Masked Beamforming in the Presence of Energy-Harvesting Eavesdroppers

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***Abstract*— This paper considers a multiple-input single-output downlink system consisting of one multiantenna transmitter, one single-antenna information receiver (IR), and multiple single-antenna energy-harvesting receivers (ERs) for simultaneous wireless information and power transfer. The design is to keep the message secret to the ERs while maximizing the information rate at the IR and meeting the energy harvesting constraints at the ERs. Technically, our objective is to optimize the information-bearing beam and artificial noise energy beam for maximizing the secrecy rate of the IR subject to individual harvested energy constraints of the ERs for the case where the ERs can collude to perform joint decoding in an attempt to illicitly decode the secret message to the IR. As a by-product, we also solve the total power minimization problem subject to secrecy rate and energy harvesting constraints. Both scenarios of perfect and imperfect channel state information (CSI) at the transmitter are addressed. For the imperfect CSI case, we study both eavesdroppers' channel covariance-based and worst case-based designs. Using semidefinite relaxation (SDR) techniques, we show that there always exists a rank-one optimal transmit covariance solution for the IR. Furthermore, if the SDR results in a higher rank solution, we propose an efficient algorithm to always construct an equivalent rank-one optimal solution. Computer simulations are carried out to demonstrate the performance of the proposed algorithms.**

***Index Terms*— Colluding eavesdroppers, energy beamforming, energy harvesting, masked beamforming.**

I. Introduction

PRACTICAL energy-constrained devices are often powered by batteries with limited lifespan. In battery-limited devices mounted at some inaccessible or difficult-to-access places, replacing or recharging the supplies usually requires high costs and is inconvenient. Practical examples include sensors embedded inside fixed structures, or inside human bodies, or in deadly environments [1]. Fortunately, in contemporary urban areas, there is a huge amount of electromagnetic energy in the environment due to numerous radio and television broadcastings. Thus a more opportunistic as well as greener alternative for powering such devices is to harvest energy from the surroundings if possible. For typical low-power applications such as sensor networks, wearable electronics etc., radio-frequency (RF) signals can be a sustainable new source for energy hunting. However, if dedicated wireless power can be transmitted, the technique can be applied for scenarios with more generous power ingestions as well.

Since RF signals that transport information can carry energy at the same time, simultaneous wireless information and power transfer (SWIPT) has attracted upsurge of interest [1]–[6]. Through SWIPT, mobile users are provided with access to both energy and data at the same time which brings enormous prospects of new application development. The concept of SWIPT was first introduced in [2]. The authors proposed a capacity-energy function to characterize the tradeoffs between the rates at which energy and reliable information can be transmitted over a single-antenna additive white Gaussian noise (AWGN) channel. The work in [2] was extended in [3] to frequency-selective channels and the optimal tradeoff between the achievable rate and the power transferred was characterized under the total power constraint. Nevertheless, it was assumed in [2] and [3] that the receiver is capable of decoding information and extracting power simultaneously from the same received signal and this assumption appears untrue for practical circuits that harvest energy from radio signals.

Conventional receiver architecture designed for information transfer is no longer optimal for SWIPT because information and power transfer operate with very different power sensitivity at the receiver (e.g., -10dBm for energy receivers (ERs) versus -60dBm for information receivers (IRs)). To facilitate wireless information and power transfer at the receiver, two practical schemes, namely, time switching (TS) and power splitting (PS) have been proposed recently [1], [4]. The scenario investigated in [1] was broadcasting from a base station (BS) to two mobile receivers taking turns for information decoding and energy harvesting (time-switching). Although the scheme in [1] simplifies the receiver design, it compromises the efficiencies of perfect SWIPT technology. Hence in [4], practical receiver architectures for point-to-point systems have been extensively investigated that enable SWIPT. In [5], SWIPT has been considered in presence of co-channel interference and the interference was utilized as a source for energy harvesting in contrast to the traditional view of taking interference as an undesired factor.

However, wireless channel is subject to signal fading. Thus, the power transfer efficiency decays drastically with the increasing transmission distance. By exploiting spatial diversity, multi-antenna techniques can be applied to combat channel fading [7]. To increase wireless power transfer efficiency in SWIPT systems, multi-antenna techniques can be useful [1]. The work in [1] has been extended to the

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case of imperfect channel state information (CSI) at the transmitter in [6], whereas multiple energy harvesting nodes were considered in [8]. However, in [6] and [8], each receiver either harvests energy or decodes information, but not both. More recently, SWIPT multicasting in multiple-input single-output (MISO) and multiple-input multiple-output (MIMO) systems were studied in [9] and [10], respectively. The authors proposed joint multicast transmit beamforming and receive PS algorithms for minimizing the transmit power of the BS subject to quality-of-service (QoS) constraints at each receiver considering both scenarios of perfect and imperfect CSI available at the BS.

Based on the fact that the IR and ER typically operate with very different power sensitivity level, a receiver-location based scheduling for information and energy transmissions has also been proposed in the SWIPT literature [1], [11], where the receivers only in closer vicinity to the transmitter are scheduled for transmitting energy. However, the scheme gives rise to a new information security vulnerability for SWIPT systems since ERs have better fading channels than IRs and thus have higher probability to overhear the information sent to the IRs [12], [13]. Therefore, the SWIPT systems need to be efficiently designed in order to guarantee information secrecy such that the legitimate user (IR) can correctly decode the confidential information, but the eavesdroppers (ERs) can retrieve almost nothing from their observations.

Recently, there has been growing interest in using multiple antennas to achieve physical-layer secrecy [14]–[17]. To make this more feasible, we usually need the IR's channel condition to be better than the eavesdroppers' which is a *conflict-of-interest* in line of energy harvesting. As a potential remedy, recent works are mainly focused on multi-antenna transmission technologies, since multiple transmit antennas provide spatial degrees of freedom to worsen the interception of the eavesdroppers. By exploiting transmit beamforming, information-bearing signals are transmitted over the direction of the legitimate user while artificially generated noise signals are directed to interfere the eavesdroppers intentionally [16], [17]. The prospect of artificial noise (AN) aided transmit beamforming in case of SWIPT is twofold. First, the AN can keep the message secure to the IR by jamming the ERs' reception. Second, the AN beams can be designed as energy beams so as to improve the amount of energy harvested by the ERs. Depending on the extent of eavesdroppers CSI available at the transmitter, different strategies can be applied to generate the optimal energy beams. If no eavesdroppers' CSI is available, then a popular design is the isotropic AN [14], [15], where the message is transmitted in the direction of the intended receiver's channel, and spatio-temporal AN is uniformly spread on the orthogonal subspace of the legitimate channel. This scheme guarantees that the IR's reception will be free from the interference by the AN, while the ERs' reception may be degraded by the AN. On the other hand, with knowledge of the eavesdroppers' CSI to some extent, one can block the eavesdroppers' interception more efficiently by generating spatially selective AN [16], [17].

To ensure that the message is delivered secretly to the IR in a SWIPT scheme even in the presence of possible eavesdropping

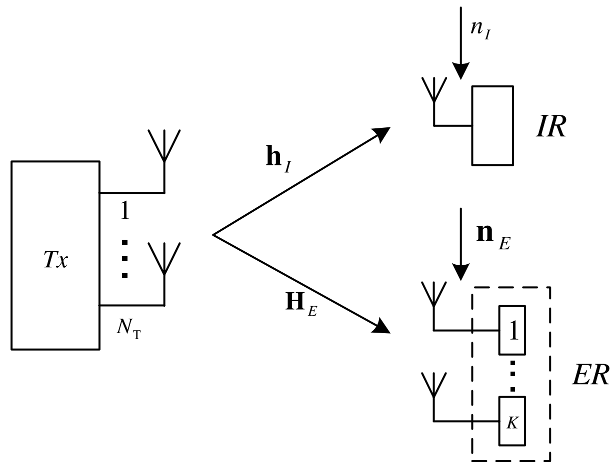

Fig. 1. A MISO SWIPT system with colluding eaves.

by the individual ERs, MISO secrecy communication schemes were studied in [12] and [13]. Two secrecy beamforming design problems have been considered in [12] namely (1) maximizing the secrecy rate subject to the total transmit power and energy harvesting constraints, and (2) maximizing the weighted sum-transferred-energy subject to the total transmit power and secrecy rate constraints. The strong assumptions in [12] were that the ERs do not collude to perform joint decoding and that the eavesdroppers' channels are perfectly known at the transmitter. However, the worst-case scenario in terms of secrecy capacity is that multiple ERs collude together to attempt to decode the data jointly such that the eavesdropping rate is maximized. Also, perfect eavesdropper CSI is impractical. Hence robust beamforming design has been considered in [13] for maximizing the harvested energy by the ERs while maintaining the signal-to-interference and noise ratio (SINR) threshold at the IR and keeping the message secure from possible eavesdropping by the ERs by suppressing their SINRs.

In this paper, we consider a secret MISO SWIPT system consisting of one multi-antenna transmitter, one single-antenna IR, and multiple single-antenna ERs, as shown in Fig. 1. Our aim is to design jointly the information and energy transmit beamforming for maximizing the secrecy rate of the IR subject to individual harvested energy constraints of the ERs for the case where the ERs may collude to perform joint decoding.[1] Note that the energy carried by the AN was subject to wastage in existing AN-aided secret communication schemes (see [14]–[17]) whereas the carried energy is reused in the proposed schemes as a potential source of energy scavenging. Since in practical multi-antenna systems, each antenna is often equipped with its own power amplifier, we impose a generalized power specification constraint in addition to the sum power constraint which provides the opportunity to specify parameters such as per-antenna or peak-power constraints etc. Finally, we consider the more practical scenario

[1]The inclusion of the colluding eavesdroppers joint beamforming in the transmit and energy beamforming design can guarantee maximum information security in the worst-case sense. The technical difficulty of secrecy communication with colluding eavesdroppers will be explained in Section III.

of imperfect CSI of the ER or both IR and ER available at the transmitter. Thus, our work is more general than the work in [12] in terms of eavesdroppers joint decoding, per-antenna power constraints, and robustness to CSI errors. As a by-product, we also solve a total power minimization problem subject to the secrecy rate and energy harvesting constraints.

Applying semidefinite relaxation (SDR) techniques, we show that there always exists a rank-one optimal transmit covariance solution for the IR, i.e., transmit beamforming is optimal for the IR. Furthermore, if the SDR results in a higher-rank solution, we propose an efficient algorithm to construct an equivalent rank-one optimal solution for both perfect and imperfect CSI cases. To tackle the imperfect CSI case, we consider both eavesdroppers' channel covariance based and worst-case based designs.

The rest of this paper is organized as follows. In Section II, the system model of a secret MISO SWIPT network with colluding ERs is introduced. The joint information and energy transmit beamformer design algorithms are developed in Section III for the case of perfect CSI and in Section IV if CSI is imperfect. Section V presents the simulation results that justify the significance of the proposed algorithms under various scenarios. Concluding remarks will be provided in Section VI.

Throughout the paper we use the following notation standards. Boldface lowercase and uppercase letters are used to represent vectors and matrices, respectively. The symbol $\mathbf{I}_n$ denotes an $n \times n$ identity matrix, $\mathbf{0}$ is a zero vector or matrix. Also, $\mathbf{A}^H$, $\mathbf{A}^\dagger$, $\mathrm{tr}(\mathbf{A})$, $\mathrm{rank}(\mathbf{A})$, and $\det(\mathbf{A})$ represent the Hermitian (conjugate) transpose, matrix pseudo inverse, trace, rank and determinant of a matrix $\mathbf{A}$; $\|\cdot\|$ and $\|\cdot\|_F$ represent the Euclidean norm and Frobenius norm, respectively; $\mathbf{A} \succeq \mathbf{0}$ ($\mathbf{A} \succ \mathbf{0}$) means that $\mathbf{A}$ is a Hermitian positive semidefinite (definite) matrix; $[\mathbf{A}]_{i,j}$ denotes the (i, j)th element of $\mathbf{A}$. The notation $\mathbf{x} \sim \mathcal{CN}(\mu, \mathbf{\Sigma})$ means that $\mathbf{x}$ is a random vector following a complex circularly symmetric Gaussian distribution with the mean vector μ and the covariance matrix of $\mathbf{\Sigma}$.

