

## DYNAMIC DEADWEIGHT LOSS IN MONOPOLISTIC AND RELATED MARKET STRUCTURES

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**Abstract:** The aim of the paper is to construct a framework in which welfare losses over time generated by alternative market structures may be estimated. An adjustment cost model of the firm under imperfect competition is developed, and consequent industry equilibria determined. A dynamic analogue of Harberger's measure of welfare loss is specified, and for the case of a monopoly industry can be expressed as a function solely of Tobin's average  $q$ . The welfare measures calculated are on the basis of the market's expectations of the future profitability of firms; such measures allow a significant additional set of data to be used to construct a forward looking measure of welfare loss, and thus augment existing measures of industry appraisal.

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## I Introduction

The aim of the paper is to construct a framework in which welfare losses *over time* generated by alternative market structures may be determined. The welfare measures proposed are on the basis of the market's expectations of the future profitability of firms; such measures allow a significant additional set of data to be used to construct a *forward* looking measure of welfare loss, and thus augment existing measures of industry appraisal. A dynamic analogue of Harberger's measure of welfare loss is derived, which for the case of a monopoly industry can be expressed as a function solely of Tobin's average  $q_a$  (Tobin(1969)). The use of firms' accounting data has constituted the major empirical source in evaluating welfare loss under different market structures since the early study of Harberger(1954); however little attempt has been made to use financial data, i.e. the market valuation of the firm, in the evaluation of welfare loss. The paper thus makes use of the relationship between market value of the firm and replacement cost as information in determining the welfare loss resulting from firm behaviour.

The majority of estimates of welfare loss that have been constructed apply to a single period of time, e.g. the yearly accounting period of the firm, in which case the information required for evaluation is usually available in a reasonably accessible form. However, welfare losses accrue over more than one period, and therefore the temporal aggregation of such losses may thus be required. At first sight this does not appear to be much of a problem, estimates of the losses for the past ten years, say, would simply require the addition of the losses over this period. In most policy exercises however, e.g. in assessing the impact of a possible merger, it is not estimates of past welfare losses which are important, but the forecast of future welfare losses.

In addition to the mechanics of computing welfare losses over time, introducing the temporal aspect brings to the forefront the question of the persistence of welfare losses. One of the standard criticisms of the monopoly welfare loss literature is that it ignores the dynamics of monopoly power. A prominent school of thought argues that the existence of monopoly power is essentially a transitory phenomena which is eliminated via increased competition from existing firms or entry of new firms. A concomitant implication of this view is that monopoly profits are an essential component of a healthy capitalist economy. It is argued that the existence of such profits serves as an indicator of excess consumer demand and a signal for the redirection of investment to a particular sector. Any attempt to regulate or restrict such profit may therefore be viewed as harmful to economic welfare.

In investigating the temporal aspect of monopoly welfare loss at least two methodologies and data sets may be suggested for use. The first looks back at the record of a particular industry; it determines whether competitive pressures have been working in this industry; and may use such information as a prediction of future behaviour. Of course, the past is not necessarily a good provider of information on future prospects. In cases where, for example, technology, tastes or the international trading environment may be quickly changing, then it may be peculiarly unfortunate to judge the future from past information. An alternative forward looking data set may then be appropriate; one such data set concerns the views of those trading in securities based on the future profit streams of companies. Evidence on the future profitability of companies may be reflected in share price movements, e.g. if a particular regulatory scheme is implemented, and the market price of the shares in those companies rose, then the regulator may use that price rise as evidence for the revision of the regulatory regime.

The question of persistence of welfare loss is an empirical question, and is of course tied to the question of the persistence of monopoly profit. The persistence of profit approach uses past data on profits in evaluating the efficiency of an industry. Recent research in this field is associated with Mueller(1977),(1986). The study edited by Mueller(1990), includes studies from the U.S., U.K., France, Germany and Japan. further studies include Connolly and Schwarz(1985) and Odagari and Yamawaki(1986).

An alternative methodology proposes the use of forward data as expressed in the stock market evaluation of the firm to the same end. The intuition behind the use of financial and accounting data in the evaluation of welfare loss may be stated as follows: the ratio between market value and the replacement cost of capital of the firm, i.e. Tobin's average  $q_a$ , can be viewed as a measure of the competitive environment in which that firm operates. For a competitive industry one would expect to find the average  $q_a$  close to unity ; i.e. in the absence of barriers to entry, firms would decide to enter and thus industry output would increase, and prices driven down to a competitive level, until  $q_a$  is equated to unity. Alternatively firms within an industry would expand output and thus engender price falls until again  $q_a$  achieved unity. Firms in non-competitive industries will however earn monopoly profits, in which case the financial markets may be expected to capitalise these rents, and the market value of the firm will exceed its replacement cost.

Although the intuition appears evident, there do remain a number of outstanding questions. To what degree is free entry a necessary requirement for industry  $q_a$  to be equated to unity ? In the absence of entry will competition amongst existing firms ensure the elimination of monopoly rents ? How is  $q_a$  related to the existence of different market structures ? To what degree may firms with differing values of  $q_a$  coexist within the same industry, or does competition ensure homogeneity of  $q_a$  across all firms ? In the present paper we focus on the problem of relating  $q_a$  to measures of deadweight loss associated

with monopoly or oligopolistic structures. Can we distinguish between static and dynamic losses associated with the existence of monopoly? Are such measures available for empirical implementation, and to what extent do they differ from existing empirical estimates of welfare loss?

The formal relationship between Tobin's  $q_a$  and indices of monopoly power including the price-cost margin or Lerner index, was developed, amongst others, by Lindenberg and Ross(1981); further work includes the papers by Salinger (1984), Smirlock, Gilligan and Marshall(1984), Connolly and Schwartz (1985) and Shepherd(1986). However rather surprisingly, no attempt appears to have been made to relate such measures to the Harberger concept of deadweight loss, or rather its dynamic equivalent.

A number of advantages may be expected to flow from the construction of a dynamic measure of welfare loss. Thus it may overcome objections to static measures of welfare loss, insofar as dynamic measures could reduce the measured impact of short run factors on welfare. Further, from a policy perspective, in deciding whether action in relation to an industry should be pursued, of primary relevance is the future behaviour of firms within that industry, and not their past actions except insofar as past actions may provide an indication of future behaviour. In this respect a forward looking measure which summarises the market view of likely pressures on a particular industry may be expected to be of some interest. Both the backward and forward looking measures of deadweight loss are perhaps best viewed as complementary analyses. The data requirements for such measures are not unduly onerous, and may be proposed as a first filter for any investigatory process concerning supernormal profits.

