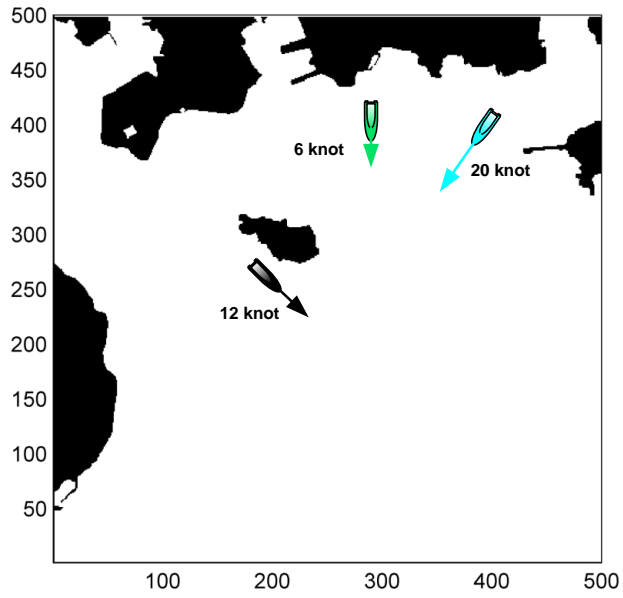


(c)

Figure 15: Evaluation results. (c) Distance errors for follower1 and follower2.



(a)



(b)

Figure 16: (a) Simulation area (Plymouth harbour). (b) Binary map of simulation area.