II. System Model

We consider a MISO downlink system for SWIPT with $K + 1$ receivers as illustrated in Fig. 1. The transmitter or BS has $N_T > 1$ transmitting antennas and each receiver has single receiving antenna. One of the receivers is an IR while other receivers are ERs. The BS performs linear transmit beamforming to send secret information to the IR. We assume that the ERs are also active users of the network but the BS trusts them only to harvest energy. By letting $\mathbf{x}$ be the transmit signal vector, the received signals at the IR and the kth ER can be modeled, respectively, as

$$y_I = \mathbf{h}_I^H \mathbf{x} + n_I, \quad (1)$$

$$y_{E,k} = \mathbf{h}_{E,k}^H \mathbf{x} + n_{E,k}, \text{ for } k = 1, \ldots, K, \quad (2)$$

where $\mathbf{h}_I$ and $\mathbf{h}_{E,k}$ are the conjugated complex channel vector between the BS and the IR and between the BS and the kth ER, respectively, $n_I \sim \mathcal{CN}(0, \sigma_I^2)$ and $n_{E,k} \sim \mathcal{CN}(0, \sigma_E^2)$ are the additive Gaussian noises at the IR and the kth ER, respectively. The BS chooses $\mathbf{x}$ as the sum of information beamforming vector $\mathbf{b}_I s_I$ and the energy-carrying AN vector $\mathbf{b}_E$ such that the baseband transmit signal vector is

$$\mathbf{x} = \mathbf{b}_I s_I + \mathbf{b}_E, \quad (3)$$

where $s_I \sim \mathcal{CN}(0, 1)$ is the confidential information-bearing signal for the IR and $\mathbf{b}_E = \sum_{i=1}^{d} \mathbf{b}_{E,i} s_{E,i}$ is the sum of $d \leq N_T$ energy beams, in which $\mathbf{b}_{E,i}$ and $s_{E,i} \sim \mathcal{CN}(0, 1)$ denote the ith energy beamforming vector and the ith energy-carrying noise signals, respectively.

Let us assume that some of the eavesdroppers (or simply Eves) cooperate and attempt to form a joint decoder, so as to improve their interception. For ease of exposition, we further assume that all the Eves are colluding. In particular, Eves are assumed to perform joint maximum SINR receive beamforming. By denoting $\mathbf{Q}_I$ as the transmit covariance and $\mathbf{Q}_E \triangleq \sum_{i=1}^{d} \mathbf{b}_{E,i} \mathbf{b}_{E,i}^H$ as the energy covariance, the mutual information (MI) between the BS and the IR is given by

$$C_I(\mathbf{Q}_I, \mathbf{Q}_E) = \log\left(1 + \frac{\mathbf{h}_I^H \mathbf{Q}_I \mathbf{h}_I}{\sigma_I^2 + \mathbf{h}_I^H \mathbf{Q}_E \mathbf{h}_I}\right), \quad (4)$$

and that between the BS and the colluded ER is given by

$$C_E(\mathbf{Q}_I, \mathbf{Q}_E) = \log\det\left(\mathbf{I}_K + \left(\sigma_E^2 \mathbf{I}_K + \mathbf{H}_E^H \mathbf{Q}_E \mathbf{H}_E\right)^{-1} \times \mathbf{H}_E^H \mathbf{Q}_I \mathbf{H}_E\right), \quad (5)$$

where it has been assumed that the ERs collude to perform joint receive beamforming so as to maximize their joint eavesdropping and the colluded Eves' conjugated channel matrix $\mathbf{H}_E$ is formed as $\mathbf{H}_E \triangleq [\mathbf{h}_{E,1}, \ldots, \mathbf{h}_{E,K}]$. Given $(\mathbf{Q}_I, \mathbf{Q}_E)$, an achievable secrecy rate is given by [14]

$$C_s = C_I(\mathbf{Q}_I, \mathbf{Q}_E) - C_E(\mathbf{Q}_I, \mathbf{Q}_E). \quad (6)$$

Note that (6) gives the perfect secrecy rate when the IR can correctly decode the confidential information at C_s bits per channel use, while the ERs can retrieve almost nothing about the secret message.

Our goal is to design the transmit and energy covariances $(\mathbf{Q}_I, \mathbf{Q}_E)$ such that maximum information secrecy can be achieved under certain power constraints. Also, the harvested power at each ER needs to be above a given threshold so that a useful level of harvested energy is reached. Hence, we formulate the following secrecy rate maximization (SRM) problem:

$$\max_{\mathbf{Q}_I, \mathbf{Q}_E} \; C_I(\mathbf{Q}_I, \mathbf{Q}_E) - C_E(\mathbf{Q}_I, \mathbf{Q}_E) \quad (7a)$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{Q}_I + \mathbf{Q}_E) \leq P_T \quad (7b)$$

$$\mathrm{tr}(\mathbf{\Upsilon}_l(\mathbf{Q}_I + \mathbf{Q}_E)) \leq p_l, \text{ for } l = 1, \ldots, L, \quad (7c)$$

$$\xi_k\left(\mathbf{h}_{E,k}^H(\mathbf{Q}_I + \mathbf{Q}_E)\mathbf{h}_{E,k}\right) \geq \eta_k, \quad \forall k, \quad (7d)$$

$$\mathbf{Q}_I \succeq \mathbf{0}, \mathbf{Q}_E \succeq \mathbf{0}, \mathbf{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \quad (7e)$$

where (7b) is the transmit sum power constraint, with $P_T > 0$ being the prescribed power budget; $\mathbf{\Upsilon}_l \succeq \mathbf{0}$ and $p_l, \forall l$, are given design parameters to accommodate more sophisticated power constraints; $\eta_k > 0$ is the minimum harvested energy protection threshold at the kth ER, and $\xi_k \in (0,1]$ is the energy conversion efficiency of the energy transducers at the kth ER that accounts for the loss in the energy transducers for converting the harvested energy to electrical energy to be stored. For convenience, we assume, without loss of generality, that $\xi_k = 1, \forall k$, in this paper. It is worth pointing out that the ERs do not need to convert the received signal from the RF band to the baseband in order to harvest the carried energy using modern energy transducers. Therefore, according to the law of energy conservation, it is assumed that the total harvested RF band power (energy normalized by the baseband symbol period) at each ER is proportional to the normalised energy of the received baseband signal.

Now we describe some application-specific scenarios where the constraint (7c) is necessary. As each antenna element in a multi-antenna system is equipped with its own power amplifier, the sum power constraint (7b) which restricts the transmit power with the expected norm of the transmit signal vector is not often suitable for practical systems. The constraint (7c) embraces more complex constraints such as per-antenna power constraint and peak power constraint as special cases. For example, to operate within the linear region of each antenna's power amplifier, one may want to limit the per-antenna peak power such that $[\mathbf{Q}_{\mathrm{I}} + \mathbf{Q}_{\mathrm{E}}]_{ll} \leq p_l$, for $l = 1, \ldots, N_{\mathrm{T}}$, where p_l is the power limit of the lth antenna. In that case, the per-antenna power constraints in (7c) can be implemented by setting $\boldsymbol{\Upsilon}_l = \mathbf{a}_l\mathbf{a}_l^H$, and $L = N_{\mathrm{T}}$, with $\mathbf{a}_l$ being the lth unit vector [17]. Similarly, for the peak power constraint, one can choose $p_l = p_{\mathrm{peak}}, \forall l$, where p_{peak} is the maximum allowable output power at each antenna. For cognitive radio networks, the constraint (7c) can also be designed to control the interference temperature to the primary users [17].

The problem (7) is highly non-convex with matrix variables and determinants. Note that if $\mathbf{Q}_{\mathrm{I}} = \mathbf{b}_{\mathrm{I}}\mathbf{b}_{\mathrm{I}}^H$ is chosen such that rank $(\mathbf{Q}_{\mathrm{I}}) \leq 1$, the transmit strategy for the confidential information is beamforming. In the following, we derive optimal beamforming strategies for both perfect and imperfect CSI cases.

III. Masked Beamforming With Perfect CSI

Let us first consider (7) for the case where the CSI of the IR and that of the possible eavesdroppers are available at the BS. This is a reasonable assumption for scenarios where the eavesdroppers are also active users of the system, and the transmitter aims to provide different services to different types of users. For such active eavesdroppers, the CSI can be estimated from the eavesdroppers' transmission. However, in the SWIPT system of our interest, it is practically reasonable to assume, as in [12], that the ERs need to assist the transmitter in obtaining their channel knowledge to design transmit and energy beamforming vectors so that their individual energy requirements can be satisfied. With knowledge of the ERs' CSI, we can block the possible eavesdropping by the ERs much more effectively by generating spatially selective AN, rather than keeping AN isotropic [17].

In order to obtain a tractable solution, let us rewrite the problem (7) as

$$\begin{aligned}
\max_{\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}},\gamma}\quad & C_{\mathrm{I}}\left(\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}}\right) - \log\gamma && \text{(8a)}\\
\text{s.t.}\quad & C_{\mathrm{E}}\left(\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}}\right) \leq \log\gamma && \text{(8b)}\\
& \mathrm{tr}\left(\mathbf{Q}_{\mathrm{I}}+\mathbf{Q}_{\mathrm{E}}\right) \leq P_{\mathrm{T}} && \text{(8c)}\\
& \mathrm{tr}\left(\boldsymbol{\Upsilon}_l\left(\mathbf{Q}_{\mathrm{I}}+\mathbf{Q}_{\mathrm{E}}\right)\right) \leq p_l,\quad \forall l, && \text{(8d)}\\
& \mathbf{h}_{\mathrm{E},k}^H\left(\mathbf{Q}_{\mathrm{I}}+\mathbf{Q}_{\mathrm{E}}\right)\mathbf{h}_{\mathrm{E},k} \geq \eta_k,\quad \forall k, && \text{(8e)}\\
& \mathbf{Q}_{\mathrm{I}} \succeq \mathbf{0}, \mathbf{Q}_{\mathrm{E}} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0},\quad \forall l, \gamma \geq 1, && \text{(8f)}
\end{aligned}$$

where γ is a slack variable that is introduced to simplify the objective function. The physical meaning of γ is that $\log\gamma$ can be interpreted as the maximal allowable MI for the colluding Eves. Hence, by adjusting γ, we can control the level of MI between the BS and the ERs, and consequently, the achievable secrecy rate. Note that there may exist circumstances where there is no feasible solution $(\mathbf{Q}_{\mathrm{I}}, \mathbf{Q}_{\mathrm{E}})$ for the problem (8); e.g., when the specification $(\gamma, \{\eta_k\})$ is set too demanding. However, it can be shown that feasibility is not a serious issue for the case where IR's instantaneous CSI is available [16].

Now by substituting (4) and (5) into (8), we can rewrite the problem (8) as

$$\begin{aligned}
\max_{\mathbf{Q}_{\mathrm{I}},\mathbf{Q}_{\mathrm{E}},\gamma}\quad & \log\left(\frac{1+\mathbf{h}_{\mathrm{I}}^H(\mathbf{Q}_{\mathrm{I}}+\mathbf{Q}_{\mathrm{E}})\mathbf{h}_{\mathrm{I}}}{\gamma\left(1+\mathbf{h}_{\mathrm{I}}^H\mathbf{Q}_{\mathrm{E}}\mathbf{h}_{\mathrm{I}}\right)}\right) && \text{(9a)}\\
\text{s.t.}\quad & \log\det\left(\mathbf{I}_K + \left(\mathbf{I}_K + \mathbf{H}_{\mathrm{E}}^H\mathbf{Q}_{\mathrm{E}}\mathbf{H}_{\mathrm{E}}\right)^{-1}\mathbf{H}_{\mathrm{E}}^H\mathbf{Q}_{\mathrm{I}}\mathbf{H}_{\mathrm{E}}\right)\\
&\qquad \leq \log\gamma && \text{(9b)}\\
& \mathrm{tr}\left(\mathbf{Q}_{\mathrm{I}}+\mathbf{Q}_{\mathrm{E}}\right) \leq P_{\mathrm{T}} && \text{(9c)}\\
& \mathrm{tr}\left(\boldsymbol{\Upsilon}_l\left(\mathbf{Q}_{\mathrm{I}}+\mathbf{Q}_{\mathrm{E}}\right)\right) \leq p_l,\quad \forall l, && \text{(9d)}\\
& \mathbf{h}_{\mathrm{E},k}^H\left(\mathbf{Q}_{\mathrm{I}}+\mathbf{Q}_{\mathrm{E}}\right)\mathbf{h}_{\mathrm{E},k} \geq \eta_k,\quad \forall k, && \text{(9e)}\\
& \mathbf{Q}_{\mathrm{I}} \succeq \mathbf{0}, \mathbf{Q}_{\mathrm{E}} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0},\quad \forall l, \gamma \geq 1, && \text{(9f)}
\end{aligned}$$

where we assume that $\sigma_{\mathrm{I}}^2 = \sigma_{\mathrm{E}}^2 = 1$ without loss of generality. The challenge in the SRM problem with colluding Eves lies in the constraint (9b) involving non-convex matrix inversion and determinant functions which are difficult to deal with. In the non-colluding Eves counterpart in [12], the constraint in (9b) is degenerated to $\frac{|\mathbf{h}_{\mathrm{E},k}^H\mathbf{b}_{\mathrm{I}}|^2}{\sum_{i=1}^d |\mathbf{h}_{\mathrm{E},k}^H\mathbf{b}_{\mathrm{E},i}|^2+\sigma_{\mathrm{E}}^2} \leq \gamma - 1$ for the kth ER, which could be eventually expressed as a convex constraint with matrix traces for fixed γ as in [12]. Unfortunately, for the colluding Eves scenario considered in this paper, this is not possible which makes (7) much more challenging compared to that with individual Eves' SINR constraints considered in [12]. Moreover, it can be analytically shown that bounding the per-Eve SINRs may not be enough to bound the colluding-Eves SINR. To circumvent this difficulty, we use the following lemma [17].