## II The Adjustment Cost Model of the Firm under Alternative Market Structures

The model of the firm is a variant of that developed by Lucas(1967) for a perfectly competitive industry; we introduce a specification which allows nesting of a number of alternative industrial structures faced by the firm; its description will be brief, the formal results being derived in Appendix 1. We assume that the firm's production function is given by the linear homogeneous function,

$$f(k, l, i) = [g(l/k) - c(i/k)]k \quad (1)$$

where  $k$  and  $l$  are the firm's capital and labour inputs, and  $i$  denotes gross investment of the firm. Net investment is defined by,

$$dk/dt = i - \delta k \quad (2)$$

where  $\delta$  is the constant depreciation rate. The production function  $f(\cdot)$  is twice continuously differentiable; with  $g' > 0$  and  $g'' < 0$ ; whilst the adjustment cost component has the properties,  $c(\delta) = 0$ ,  $c'(\delta) = 0$ ,  $c' > 0$  and  $c'' > 0$ . These assumptions imply that if the growth of the firm is zero, then the firm faces no

adjustment costs. Gross investment,  $i$ , may be positive or negative; whilst of course,  $k \geq 0$ ,  $l \geq 0$ .

Our attention in this paper will be restricted to steady state paths and hence we assume that the firm has constant expectations regarding future values of the product price,  $p$ ; the price of gross investment,  $v$ ; the wage rate,  $w$ ; and the interest rate,  $r$ .

The firm is assumed to maximize the present value of its cash flow by appropriate choice of  $i, l$ ; i.e.

$$V(k, 0) = \max_{i_s, l_s} \int_{s=0}^{s=\infty} [p_s f(k_s, l_s, i_s) - w_s l_s - v_s i_s] e^{-rs} ds \quad (3)$$

subject to the capital accumulation equation (2), and the initial capital stock,  $k(0) = k_0$ .

In addition the firm is assumed to face a known market demand function for the homogeneous product for the industry in which it participates given by,

$$Q(p, t) = D(p)e^{\lambda t} \quad (4)$$

where  $\lambda \geq 0$ , denotes the growth rate of demand.  $D(p)$  is assumed to have constant price elasticity.

In the case where the firm is not a monopoly we define the firm's beliefs about the response of other firms to its own output changes is assumed to be reflected in the function,

$$\frac{\partial(Y - y_i)}{\partial y_i} = \varepsilon \frac{(Y - y_i)}{y_i} \quad (5)$$

where  $\varepsilon$  is some constant (values of which will be specified below), and  $Y = \sum_i y_i$  denotes total market production.

As shown in Appendix 1, the solution for  $V(k, 0)$  is given by,

$$V(k, 0) = V(k) = \pi_t B_t(w/\pi, v/\pi, p/\pi, \pi, r, \delta, \eta_p) k_t \quad (6)$$

where  $B_t > 0$ , and where,

$$\pi_t = p_t [1 - \{1/\eta_p\} \{(1 - \varepsilon)(y_t/Y_t) + \varepsilon\}] \quad (7)$$

with  $\eta_p$  defined as the price elasticity of market demand.

If  $\varepsilon = 1$  then we have the standard "perfect collusion" case with,

$$\pi_t = p_t [1 - \{1/\eta_p\}] \quad (8)$$

If  $\varepsilon = 0$ , then we have the "Cournot" case,

$$\pi_t = p_t [1 - \{s_i/\eta_p\}] \quad (9)$$

The perfectly competitive case applies if,

$$\varepsilon = -(y_i/(Y - y_i)) \quad (10)$$

in which case,

$$\pi_t = p_t \quad (11)$$

The perfectly competitive case also of course applies if  $\eta_p \rightarrow \infty$ . The monopoly case arises when  $y_i = Y$  which is equivalent to the perfect collusion case.

The first order conditions for an interior optimum are given by,

$$pg'(w/\pi) = w \quad (12)$$

$$\frac{\partial V(k)}{\partial k} = \pi B_t(\cdot)(1 - \frac{\eta_B}{\eta_p}) = \pi c'_t(i_t/k_t) + v \quad (13)$$

Equation (12) states that the value of the marginal product of labour must equal it's price; and equation (13) that the demand price of capital must equal the current cost of an additional unit of gross investment.

Inverting (12) and (13) we may solve for  $i$  and  $l$  as,

$$l_t/k_t = D^l(w/p) \quad (14)$$

$$i_t/k_t = D^i[B_t(\cdot)(1 - \frac{\eta_B}{\eta_p}) - \frac{v}{\pi_t}] \quad (15)$$

In particular the growth function for the firm may be written as,

$$dk/dt = g^k(w/\pi, v/\pi, p/\pi, \pi, r, d, \eta_p)k \quad (16)$$

where,

$$g^k(\cdot) = D^i - \delta \quad (17)$$

The present value of the firm can be written in a manner particularly apposite to the analysis of welfare loss. From (13), let

$$\mu_t = \pi c'_t(i_t/k_t) + v = \frac{\partial V(k)}{\partial k} \quad (18)$$

i.e.  $\mu_t$  denotes the per unit cost of capital inclusive of any adjustment cost at time  $t$ , and this is equated to the marginal value of an investment at time  $t$ , which is given by the present discounted marginal contribution to the present value of the firm,  $\partial V(k)/\partial k$ .

In the appendix we then show that,

$$V(k, t) = \mu_t k + \int_{s=t}^{s=\infty} (\mu_s - v_s) k_s e^{-r(s-t)} ds + \int_{s=t}^{s=\infty} (R - kR_k) e^{-r(s-t)} ds \quad (19)$$

where,  $\dot{k} = dk/dt$  and

$$R = p_s f(k_s, l_s, \dot{k}_s + dk) - w_s l_s \quad (20)$$

i.e. revenue net of variable costs.

