Congestion Aware Routing in Nonlinear Elastic Optical Networks

Seb J. Savory, *Senior Member, IEEE*

***Abstract*— In elastic optical networks, digital coherent transceivers modify their symbol rate, modulation format, and forward error correction to best serve the network demands. In a nonlinear elastic optical network, these parameters are inherently coupled with the routing algorithm. We propose to use congestion aware routing in a nonlinear elastic optical network and demonstrate its efficacy for the NSFNET reference network (14 nodes, 22 links). The network is sequentially loaded with 100 GbE demands until a demand becomes blocked, this procedure being repeated 10 000 times to estimate the network blocking probability (NBP). Three routing algorithms are considered: 1) shortest path routing; 2) simple congestion aware algorithm; and 3) weighted congestion aware routing algorithm with 50, 25, 12.5, and 6.25 GHz resolution flexgrids. For NBP = 1% using a 50 GHz grid, congestion aware routing doubles the network capacity compared with the shortest path routing. When congestion aware routing is combined with a 6.25 GHz resolution flexgrid, a fivefold increase in network capacity is afforded.**

***Index Terms*— Optical fiber communication, networks, routing, adaptive modulation, extreme value distributions.**

I. INTRODUCTION

AS OPTICAL networks move towards being able to cope with elastic demands the traditional challenge of routing and wavelength assignment becomes replaced by that of routing, modulation and spectrum allocation (RMSA) [1], [2]. The high dimensionality of the RMSA problem can however be simplified due to the coupling between the dimensions caused by the fiber nonlinearities. Thus in a nonlinear optical network the RMSA becomes primarily a routing problem, with the assignment of modulation format and forward error correction (FEC) depending on the signal to noise available for a given route and the optical spectrum to be assigned depending on the required data rate [3]. Given the increased importance of the routing algorithm for the nonlinear RMSA, in this letter we investigate the impact of congestion aware routing algorithms and quantify their performance and efficacy in delaying the onset of network blocking for the NSFNET topology.

II. UNDERLYING ASSUMPTIONS IN THE PROPOSED MODEL

In order to facilitate an investigation of congestion aware routing we make the following assumptions in our analysis:

1. The client side data rate is fixed (herein we restrict our consideration to 100 GbE assumed to be 104 Gbit/s including framing plus an overhead for FEC).
2. Transceivers can vary modulation format and FEC.
3. Channels are Nyquist shaped, having a rectangular spectrum of width equal to the symbol rate.
4. There are negligible guard bands between channels.
5. The network employs single mode fiber and is periodically amplified by a lumped erbium doped fiber amplifier (EDFA) with a bandwidth of 5 THz.
6. The spacing between amplifiers is fixed throughout the network (herein we consider 100 km span length).
7. No optical dispersion compensation is employed and the fiber plant is the same over the entire network.
8. The Gaussian noise model [4], [5] is valid and the nonlinear interference (NLI) adds incoherently such that the total NLI is proportional to the path length.
9. Primary source of noise is from the EDFA within the links and that losses at the nodes may be neglected.
10. In the link where blocking occurs the spectral utilization is sufficiently high that the nonlinear impairments correspond to 100% spectral utilization.
11. We consider the network to become blocking at the point when the first blocked demand occurs.

III. PROPOSED ALGORITHM FOR NONLINEAR RMSA

The proposed algorithm for RMSA in a nonlinear elastic network utilizing Nyquist pulse shaping is as follows:

1. Determine the optimum signal power spectral density given the fiber and amplifier parameters.
2. For a pair of nodes, select the shortest path that avoids the link with the highest spectral usage (determined by measuring the total optical power which is proportional to spectral usage).
3. For this path determine the total number of amplifier spans (100 km herein) in order to determine the received signal to noise ratio (SNR).
4. For this SNR, determine the maximum net spectral efficiency (NSE) based on known relationship between SNR and NSE for a range of polarization division multiplexed formats with Nyquist spectra where variable rate FEC is also included.
5. Finally determine the gross symbol rate and assign spectrum to serve the demand between the two nodes.