Lemma 1: The following implication holds

$$\log\det\left(\mathbf{I} + \left(\mathbf{I}+\mathbf{G}^H\mathbf{Q}_{\mathrm{E}}\mathbf{G}\right)^{-1}\mathbf{G}^H\mathbf{Q}_{\mathrm{I}}\mathbf{G}\right) \leq \log\gamma \quad \text{(10a)}$$

$$\Longrightarrow (\gamma - 1)(\mathbf{I}+\mathbf{G}^H\mathbf{Q}_{\mathrm{E}}\mathbf{G}) - \mathbf{G}^H\mathbf{Q}_{\mathrm{I}}\mathbf{G} \succeq \mathbf{0} \quad \text{(10b)}$$

for any $\mathbf{G} \in \mathbb{C}^{N_{\mathrm{T}}\times K}$, $\mathbf{Q}_{\mathrm{I}} \succeq \mathbf{0}$, and $\mathbf{Q}_{\mathrm{E}} \succeq \mathbf{0}$. Also, (10a) and (10b) are equivalent if $\mathrm{rank}\big(\mathbf{Q}_{\mathrm{I}}\big) \leq 1$.

Proof: See Appendix A. □

Lemma 1 indicates that (10b) is a relaxation of (10a) that yields a larger feasible solution set. Therefore, replacing (9b) by (10b) will achieve a higher secrecy rate for the SRM problem (9). In addition, such a replacement makes no difference in terms of secrecy rate if $\text{rank}(\mathbf{Q}_\text{I}) \leq 1$. The merit of *Lemma 1* is that it helps us get rid of the troublesome matrix inversion and determinant and (10b) is a convex linear matrix inequality (LMI) for any given β. Let us now replace (9b) by (10b) and reformulate the relaxed SRM problem as

$$\max_{\mathbf{Q}_\text{I},\mathbf{Q}_\text{E},\gamma} \quad \log\left(\frac{1+\mathbf{h}_\text{I}^H(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})\mathbf{h}_\text{I}}{\gamma\,(1+\mathbf{h}_\text{I}^H\mathbf{Q}_\text{E}\mathbf{h}_\text{I})}\right) \tag{11a}$$

$$\text{s.t.} \quad (\gamma-1)\left(\mathbf{I}_K+\mathbf{H}_\text{E}^H\mathbf{Q}_\text{E}\mathbf{H}_\text{E}\right) \succeq \mathbf{H}_\text{E}^H\mathbf{Q}_\text{I}\mathbf{H}_\text{E} \tag{11b}$$

$$\text{tr}\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E}) \leq P_\text{T} \tag{11c}$$

$$\text{tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})) \leq p_l, \quad \forall l, \tag{11d}$$

$$\mathbf{h}_{\text{E},k}^H\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})\,\mathbf{h}_{\text{E},k} \geq \eta_k, \quad \forall k, \tag{11e}$$

$$\mathbf{Q}_\text{I} \succeq \mathbf{0}, \mathbf{Q}_\text{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \gamma \geq 1. \tag{11f}$$

Interestingly, based on *Lemma 1*, we can readily describe the following theorem.

Theorem 1: For any given feasible γ, the optimal solution of the relaxed problem (11) is also optimal for the problem (9), if $\text{rank}(\mathbf{Q}_\text{I}) \leq 1$, yielding the same optimal value.

Proof: See Appendix B. □

The intuition given by *Theorem 1* is the solution equivalence between the problems (11) and (9) when $\text{rank}(\mathbf{Q}_\text{I}) \leq 1$, Unfortunately, it is not straightforward to show that the relaxed problem (11) is indeed tight, i.e., $\text{rank}(\mathbf{Q}_\text{I}) \leq 1$. Alternatively, we reformulate (11) as a two-step maximization problem:

$$\max_{\gamma}\begin{cases}\max\limits_{\mathbf{Q}_\text{I},\mathbf{Q}_\text{E}} & \log\left(\frac{1+\mathbf{h}_\text{I}^H(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})\mathbf{h}_\text{I}}{\gamma\,(1+\mathbf{h}_\text{I}^H\mathbf{Q}_\text{E}\mathbf{h}_\text{I})}\right) \quad (12\text{a})\\ \text{s.t.} & (\gamma-1)\left(\mathbf{I}_K+\mathbf{H}_\text{E}^H\mathbf{Q}_\text{E}\mathbf{H}_\text{E}\right) \succeq \mathbf{H}_\text{E}^H\mathbf{Q}_\text{I}\mathbf{H}_\text{E} \quad (12\text{b})\\ & \text{tr}\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E}) \leq P_\text{T} \quad (12\text{c})\\ & \text{tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})) \leq p_l, \quad \forall l, \quad (12\text{d})\\ & \mathbf{h}_{\text{E},k}^H\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})\,\mathbf{h}_{\text{E},k} \geq \eta_k, \quad \forall k, \quad (12\text{e})\\ & \mathbf{Q}_\text{I} \succeq \mathbf{0}, \mathbf{Q}_\text{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l. \quad (12\text{f})\end{cases}$$

Then we proceed to find the optimal solution in two steps: First, we obtain an optimal $\mathbf{Q}_\text{I}$ for the inner maximization problem of (12) for any given feasible γ such that rank $(\mathbf{Q}_\text{I}) \leq 1$; second, we perform a one-dimensional line search over γ that leads to the optimal solution of the problem (11). The detailed procedure of finding such an optimal γ will be explained later.

Let us now consider the following secrecy rate constrained (SRC) power minimization problem which is a variation of the inner maximization problem in (12). The reason for considering the SRC problem is threefold. First, the SRC formulation itself is interesting from the practical viewpoint. Second, the SRC problem is comparatively easier to analyze than the inner maximization problem in (12). Third, we can establish a solution correspondence between the SRC and the inner SRM maximization problems.

A. SRC Power Minimization Problem

Given the optimal objective value C_I^* of the inner maximization problem in (12), the corresponding SRC power minimization problem can be formulated as

$$\min_{\mathbf{Q}_\text{I},\mathbf{Q}_\text{E}} \quad \text{tr}\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E}) \tag{13a}$$

$$\text{s.t.} \quad \log\left(\frac{1+\mathbf{h}_\text{I}^H(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})\mathbf{h}_\text{I}}{\gamma\,(1+\mathbf{h}_\text{I}^H\mathbf{Q}_\text{E}\mathbf{h}_\text{I})}\right) \geq C_\text{I}^* \tag{13b}$$

$$(\gamma-1)\left(\mathbf{I}_K+\mathbf{H}_\text{E}^H\mathbf{Q}_\text{E}\mathbf{H}_\text{E}\right) \succeq \mathbf{H}_\text{E}^H\mathbf{Q}_\text{I}\mathbf{H}_\text{E} \tag{13c}$$

$$\text{tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})) \leq p_l, \quad \forall l, \tag{13d}$$

$$\mathbf{h}_{\text{E},k}^H\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})\,\mathbf{h}_{\text{E},k} \geq \eta_k, \quad \forall k, \tag{13e}$$

$$\mathbf{Q}_\text{I} \succeq \mathbf{0}, \mathbf{Q}_\text{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l. \tag{13f}$$

The SRC problem (13) aims to minimize the total transmit power subject to a minimum required secrecy rate C_I^*. All the other constraints of the inner maximization problem in (12) remain unchanged.

Theorem 2: Any optimal solution to the SRC power minimization problem (13) is also optimal to the inner maximization problem in (12).

Proof: See Appendix C. □

Theorem 2 proves the solution equivalence between the SRC problem (13) and the inner maximization problem in (12). Now the remaining task is to show that the SRC problem (13) has an optimal solution with $\text{rank}(\mathbf{Q}_\text{I}) \leq 1$. If the optimal solution of (13) can be proven to be of rank-one, then we will be able to infer immediately that the relaxation (11) is in fact tight.

Denoting $\beta \triangleq 1-\gamma\,2^{C_\text{I}^*}$, the problem (13) can be recast into a semidefinite program (SDP) as

$$\min_{\mathbf{Q}_\text{I},\mathbf{Q}_\text{E}} \quad \text{tr}\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E}) \tag{14a}$$

$$\text{s.t.} \quad \text{tr}\left(\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H(\mathbf{Q}_\text{I}+\beta\mathbf{Q}_\text{E})\right)+\beta \geq 0 \tag{14b}$$

$$(\gamma-1)\left(\mathbf{I}_K+\mathbf{H}_\text{E}^H\mathbf{Q}_\text{E}\mathbf{H}_\text{E}\right) \succeq \mathbf{H}_\text{E}^H\mathbf{Q}_\text{I}\mathbf{H}_\text{E} \tag{14c}$$

$$\text{tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})) \leq p_l, \quad \forall l, \tag{14d}$$

$$\text{tr}\left(\mathbf{H}_{\text{E},k}\,(\mathbf{Q}_\text{I}+\mathbf{Q}_\text{E})\right) \geq \eta_k, \quad \forall k, \tag{14e}$$

$$\mathbf{Q}_\text{I} \succeq \mathbf{0}, \mathbf{Q}_\text{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \tag{14f}$$

where $\mathbf{H}_{\text{E},k} \triangleq \mathbf{h}_{\text{E},k}\mathbf{h}_{\text{E},k}^H$. The problem (14) is a convex SDP problem whose globally optimal solution can be efficiently found by the existing disciplined convex programming toolboxes such as SeDuMi [18] and CVX [19]. Interestingly, the following theorem states that for a practically representative class of problem instances, the SDR (14) always yields a rank-one transmit beamforming solution.

Theorem 3: Suppose that the SDP problem (14) is feasible for $C_\text{I}^* > 0$. There always exists an optimal solution $(\mathbf{Q}_\text{I}, \mathbf{Q}_\text{E})$ such that $\text{rank}(\mathbf{Q}_\text{I}) = 1$.

Proof: See Appendix D. □

The rank-one optimality of the SRC problem (14) is obtained by examining the Karush-Kuhn-Tucker (KKT) conditions of the problem, where we prove that any SDR optimal solution has to have a rank-one $\mathbf{Q}_\text{I}$. Now, combining the results in *Theorems 1–3*, we can conclude that a rank-one optimal $\mathbf{Q}_\text{I}$ for the problem (11) (and hence, for (7)) can always be constructed algorithmically.