In long run equilibrium, when the firm achieves its optimal capital stock, adjustment costs disappear, i.e.  $\mu_t = \nu_t$ , and we have the result established by Lindenberg and Ross(1981),

$$V(k, t) = \nu_t k + \int_{s=t}^{s=\infty} (R - kR_k) e^{-r(s-t)} ds \quad (21)$$

whilst in general the cost of achieving a capital stock of size  $k$ , is defined by,

$$CK_t = \mu_t k + \int_{s=t}^{s=\infty} (\mu_s - v_s) \dot{k} e^{-r(s-t)} ds \quad (22)$$

and so,

$$V(k, t) = CK_t + \int_{s=t}^{s=\infty} (R - kR_k) e^{-r(s-t)} ds \quad (23)$$

Since  $R$  denotes revenue net of variable costs, it follows that  $R - kR_k$  is revenue minus long-run costs, and so,

$$V(k, t) = CK_t + \int_{s=t}^{s=\infty} (p - AC) y_s e^{-r(s-t)} ds \quad (24)$$

and with Tobin's average  $q_a$  defined by,

$$q_a = V(k, t)/CK_t \quad (25)$$

and with  $AC = MC$  in a constant returns to scale industry ,we have,

$$q_a = 1 + \frac{1}{CK_t} \int_{s=t}^{s=\infty} (p - MC) y_s e^{-r(s-t)} ds \quad (26)$$

### III Equilibrium Industry Structure

In the preceding section we have derived equations for growth and valuation of the firm under a variety of market circumstances. In the present section we consider the equilibrium price established by such firms.

#### (i) *The Monopoly Industry*

The simplest case to consider is that of the monopoly industry. The firm may be viewed as starting with an historically given level of capital stock; and given its factor prices and knowledge of the demand curve which it faces, produces its optimal level of output. The firm then makes the decision whether to expand

or contract by increasing or reducing the level of the capital stock employed. Long run equilibrium is then established by a configuration of prices such that,

$$g^k(\cdot) = 0 \quad (27)$$

or equivalently,

$$\pi_t B(\cdot) \left(1 - \frac{\eta_B}{\eta_p}\right) = v \quad (28)$$

where,

$$\pi_t = p_t \left(1 - \frac{1}{\eta_p}\right) \quad (29)$$

Long run monopoly price as a proportion of long-run competitive price can thus easily be determined. Let  $\pi_t^g$  be the value which solves,

$$g^k(w/\pi_t^g, v/\pi_t^g, p_t/\pi_t^g, \pi_t^g, r, \delta, \eta_p) = \lambda \quad (30)$$

We must therefore have,

$$\pi_t^g = p_t \left(1 - \frac{1}{\eta_p}\right) \quad (31)$$

and so,

$$\frac{p_t - \pi_t^g}{p_t} = \frac{1}{\eta_p} \quad (32)$$

i.e. the generalization of the usual Lerner relationship, where the "perfectly competitive price" is replaced by that price at which the growth in industry supply is equated to the growth in industry demand.

*(ii) Generalized Cournot with Identical Costs and Zero Entry*

The second case considered is that of an industry with  $n(\geq 2)$  firms, with zero entry into the industry. We assume that all firms have identical cost structures, but may differ in terms of their current capital stock.

The growth equation for each firm is given by,

$$\frac{dk_i}{dt} = g_i^k(w/\pi_{it}, v/\pi_{it}, p_t/\pi_{it}, \pi_{it}, r, \delta, \eta_p) k_i \quad (33)$$

with,

$$\pi_{it} = p_t [1 - \{1/\eta_p\} \{\varepsilon + (1 - \varepsilon) s_{it}\}] \quad (34)$$

We note that,  $\partial g_i^k / \partial s_i < 0$ .

Industry equilibrium, i.e. stable price and output, requires either that the growth rate of all firms is zero, or alternatively that the positive growth of some

firms is offset by the negative growth of others. The latter possibility cannot be viewed as a long run equilibrium outcome; since positive growth implies increased market share and negative growth decreased market share; market shares of all firms must eventually converge to the same value, i.e.  $s_i = (1/n)$ .

Long run equilibrium price is therefore that  $p$  which satisfies (33), for  $dk_i/dt = \lambda k_i$ ,  $i = 1, \dots, n$ ; with,

$$\pi_{it}^g = p_t [1 - \{1/\eta_p\} \{\varepsilon + (1 - \varepsilon)(1/n)\}] \quad (35)$$

i.e. the price cost margin,

$$\frac{p_t - \pi_t^g}{p_t} = \{1/\eta_p\} \{\varepsilon + (1 - \varepsilon)(1/n)\} \quad (36)$$

*(iii) Monopolistic Competition*

The case of monopolistic competition arises when free entry is allowed in the case of the generalized Cournot model. Free entry implies that new entrants are attracted into the industry as long as supernormal profits are possible; after entering the industry we assume they have the same costs and conjectured variations as existing firms. Applying this notion of free entry to case (ii) above would imply that the number of firms  $n \rightarrow \infty$ . A modified free entry condition utilised in the literature (see e.g. Stern(1987)) is to assume a fixed cost of entry, and thus impose a zero profit condition on the industry of the form,

$$(p^m - \pi^g)Y - Kn^* = 0 \quad (37)$$

where  $K$  is the fixed entry price per firm. Subst. (36) in (37), we thus have,

$$\{1/\eta_p\} \{\varepsilon + (1 - \varepsilon)(1/n)\} p^m Y - Kn^* = 0 \quad (38)$$

and hence (36),(37) and the demand function (4) serve to determine the three unknowns  $p$ ,  $Y$  and  $n$ . In a dynamic context (37) may be replaced by,

$$\int_{s=t}^{s=\infty} (p_t^m - \pi_t^g) Y_t ds = Kn^* \quad (39)$$

again serving to determine the unknowns  $p$ ,  $Y$  and  $n$ .

*(iv) Generalized Cournot with Non-Identical Cost Structures*

The fourth case we consider is where we have a generalized Cournot with firms having non-identical cost structures. We shall assume the existence of  $m > 2$  firms, some of which will find it not to their advantage to enter the industry; the number of active firms,  $n < m$ , will be determined by equilibrium price and the relative cost structures.

Consider the set of growth rates,

$$g_i^k(w/\pi_{it}, v/\pi_{it}, p_t/\pi_{it}, \pi_{it}, r, \delta, \eta_p) \quad i = 1, \dots, m \quad (40)$$

Let  $\pi_{it}^*$ ,  $i = 1, \dots, m$  denote the ordered set of values of  $\pi_{it}$  which ensure that  $g_i^k = \lambda$ , for all  $i$ , where the values of  $\pi_{it}$  are ordered from the lowest to the highest,

$$\pi_{1t}^* \leq \pi_{2t}^* \leq \dots \leq \pi_{mt}^* \quad (41)$$

the value  $\pi_{1t}^*$  is thus associated with the most efficient firm and  $\pi_{mt}^*$  with the least efficient.

