Online Dictionary Learning with Group Structure Inducing Norms

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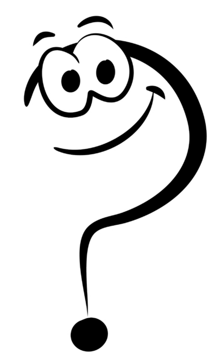
1. Introduction

- Sparse coding.
- Structured sparsity (e.g., disjunct groups, trees): increased performance in several applications.
- **Our goal**: develop a dictionary learning method, which
  - enables general overlapping group structures,
  - is online: fast, memory efficient, adaptive,
  - applies non-convex sparsity inducing regularization:
    - fewer measurements,
    - weaker conditions on the dictionary,
    - robust (w.r.t. noise, compressibility).
  - can deal with missing information.

Current approaches can exhibit two of these features at most.



2. Problem

Task:

- Group structure inducing on the hidden representation $\boldsymbol{\alpha}$ through regularization:

$$\Omega(\boldsymbol{\alpha}) = \|(\|\boldsymbol{\alpha}_G\|_2)_{G\in\mathcal{G}}\|_\eta, \tag{1}$$
$$\Omega(\boldsymbol{\alpha}) = \|(\|\mathbf{d}^G \circ \boldsymbol{\alpha}\|_2)_{G\in\mathcal{G}}\|_\eta, \tag{2}$$
$$\Omega(\boldsymbol{\alpha}) = \|(\|\mathbf{A}^G \boldsymbol{\alpha}\|_2)_{G\in\mathcal{G}}\|_\eta, \quad \eta \in (0,2). \tag{3}$$

- Approximate on the observed coordinates ($\mathbf{x}_O$) using dictionary $\mathbf{D}$:

$$\frac{1}{2}\|\mathbf{x}_O - \mathbf{D}_O\boldsymbol{\alpha}\|_2^2. \tag{4}$$

- Loss for a fixed observation ($\kappa > 0$):

$$l(\mathbf{x}_O, \mathbf{D}_O) = \min_{\boldsymbol{\alpha}}\left[\frac{1}{2}\|\mathbf{x}_O - \mathbf{D}_O\boldsymbol{\alpha}\|_2^2 + \kappa\Omega(\boldsymbol{\alpha})\right]. \tag{5}$$

- Goal: minimize online the average loss of the dictionary

$$\min_{\mathbf{D}} f_t(\mathbf{D}) := \frac{1}{t}\sum_{i=1}^{t} l(\mathbf{x}_{O_i}, \mathbf{D}_{O_i}). \tag{6}$$

- Possible dictionary/representation constraints:
  - $\mathbf{D} \in \mathcal{D} = \times_{i=1}^{d_\alpha}\mathcal{D}_i \subseteq \mathbb{R}^{d_x}$: closed, convex, and bounded.
  - $\boldsymbol{\alpha} \in \mathcal{A} \subseteq \mathbb{R}^{d_\alpha}$: convex, closed.

3. Special cases

$O_i = \{1,\ldots,d_x\}$ $(\forall i)$: fully observed OSDL task.

Special cases for $\mathcal{G}$:

'Traditional' sparse dictionary	$\mathcal{G}$ = $\{\{1\},\{2\},\ldots,\{d_\alpha\}\}$.
Hierarchical dictionary	$\mathcal{G}$ = descendants of the nodes.
Grid adopted dictionary	$\mathcal{G}$ = nearest neighbors of the nodes.
Group Lasso	$\mathcal{G}$ = partition.
Elastic net	$\mathcal{G}$ = singletons and $\{1,\ldots,d_\alpha\}$.
Contiguous, nonzero representations	$\mathcal{G}$ = intervals.

Special cases for $\mathcal{D},\mathcal{A}$:

'Traditional' setting	ℓ_2 constrained $\mathbf{D}$.
Structured NMF	non-negative $\mathbf{D}$ and $\boldsymbol{\alpha}$.
Structured mixture-of-topics	ℓ_1 constained $\mathbf{D}$, non-negative $\mathbf{D}$, $\boldsymbol{\alpha}$.
'Hard' representation constraints	group norm/elastic net/fused Lasso constrained $\boldsymbol{\alpha}$.
Double structured dictionaries	group norm constraints to $\boldsymbol{\alpha}$ *and* $\mathbf{D}$.