Manuscript received March 5, 2014; revised March 19, 2014; accepted March 20, 2014. Date of publication March 21, 2014; date of current version April 24, 2014. This work was supported by the EPSRC Programme Grant UNLOC (EP/J017582/1).

The author is with the Department of Electronic and Electrical Engineering, University College London, London WC1E 7JE, U.K. (e-mail: s.savory@ucl.ac.uk).

Color versions of one or more of the figures in this letter are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LPT.2014.2314438

IV. MODELING FIBER NONLINEARITIES

While numerous models exist for fiber nonlinearities, we seek a model that captures the salient features of the nonlinear impairments, but is simple enough to permit complex network studies. As in [4] we turn to the Gaussian noise model [5] that has recently been shown to be accurate for uncompensated links using digital coherent transceivers [6] as is likely to occur in future nonlinear elastic optical networks. Using the Gaussian noise model it can be shown [7], [8] that if the total available spectrum B is modulated[1] over a single span having an attenuation coefficient of α and effective length $1/\alpha$, the optimum power spectral density (S_{sig}) is given by[2]

$$S_{sig} = \sqrt[3]{\frac{27\pi|\beta_2|\alpha S_{ASE}}{16\gamma^2 \ln\left(\frac{2B^2\pi^2|\beta_2|}{3\alpha}\right)}} \tag{1}$$

where β_2 is the dispersion coefficient and $S_{ASE} = 2n_{sp}h\nu\,(G-1)$ is the power spectral density of the amplified spontaneous emission (ASE) noise (where G is the amplifier gain, $h\nu$ is the photon energy and n_{sp} is the population inversion factor). Assuming incoherent addition of the noise over N identical spans the resulting SNR is

$$SNR = \frac{2S_{sig}}{3NS_{ASE}} \tag{2}$$

Rather than deal in the abstract let us consider a specific fiber type such as standard single mode fiber with an attenuation of 0.22 dB/km, nonlinear coefficient $\gamma = 1.3$ W^{-1}km^{-1} and chromatic dispersion of 16.7 ps/nm/km. Assuming the amplifiers have a noise figure of 5 dB with the span length between amplifiers being 100 km then 27 mW/THz is the optimal power density over the 5 THz bandwidth. As such the fiber nonlinearities can be accounted for by limiting the power spectral density to 27 mW/THz. Given that the SNR after the first span is 24.5 dB after N spans the SNR in decibels is

$$SNR_{dB} = 24.5 - 10\log_{10}(N) \tag{3}$$

V. OPTIMAL MODULATION FORMAT

Shannon gives a relationship between the spectral efficiency and the linear SNR such that for a polarization multiplexed format the net spectral efficiency (NSE) is given by [10]

$$NSE = 2\log_2(1+SNR) \tag{4}$$

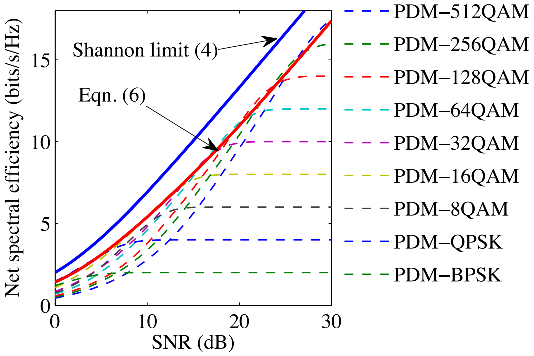

Fig. 1. Net spectral efficiency (NSE) versus SNR for various PDM-QAM formats. As the available SNR decreases, the net spectral efficiency also decreases requiring a higher symbol rate for a given data rate.

Since the Shannon limit does not indicate the modulation format or the FEC coding overhead that should be employed, we seek an alternative approximate bound as to what might be realizable in practice. In order to consider this for polarization division multiplexed quadrature amplitude modulation (PDM-QAM) constellations we determine using analytical expressions from [11] combined with direct simulation of the performance in the presence of additive white Gaussian noise the bit error rate (BER) as a function of SNR. Rather than assume soft decoding we conservatively use the hard decision decoding bound for the binary symmetric channel[3] such that if $p_b = BER$, then the maximum code rate r is given by [10]

$$r = 1 + p_b\log_2(p_b) + (1-p_b)\log_2(1-p_b) \tag{5}$$

from this the NSE as a function of SNR can be obtained for a given cardinality of QAM to give Fig. 1. We note that for a terrestrial core optical transmission network the SNR will typically be in the region of 5 to 25 dB for the fiber and amplifier parameters previously discussed corresponding distances ranging from approximately 100 to 10000 km.