The equilibrium price established ensures that the growth rate of the most efficient firm is zero, i.e.,

$$p_t [1 - \{1/\eta_p\} \{\varepsilon + (1 - \varepsilon)(s_1)\}] = \pi_{1t}^* \quad (42)$$

The market share of the most efficient firm is thus,

$$s_1 = \frac{\eta_p}{(1 - \varepsilon)} (\pi_{1t}^*/p_t) - \frac{\varepsilon \eta_p}{1 - \varepsilon} \quad (43)$$

Either  $s_1 = 1$ , in which case we have the monopoly solution considered above, or  $s_1 < 1$ , in which case further firms may coexist in the industry; in such cases the shares of these firms are given by,

$$s_i = \frac{\eta_p}{(1 - \varepsilon)} (\pi_{it}^*/p_t) - \frac{\varepsilon \eta_p}{1 - \varepsilon} \quad i = 2, \dots, m \quad (44)$$

for  $s_i > 0$ .

We also have the normalization condition,

$$\sum_i s_i = 1 \quad (45)$$

We thus have  $(m + 1)$  equations to determine the  $(m + 1)$  unknowns, i.e. the  $m$  shares (some of which may be zero), and the equilibrium price. With the equilibrium price determined, the demand function allows us to determine equilibrium output.

Summing over the  $n$  equations where  $s_i > 0$ , we have the result for the average price cost margin,

$$\frac{p_t - (\sum_n \pi_{it}^g/n)}{p_t} = \{1/\eta_p\} \{\varepsilon + (1 - \varepsilon)(1/n)\} \quad (46)$$

which for the case of identical firms gives the result noted in case (iii) above.

#### IV Measures of Welfare Loss

In the present section we derive measures of welfare loss associated with different market structures. The baselines against which welfare losses are measured are the price and output associated with a perfectly competitive industry, i.e. when all firms are price takers. The measurement of welfare loss is viewed

within the context of a partial equilibrium analysis by industry, an approach followed by the majority of previous investigators. The novel aspect of this section is to consider explicitly the "dynamic" deadweight loss, and to relate such measures to financial indicators of the firms' economic position, in particular Tobin's  $q$  ratio.

(i) Monopoly

The standard measure of welfare loss due to monopoly in a static partial equilibrium framework is the "Harberger triangle", i.e.,

$$DWL = \frac{1}{2}(p_m - p_m)(y_c - y_m) \quad (47)$$

In terms of the future existence of the monopoly therefore, we can assess the total discounted deadweight loss as being defined by,

$$DDWL = \int_{s=t}^{s=\infty} \frac{1}{2}(p_m - p_m)(y_c - y_m)e^{-r(s-t)} ds \quad (48)$$

The identity,

$$(p_m - p_m)(y_c - y_m) = (P^2/R)\eta_p \quad (49)$$

where  $P = (p_m - p_c)y$ ,  $R = p_m y_m$ , and  $\eta_p = (p/y)(dy/dp)$ , is well known, and hence,

$$DDWL = \int_{s=t}^{s=\infty} \frac{1}{2} \frac{P^2}{R} \eta_p e^{-r(s-t)} ds \quad (50)$$

If in each period the industry is in equilibrium and the monopoly maximizes profit, then the profit-sales ratio,  $P/R$ , is equal to the reciprocal of the price elasticity of demand,  $\eta_p$ , and so,

$$DDWL = \int_{s=t}^{s=\infty} \frac{1}{2} P e^{-r(s-t)} ds + \int_{s=t}^{s=\infty} \frac{1}{2} (p_m - p_m) y_m e^{-r(s-t)} ds \quad (51)$$

However in Section II we have shown that,

$$V(k, t) = CK_t + \int_{s=t}^{s=\infty} (p - MC) y_s e^{-r(s-t)} ds \quad (52)$$

With Tobin's average  $q_a$  defined by,

$$q_a = \frac{V(k, t)}{CK_t} \quad (53)$$

we thus have, where  $MC = p_c$  and  $p = p_m$ ,

$$\int_{s=t}^{s=\infty} (p_m - p_c)y_s e^{-r(s-t)} ds = CK_t(q_a - 1) = V(k, t) \frac{q_a - 1}{q_a} \quad (54)$$

and the immediate identification,

$$\frac{DDWL_t}{V(k, t)} = \frac{1}{2} \frac{q_a - 1}{q_a} \quad (55)$$

i.e. dynamic deadweight loss expressed as a proportion of the current valuation of the firm.

(ii) Generalized Cournot Industry

The next case we consider is *DDWL* for the generalized Cournot industry with identical cost structures across firms. From (48) using the price cost margin given by (36) we have the industry *DDWL* defined by,

$$DDWL = \int_{s=t}^{s=\infty} \frac{1}{2} (p_m - p_m) n y_i \{1/\eta_p\} \{\varepsilon + (1 - \varepsilon)(1/n)\} e^{-r(s-t)} ds \quad (56)$$

where  $y_i$  is the output of the  $i$ th firm; since all firms are symmetric, industry output is  $y_i n$ , where  $n$  is the number of firms in the industry. Factorising (50) and letting  $V_i(k, t)$  be the valuation of the  $i$ th firm, we have,

$$\frac{DDWL_{ind}}{V_{ind}(k, t)} = \frac{1}{2} \frac{q_a - 1}{q_a} \{\varepsilon + (1 - \varepsilon)(1/n)\} \quad (57)$$

where  $V_{ind}(k, t) = \sum_i V_i(k, t)$ , and  $q_a = q_a^i$ , the average  $q_a$  ratio, identical across all firms.

(iii) Cournot Industry with Differentiated Cost Structure

The derivation of *DDWL* for the case of non-symmetric cost structures across firms is a little more complex. As the competitive baseline for this industry, we shall assume that all firms have the cost structure of the most efficient firm, and are all price takers.

The price cost margin, relative to the lowest cost firm, is given by,

$$\frac{p - \pi_1^*}{p} \quad (58)$$

where  $\pi_1^*$  is  $\min[\pi_1^*, \pi_2^*, \dots, \pi_m^*]$  which satisfy,

$$g_i^k(w/\pi_{it}, v/\pi_{it}, p_t/\pi_{it}, \pi_{it}, r, \delta, \eta_p) = 0 \quad (59)$$

$p$  is determined from the condition (46), which on rearranging gives,

$$p = \frac{\bar{\pi}}{1 - \frac{1}{\eta_p} \left( \varepsilon + \frac{(1-\varepsilon)}{\varepsilon} \right)} \quad (60)$$

where  $\bar{\pi} = \Sigma \pi_i / n$ .