B. SDP-Based Solution to the SRM Problem

In the previous sub-section, we have proved that we can obtain a rank-one solution for the problem (11) through solving the SRC problem (14). In this sub-section, we concentrate on developing an SDP-based solution for the problem (11). Since we assume that the optimal secrecy rate satisfies $C_s^* \geq 0$, we have from the objective function of the problem (8) that

$$\gamma \leq 1 + \frac{\mathbf{h}_I^H \mathbf{Q}_I \mathbf{h}_I}{\sigma_I^2 + \mathbf{h}_I^H \mathbf{Q}_E \mathbf{h}_I} \leq 1 + \mathbf{h}_I^H \mathbf{Q}_I \mathbf{h}_I \leq 1 + \mathrm{tr}(\mathbf{Q}_I)\|\mathbf{h}_I\|^2 \leq 1 + P_T\|\mathbf{h}_I\|^2. \quad (15)$$

Note that in the last inequality, we have used the sum power constraint in (11c) to conclude that $\mathrm{tr}(\mathbf{Q}_I) \leq P_T$. Now we can rewrite the two-step SRM problem (12) as

$$\gamma^* \triangleq \max_{\gamma} \quad \mathcal{G}(\gamma) \quad (16a)$$

$$\text{s.t.} \quad 0 \leq \gamma \leq 1 + P_T\|\mathbf{h}_I\|^2, \quad (16b)$$

where $\log \gamma^* = C_s^*$, and

$$\mathcal{G}(\gamma) \triangleq \max_{\mathbf{Q}_I, \mathbf{Q}_E} \frac{1 + \mathbf{h}_I^H(\mathbf{Q}_I + \mathbf{Q}_E)\mathbf{h}_I}{\gamma(1 + \mathbf{h}_I^H \mathbf{Q}_E \mathbf{h}_I)} \quad (17a)$$

$$\text{s.t.} \quad (\gamma - 1)\big(\mathbf{I}_K + \mathbf{H}_E^H \mathbf{Q}_E \mathbf{H}_E\big) \succeq \mathbf{H}_E^H \mathbf{Q}_I \mathbf{H}_E \quad (17b)$$

$$\mathrm{tr}(\mathbf{Q}_I + \mathbf{Q}_E) \leq P_T \quad (17c)$$

$$\mathrm{tr}(\boldsymbol{\Upsilon}_l(\mathbf{Q}_I + \mathbf{Q}_E)) \leq p_l, \quad \forall l, \quad (17d)$$

$$\mathrm{tr}\big(\mathbf{h}_{E,k}\mathbf{h}_{E,k}^H(\mathbf{Q}_I + \mathbf{Q}_E)\big) \geq \eta_k, \quad \forall k, \quad (17e)$$

$$\mathbf{Q}_I \succeq \mathbf{0}, \mathbf{Q}_E \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l. \quad (17f)$$

The SDR problem (17) is a quasi-convex problem, due to the linear fractional structure of its objective function (17a). A common practice to solving this kind of quasi-convex problems is to apply a linear searching (e.g. bisection search) technique, where the globally optimal solution is successively searched by solving a sequence of SDPs [20]. Here we develop a simpler but efficient alternative to solving (17) by linear searching. By using the Charnes-Cooper transformation [21], we can equivalently express (17) as

$$\mathcal{G}(\gamma) \triangleq \max_{\breve{\mathbf{Q}}_I, \breve{\mathbf{Q}}_E, \nu} \quad \nu + \mathbf{h}_I^H(\breve{\mathbf{Q}}_I + \breve{\mathbf{Q}}_E)\mathbf{h}_I \quad (18a)$$

$$\text{s.t.} \quad \gamma\left(\nu + \mathbf{h}_I^H \breve{\mathbf{Q}}_E \mathbf{h}_I\right) = 1 \quad (18b)$$

$$(\gamma - 1)\left(\nu \mathbf{I}_K + \mathbf{H}_E^H \breve{\mathbf{Q}}_E \mathbf{H}_E\right) \succeq \mathbf{H}_E^H \breve{\mathbf{Q}}_I \mathbf{H}_E \quad (18c)$$

$$\mathrm{tr}\left(\breve{\mathbf{Q}}_I + \breve{\mathbf{Q}}_E\right) \leq \nu P_T \quad (18d)$$

$$\mathrm{tr}\left(\boldsymbol{\Upsilon}_l\left(\breve{\mathbf{Q}}_I + \breve{\mathbf{Q}}_E\right)\right) \leq \nu p_l, \quad \forall l, \quad (18e)$$

$$\mathrm{tr}\left(\mathbf{h}_{E,k}\mathbf{h}_{E,k}^H\left(\breve{\mathbf{Q}}_I + \breve{\mathbf{Q}}_E\right)\right) \geq \nu \eta_k, \quad \forall k, \quad (18f)$$

$$\breve{\mathbf{Q}}_I \succeq \mathbf{0}, \breve{\mathbf{Q}}_E \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \nu > 0, \quad (18g)$$

where we have introduced a change of variables such that $\breve{\mathbf{Q}}_I = \nu \mathbf{Q}_I, \breve{\mathbf{Q}}_E = \nu \mathbf{Q}_E$ and (18b) is additionally introduced to fix the denominator of the objective function in (17a) without loss of generality. The merit of (18) is that the objective function (18a) is now linear in place of a fractional one (17a). The proof of the solution equivalence of the problems (17) and (18) can be easily obtained by following the argument in [22]. The SDP (18) can be efficiently solved by existing convex programming toolboxes [18], [19].

Now that the optimal γ lies in the interval $[0, (1 + P_T\|\mathbf{h}_I\|^2)]$, the single-variable optimization problem (16) can be efficiently solved by conducting a one-dimensional linear search over γ, and choosing the one that leads to the maximum $\mathcal{G}(\gamma)$ as an optimal solution of (16). There are many one-dimensional search algorithms for solving optimization problems like (16). In practice, we use either uniform sampling or the golden section search algorithm [23] to obtain an acceptable optimal solution [17].

Once the problem (16) has been solved, one can easily recover $\mathbf{Q}_I$ and $\mathbf{Q}_E$ through the transformation $\breve{\mathbf{Q}}_I = \nu \mathbf{Q}_I, \breve{\mathbf{Q}}_E = \nu \mathbf{Q}_E$. If $\mathrm{rank}\big(\mathbf{Q}_I\big) \leq 1$, then $(\mathbf{Q}_I, \mathbf{Q}_E)$ is the optimal solution to the problem (11). Otherwise, we can obtain a rank-one solution through solving the problem (14).

Here, let us describe a rank-one solution construction procedure for the problem (18) based on the Lagrangian duality. Since (14) is convex and satisfies the Slater's condition, its duality gap is zero. The Lagrangian of the problem (18) can be expressed as

$$\mathcal{L} \triangleq \mathrm{tr}\left(\left(\mathbf{h}_I\mathbf{h}_I^H - \mathbf{H}_E\boldsymbol{\Phi}\mathbf{H}_E^H + \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{E,k} - \sum_{l=1}^{L}\lambda_{g,l}\boldsymbol{\Upsilon}_l - \lambda_p\mathbf{I}_{N_T}\right)\breve{\mathbf{Q}}_I\right) + \mathrm{tr}\left(\left((1-\gamma\lambda_e)\mathbf{h}_I\mathbf{h}_I^H + (\gamma - 1)\mathbf{H}_E\boldsymbol{\Phi}\mathbf{H}_E^H + \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{E,k} - \sum_{l=1}^{L}\lambda_{g,l}\boldsymbol{\Upsilon}_l - \lambda_p\mathbf{I}_{N_T}\right)\breve{\mathbf{Q}}_E\right) + \lambda_e + \left(1 - \lambda_e\gamma + (\gamma - 1)\,\mathrm{tr}(\boldsymbol{\Phi}) + \lambda_p P_T + \sum_{l=1}^{L}\lambda_{g,l}p_l - \sum_{k=1}^{K}\lambda_{e,k}\eta_k\right)\nu, \quad (19)$$

where $\lambda_e \geq 0, \boldsymbol{\Phi} \succeq \mathbf{0}, \lambda_p \geq 0, \lambda_{g,l} \geq 0, \forall l, \lambda_{e,k} \geq 0, \forall k$, are the dual variables associated with the constraints (18b)–(18f), respectively. Let us define

$$\mathbf{B} \triangleq -\mathbf{H}_E\boldsymbol{\Phi}\mathbf{H}_E^H + \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{E,k} - \sum_{l=1}^{L}\lambda_{g,l}\boldsymbol{\Upsilon}_l - \lambda_p\mathbf{I}_{N_T}, \quad (20)$$

$$\mathbf{D} \triangleq (1-\gamma\lambda_e)\mathbf{h}_I\mathbf{h}_I^H + (\gamma - 1)\mathbf{H}_E\boldsymbol{\Phi}\mathbf{H}_E^H + \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{E,k} - \sum_{l=1}^{L}\lambda_{g,l}\boldsymbol{\Upsilon}_l - \lambda_p\mathbf{I}_{N_T}, \quad (21)$$

$$\mathbf{A} \triangleq \mathbf{B} + \mathbf{h}_I\mathbf{h}_I^H, \quad (22)$$

and denote $r_B \triangleq \mathrm{rank}(\mathbf{B})$ as the rank of $\mathbf{B}$. Further, define $\boldsymbol{\Psi} \triangleq \left[\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \ldots, \boldsymbol{\psi}_{N_T - r_B}\right]$ as the orthogonal basis for the null space of $\mathbf{B}$, where $\boldsymbol{\Psi} = \mathbf{0}$ if $r_B = N_T$, and a unit-norm vector $\boldsymbol{\upsilon}$ such that $\boldsymbol{\upsilon}^H\boldsymbol{\Psi} = \mathbf{0}$. Then using the KKT conditions, *Proposition 1* constructs a rank-one solution to (18), and hence (11).

Proposition 1:

a) Let $(\breve{\mathbf{Q}}_{\rm I}^*, \breve{\mathbf{Q}}_{\rm E}^*)$ denote the optimal solution to the problem (18). Then $\breve{\mathbf{Q}}_{\rm I}^*$ can be expressed in the form as $\breve{\mathbf{Q}}_{\rm I} = \sum_{i=1}^{N_{\rm T}-r_{\rm B}} a_i \boldsymbol{\psi}_i \boldsymbol{\psi}_i^H + b\boldsymbol{v}\boldsymbol{v}^H$, where $a_i \geq 0, \forall i, b > 0$.

b) If there exists at least an i such that $a_i > 0$, then the following solution

$$\breve{\mathbf{Q}}_{\rm I}^* = b\boldsymbol{v}\boldsymbol{v}^H \tag{23}$$

$$\breve{\mathbf{Q}}_{\rm E}^* = \mathbf{Q}_{\rm E} + \sum_{i=1}^{N_{\rm T}-r_{\rm B}} a_i \boldsymbol{\psi}_i \boldsymbol{\psi}_i^H \tag{24}$$

to problem (18) is also optimal with rank $(\mathbf{Q}_{\rm I}) = 1$ [12].

Proof: See Appendix E. □

Note that to deliver the required wireless energy to the ER and at the same time interfere with it from eavesdropping the IR's message, energy beam (i.e., AN) is in general needed according to *Proposition 1*. On the other hand, for SRM problems without involving the energy harvesting constraints (see [14], [16], [17]), AN beams may not be necessary for optimal secrecy in certain scenarios, e.g. when $K = 1$.

IV. Robust Beamforming for Secrecy SWIPT

In the previous section, it is assumed that the instantaneous CSI of all the receivers is available at the transmitter. However, in practical wireless communication systems, perfect CSI may not always be available and an important issue is how to robustify a secure transmit design in the presence of imperfect CSI. As a consequence, our next exertion is to extend the optimization algorithms developed in the last section to the case where the transmitter does not have perfect CSI knowledge about the receivers' channels. The interest here is in active Eves cases, where the BS has knowledge of Eves' CSI to a certain extent so that $\mathbf{Q}_{\rm E}$ can be designed spatially non-isotropically to interfere Eves selectively.

Robust design strategies can be broadly categorized into statistical and worst-case based designs. Depending on the scenarios, statistics-based designs may adopt criteria such as Bayesian-based, outage probability-based and correlation-based schemes. Due to space limitations, we consider correlation-based and worst-case based designs in this paper. In particular, we consider the case of correlation based CSI of all receivers in a second-order statistics sense and the worst-case based robust formulation under norm-bounded Eves' CSI uncertainties assuming perfect CSI of the IR.