Over the region of interest shown in Fig. 1 the NSE that can be realized with PDM-QAM and optimal hard FEC can be approximately bounded by the following expression[4]

$$NSE \approx 2\log_2\left(1 + SNR\left[\frac{210+9SNR}{325+22SNR}\right]\right) \tag{6}$$

If the optimum launch power spectral density is used then the SNR is uniquely defined by the route though the network. Hence knowing the SNR then using the approximate realizable bound (6) this then defines the appropriate amount of spectrum that should be assigned in an elastic network.

VI. ROUTING ALGORITHMS

We consider our benchmark as shortest path (SP) routing with first fit allocation of the optical spectrum, and two congestion aware (CA) variants of shortest path routing that are:

CA1. Selects the shortest path that avoids the fiber link that is most congested, implemented with Dijkstra's algorithm on the graph where the edge weight for the most congested path has been replaced by infinity.

CA2. The shortest path through a weighted network, where the weight of an edge joining nodes i and j is given by $W_{ij} = L_{ij}/\eta_{ij}$ where L_{ij} is the physical length and η_{ij} is the proportion of the total spectrum which is still available on that edge.

Since the system operates with a constant power spectral density the spectral usage is proportional to the total optical

---

[1] This is based on the assumption that in the link where blocking first occurs the spectrum will be sufficiently close to fully utilized. As such the nonlinear interference noise density corresponds to $B \approx 5$ THz, similar to the local optimum global optimum Nyquist (LOGON) strategy proposed in [8].

[2] This follows from [9] with an asymptotically large total modulated bandwidth $B \approx 5$ THz, with the peak power spectral density converging to the spectrally averaged integrated nonlinear noise power indicating the validity of the white noise approximation. Replacing the effective length by $1/\alpha$ gives an error of ≤ 0.05 dB in the launch power for span lengths of 80 km or more.

[3] We note that the latest state of the art transceivers employing soft FEC are close to this bound, e.g. with a 15% FEC overhead, hard FEC limit is a BER = 0.018, c.f. BER = 0.019 reported for an implemented soft FEC [12].

[4] For SNR from −30 to 50 dB this expression, being a minimax fit with a Pade approximant, gives the NSE with an accuracy of better than 5%.

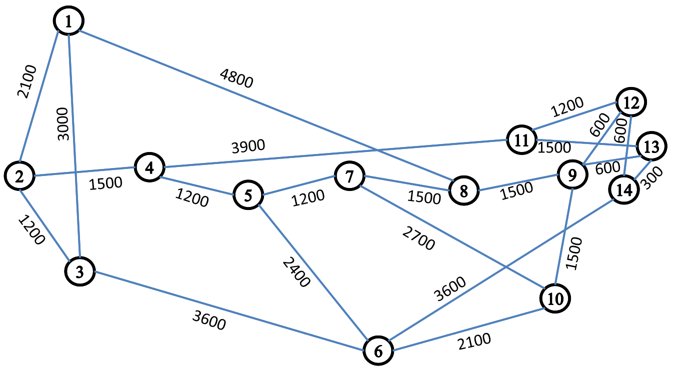

Fig. 2. NSFNET topology with the lengths in km marked on links.

power in any link making congestion a parameter that is straightforward to measure for an installed network.

VII. ESTIMATING THE NETWORK BLOCKING PROBABILITY

A. Sequential Loading of the Network

In order to estimate the network blocking probability (NBP), we follow the methodology of [13], sequentially loading the network with bi-directional demands between randomly selected pairs of nodes in the network. In this letter for simplicity we restrict our discussion to uniformly generated traffic with a demand granularity of 100 GbE, each of which is independently optically routed through the network. The point at which the path assigned by the routing algorithm cannot physically be optically routed by the network is considered blocking and the number of demands recorded. This procedure is then repeated 10000 times to build up the statistical behavior of the network blocking probability. From this we calculate the probability that blocking is occurring within the network for a given network load.