Subst. (60) in (58), we thus have,

$$\frac{p - \pi_1^*}{p} = 1 - \frac{\pi_1^*}{\bar{\pi}} \left[ 1 - \frac{1}{\eta_p} \left( \varepsilon + \frac{(1 - \varepsilon)}{\varepsilon} \right) \right] \quad (61)$$

Initially assume that all  $n$  firms are equally efficient; with the price cost margin given by (61), we may calculate the  $DDWL$  by subst (61) in (50), to get,

$$DDWL_{ind} = nV_1(k, t) \left\{ \frac{1}{2} \frac{q_a - 1}{q_a} \right\} \left\{ 1 - \frac{\pi_1^*}{\bar{\pi}} \left[ 1 - \frac{1}{\eta_p} \left\{ \varepsilon + (1 - \varepsilon)(1/n) \right\} \right] \right\} \{\eta_p\} \quad (62)$$

However, (62) only constitutes the  $DDWL$  with equally efficient firms; in addition we have the deadweight loss associated with the existence of inefficient firms within the industry. In the case where all firms are equally efficient, we gross up  $V_1(k, t)$  to equal industry profit; i.e. to  $V_1(k, t)/s_1$ . If we then subtract the actual values of all firms, including the most efficient, we are then left with the resulting loss of value through non-efficient allocation of output;

$$V_1(k, t)(1/s_1) - \sum_i V_i(k, t) \quad (63)$$

Adding (63) to (62) gives the total  $DDWL$  for the industry, and hence dividing  $DDWL_{ind}$  by  $V_{ind}(k, t) = \sum_i V_i(k, t)$ , we arrive at,

$$\frac{DDWL_{ind}}{V_{ind}(k, t)} = \left[ \frac{1}{2} \frac{q_a - 1}{q_a} \right] \left[ 1 - \frac{\pi_{\min}}{\bar{\pi}} \left[ 1 - \frac{1}{\eta_p} \left( \varepsilon + \frac{1 - \varepsilon}{n} \right) \right] \right] \{\eta_p\} + \frac{1}{s_1} \frac{V_1}{V_{ind}} - 1 \quad (64)$$

The results for the above and other industry structures are summarised in Table 1.

*Table 1. Measures of Dynamic Deadweight Loss*

Case 1, Monopoly

$$P/R = 1/\eta_p \quad (65)$$

$$\frac{DDWL_t}{V(k, t)} = \frac{1}{2} \frac{q_a - 1}{q_a} \quad (66)$$

Case 2, Generalized Cournot, Zero Entry, Identical Firms

$$P/R = (1/\eta_p) \left( \varepsilon + \frac{1 - \varepsilon}{n} \right) \quad (67)$$

$$\frac{DDWL_{ind}}{V_{ind}(k, t)} = \frac{1}{2} \frac{q_a - 1}{q_a} \left\{ \varepsilon + (1 - \varepsilon)(1/n) \right\} \quad (68)$$

Case 3, Generalized Cournot, Free Entry, Identical Firms

As Case 2 with  $n = n^*$  determined by (32).

Case 4, Generalized Cournot, Non-Identical Firms

$$\frac{p - \pi_{\min}^*}{p} = 1 - \frac{\pi_{\min}^*}{\pi} \left[ 1 - \frac{1}{\eta_p} \left( \varepsilon + \frac{1 - \varepsilon}{n} \right) \right] \quad (69)$$

$$\frac{DDWL_{ind}}{V_{ind}(k, t)} = \left[ \frac{1}{2} \frac{q_a - 1}{q_a} \right] \left[ 1 - \frac{\pi_{\min}}{\pi} \left[ 1 - \frac{1}{\eta_p} \left( \varepsilon + \frac{1 - \varepsilon}{n} \right) \right] \{ \eta_p \} \right] + \frac{1}{s_1} \frac{V_1}{V_{ind}} - 1 \quad (70)$$

The values for  $DDWL$  in this section are based on the assumption of long-run industry equilibrium. In disequilibrium the values of the price cost margin are not known, however assuming that  $(P/R)\eta_p$  is bounded in each time period; and that the integral  $\int P e^{-r(s-t)} dt$  is bounded on the interval  $[t, \infty)$  then we may use the mean value theorem to show that,

$$\left[ \frac{1}{2} \frac{q_a - 1}{q_a} \right] \text{Min} \left[ \frac{P}{R} \eta_p, 0 \right] \leq \frac{DDWL_t}{V(k, t)} \leq \left[ \frac{1}{2} \frac{q_a - 1}{q_a} \right] \text{Max} \left[ \frac{P}{R} \eta_p \right] \quad (71)$$

Thus, for example, for firms which are below their equilibrium size, in the monopoly case for which,  $P/R > (1/\eta_p)$ , the correct measure of  $DDWL_t/V(k, t)$  will be understated using  $\left[ \frac{1}{2} \frac{q_a - 1}{q_a} \right]$ . It should also be noted that  $DDWL$  will in general understate welfare loss for any period during which  $p < MC$ , even though  $q_a > 1$ .

## V Static and Dynamic Welfare Loss: Implementation and Comparison

The calculation of  $DDWL$  for monopoly industries simply requires knowledge of Average  $q$  for the monopoly firm; the case of oligopolistic industries requires additional information regarding market shares and valuation of each of the firms. We shall confine the discussion in this section to the monopoly case.

The first question we have to ask is whether the methodology proposed in the preceding sections leads to reasonable estimates of welfare loss. What are typical values of  $DDWL/V(k, t)$  as defined by (66); and how do such values compare with other estimates of  $DWL$  compiled by alternative means.

The median values of  $q$  for the sample constructed by Lindenberg and Ross (1981) was 1.25 with a mean value of 1.52; thus using eq.(66)  $DDWL/V(k, t) = 0.1$  for the median firm, i.e.e an initial estimate of 10% of market capitalization of the median firm is the cumulated discounted welfare loss associated with the monopoly position for such firms. Such a measure is difficult to compare with

the standard measures of deadweight loss which are expressed as percentages of current output or sales.

In order to make a meaningful comparison of *DDWL* with current measures of static deadweight loss (*SDWL*), we therefore have to bring either *SDWL* or *DDWL* to a common base. The method of comparison we propose in this section is to construct the hypothetical *SDWL* measure associated with a given *DDWL*, under the assumption that demand and output were growing at a constant rate  $g$ . Then, from equation (51) we have the immediate identification,

$$DDWL = SDWL/(r - g) \tag{72}$$

Now from (55),

$$DDWL = \frac{1}{2} \frac{q_a - 1}{q_a} V(k, t) = \frac{1}{2} (q_a - 1) vK \tag{73}$$

and so,

$$SDWL/pY = \frac{1}{2} (r - g) (q_a - 1) \frac{vK}{pY} \tag{74}$$

i.e. the implied welfare loss as a proportion of the firms revenues.