A. Correlation Based Robust Design

For the correlation-based CSI case, the channel covariances characterize the uncertainty to Eves' channels in a second-order statistics sense. Suppose that the BS-to-IR channel $\mathbf{h}_{\rm I}$ is random with mean $\bar{\mathbf{h}}_{\rm I}$ and covariance $\sigma_{\mathbf{h}_{\rm I}}^2 \mathbf{I}_{N_{\rm T}}$. Let the correlation matrix of the BS-to-IR channel be

$$\mathbf{R}_{\rm I} = {\rm E}\left\{\mathbf{h}_{\rm I}\mathbf{h}_{\rm I}^H\right\} = \bar{\mathbf{h}}_{\rm I}\bar{\mathbf{h}}_{\rm I}^H + \sigma_{\mathbf{h}_{\rm I}}^2 \mathbf{I}_{N_{\rm T}} \tag{25}$$

and that with the kth ER be

$$\mathbf{R}_{{\rm E},k} = {\rm E}\left\{\mathbf{h}_{{\rm E},k}\mathbf{h}_{{\rm E},k}^H\right\} = \bar{\mathbf{h}}_{{\rm E},k}\bar{\mathbf{h}}_{{\rm E},k}^H + \sigma_{\mathbf{h}_{{\rm E},k}}^2 \mathbf{I}_{N_{\rm T}}, \quad \forall k, \tag{26}$$

where $\bar{\mathbf{h}}_{{\rm E},k}$ and $\sigma_{\mathbf{h}_{{\rm E},k}}^2 \mathbf{I}_{N_{\rm T}}$ being the mean and covariance of $\mathbf{h}_{{\rm E},k}$, respectively. It will be assumed in the sequel that $\mathbf{R}_{\rm I}$ and $\mathbf{R}_{{\rm E},k}, \forall k$, are known perfectly to the BS. In the worst-case event of $\mathbf{R}_{{\rm E},k} = \sigma_{\mathbf{h}_{{\rm E},k}}^2 \mathbf{I}_{N_{\rm T}}$, the physical meaning is that we have no information available at the BS about the channel direction of that ER. However, it is not straightforward to solve the problem (9) for the correlation based CSI case with colluding Eves due to the constraint (9b) with the complicated determinant function.

For ease of exposition, assume for the moment that the energy harvesting Eves do not collude together. Thus, the SINR at the IR is given by

$$\text{SINR}_{\rm I} = \frac{{\rm tr}\,(\mathbf{Q}_{\rm I}\mathbf{R}_{\rm I})}{{\rm tr}\,(\mathbf{Q}_{\rm E}\mathbf{R}_{\rm I}) + \sigma_{\rm I}^2}, \tag{27}$$

and that at the kth Eve is given by

$$\text{SINR}_k = \frac{{\rm tr}\left(\mathbf{Q}_{\rm I}\mathbf{R}_{{\rm E},k}\right)}{{\rm tr}\left(\mathbf{Q}_{\rm E}\mathbf{R}_{{\rm E},k}\right) + \sigma_{\rm E}^2}, \ \text{for } k = 1, \ldots, K. \tag{28}$$

Accordingly, the corresponding SRM problem can be formulated as

$$\max_{\mathbf{Q}_{\rm I},\mathbf{Q}_{\rm E}} \ \min_k \ \log\left(1 + \frac{{\rm tr}\,(\mathbf{Q}_{\rm I}\mathbf{R}_{\rm I})}{{\rm tr}\,(\mathbf{Q}_{\rm E}\mathbf{R}_{\rm I}) + \sigma_{\rm I}^2}\right) - \log\left(1 + \frac{{\rm tr}\left(\mathbf{Q}_{\rm I}\mathbf{R}_{{\rm E},k}\right)}{{\rm tr}\left(\mathbf{Q}_{\rm E}\mathbf{R}_{{\rm E},k}\right) + \sigma_{\rm E}^2}\right) \tag{29a}$$

$$\text{s.t.} \quad {\rm tr}\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E}) \leq P_{\rm T} \tag{29b}$$

$${\rm tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})) \leq p_l, \ \text{for } l = 1, \ldots, L, \tag{29c}$$

$${\rm tr}\left(\mathbf{R}_{{\rm E},k}\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})\right) \geq \eta_k, \quad \forall k, \tag{29d}$$

$$\mathbf{Q}_{\rm I} \succeq \mathbf{0}, \mathbf{Q}_{\rm E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l. \tag{29e}$$

Note that due to the monotonicity of the log function, any optimal solution $(\mathbf{Q}_{\rm I}^*, \mathbf{Q}_{\rm E}^*)$ of (29) is also optimal for the following problem:

$$\max_{\mathbf{Q}_{\rm I},\mathbf{Q}_{\rm E}} \ \frac{{\rm tr}\,(\mathbf{Q}_{\rm I}\mathbf{R}_{\rm I})}{{\rm tr}\,(\mathbf{Q}_{\rm E}\mathbf{R}_{\rm I}) + \sigma_{\rm I}^2} \tag{30a}$$

$$\text{s.t.} \quad \frac{{\rm tr}\left(\mathbf{Q}_{\rm I}\mathbf{R}_{{\rm E},k}\right)}{{\rm tr}\left(\mathbf{Q}_{\rm E}\mathbf{R}_{{\rm E},k}\right) + \sigma_{\rm E}^2} \leq \gamma, \quad \text{for } k = 1, \ldots, K, \tag{30b}$$

$${\rm tr}\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E}) \leq P_{\rm T} \tag{30c}$$

$${\rm tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})) \leq p_l, \quad \text{for } l = 1, \ldots, L, \tag{30d}$$

$${\rm tr}\left(\mathbf{R}_{{\rm E},k}\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})\right) \geq \eta_k, \quad \forall k, \tag{30e}$$

$$\mathbf{Q}_{\rm I} \succeq \mathbf{0}, \mathbf{Q}_{\rm E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \tag{30f}$$

with $\gamma = \max_k \frac{{\rm tr}\left(\mathbf{Q}_{\rm I}^*\mathbf{R}_{{\rm E},k}\right)}{{\rm tr}\left(\mathbf{Q}_{\rm E}^*\mathbf{R}_{{\rm E},k}\right) + \sigma_{\rm E}^2} > 0$. The above formulation offers the best possible SINR for the IR, while keeping the per-Eve SINR below a known threshold and the harvested power above a meaningful level under certain power constraints. The problem (30) always has a feasible solution in general. However, in some *not-that-practical* instances; e.g., the threshold specifications are too demanding, it is possible that the best SINR of the IR found by (30) is lower than γ resulting in negative secrecy rate. Under such circumstances, the system operator should consider relaxing the constraints to yield a reasonable SINR at the IR.

Let $\mathcal{G}_1(\gamma)$ denote the optimal value of (30) for a given $\gamma > 0$. Then it can be shown similar to the perfect CSI case that the optimal value of (7) is the same as that of the problem:

$$\max_{0 \le \gamma \le 1+P_\mathrm{T}\mathrm{tr}(\mathbf{R}_\mathrm{I})} \log\left(\frac{1+\mathcal{G}_1(\gamma)}{1+\gamma}\right). \tag{31}$$

The SDR problem (30) is a quasi-convex problem whose globally optimal solution can be determined by applying linear searching methods (e.g. bisection search). Here we develop a simpler alternative to solving (30) by linear searching. By using the Charnes-Cooper transformation [21], we can equivalently express (30) as an SDP

$$\max_{\breve{\mathbf{Q}}_\mathrm{I},\breve{\mathbf{Q}}_\mathrm{E},\alpha} \ \mathrm{tr}\left(\breve{\mathbf{Q}}_\mathrm{I}\mathbf{R}_\mathrm{I}\right) \tag{32a}$$

$$\text{s.t.} \quad \mathrm{tr}\left(\breve{\mathbf{Q}}_\mathrm{E}\mathbf{R}_\mathrm{I}\right) + \alpha\sigma_\mathrm{I}^2 = 1 \tag{32b}$$

$$\mathrm{tr}\left(\breve{\mathbf{Q}}_\mathrm{I}\mathbf{R}_{\mathrm{E},k}\right) \le \gamma\left(\mathrm{tr}\left(\breve{\mathbf{Q}}_\mathrm{E}\mathbf{R}_{\mathrm{E},k}\right) + \alpha\sigma_\mathrm{E}^2\right), \quad \text{for } k = 1,\ldots,K, \tag{32c}$$

$$\mathrm{tr}\left(\breve{\mathbf{Q}}_\mathrm{I} + \breve{\mathbf{Q}}_\mathrm{E}\right) \le \alpha P_\mathrm{T} \tag{32d}$$

$$\mathrm{tr}\left(\boldsymbol{\Upsilon}_l\left(\breve{\mathbf{Q}}_\mathrm{I} + \breve{\mathbf{Q}}_\mathrm{E}\right)\right) \le \alpha p_l, \quad \text{for } l = 1,\ldots,L, \tag{32e}$$

$$\mathrm{tr}\left(\mathbf{R}_{\mathrm{E},k}\left(\breve{\mathbf{Q}}_\mathrm{I} + \breve{\mathbf{Q}}_\mathrm{E}\right)\right) \ge \alpha\eta_k, \quad \forall k, \tag{32f}$$

$$\breve{\mathbf{Q}}_\mathrm{I} \succeq \mathbf{0}, \breve{\mathbf{Q}}_\mathrm{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \alpha > 0, \tag{32g}$$

where we have defined $\alpha = \frac{1}{\mathrm{tr}(\mathbf{Q}_\mathrm{E}\mathbf{R}_\mathrm{I})+\sigma_\mathrm{I}^2}$, $\breve{\mathbf{Q}}_\mathrm{I} = \alpha\mathbf{Q}_\mathrm{I}$, and $\breve{\mathbf{Q}}_\mathrm{E} = \alpha\mathbf{Q}_\mathrm{E}$. Now we can obtain the optimal solution to the problem (30) by solving the problem (32) such that $\mathcal{G}_1(\gamma)$ becomes optimal. Similar to the perfect CSI case, we describe the following proposition using KKT conditions.

Proposition 2: A rank-one optimal $\breve{\mathbf{Q}}_\mathrm{I}$ can always be constructed for the problem (32).

Proof: The proof follows similar arguments as that for the proof of *Proposition 1* in the non-robust counterpart and is thus omitted her for brevity. □

Let us now turn to the more challenging case of colluding Eves performing joint maximum SINR receive beamforming. The maximum receive SINR achieved by colluding Eves can be defined as

$$\mathrm{SINR}_\mathrm{ce} = \max_{\mathbf{w}\neq\mathbf{0}} \frac{\mathrm{E}\left\{|\ \mathbf{w}^H\mathbf{H}_\mathrm{E}^H\mathbf{b}_\mathrm{I}s_\mathrm{I}\ |^2\right\}}{\mathrm{E}\left\{|\ \mathbf{w}^H\mathbf{H}_\mathrm{E}^H\mathbf{b}_\mathrm{E} + \mathbf{n}_\mathrm{E}\ |^2\right\}}, \tag{33}$$

where $\mathbf{w}$ denotes the joint receive beamforming vector of the colluding Eves. Let us now denote the linear matrix function $\mathcal{M}(\cdot)$ whose (k,l)th entry is given by

$$[\mathcal{M}(\mathbf{Q})]_{(k,l)} = \mathrm{tr}\left(\mathbf{Q}\mathbf{R}_{\mathrm{E},(l,k)}\right) \tag{34}$$

with $\mathbf{R}_{\mathrm{E},(l,k)} = \mathrm{E}\{\mathbf{h}_{\mathrm{E},l}\mathbf{h}_{\mathrm{E},k}^H\}$. Thus, the SINR in (33) can be expressed as

$$\mathrm{SINR}_\mathrm{ce} = \max_{\mathbf{w}\neq\mathbf{0}} \frac{\mathbf{w}^H\mathcal{M}(\breve{\mathbf{Q}}_\mathrm{I})\mathbf{w}}{\mathbf{w}^H\left(\mathcal{M}(\breve{\mathbf{Q}}_\mathrm{E}) + \boldsymbol{\Sigma}^2\right)\mathbf{w}}, \tag{35}$$

where $\boldsymbol{\Sigma} = \sigma_\mathrm{E}\mathbf{I}_K$. The difficulty for the colluding Eves' case is that bounding the per-Eve SINRs may not be sufficient to bound the colluding-Eve SINR. However, there is a simple remedy that can bound $\mathrm{SINR}_\mathrm{ce}$ below a known threshold by bounding the per-Eve SINRs as described in the following proposition [16].

Proposition 3: Adding the following constraint

$$\boldsymbol{\Sigma}^{-1}\mathcal{M}(\breve{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1} \succeq \frac{1}{K}\mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\breve{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1}\right)\mathbf{I}_K \tag{36}$$

to the problem (32) satisfies the colluding Eves SINR constraint for any feasible solution, i.e.,

$$\mathrm{SINR}_\mathrm{ce} \le K\tau. \tag{37}$$

Proof: See Appendix F. □

As a result, the maximum allowable colluding-Eve SINR is bounded below $K\tau$ indirectly according to *Proposition 3*. Note that the SDR of the modified design is identical to that of the original problem (30) with the inclusion of the constraint (36). It is obvious that the resultant SDR is a convex SDP, since (36) is convex, which can be solved by existing solvers [18], [19], and it can be proved that the SDR rank-one optimality described in *Proposition 2* still holds for the amended SDR. Thus, any optimal solution to problem (30) including the new constraint (36) is optimal for the colluding Eves scenario.