B. Statistical Analysis of Network Blocking Probabilities

Herein we focus on the number of demands that cause blocking within the network in 1% of occasions. Given that the network blocking occurs when blocking occurs in the most congested link, in essence we are concerned with the distribution of a minimum. We therefore propose to analyze the networking blocking probability using a generalized extreme value distribution whose cumulative distribution function (CDF) may be expressed as [14]:

$$F\left(x|k,\mu,\sigma\right) = \exp\left[-\left(1+k\frac{(x-\mu)}{\sigma}\right)^{-\frac{1}{k}}\right] \tag{7}$$

where k, μ and σ are the shape, location and scale parameters of the distribution, all of which may be determined from a given data set using maximum likelihood estimation.

VIII. APPLICATION TO AN EXEMPLAR OPTICAL NETWORK

So as to quantify the benefit afforded in an optical network by using the algorithm proposed in this letter, we consider a mesh topology the NSFNET shown in Fig. 2, having 14 nodes, 22 edges using the same path lengths as per [15].

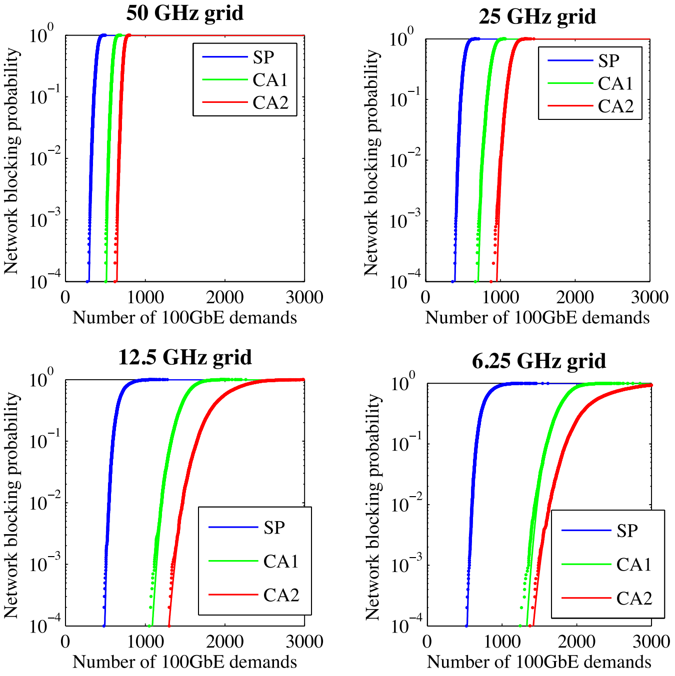

Fig. 3. Network blocking probability for 50 GHz grid (top left), 25 GHz grid (top right), 12.5 GHz grid (bottom left) and 6.25 GHz grid (bottom right). In all cases the congestion aware algorithms (CA1 and CA2 detailed in section VI) offer improved performance compared with shortest path (SP). In all cases for network blocking probabilities greater than 1%, the extreme value distribution is indistinguishable from the observed points (denoted as dots).

Of particular relevance for the SNR based RMSA is that the shortest and longest shortest path between nodes are 300 km and 7800 km respectively such that we expect the SNR to vary between 5.6 dB and 19.7 dB. We consider four possible grid options, namely 50, 25, 12.5 and 6.25 GHz grids, in order to assess the impact of this on the performance. In all cases it is assumed the signal may occupy multiple slots should the available SNR dictate this. We also consider the three routing algorithms, shortest path (SP), a simple congestion aware routing algorithm (CA1) and a weighted congestion aware routing algorithm (CA2) detailed in section VI.

Fig. 3 indicates excellent agreement between the observed network blocking probabilities and the fitted generalized extreme value distribution (7) validating our hypothesis that we are concerned with the distribution of a minimum.