In table 2 we have constructed estimates of implied *SDWL*, for various values of  $(r - g)$  and  $q_a$ ; we have taken the capital/sales ratio as 5.

Table 2. Values of implied SDWL

$q$	1.10	1.25	1.50	1.75	2.00	5.00
$r - g$						
0.01	0.25	0.63	1.25	1.88	2.50	10.00
0.02	0.50	1.26	2.50	3.36	5.00	20.00
0.03	0.75	1.89	3.75	5.64	7.50	30.00
0.04	1.00	2.52	5.00	7.52	10.00	40.00
0.05	1.25	3.15	6.25	9.38	12.58	50.00

Of course  $V(k, t)$ ,  $r$ ,  $g$ , and the capital-sales ratio are not independent variables, and therefore the implied percentage losses must be treated with care. However, it would thus appear that the present methodology would lead to estimates of welfare loss which lie within the range of estimates commonly found in studies of welfare loss using more traditional methods of estimation

As a further example let us take some plausible values for the parameters suggested by Salinger(1984) in a different context; he proposed a rate of interest of 0.08, a growth rate of .03, and a  $(vK/pY)$  ratio of 5.0, we supplement these figures with Lindenberg and Ross's median value of  $q_a = 1.25$ ; then subst in (62) we have,  $SDWL/pY = 0.031$ , i.e.e 3.1 per cent of the value of output; this lies on the range of welfare losses quoted in the recent paper by Daskin(1991).

Similarity between the calculated welfare rankings, on average, however need not imply similarity in rankings of *SDWL* and *DDWL* by firm. Quite clearly

firms with low *SDWL*, may have high *DDWL* if the market believes that the firm is likely to gain from anticipated future monopoly profits; conversely for a firm with high *DDWL*, in which the market anticipates these gains to be eroded over future time periods. A brief example showing differences in such rankings is now provided.

A substantive amount of empirical data on average  $q_a$  may be found in the literature, mainly in relation to studies of investment behaviour. Unfortunately, most of this data is unsuitable for use in studies of monopoly welfare loss, relating either to economy wide estimates of  $q_a$ , or to industry averages; only if all firms have the same value of  $q_a$  could such estimates be used to estimate welfare loss. However the firm estimates used by Lindenberg and Ross(1981), may be used to advantage in the estimation of *DDWL*. In *Table 2* we have utilised data for a common set of firms for which information is provided in the studies by Cowling and Mueller(1978) and Lindenberg and Ross(1981), in order to compare *DWL* and *DDWL*.

Unfortunately, comparability for the two published data sets was restricted to the ten firms indicated below. What is perhaps surprising is the rather weak relationship between *DWL* and *DDWL*. The Spearman rank-difference correlation statistic has a value of  $r = +0.5272$ , with a  $t$  value of 1.755; which although of the right sign is not significant at  $t_{0.95}$  although significant at  $t_{0.90}$ . Of course, this weak relationship may be a result of the disparate time periods over which averages were taken; however the weak relationship between  $q_a$  and the Lerner index ( $P/R$ ) was noted by Lindenberg and Ross(1981) for their entire sample of 256 firms.

*Table A2.1. Estimates of Dynamic Deadweight Loss and Deadweight Loss*

	$DDWL/V(k, t)$	$DWL/R$
	$\frac{1}{2} \frac{(q_a - 1)}{q_a} (100)$	$\frac{1}{2} \frac{P}{R} (100)$
<i>Minnesota Mining and Manufacturing Co.</i>	39.37(1)	6.56(5)
<i>International Business Machines Corp.</i>	38.12(2)	7.31(4)
<i>Gillette Co.</i>	37.25(3)	8.45(1)
<i>E.I. Du Pont De Nemours</i>	29.76(4)	8.06(2)
<i>General Electric Co.</i>	25.85(5)	1.56(8)
<i>Sears Roebuck &amp; Co.</i>	25.49(6)	0.69(10)
<i>R.J.Reynolds</i>	23.68(7)	7.39(3)
<i>General Motors Corp.</i>	18.55(8)	5.82(6)
<i>American Cyanamid Co.</i>	15.75(9)	3.44(7)
<i>Exxon Corp.</i>	2.38(10)	1.04(9)

*Source: Recalculated from data published in Lindenberg and Ross(1981) and, Cowling and Mueller(1978). DDWL, based on average q for the firms concerned, 1960-77; DWL based on firm data, average 1963-66.*

If further study confirms the weak relationship between *DDWL* and *DWL*

it would tend to suggest that static  $DWL$  is not a particularly useful indicator of long-run welfare losses resulting from monopoly; as noted in the text firms with low  $DWL$ , may have high  $DDWL$  if the market believes that the firm is likely to gain from anticipated future monopoly profits; conversely for a firm with high  $DWL$ , in which the market anticipates these gains to be eroded over future time periods. Direct comparison of the two data sets may better be performed by use of the analysis suggested above, by taking account of the capital/sales ratio for the different firms a closer congruence in the rankings may be obtained. However, for the moment this is left as an open question.

## VI Conclusion

The aim of this paper has been to construct a framework in which welfare losses over time generated by alternative market structures may be estimated. An adjustment cost model of the firm under imperfect competition was developed, and consequent industry equilibria determined. A dynamic analogue of Harberger's measure of welfare loss was specified, and for the case of a monopoly industry, such a measure could be expressed as a function solely of Tobin's average  $q_a$ . In the case of more general oligopolistic structures, additional information including knowledge of average  $q_a$  ratios and market share information for each firm was required before  $DDWL$  could be determined.

The relationship between static and dynamic deadweight loss was noted, and implied values of  $SDWL$  for certain ranges of parameter values established. It appeared that such computed  $SDWL$  for "reasonable" parameter values lay well within the bounds of welfare loss established by alternative methodologies; we should remember that estimates quoted in existing studies, e.g. Cowling and Mueller(1978) and Daskin(1991) are based on *ex post* information of cost structures and pricing, whereas the estimates in this paper based on  $q_a$  refer to future expectations regarding such cost and pricing structures. Thus we stress that the  $DDWL$  measures calculated are on the basis of the market's expectations of the future profitability of these firms; if the market is wrong then so are the estimates of the  $DDWL$ . The measures of  $DDWL$  thus constructed, and others that might be proposed we argue allow a significant additional set of data to be used to construct a forward looking measure of welfare loss, and thus augment existing measures of industry appraisal.