B. Worst-Case Based Robust Design

In this sub-section, we develop robust algorithm for the SRM problem with energy harvesting Eves in the case of erroneous CSI which uses the concept of worst-case design. We assume that the BS has incomplete knowledge of Eves' channels while the IR's channel is perfectly known.

We consider a deterministic model for the imperfect CSI of the Eves' channels. To model imperfect CSI, we assume that the actual channels $\mathbf{h}_{\mathrm{E},k}$, for $k = 1,\ldots,K$, lie in the neighbourhood of the estimated channels $\hat{\mathbf{h}}_{\mathrm{E},k}$, for $k = 1,\ldots,K$, available at the transmitter. Thus, the actual channels are modeled as

$$\mathbf{h}_{\mathrm{E},k} = \hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_k, \ \text{for } k = 1,\ldots,K, \tag{38}$$

where $\boldsymbol{\delta}_k$, for $k = 1,\ldots,K$, represent the channel uncertainties. These uncertainties are assumed to be deterministic unknowns with bounds on their magnitudes such that

$$\|\boldsymbol{\delta}_k\|_2 = \|\mathbf{h}_{\mathrm{E},k} - \hat{\mathbf{h}}_{\mathrm{E},k}\|_2 \le \varepsilon, \ \text{for some } \varepsilon \ge 0. \tag{39}$$

Accordingly, the colluding Eves' channel becomes $\mathbf{H}_\mathrm{E} = \hat{\mathbf{H}}_\mathrm{E} + \boldsymbol{\Delta}$ with $\|\boldsymbol{\Delta}\|_\mathrm{F} = \|\mathbf{H}_\mathrm{E} - \hat{\mathbf{H}}_\mathrm{E}\|_\mathrm{F} \le \varepsilon$.

The value of ε depends on the accuracy of channel estimation. Higher training signal-to-noise ratio (SNR) provides better CSI estimates, and smaller ε. Thus, the robust formulation of (7) becomes

$$\max_{\mathbf{Q}_\mathrm{I},\mathbf{Q}_\mathrm{E}} \ \min_{\|\boldsymbol{\Delta}_k\|_\mathrm{F}\le\varepsilon_k} \left\{C_\mathrm{I}(\mathbf{Q}_\mathrm{I},\mathbf{Q}_\mathrm{E}) - \hat{C}_\mathrm{E}(\mathbf{Q}_\mathrm{I},\mathbf{Q}_\mathrm{E})\right\} \tag{40a}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{Q}_\mathrm{I} + \mathbf{Q}_\mathrm{E}) \le P_\mathrm{T} \tag{40b}$$

$$\mathrm{tr}(\boldsymbol{\Upsilon}_l(\mathbf{Q}_\mathrm{I} + \mathbf{Q}_\mathrm{E})) \le p_l, \text{for } l = 1,\ldots,L, \tag{40c}$$

$$\min_{\|\boldsymbol{\delta}_k\|_2\le\varepsilon_k} \left(|(\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_k)^H\mathbf{b}_\mathrm{I}|^2 + |(\hat{\mathbf{h}}_{\mathrm{E},k} + \boldsymbol{\delta}_k)^H\mathbf{b}_\mathrm{E}|^2\right) \ge \eta_k, \quad \forall k, \tag{40d}$$

$$\mathbf{Q}_\mathrm{I} \succeq \mathbf{0}, \mathbf{Q}_\mathrm{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \tag{40e}$$

where $\hat{C}_{\rm E}(\mathbf{Q}_{\rm I},\mathbf{Q}_{\rm E}) \triangleq \max_{\mathbf{H}_{\rm E}\in\mathcal{H}} \log\det\big(\mathbf{I}_{N_{\rm T}} + \big(\mathbf{I}_{N_{\rm T}} + \mathbf{H}_{\rm E}^H\mathbf{Q}_{\rm E} \times \mathbf{H}_{\rm E}\big)^{-1}\mathbf{H}_{\rm E}^H\mathbf{Q}_{\rm I}\mathbf{H}_{\rm E}\big)$ is the colluding Eves' largest possible MI among the set of all possible CSIs $\mathcal{H}$ with $\mathcal{H} \triangleq \left\{\mathbf{H}_{\rm E}|\mathbf{H}_{\rm E} = \hat{\mathbf{H}}_{\rm E} + \mathbf{\Delta}, \|\mathbf{\Delta}\|_{\rm F} \le \varepsilon\right\}$. Note that statistical information about the channel error vectors is not required in this approach, and the minimal knowledge of the upper-bound of channel error vector norms is sufficient. Like the perfect CSI case, we can rewrite problem (40) as

$$\max_{\mathbf{Q}_{\rm I},\mathbf{Q}_{\rm E},\gamma} \; C_{\rm I}(\mathbf{Q}_{\rm I},\mathbf{Q}_{\rm E}) - \log\gamma \tag{41a}$$

$$\text{s.t.} \quad \max_{\mathbf{H}_{\rm E}\in\mathcal{H}} \log\det\left(\mathbf{I}_K + \left(\mathbf{I}_K + \mathbf{H}_{\rm E}^H\mathbf{Q}_{\rm E}\mathbf{H}_{\rm E}\right)^{-1}\mathbf{H}_{\rm E}^H\mathbf{Q}_{\rm I} \times \mathbf{H}_{\rm E}\right) \le \log\gamma \tag{41b}$$

$$\text{tr}\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E}) \le P_{\rm T} \tag{41c}$$

$$\text{tr}\,(\mathbf{\Upsilon}_l(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})) \le p_l, \forall l, \tag{41d}$$

$$\min_{\|\boldsymbol{\delta}_k\|_2\le\varepsilon_k} \left(|(\hat{\mathbf{h}}_{{\rm E},k} + \boldsymbol{\delta}_k)^H\mathbf{b}_{\rm I}|^2 + |(\hat{\mathbf{h}}_{{\rm E},k} + \boldsymbol{\delta}_k)^H \times\mathbf{b}_{\rm E}|^2\right) \ge \eta_k, \quad \forall k, \tag{41e}$$

$$\mathbf{Q}_{\rm I} \succeq \mathbf{0}, \mathbf{Q}_{\rm E} \succeq \mathbf{0}, \mathbf{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \gamma \ge 1. \tag{41f}$$

Note that in the constraints (41b) and (41e), there are infinite number of inequalities with respect to (w.r.t.) $\mathbf{h}_{{\rm E},k}$ to satisfy, which makes the worst-case based design particularly challenging. By using *Lemma 1*, we have the following relaxation for (41b)

$$\log\det\left(\mathbf{I} + (\mathbf{I} + \mathbf{G}^H\mathbf{Q}_{\rm E}\mathbf{G})^{-1}\mathbf{G}^H\mathbf{Q}_{\rm I}\mathbf{G}\right) \le \log\gamma, \quad \forall\mathbf{H}_{\rm E} \in \mathcal{H}, \tag{42a}$$

$$\Longrightarrow (\gamma - 1)(\mathbf{I} + \mathbf{G}^H\mathbf{Q}_{\rm E}\mathbf{G}) - \mathbf{G}^H\mathbf{Q}_{\rm I}\mathbf{G} \succeq \mathbf{0}, \ \forall\mathbf{H}_{\rm E} \in \mathcal{H}. \tag{42b}$$

Recall that the relaxation (42b) is in fact tight if $\text{rank}(\mathbf{Q}_{\rm I}) \le 1$. Our goal is to reformulate problem (41) as a tractable convex problem and then prove that the relaxation in (41) is indeed tight for the worst-case based CSI case by proving the rank-one structure of $\mathbf{Q}_{\rm I}$. To make the robust problem (41) more tractable to analyze and solve, we first transform the robust constraints in (41b) and (41e) into LMIs using advanced matrix inequality results in the optimization literature.

Towards this end, we apply the so-called $\mathcal{S}$-procedure [20] to transform the constraint (41e) into an LMI. For completeness, the $\mathcal{S}$-procedure is presented in *Lemma 2* below.

*Lemma 2 (**$\mathcal{S}$-Procedure**):* Let $f_i(\mathbf{x}), i = 1, 2$, be defined as

$$f_i(\mathbf{x}) = \mathbf{x}^H\mathbf{A}_i\mathbf{x} + 2\text{Re}\left\{\mathbf{b}_i^H\mathbf{x}\right\} + c_i$$

where $\mathbf{A}_i \in \mathcal{C}^{n\times n}, \mathbf{b}_i \in \mathcal{C}^n, c_i \in \mathcal{R}$. The implication $f_1(\mathbf{x}) \le 0 \Rightarrow f_2(\mathbf{x}) \le 0$ holds if and only if there exists $\mu \ge 0$ such that

$$\mu\begin{bmatrix}\mathbf{A}_1 & \mathbf{b}_1\\ \mathbf{b}_1^H & c_1\end{bmatrix} - \begin{bmatrix}\mathbf{A}_2 & \mathbf{b}_2\\ \mathbf{b}_2^H & c_2\end{bmatrix} \succeq \mathbf{0}$$

provided that there exists a point $\hat{\mathbf{x}}$ such that $f_1(\hat{\mathbf{x}}) < 0$.

To apply the $\mathcal{S}$-procedure, we re-express (41e) as

$$\boldsymbol{\delta}_k^H\boldsymbol{\delta}_k \le \varepsilon^2 \Rightarrow \boldsymbol{\delta}_k^H(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})\boldsymbol{\delta}_k + 2\text{Re}\{\hat{\mathbf{h}}_{{\rm E},k}^H(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})\boldsymbol{\delta}_k\} + \hat{\mathbf{h}}_{{\rm E},k}^H(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})\hat{\mathbf{h}}_{{\rm E},k} - \eta_k \ge 0. \tag{43}$$

According to *Lemma 2*, (43) holds if and only if there exists $\mu_k \ge 0, \forall k$, such that (44) (shown at the bottom of the page) holds.

In order to transform the colluding Eves SINR constraint (41b) into a tractable convex LMI, we use the following lemma [24].

Lemma 3: Let $f(\mathbf{X}) = \mathbf{X}^H\mathbf{A}\mathbf{X} + \mathbf{X}^H\mathbf{B} + \mathbf{B}^H\mathbf{X} + \mathbf{C}$, and $\mathbf{D} \succeq \mathbf{0}$. The following equivalence holds:

$$f(\mathbf{X}) \succeq \mathbf{0}, \forall\mathbf{X} \in \left\{\mathbf{X}|\text{tr}(\mathbf{D}\mathbf{X}\mathbf{X}^H) \le 1\right\} \Longleftrightarrow \begin{bmatrix}\mathbf{C} & \mathbf{B}^H\\ \mathbf{B} & \mathbf{A}\end{bmatrix} - \mu\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{D}\end{bmatrix} \succeq \mathbf{0} \tag{45}$$

for some $\mu \ge 0$.

Using *Lemma 3*, we describe the following proposition.

Proposition 4: The following implication holds

$$\log\det\left(\mathbf{I} + (\mathbf{I} + \mathbf{G}^H\mathbf{Q}_{\rm E}\mathbf{G})^{-1}\mathbf{G}^H\mathbf{Q}_{\rm I}\mathbf{G}\right) \le \log\beta, \quad \forall\mathbf{H}_{\rm E} \in \mathcal{H}, \tag{46a}$$

$$\Longrightarrow \mathbf{\Gamma}_{\rm ce}(\gamma, \mathbf{Q}_{\rm I}, \mathbf{Q}_{\rm E}, \mu_{\rm ce}) \succeq \mathbf{0}, \ \forall\mathbf{H}_{\rm E} \in \mathcal{H}, \tag{46b}$$

for some $\mu_{\rm ce} \ge 0$ where $\mathbf{\Gamma}_{\rm ce}(\gamma, \mathbf{Q}_{\rm I}, \mathbf{Q}_{\rm E}, \mu_{\rm ce})$ is defined in (47) at the bottom of the page. Moreover, (46a) and (46b) are equivalent if $\text{rank}(\mathbf{Q}_{\rm I}) \le 1$.