As can be seen in Table I, the two congestion aware routing algorithms significantly increase the capacity for a 1% network blocking probability for all of the frequency grids considered, with the maximum benefit afforded by combining a flexgrid with congestion aware routing. For the 6.25 GHz flexgrid with a weighted congestion aware routing algorithm (CA2), the network capacity is 1744 demands each of 100 GbE, compared to just 328 demands with standard shortest path routing with a 50 GHz grid. These results indicate the benefit that might be obtained in a sequentially loaded optical network, mirroring the current operation of installed optical networks [16]. While dynamic optical networks have not been studied, the results for the sequentially loaded network indicate that this would warrant further investigation.

For the results obtained, one of the striking features contained within Table I, is the significant increase in the

TABLE I

PERFORMANCE CHARACTERISTICS OF THE ROUTING ALGORITHMS

Grid (GHz)	*Routing Algorithm*	$\overline{L}$ *(km)*	σ_L *(km)*	$\max\{L\}$ *(km)*	*Number of 100 GbE demands for NBP=1%*
50	SP	3986	2048	7800	328
50	CA1	4432	2337	10200	541
50	CA2	4340	2375	19500	674
25	SP	3986	2047	7800	459
25	CA1	4436	2342	10200	802
25	CA2	4377	2435	19500	1012
12.5	SP	3986	2048	7800	572
12.5	CA1	4425	2332	10200	1265
12.5	CA2	4417	2443	18300	1513
6.25	SP	3988	2049	7800	653
6.25	CA1	4430	2336	10200	1558
6.25	CA2	4410	2440	18300	1744

$\overline{L}, \sigma_L$ and $\max\{L\}$ denote the average, standard deviation and maximum path length respectively for the three different routing algorithms and four frequency grids considered.

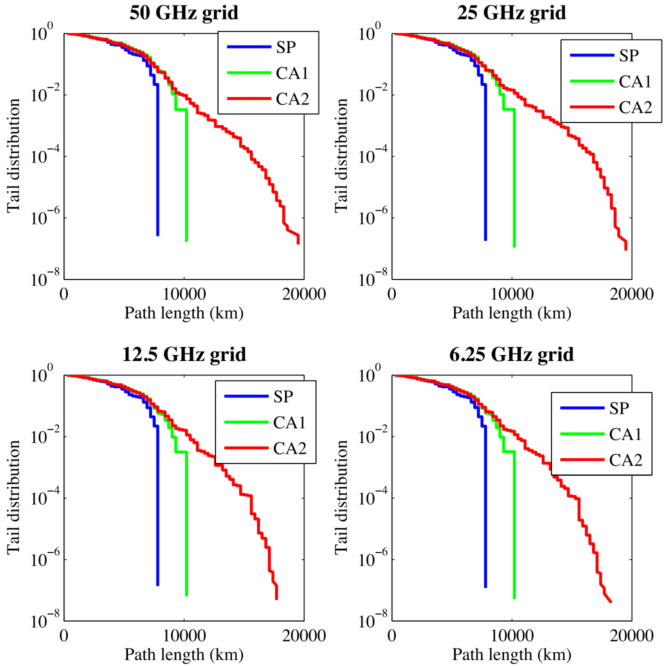

Fig. 4. Impact of routing algorithm on the tail distribution of path length for 50 GHz grid (top left), 25 GHz grid (top right), 12.5 GHz grid (bottom left) and 6.25 GHz grid (bottom right).

maximum path length when using CA2. To investigate this further in Fig. 4 we plot the tail distribution[5] for the path length, with each of the frequency grids and routing algorithms.

As can be seen in Fig. 4 for both of the congestion aware routing algorithms, in all cases more than 5% of all routed path lengths exceed the longest shortest path of 7800 km highlighting the additional resources congestion aware routing requires. Nevertheless, for a given network blocking probability, it is evident that by using more resources to route traffic away from congestion, the number of 100 GbE demands which can be served increases significantly.

IX. CONCLUSION

Congestion aware routing has been investigated in nonlinear elastic optical networks and shown to be effective for the reference NSFNET topology. We observe that the network blocking probability (NBP) follows a generalized extreme value distribution, allowing robust estimates of the load for a given NBP to be obtained. When NSFNET is sequentially loaded with 100 GbE demands the proposed algorithm with a 6.25 GHz flexgrid, allows the network to support 1744 demands compared to 328 demands using a fixed 50 GHz grid with shortest path routing for NBP = 1%. The congestion aware routing algorithms investigated resulted in longer average paths, with 5% of all routes exceeding the maximum shortest path in order to increase the overall network capacity.