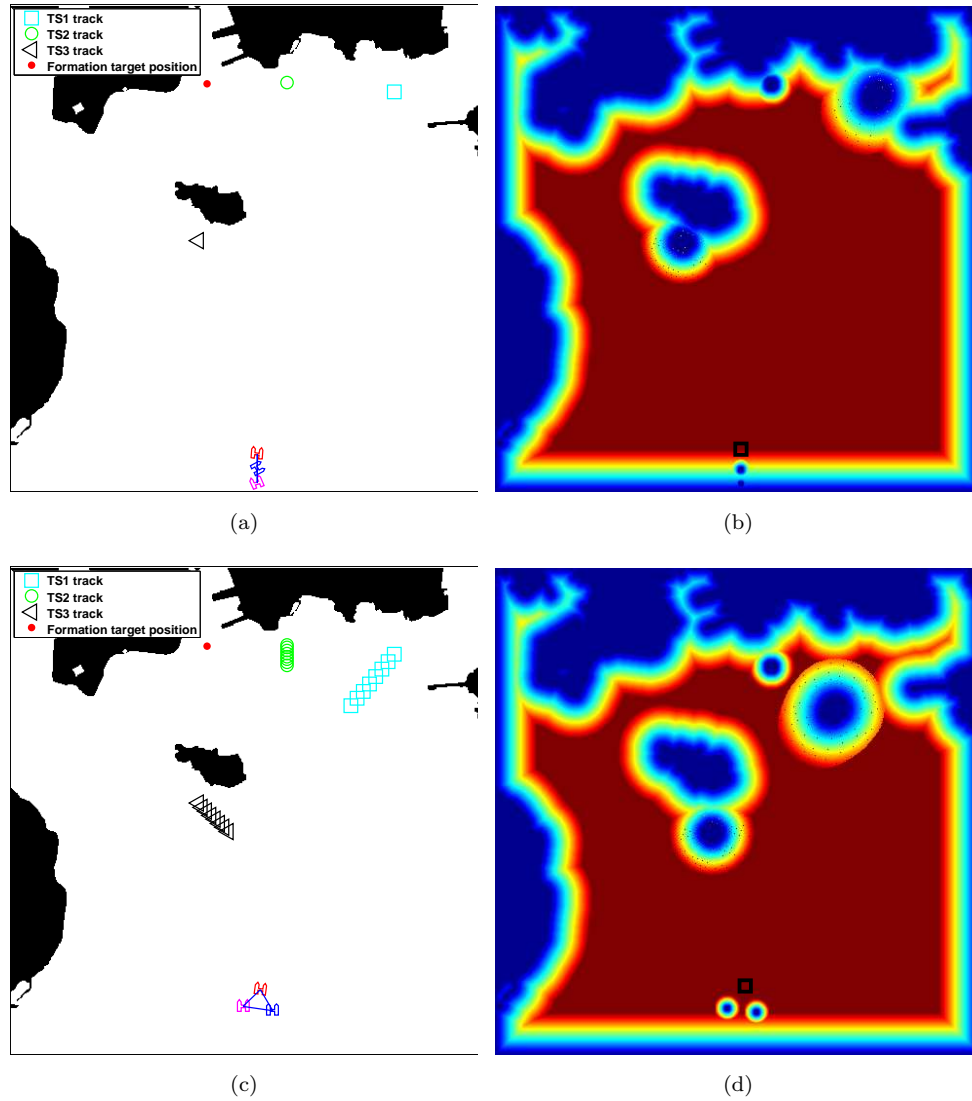


Figure 17: Formation movement sequences and corresponding potential maps. (a)-(b) Time step = 1. (c)-(d) Time step = 8.

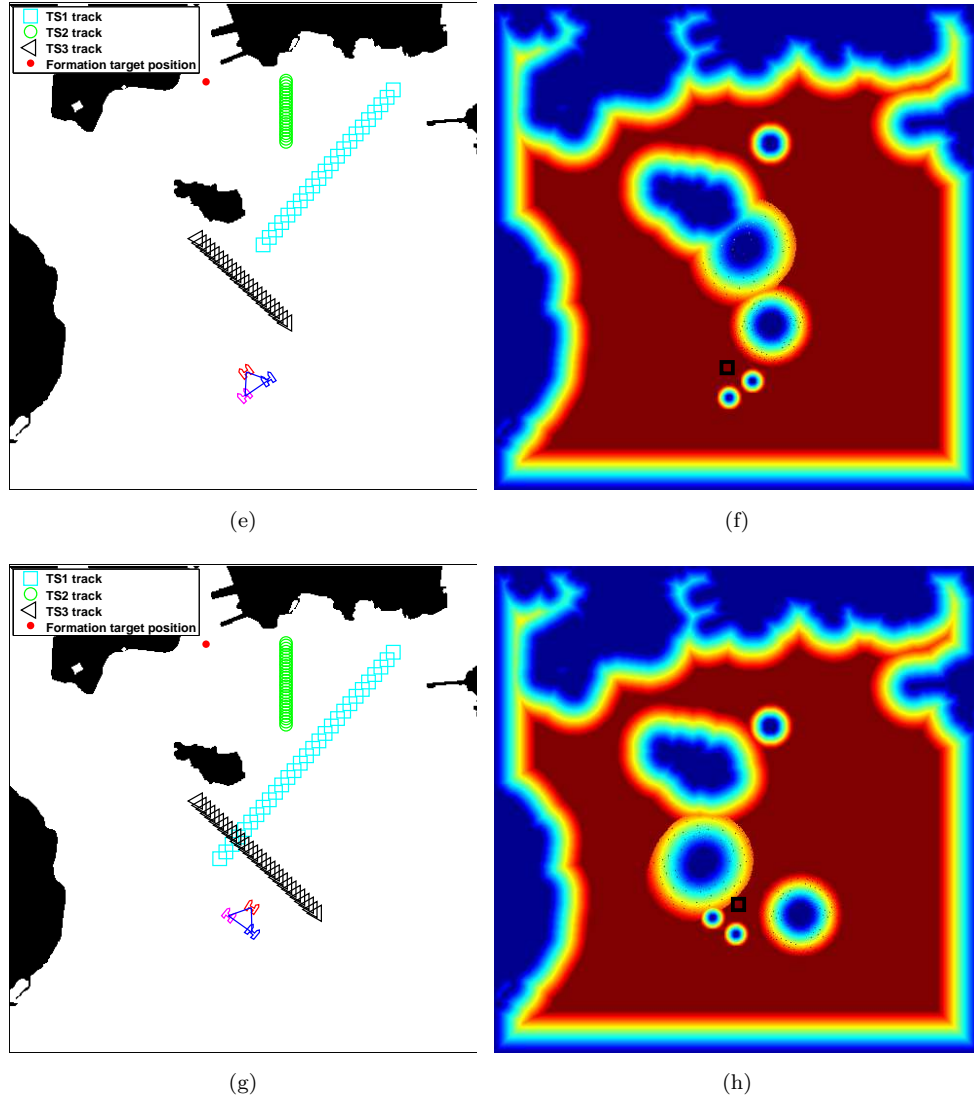


Figure 17: Formation movement sequences and corresponding potential maps. (e)-(f) Time step = 22. (g)-(h) Time step = 29.