Special cases for $\{\mathbf{A}^G\}_{G\in\mathcal{G}}$:

Fused Lasso	$\Omega(\boldsymbol{\alpha})$ = $\sum_{j=1}^{d_\alpha-1}\|\alpha_{j+1}-\alpha_j\|$
Graph-guided fusion penalty	$\Omega(\boldsymbol{\alpha})$ = $\sum_{e=(i,j)\in E:i<j} w_{ij}\|\alpha_i - v_{ij}\alpha_j\|$
Linear trend/polynomial filtering	$\Omega(\boldsymbol{\alpha})$ = $\sum_{j=2}^{d_\alpha-1}\|-\alpha_{j-1}+2\alpha_j-\alpha_{j+1}\|$
Generalized Lasso penalty	$\Omega(\boldsymbol{\alpha})$ = $\\|\mathbf{A}\boldsymbol{\alpha}\\|_1$
Total variation	$\Omega(\boldsymbol{\alpha})$ = $\sum_{i=1}^{d_1}\sum_{j=1}^{d_2}\\|(\nabla\boldsymbol{\alpha})_{ij}\\|_2$

4. Optimization

Online optimization of dictionary $\mathbf{D}$ through alternations:

1. $(\mathbf{x}_{O_t}, \mathbf{D}_{t-1}, O_t) \mapsto \boldsymbol{\alpha}_t$:

$$\boldsymbol{\alpha}_t = \underset{\boldsymbol{\alpha}\in\mathcal{A}}{\operatorname{argmin}}\left[\frac{1}{2}\|\mathbf{x}_{O_t} - (\mathbf{D}_{t-1})_{O_t}\boldsymbol{\alpha}\|_2^2 + \kappa\Omega(\boldsymbol{\alpha})\right]. \tag{7}$$

Solution idea: iterated reweighted least squares using the variational property of $\|\cdot\|_\eta$.

2. $\{\boldsymbol{\alpha}_i\}_{i=1}^t \mapsto \mathbf{D}_t$ by means of quadratic optimization:

$$\hat{f}_t(\mathbf{D}_t) = \min_{\mathbf{D}\in\mathcal{D}} f_t(\mathbf{D}, \{\boldsymbol{\alpha}_i\}_{i=1}^t). \tag{8}$$

Solution idea:

- block-coordinate descent optimization: update column $\mathbf{d}_j$, while keeping the others fixed,
- statistics of the cost $\hat{f}_t$ can be efficiently updated online (matrix recursions).

5. Numerical experiments

5.1 Inpainting of natural images

We focused on the following questions:

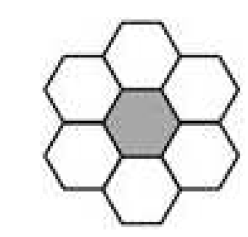
- structured (toroid) vs. unstructured dictionary for inpainting,
- efficiency in case of missing observations,
- inpainting of *full images* using dictionaries learned on partially observed patches.

First experiment (complete observation):

- increasing neighbor size = decreasing MSE.
- $r = 3$: $13-19\%$ improvement compared to the unstructured case ($r = 0$).

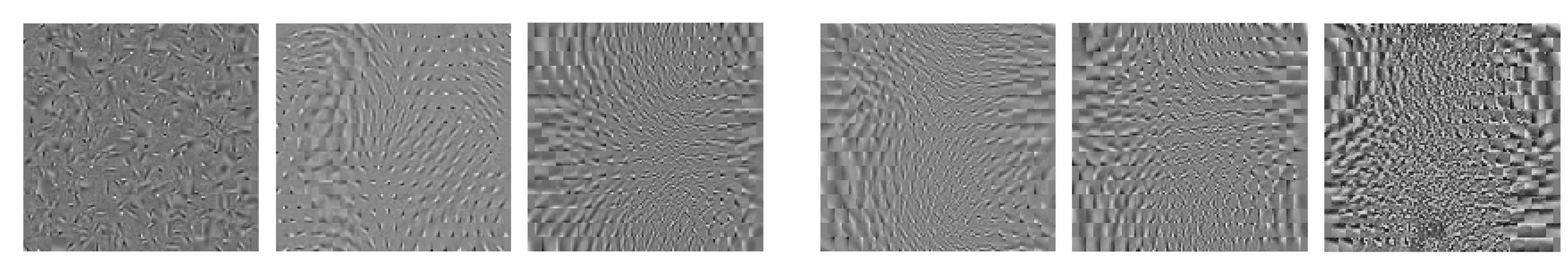
Second experiment (neighbor size: $r = 3$, missing pixels: $p_{tr} \le 0.9$):

- Up to about $p_{tr} = 0.7$: MSE grows slowly.
- $\mathbf{D}$-s in Fig. 1(d)-(f).
- For $p_{tr} = 0.9$, MSE still relatively small, see Fig. 2(a).



Third experiment (neighbor size: $r = 3$, missing pixels: $p_{tr} = 0.5$):

- Task: inpainting of a *full* unseen image.
- Result: sliding average, Fig. 2(a), $p_{test}^{val} = 0.7$, PSNR = 29 dB.