Proof: See Appendix G. □

Note that according to *Proposition 4*, (46b) is a single convex LMI (in place of infinitely many) for given γ. Now we replace the constraints (41b) and (41e) by (46b) and (44), respectively, to obtain a relaxed problem given as follows:

$$\max_{\mathbf{Q}_{\rm I},\mathbf{Q}_{\rm E},\gamma,\mu_{\rm ce},\mu_k} \; C_{\rm I}(\mathbf{Q}_{\rm I},\mathbf{Q}_{\rm E}) - \log\gamma \tag{48a}$$

$$\text{s.t.} \quad \mathbf{\Gamma}_{\rm ce}(\gamma, \mathbf{Q}_{\rm I}, \mathbf{Q}_{\rm E}, \mu_{\rm ce}) \succeq \mathbf{0} \tag{48b}$$

$$\text{tr}\,(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E}) \le P_{\rm T} \tag{48c}$$

$$\text{tr}\,(\mathbf{\Upsilon}_l(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})) \le p_l, \quad \forall l, \tag{48d}$$

$$\mathbf{\Theta}(\mathbf{Q}_{\rm I}, \mathbf{Q}_{\rm E}, \mu_k) \succeq \mathbf{0}, \quad \forall k, \tag{48e}$$

$$\mathbf{Q}_{\rm I} \succeq \mathbf{0}, \mathbf{Q}_{\rm E} \succeq \mathbf{0}, \mathbf{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \tag{48f}$$

$$\gamma \ge 1, \mu_{\rm ce} \ge 0, \mu_k \ge 0, \quad \forall k. \tag{48g}$$

$$\mathbf{\Theta}(\mathbf{Q}_{\rm I}, \mathbf{Q}_{\rm E}, \mu_k) \triangleq \begin{bmatrix}\mu_k\mathbf{I}_{N_s} + \mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E} & (\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})\hat{\mathbf{h}}_{{\rm E},k}\\ \hat{\mathbf{h}}_{{\rm E},k}^H(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E}) & \hat{\mathbf{h}}_{{\rm E},k}^H(\mathbf{Q}_{\rm I} + \mathbf{Q}_{\rm E})\hat{\mathbf{h}}_{{\rm E},k} - \eta_k - \mu_k\varepsilon^2\end{bmatrix} \succeq \mathbf{0} \tag{44}$$

$$\mathbf{\Gamma}_{\rm ce}(\gamma, \mathbf{Q}_{\rm I}, \mathbf{Q}_{\rm E}, \mu_{\rm ce}) \triangleq \begin{bmatrix}(\gamma - 1 - \mu_{\rm ce})\mathbf{I}_K + \hat{\mathbf{H}}_{\rm E}^H((\gamma - 1)\mathbf{Q}_{\rm E} - \mathbf{Q}_{\rm I})\hat{\mathbf{H}}_{\rm E} & \hat{\mathbf{H}}_{\rm E}^H((\gamma - 1)\mathbf{Q}_{\rm E} - \mathbf{Q}_{\rm I})\\ ((\gamma - 1)\mathbf{Q}_{\rm E} - \mathbf{Q}_{\rm I})\hat{\mathbf{H}}_{\rm E} & (\gamma - 1)\mathbf{Q}_{\rm E} - \mathbf{Q}_{\rm I} + \frac{\mu_{\rm ce}}{\varepsilon^2}\mathbf{I}_{N_{\rm T}}\end{bmatrix} \tag{47}$$

Similar to the problem (12) in the perfect CSI case, the optimal $(\mathbf{Q}_\text{I}, \mathbf{Q}_\text{E})$ for the problem (48) can be obtained for given feasible γ through solving the following robust SRC (RSRC) problem:

$$\min_{\mathbf{Q}_\text{I},\mathbf{Q}_\text{E}} \quad \text{tr}\,(\mathbf{Q}_\text{I} + \mathbf{Q}_\text{E}) \tag{49a}$$

$$\text{s.t.} \quad \text{tr}\left(\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H(\mathbf{Q}_\text{I} + \beta\mathbf{Q}_\text{E})\right) + \beta \geq 0 \tag{49b}$$

$$\boldsymbol{\Gamma}_\text{ce}\,(\gamma, \mathbf{Q}_\text{I}, \mathbf{Q}_\text{E}, \mu_\text{ce}) \succeq \mathbf{0} \tag{49c}$$

$$\text{tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_\text{I} + \mathbf{Q}_\text{E})) \leq p_l, \quad \forall l, \tag{49d}$$

$$\boldsymbol{\Theta}\,(\mathbf{Q}_\text{I}, \mathbf{Q}_\text{E}, \mu_k) \succeq \mathbf{0}, \quad \forall k, \tag{49e}$$

$$\mathbf{Q}_\text{I} \succeq \mathbf{0}, \mathbf{Q}_\text{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l, \tag{49f}$$

$$\gamma \geq 1, \mu_\text{ce} \geq 0, \mu_k \geq 0, \quad \forall k, \tag{49g}$$

where $\beta \triangleq 1 - \gamma 2^{C_\text{I}^*}$ and C_I^* is the optimal objective value of (48) for given feasible γ. As for the perfect CSI case, we describe the following theorem regarding the tightness of the relaxation in (48).

Theorem 4: Suppose that the problem (49) is feasible for $C_\text{I}^* > 0$. There exists an optimal solution of the problem (49) for which $\text{rank}(\mathbf{Q}_\text{I}) \leq 1$ and the solution is also optimal for the problem (41) achieving the optimal secrecy rate.

Proof: The proof follows similar arguments as that for *Theorem 2* and *Theorem 3* for the perfect-CSI counterpart and is thus omitted for brevity. □

Theorem 4 can be seen as a generalization of the perfect CSI counterpart. Note that (48) can also be solved by using the SDP-based line search method described in Section III-B for the perfect CSI case.

V. Simulation Results

In this section, we study the performance of the proposed algorithms in MISO secrecy SWIPT systems through numerical simulations. It was considered that the transmitter (or the BS) which is equipped with N_T antennas sends a secret message to a single-antenna IR and there are K single-antenna ERs. The secret message intended for the IR is at risk of being decoded at the ERs. Both perfect and imperfect CSI cases were evaluated. For simplicity, it was assumed that $\eta_k = \eta$, $\forall k$, and $\sigma_\text{I}^2 = \sigma_\text{E}^2 = 1$. Also, in all simulations, we imposed the sum power constraint only. We simulated a flat Rayleigh fading environment where the channel vectors have entries with zero mean and variance $1/N_\text{T}$. All the channel vectors are assumed to be random with correlation matrices given by

$$\mathbf{R}_\text{I} = \rho\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H + (1-\rho)\frac{\mathbf{I}_{N_\text{T}}}{N_\text{T}}, \tag{50}$$

$$\mathbf{R}_{\text{E},k} = \rho\mathbf{h}_{\text{E},k}\mathbf{h}_{\text{E},k}^H + (1-\rho)\frac{\mathbf{I}_{N_\text{T}}}{N_\text{T}}, \text{ for } k = 1, \ldots, K, \tag{51}$$

where $\rho \in [0, 1]$ defines the level of channel uncertainty ($\rho = 1$ corresponds to the case of perfect CSI while $\rho = 0$ means that there is no knowledge of the channel directions at all). The channel vectors $\mathbf{h}_\text{I}$ and $\mathbf{h}_{\text{E},k}, \forall k$, are isotropically distributed on a unit sphere. In the simulations, we chose $\rho = 0.7$, $N_\text{T} = 5$, and $K = 3$ unless explicitly mentioned. For the worst-case based design, the error vectors are uniformly and randomly generated in a sphere centered at zero with the radius $\varepsilon = 0.2$. All simulation results were averaged over 500 independent channel realizations unless specified otherwise.

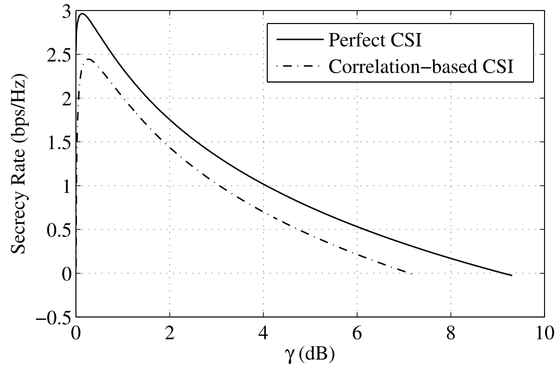

Fig. 2. Unimodal objective function of (12) versus γ for the case with $N_\text{T} = 5$, $K = 3$, and $\eta = -10$ (dB).

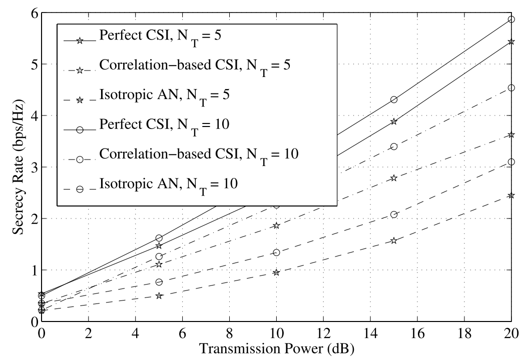

Fig. 3. Secrecy rate versus sum power P_T with $\eta = -5$ (dB) and $K = 3$.

In Fig. 2, we illustrate the two-stage optimization procedure of the problem (12) for both perfect and imperfect CSI cases. We plot the objective function versus γ for $P_\text{T} = 10$ (dB) and $\eta = -10$ (dB). We see that in this particular setup there is only one maximum point of the objective function which justifies its quasi-convex (unimodal) nature and the global optimality of the proposed algorithms. Golden section search technique is applied to find the optimal objective value wherever required.

In the next example, we compare the secrecy rate performance of the proposed active Eves algorithms with that of the existing algorithms considering passive Eves [14]. For the passive Eves scenario, we consider isotropic beamforming to obtain the energy covariance matrix $\mathbf{Q}_\text{I}$ such that the beamforming vector lies in the null space of $\mathbf{h}_\text{I}$ [14]. In particular, we choose

$$\mathbf{Q}_\text{I} = \frac{P_\text{T}}{2\|\mathbf{h}_\text{I}\|^2}\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H, \text{ and } \mathbf{Q}_\text{E} = \frac{P_\text{T}}{2}\frac{\mathbf{H}^\perp}{\|\mathbf{H}^\perp\|_\text{F}^2}, \tag{52}$$

where $\mathbf{H}^\perp = \mathbf{I}_{N_\text{T}} - \frac{\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H}{\|\mathbf{h}_\text{I}\|^2}$ denotes the orthogonal complement projector of $\mathbf{h}_\text{I}$. Note that the isotropic AN design uses half of the transmit power to transmit the confidential information and uses the remaining half to transmit the AN on the null space of $\mathbf{h}_\text{I}$ isotropically. Results in Fig. 3 illustrate the fact that the secrecy rate increases with increasing budget

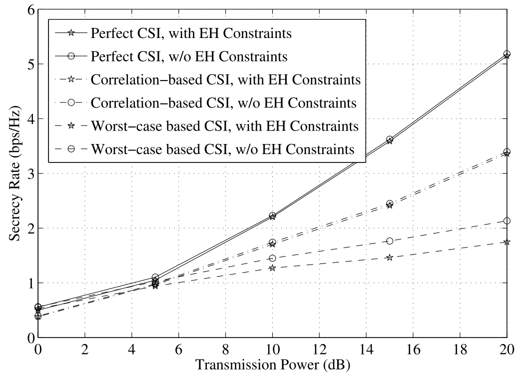

Fig. 4. Secrecy rate versus sum power P_{T} with $N_{\mathrm{T}} = 5$, $K = 3$, and $\eta = -10$ (dB).

of transmit power. Essencially, if the CSI of the ERs is available at the transmitter to some extent (e.g., channel statistics), better secrecy rates can be achieved through the proposed algorithms. The reason is that with some knowledge of ERs' CSI, their reception can be blocked much more effectively by generating spatially selective AN, rather than keeping AN isotropic. The more extent of ERs' CSI available, the higher the achievable secrecy rate. Also, when the number of transmit antennas N_{T} increases from 5 to 10, the secrecy rate performance of all the algorithms increases which is due to the increased spatial diversity provided by the additional transmit antennas.