## Appendix 1. Optimal Behaviour of the Firm under Imperfect Competition

The problem may be formally stated as,

$$\max_{i_s, l_s} \int_{s=0}^{s=\infty} c_s e^{-rs} ds \quad (75)$$

subject to the capital accumulation equation,

$$\frac{dk}{dt} = i - dk \quad (76)$$

where,

$$c_s = p_s \{g(l_s/k_s) - c(i_s/k_s)\} k_s - w_s l_s - v_s i_s \quad (77)$$

denotes the cash flow of the firm at time  $s$ .

The firm's beliefs about the response of other firms to its own output change is assumed to be reflected in the function,

$$\frac{\partial(y - y_i)}{\partial y_i} = \varepsilon \frac{(Y - y_i)}{y_i} \quad (78)$$

We define the Optimal Value Function as,

$$V(k(t), t) = \max_{i_s, l_s} \int_{s=0}^{s=\infty} c_s e^{-r(s-t)} ds \quad (79)$$

which may be written as,

$$V(k(t), t) = \max_{i_s, l_s} \int_{s=0}^{s=t+\Delta t} c_s e^{-r(s-t)} ds + V(k(t + \Delta t), t + \Delta t) \quad (80)$$

Now from the Mean Value Theorem there is some point  $t^* \in [t, t + \Delta t]$  such that,

$$e^{-rt^*} c_t = \frac{1}{(t + \Delta t) - t} \int_{s=t}^{s=t+\Delta t} e^{-r(s-t)} c_s ds \quad (81)$$

Further, expanding  $V(k(t + \Delta t), t + \Delta t)$  in a Taylor series about the point  $(k(t), t)$ , we have,

$$\begin{aligned} V(k(t + \Delta t), t + \Delta t) &= V(k(t), t) + \frac{\partial V(k(t), t)}{\partial t} \Delta t \\ &+ \frac{\partial V(k(t), t)}{\partial k} (k(t + \Delta t) - k(t)) + o(\Delta t) \end{aligned} \quad (82)$$

Therefore subst. (81) and (82) in (80) we have,

$$\begin{aligned}
V(k(t), t) = & \max_{i_s, l_s} \left[ \int_{s=0}^{s=t+\Delta t} c_s e^{-r(s-t)} ds + V(k(t), t) + \frac{\partial V(k(t), t)}{\partial t} \Delta t \right. \\
& \left. + \frac{\partial V(k(t), t)}{\partial k} (k(t + \Delta t) - k(t)) + o(\Delta t) \right] \quad (83)
\end{aligned}$$

Cancelling  $V(k(t), t)$  from both sides of (1.8) , dividing through by  $\Delta t$  and then letting  $\Delta t \rightarrow 0$  , we have,

$$0 = \max_{i_t, l_t} \left[ c_t e^{-rt} + \frac{\partial V(k(t), t)}{\partial t} + \frac{\partial V(k(t), t)}{\partial k} u_g \right] \quad (84)$$

the fundamental differential equation of dynamic programming (see e.g. Dreyfus(1965)), where,

$$u_g = \lim_{\Delta t \rightarrow 0} [k(t + \Delta t) - k(t)]/\Delta t = i_t - \delta k_t. \quad (85)$$

The equation (84) has to be solved for  $V(k, t)$  subject to the terminal boundary condition,

$$\lim_{t \rightarrow \infty} V(k, t) = 0 \quad (86)$$

An evident trial solution is of the form,

$$V(k, t) = e^{-rt} V(k) \quad (87)$$

which subst. in (84) together with the definitions for  $c_t$  and  $u_g$  gives,

$$\max_{i_t, l_t} \left[ p_t f(k, l, i) - w l_t - v i_t - r V(k) + \frac{\partial V(k(t), t)}{\partial k} (i_t - \delta k_t) \right] = 0 \quad (88)$$

The first order conditions for a maximum w.r.t.  $l$  and  $i$  are respectively,

$$p_t \left[ 1 - \frac{s_i}{\eta} \frac{\partial y}{\partial f} \right] g'(l_t/k_t) - w = 0 \quad (89)$$

$$p_t \left[ 1 - \frac{s_i}{\eta} \frac{\partial y}{\partial f} \right] c'(i_t/k_t) - v + \frac{\partial V(k)}{\partial k} = 0 \quad (90)$$

Letting,

$$\pi_t = p_t \left[ 1 - \frac{s_i}{\eta} \frac{\partial y}{\partial f} \right] \quad (91)$$

Solving (89),(90) for  $l_t, i_t$  we therefore have,

$$l_t = k_t (g')^{-1} \left( \frac{w}{\pi} \right) = k_t D^1(w/\pi) \quad (92)$$

$$i_t = k_t(c')^{-1}[(\frac{\partial V(k)}{\partial k} - v)/\pi] = k_t D^2((\frac{\partial V(k)}{\partial k} - v)/\pi) \quad (93)$$

Subst. (92),(93) in (88) we therefore have to solve for  $V(k)$

$$\begin{aligned} & p_t [g\{D^1(w/\pi)\} - cD^2((\frac{\partial V(k)}{\partial k} - v)/\pi)]k_t - w\{D^1(w/\pi)\}k_t \\ & -vD^2((\frac{\partial V(k)}{\partial k} - v)/\pi)k_t - rV(k) + \frac{\partial V(k(t), t)}{\partial k} (D^2((\frac{\partial V(k)}{\partial k} - v)/\pi) - \delta)k_t] = 0 \end{aligned} \quad (94)$$

As a trial solution we let,

$$V(k) = \pi_t B_t k_t \quad (95)$$

subst. in (94), dividing through by  $\pi_t k_t$  and regrouping terms, we get the equation which implicitly defines  $B_t$  as,

$$\begin{aligned} & \frac{p}{\pi} [g\{D^1(w/\pi)\} - cD^2((B_t(1 - \frac{\eta_B}{\eta_p}) - (v/\pi))] - \frac{w}{\pi} \{D^1(w/\pi)\} \\ & + (B_t(1 - \frac{\eta_B}{\eta_p}) - \frac{v}{\pi})D^2((B_t(1 - \frac{\eta_B}{\eta_p}) - (v/\pi)) - B_t(r + (1 - \frac{\eta_B}{\eta_p})\delta) = 0 \end{aligned} \quad (96)$$

where  $\eta_B = (p_t/\pi_t B_t)d(\pi_t B_t)/dp_t$ ,  $\eta_p = (p_t/y_t)d(y_t)/dp_t$ . (96) is a first order differential equation in  $B_t$  w.r.t.  $p$ , and as may be seen, the only independent variables are the price ratios,  $w/\pi, v/\pi, p/\pi$  the interest and depreciation rates  $r, \delta$ ;  $\pi$  and the price elasticity  $\eta_p$ . Therefore, given existence, the implicit function must be of the form,

$$B_t = B_t(w/\pi, v/\pi, p/\pi, r, \delta, \pi, \eta_p). \quad (97)$$

The investment behaviour of the firm has a clear interpretation in terms of Tobin's average and marginal q ratio.