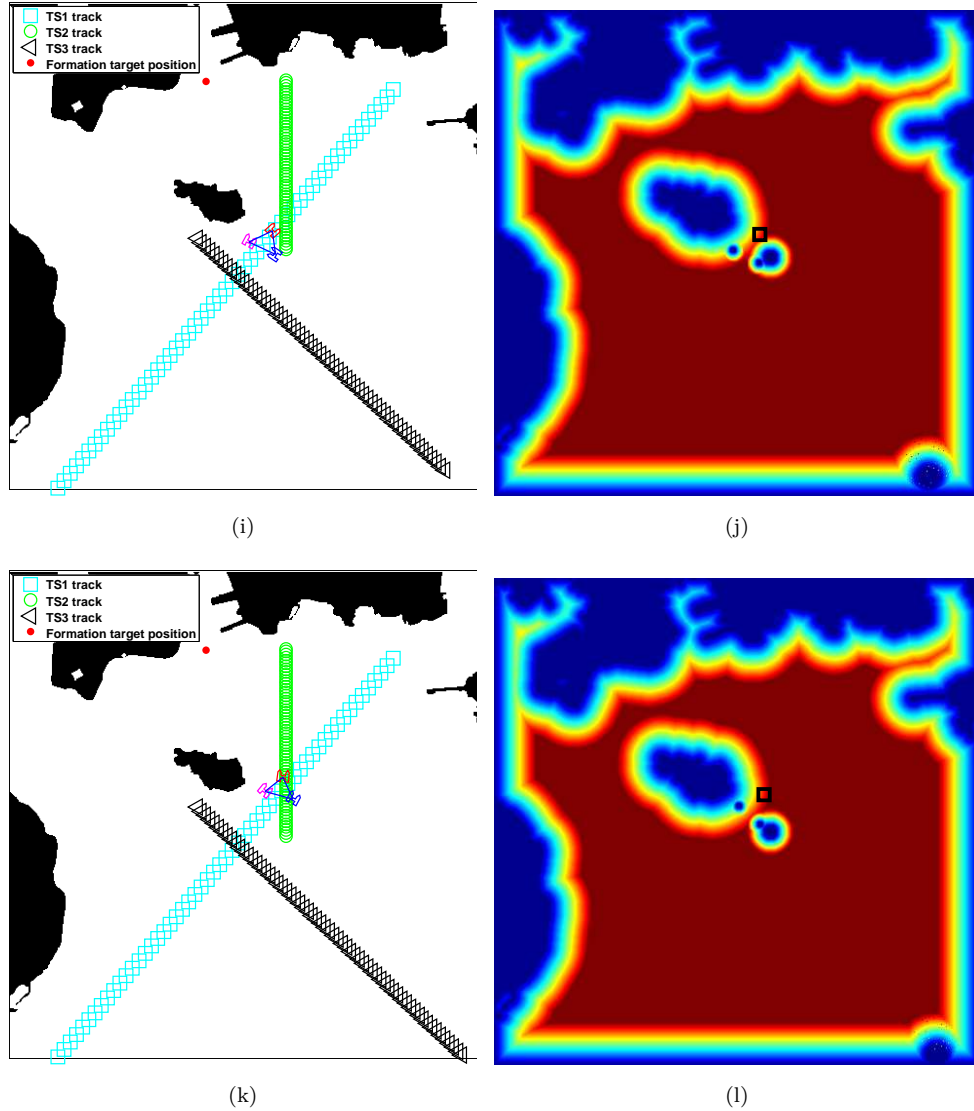


Figure 17: Formation movement sequences and corresponding potential maps. (i)-(j) Time step = 59. (k)-(l) Time step = 65.