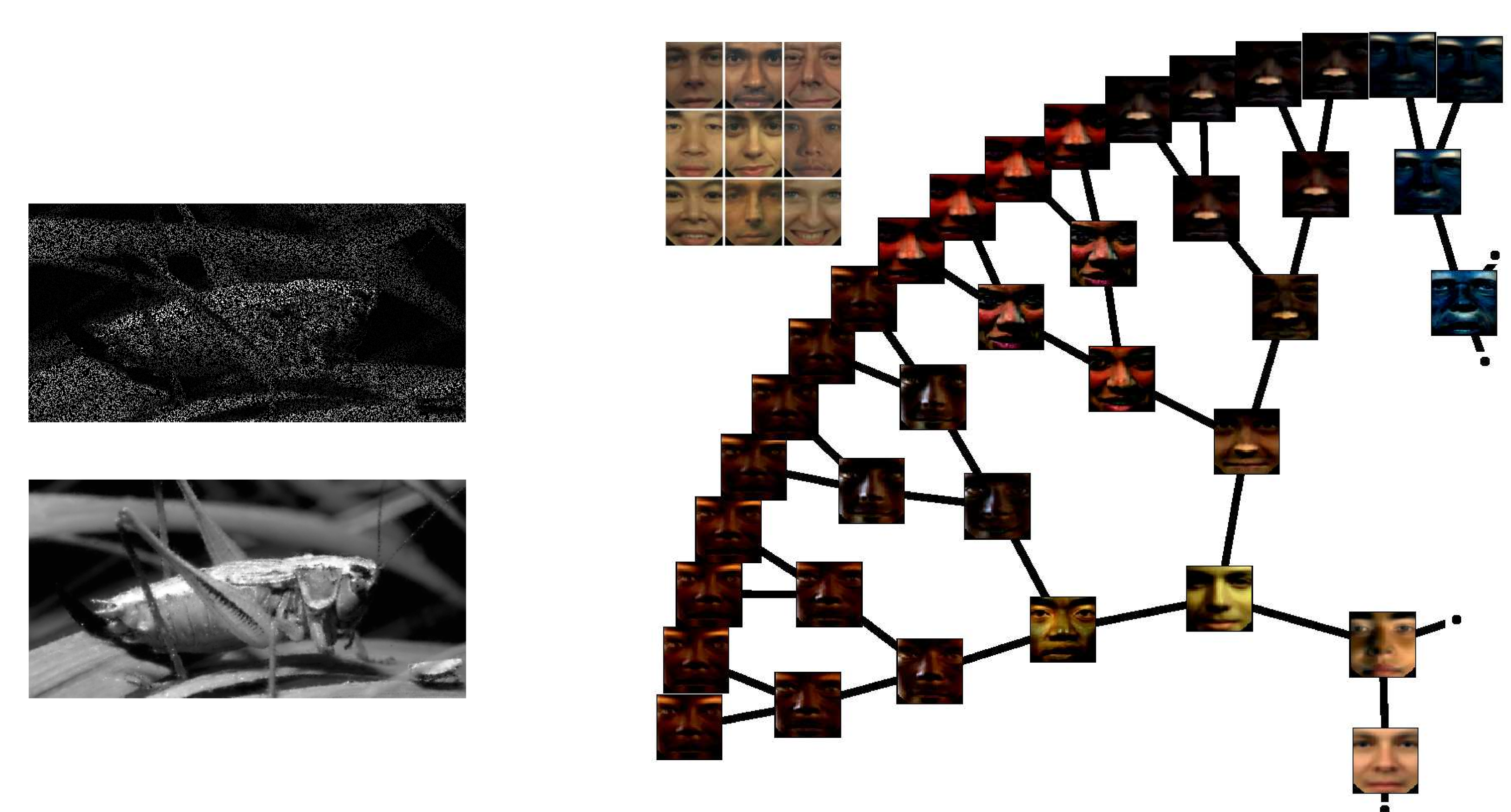


(a) (b) (c) (d) (e) (f)

Figure 1: *Group-structured* $\mathbf{D}$*-s. (a)-(c): complete; increasing neighbor size* ($r = 0, 2, 3$)*. (d)-(f): increasing incompleteness* ($p_{tr} = 0, 0.1, 0.5$)*.*

5.2 Online structured non-negative matrix factorization on faces

- Online, $\mathcal{G}$-NMF: special case of OSDL.
- Illustration: color FERET large-scale (140×120) facial dataset.
- $\mathcal{G}$: complete, 8-level binary tree ($d_\alpha = 255$).



(a) (b)

Figure 2: *(a): full image inpainting illustration; top: observed, bottom: estimated. (b): structured NMF dictionary, training samples at the upper left corner.*

5.3 Collaborative Filtering

- Joke recommendation (Jester): 100 jokes $\times$ $73,421$ users.
- Observation: $\mathbf{x}_{O_t}$ = ratings of the t^{th} user.
- Baseline: best known RMSE = 4.1123 (item neighbor), 4.1229 (unstructured dictionary, $d_\alpha = 100$).
- Result: toroid $\mathcal{G}$ ($d_\alpha = 100$) $\rightarrow$ RMSE = 4.0774, hierarchical ($d_\alpha = 15$) $\rightarrow$ 4.1220.

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