ACKNOWLEDGEMENTS

The author thanks David Ives, Andrew Lord, Polina Bayvel and Benn Thomsen for stimulating discussions and their comments on early drafts of this manuscript.

REFERENCES

[1] O. Gerstel, M. Jinno, A. Lord, and S. J. B. Yoo, "Elastic optical networking: A new dawn for the optical layer?" *IEEE Commun. Mag.*, vol. 50, no. 2, pp. s12–s20, Feb. 2012.

[2] L. Velasco, M. Klinkowski, M. Ruiz, and J. Comellas, "Modeling the routing and spectrum allocation problem for flexgrid optical networks," *Photon. Netw. Commun.*, vol. 24, no. 3, pp. 177–186, 2012.

[3] M. Jinno *et al.*, "Distance-adaptive spectrum resource allocation in spectrum-sliced elastic optical path network," *IEEE Commun. Mag.*, vol. 48, no. 8, pp. 138–145, Aug. 2010.

[4] M. Angelou, P. N. Ji, I. Tomkos, and T. Wang, "QoT prediction for core networks with uncompensated coherent transmission," in *Proc. OECC/PS* Jul. 2013, pp. 1–2, paper TuQ3-4.

[5] A. Splett, C. Kurtzke, and K. Petermann, "Ultimate transmission capacity of amplified optical fiber communication systems taking into account fiber nonlinearities," in *Proc. ECOC*, 1993, pp. 41–44.

[6] P. Poggiolini, A. Carena, V. Curri, G. Bosco, and F. Forghieri, "Analytical modeling of nonlinear propagation in uncompensated optical transmission links," *IEEE Photon. Technol. Lett.*, vol. 23, no. 11, pp. 742–744, Jun. 1, 2011.

[7] P. Poggiolini, "The GN model of non-linear propagation in uncompensated coherent optical systems," *J. Lightw. Technol.*, vol. 30, no. 24, pp. 3857–3879, Dec. 15, 2012.

[8] P. Poggiolini *et al.*, "The LOGON strategy for low-complexity control plane implementation in new-generation flexible networks," in *Proc. OFC*, Mar. 2013, pp. 1–3, paper OW1H.3.

[9] S. J. Savory, "Approximations for the nonlinear self-channel interference of channels with rectangular spectra," *IEEE Photon. Technol. Lett.*, vol. 25, no. 10, pp. 961–964, May 15, 2013.

[10] C. E. Shannon, "A mathematical theory of communication," *Bell Syst. Tech. J.*, vol. 27, no. 3, pp. 379–423, 1948.

[11] K. Cho and D. Yoon, "On the general BER expression of one- and two-dimensional amplitude modulations," *IEEE Trans. Commun.*, vol. 50, no. 7, pp. 1074–1080, Jul. 2002.

[12] L. E. Nelson *et al.*, "A robust real-time 100G transceiver with soft-decision forward error correction," *J. Opt. Commun. Netw.*, vol. 4, no. 11, pp. B131–B141, 2012.

[13] P. Wright, A. Lord, and S. Nicholas, "Comparison of optical spectrum utilization between flexgrid and fixed grid on a real network topology," in *Proc. OFC*, 2012, pp. 1–3, paper OTh3B.5.

[14] S. Coles, *An Introduction to Statistical Modeling of Extreme Values*. Berlin, Germany: Springer-Verlag, 2001.

[15] A. Nag, M. Tornatore, and B. Mukherjee, "Optical network design with mixed line rates and multiple modulation formats," *J. Lightw. Technol.*, vol. 28, no. 4, pp. 466–475, Feb. 15, 2010.

[16] A. Mitra, A. Lord, S. Kar, and P. Wright, "Effect of link margin and frequency granularity on the performance of a flexgrid optical network," in *Proc. ECOC*, 2013, pp. 1–6, paper We.2.E.3.

---

[5] The tail distribution, also known as the complementary CDF is the probability that a variable (e.g. path length) is exceeded.