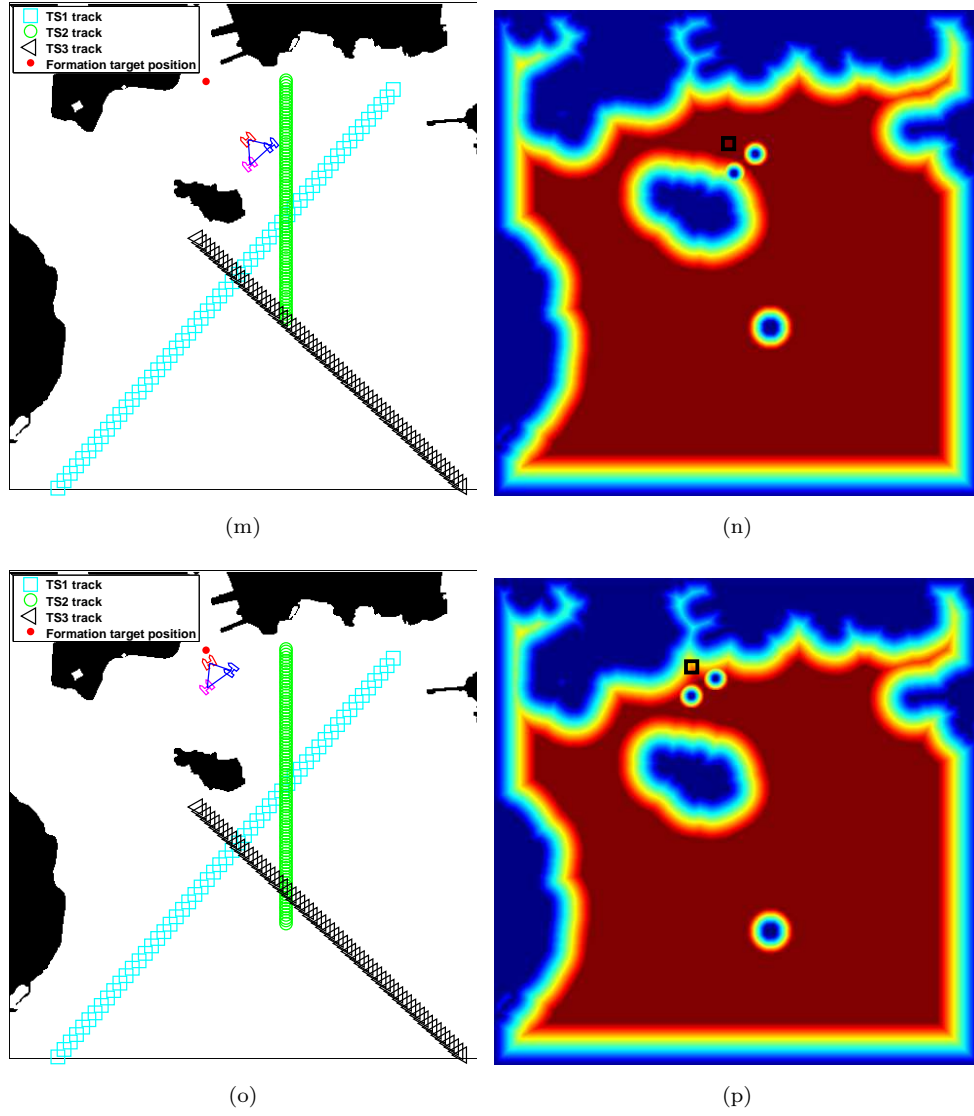
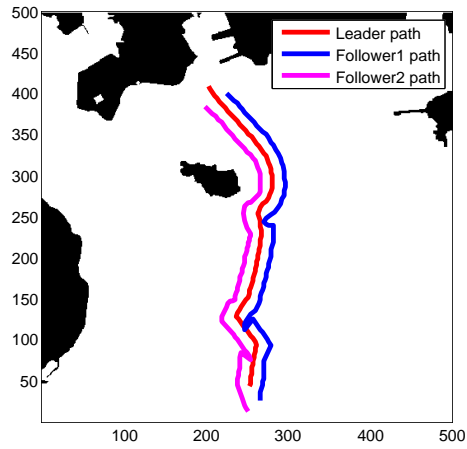
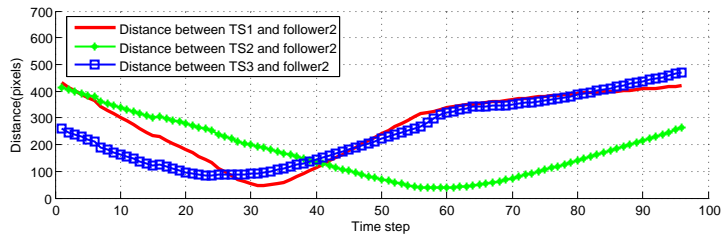
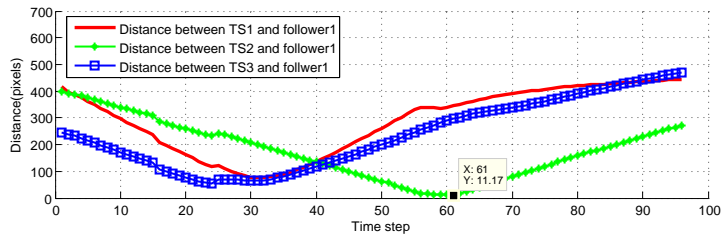
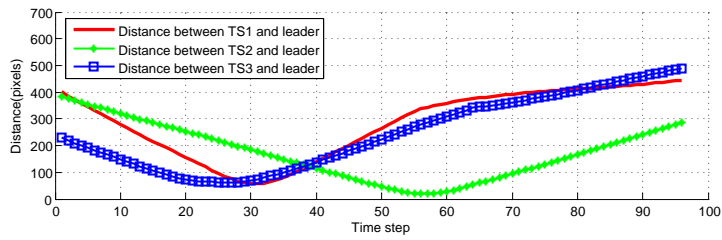