$$\text{Average } q = q_a = V(k)/vk = \pi B/v \quad (98)$$

$$\text{Marginal } q = q_m = [\partial V(k)/\partial k]/v = \pi B(1 - \frac{\eta_B}{\eta_p})/v \quad (99)$$

Thus,

$$i_t = k_t D^2((B_t(1 - \frac{\eta_B}{\eta_p}) - (v/\pi)) = k_t D^2(\frac{v}{\pi}(q_m - 1)) \quad (100)$$

If  $q_m = 1$ , then  $i_t/k_t = D^2(0)$ ; i.e.  $c'(i_t/k_t) = 0$ , and hence  $i_t = \delta k_t$ . Thus positive growth of firms can only result when  $q_m$  exceeds unity. If  $q_m \geq 1$ , then from (98),(99) we see that  $q_a > 1$ ; in empirical work average  $q_a$ 's below 1 are quite often found; in these cases the theory predicts that these firms would not

be increasing their productive capacity. We do not consider these cases in this paper.

Note that,

$$q_m = q_a \left(1 - \frac{\eta_B}{\eta_p}\right) \quad (101)$$

as  $\eta_p \rightarrow \infty$ , so does  $q_m \rightarrow q_a$ . This result may be contrasted with the solution proposed by Hayashi(1982) in which, using notation of the present paper,

$$q_m = q_a - (1/vk) \int_0^\infty (py/\eta_p) \exp[-\int_0^t r ds] dt \quad (102)$$

a relationship which requires knowledge of future revenue streams ( $py$ ) not embodied in  $q_m$  or  $q_a$ . Since  $q_m$  is used in the estimation of investment demand equations, (1.31) has the advantage of being related to current rather than anticipated data.

The above theory may be related to the familiar static model of the firm as follows. From (13), let

$$\mu_t = \pi c'(i_t/k_t) + v_t = \frac{\partial V(k)}{\partial k} \quad (103)$$

i.e.  $\mu_t$  denotes the per unit cost of capital inclusive of any adjustment cost at time  $t$ ; and this is equated to the marginal value of an investment at time  $t$ , which is given by the present discounted marginal contribution to the present value of the firm,  $\partial V(k)/\partial k$ . Subst. for  $i_t$  from (2) in (3), we have,

$$V(k, t) = \max_{i_s, l_s} \int_{s=t}^{s=\infty} [p_s f(k_s, l_s, \dot{k}_s + \delta k_s) - w_s l_s - v_s (k_s + \delta k_s)] e^{-r(s-t)} ds \quad (104)$$

where  $\dot{k} = dk/dt$ . Letting,

$$R = p_s f(k_s, l_s, \dot{k}_s + \delta k_s) - w_s l_s \quad (105)$$

i.e. revenue net of variable costs, we have, diff. (104)

$$\frac{\partial V(k)}{\partial k} = \int_{s=t}^{s=\infty} \left[ \frac{\partial R}{\partial k} - v_s \delta \right] e^{-r(s-t)} ds = \mu_t \quad (106)$$

and so differentiating (106) w.r.t.  $t$ , we have,

$$\dot{\mu}_t = r\mu_t + v_t \delta - \frac{\partial R}{\partial k} \quad (107)$$

where  $\dot{\mu}_t = d\mu_t/dt$ . Now consider the integral,

$$\int_{s=t}^{s=\infty} \left[ k \frac{\partial R}{\partial k} - v_s i \right] e^{-r(s-t)} ds \quad (108)$$

From (107),

$$\frac{\partial R}{\partial k} = r\mu_t + v_t\delta - \dot{\mu}_t \quad (109)$$

and so subst. in (108),

$$\begin{aligned} & \int_{s=t}^{s=\infty} [r\mu_s + v_s\delta - \dot{\mu}_s]k - v_s(\dot{k} + \delta k)]e^{-r(s-t)} ds \\ &= \int_{s=t}^{s=\infty} [r\mu_s k - \dot{\mu}_s k - v_s \dot{k}]e^{-r(s-t)} ds \end{aligned} \quad (110)$$

Now,

$$\mu_t = \pi c'(i_t/k_t) + v_t = a + v_t \quad (111)$$

where we let  $a = \pi c'(i_t/k_t)$ .

Now subst. (111) in (110) we have,

$$\int_{s=t}^{s=\infty} [r\mu_s k - \frac{\partial(\mu k)}{\partial s}]e^{-r(s-t)} ds + \int_{s=t}^{s=\infty} [a\dot{k}]e^{-r(s-t)} ds \quad (112)$$

and so integrating by parts,

$$\begin{aligned} & r \int_{s=t}^{s=\infty} [\mu_s k]e^{-r(s-t)} ds - [[\mu_s k]e^{-r(s-t)}]_t^\infty \\ &+ \int_{s=t}^{s=\infty} [a\dot{k}]e^{-r(s-t)} ds - r \int_{s=t}^{s=\infty} [\mu_s k]e^{-r(s-t)} ds \end{aligned} \quad (113)$$

which may be simplified to,

$$\mu_t k + \int_{s=t}^{s=\infty} [a\dot{k}]e^{-r(s-t)} ds \quad (114)$$

Now,

$$V(k, t) = \int_{s=t}^{s=\infty} (R - v_s i_s) e^{-r(s-t)} ds = \int_{s=t}^{s=\infty} (kR_k - v_s i_s) e^{-r(s-t)} ds + \int_{s=t}^{s=\infty} (R - kR_k) e^{-r(s-t)} ds \quad (115)$$

and so subst. from (111), (114), we have,

$$V(k, t) = \mu_t k + \int_{s=t}^{s=\infty} (\mu_s - v_s) \dot{k} e^{-r(s-t)} ds + \int_{s=t}^{s=\infty} (R - kR_k) e^{-r(s-t)} ds \quad (116)$$

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