Figure 17: Formation movement sequences and corresponding potential maps. (m)-(n) Time step = 83. (o)-(p) Time step = 97.

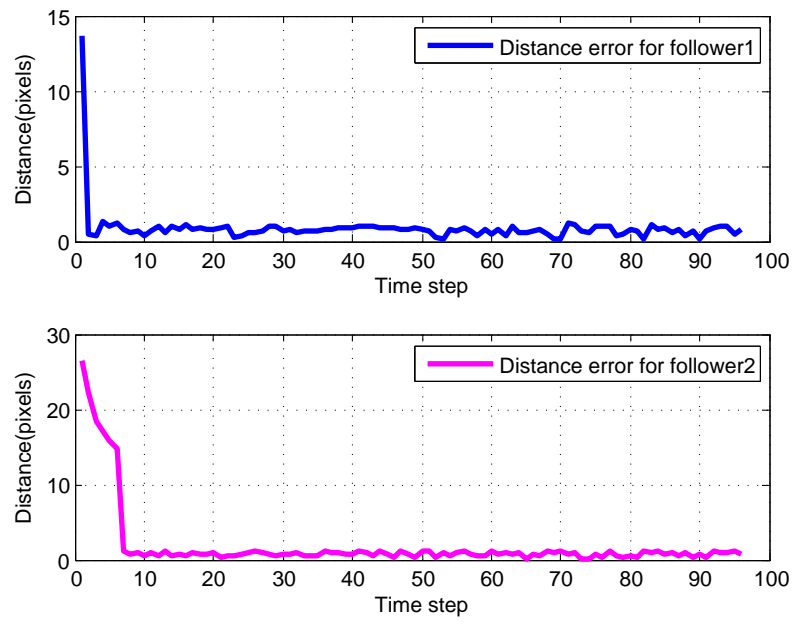


(a)



(b)

Figure 18: Evaluation results. (a) Trajectories for formation. (b) Distance between target ships and each USV in formation.



(c)

Figure 18: Evaluation results. (c) Distance errors for follower1 and follower2.