Results in Fig. 4 show the achievable secrecy rates for the proposed non-robust and robust algorithms with ($\eta = -10$ (dB)) and without energy harvesting constraints. As we can see, secrecy rate increases with the increase in the transmit power for all the algorithms. Fig. 4 also reveals that in order to satisfy the energy harvesting requirements of the ERs, one may need to sacrifice the secrecy rate performance. However, the performance loss due to incorporating the energy harvesting ("EH" in the figure) constraints with threshold $\eta = -10$ (dB) is negligible for the perfect and correlation-based CSI cases whereas the loss in the worst-case based design is higher. The results also reveal that we can obtain secrecy close to that without considering the energy harvesting constraints while guaranteeing the minimum harvested energey at a meaningful level. Also, the correlation-based design yields better secrecy rate than the worst-case based design.

In Fig. 5, we analyze the rate-energy regions of the proposed algorithms. Note that in general the rate-energy region for the scenario is a $(K + 1)$-dimensional region. However, with all ERs having identical energy constraints, i.e., $\eta_k = \eta$, $\forall k$, the rate-energy region reduces to a two-dimensional region. Thus by solving (7) with $\eta_k = \eta$, $\forall k$, and changing the values of η, we can characterize the boundary of the resulting rate-energy region. Fig. 5 compares the rate-energy regions achieved by the proposed algorithms in a random channel realization for $P_{\mathrm{T}} = 10$ (dB), $N_{\mathrm{T}} = 5$ and $K = 3$. As a benchmark, we also plot the rate-energy region of a system without secrecy constraints. Since the benchmark scheme does not need to ensure information secrecy, it achieves the largest region. Obviously, the perfect CSI case achieves the best rate-energy trade-offs among the proposed secrecy algorithms. Note that

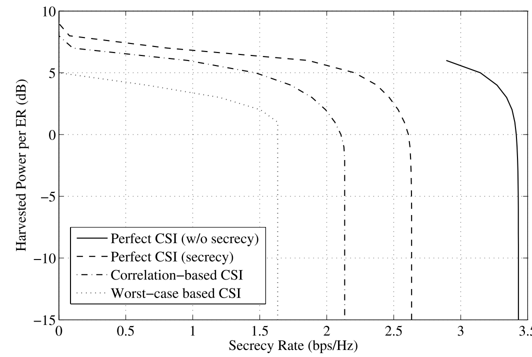

Fig. 5. Achievable rate-energy region by the proposed algorithms with $P_{\mathrm{T}} = 10$ (dB), $N_{\mathrm{T}} = 5$ and $K = 3$.

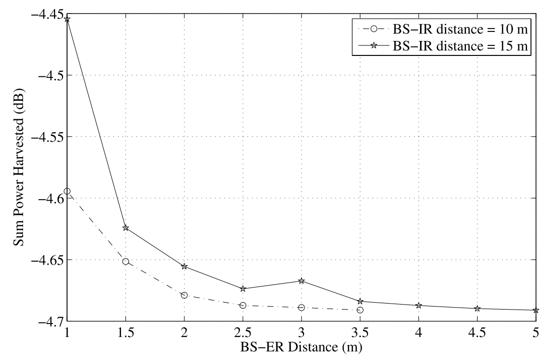

Fig. 6. Effect of distance on harvested energy with $P_{\mathrm{T}} = 20$ (dB), $N_{\mathrm{T}} = 5$ and $K = 3$.

the minimum harvested power by each ER is 2.5 (dB) at a secrecy rate of 2.4 (bps/Hz) if the full CSI of the Eves is available at the BS.

Finally, we show the effect of IR's and ERs' distance from the BS on harvestable energy in Fig. 6. The TGn path loss model [25] for indoor communications is adopted with a path loss exponent of 3.5. It can be seen from Fig. 6 that the sum harvested power keeps decreasing as the average BS-to-ERs distance increases. However, if the BS-to-IR distance increases, the sum harvested power increases up to a certain point. This is supported by the constraint (7d) since the BS needs to transmit information signals with higher power if the IR is located at a larger distance and the ERs harvest energy from both information and energy signals. Note that after an average BS-to-ERs distance of 3.5 meters, the harvested power becomes almost linear.

VI. Conclusions

In this paper, we investigated the SRM problem in MISO systems for SWIPT and proposed transmit and energy beamforming algorithms for both perfect and imperfect CSI cases, utilizing SDR techniques. We also proposed an optimal transmit power minimization algorithm subject to secrecy rate and energy harvesting constraints. In particular, we considered the case where the energy harvesting eavesdroppers collude together in order to maximize their joint interception and illustrated that a rank-one transmit covariance matrix can always be constructed algorithmically.

Appendix

A. Proof of Lemma 1

Let us first describe the following lemma that we will use in this proof.

Lemma 4 ([22]): For any positive semidefinite matrix $\mathbf{A}$, it holds true that

$$\det(\mathbf{I}+\mathbf{A}) \geq 1+\mathrm{tr}(\mathbf{A}) \tag{53}$$

and that the equality in (53) holds if and only if $\mathrm{rank}(\mathbf{A}) \leq 1$.

Now denoting $\mathbf{U} \triangleq (\mathbf{I}+\mathbf{G}^H\mathbf{Q}_\mathrm{E}\mathbf{G})^{-1}$ and using the basic matrix result $\det(\mathbf{I}+\mathbf{AB}) = \det(\mathbf{I}+\mathbf{BA})$, we rewrite the inequality in (10a) as

$$\log\det\left(\mathbf{I}+(\mathbf{I}+\mathbf{G}^H\mathbf{Q}_\mathrm{E}\mathbf{G})^{-1}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\right) \leq \log\gamma \tag{54a}$$

$$\iff \det\left(\mathbf{I}+\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}}\right) \leq \gamma. \tag{54b}$$

Applying *Lemma 4*, we have

$$\det\left(\mathbf{I}+\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}}\right) \geq 1+\mathrm{tr}\left(\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}}\right). \tag{55}$$

Combining (54a) and (55), we get

$$\log\det\left(\mathbf{I}+(\mathbf{I}+\mathbf{G}^H\mathbf{Q}_\mathrm{E}\mathbf{G})^{-1}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\right) \leq \log\gamma \tag{56a}$$

$$\implies \mathrm{tr}\left(\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}}\right) \leq \gamma-1. \tag{56b}$$

Since $\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}} \succeq \mathbf{0}$, and $\mathrm{tr}(\mathbf{A}) \geq \lambda_{\max}(\mathbf{A})$ holds for any $\mathbf{A} \succeq \mathbf{0}$, we have

$$\log\det\left(\mathbf{I}+(\mathbf{I}+\mathbf{G}^H\mathbf{Q}_\mathrm{E}\mathbf{G})^{-1}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\right) \leq \log\gamma \tag{57a}$$

$$\implies \lambda_{\max}\left(\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}}\right) \leq \gamma-1, \tag{57b}$$

$$\iff \mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}} \preceq (\gamma-1)\mathbf{I}, \tag{57c}$$

$$\iff (\gamma-1)\mathbf{U} \succeq \mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}. \tag{57d}$$

The first part of *Lemma 1* is thus proved.

By *Lemma 4*, the equality in (55) holds if $\mathrm{rank}(\mathbf{Q}_\mathrm{I}) \leq 1$. This is because $\mathrm{rank}(\mathbf{Q}_\mathrm{I}) \leq 1$ implies $\mathrm{rank}(\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}}) \leq 1$. Since $\mathrm{rank}(\mathbf{A}) \leq 1$ for $\mathbf{A} \succeq \mathbf{0}$ indicates that $\mathbf{A} = \mathbf{a}\mathbf{a}^H$ for some vector $\mathbf{a}$, we can conclude that $\mathbf{U}^{-\frac{1}{2}}\mathbf{G}^H\mathbf{Q}_\mathrm{I}\mathbf{G}\mathbf{U}^{-\frac{1}{2}} = \mathbf{q}\mathbf{q}^H$ for some vector $\mathbf{q}$. Therefore, (57b) can be reexpressed as $\lambda_{\max}(\mathbf{q}\mathbf{q}^H) \leq \gamma-1$, which is equivalent to the right-hand side of (56b) by noting that $\lambda_{\max}\left(\mathbf{q}\mathbf{q}^H\right) = \mathrm{tr}\left(\mathbf{q}\mathbf{q}^H\right)$. □

B. Proof of Theorem 1

Let $f_\gamma(\mathbf{Q}_\mathrm{I}, \mathbf{Q}_\mathrm{E})$ denote the objective function of (9) (or (11)) for a particular γ, and $(\bar{\mathbf{Q}}_\mathrm{I}, \bar{\mathbf{Q}}_\mathrm{E})$ and $(\tilde{\mathbf{Q}}_\mathrm{I}, \tilde{\mathbf{Q}}_\mathrm{E})$ be the corresponding optimal solutions of problems (9) and (11), respectively. We assume that $\mathrm{rank}(\mathbf{Q}_\mathrm{I}) \leq 1$. Since according to *Lemma 1*, (11) is a relaxation of (9), we have

$$f_\gamma(\tilde{\mathbf{Q}}_\mathrm{I}, \tilde{\mathbf{Q}}_\mathrm{E}) \geq f_\gamma(\bar{\mathbf{Q}}_\mathrm{I}, \bar{\mathbf{Q}}_\mathrm{E}). \tag{58}$$

On the other hand, the condition $\mathrm{rank}(\mathbf{Q}_\mathrm{I}) \leq 1$ implies that $(\tilde{\mathbf{Q}}_\mathrm{I}, \tilde{\mathbf{Q}}_\mathrm{E})$ is also a feasible solution of the problem (9), owing to the equivalence condition in *Lemma 1*. As a result, we have

$$f_\gamma(\tilde{\mathbf{Q}}_\mathrm{I}, \tilde{\mathbf{Q}}_\mathrm{E}) \leq f_\gamma(\bar{\mathbf{Q}}_\mathrm{I}, \bar{\mathbf{Q}}_\mathrm{E}). \tag{59}$$

Combining the above two inequalities, we conclude that $f_\gamma(\tilde{\mathbf{Q}}_\mathrm{I}, \tilde{\mathbf{Q}}_\mathrm{E}) = f_\gamma(\bar{\mathbf{Q}}_\mathrm{I}, \bar{\mathbf{Q}}_\mathrm{E})$, i.e., $(\tilde{\mathbf{Q}}_\mathrm{I}, \tilde{\mathbf{Q}}_\mathrm{E})$ is also optimal for (9).

Thus we have established a solution correspondence between (9) and (11) for any given feasible γ, which includes the optimal γ as well. Subsequently, the results in *Theorem 1* are obtained. □

C. Proof of Theorem 2

For ease of exposition, we rewrite the inner maximization problem of (12) here as

$$\max_{\mathbf{Q}_\mathrm{I},\mathbf{Q}_\mathrm{E}} \quad \log\left(\frac{1+\mathbf{h}_\mathrm{I}^H(\mathbf{Q}_\mathrm{I}+\mathbf{Q}_\mathrm{E})\mathbf{h}_\mathrm{I}}{\gamma\,(1+\mathbf{h}_\mathrm{I}^H\mathbf{Q}_\mathrm{E}\mathbf{h}_\mathrm{I})}\right) \tag{60a}$$

$$\text{s.t.} \quad (\gamma-1)\left(\mathbf{I}_{N_\mathrm{T}}+\mathbf{H}_\mathrm{E}^H\mathbf{Q}_\mathrm{E}\mathbf{H}_\mathrm{E}\right) \succeq \mathbf{H}_\mathrm{E}^H\mathbf{Q}_\mathrm{I}\mathbf{H}_\mathrm{E} \tag{60b}$$

$$\mathrm{tr}\,(\mathbf{Q}_\mathrm{I}+\mathbf{Q}_\mathrm{E}) \leq P_\mathrm{T} \tag{60c}$$

$$\mathrm{tr}\,(\boldsymbol{\Upsilon}_l\,(\mathbf{Q}_\mathrm{I}+\mathbf{Q}_\mathrm{E})) \leq p_l, \quad \forall l, \tag{60d}$$

$$\mathbf{h}_{\mathrm{E},k}^H\,(\mathbf{Q}_\mathrm{I}+\mathbf{Q}_\mathrm{E})\,\mathbf{h}_{\mathrm{E},k} \geq \eta_k, \quad \forall k, \tag{60e}$$

$$\mathbf{Q}_\mathrm{I} \succeq \mathbf{0}, \mathbf{Q}_\mathrm{E} \succeq \mathbf{0}, \boldsymbol{\Upsilon}_l \succeq \mathbf{0}, \quad \forall l. \tag{60f}$$

Let $(\bar{\mathbf{Q}}_\mathrm{I}, \bar{\mathbf{Q}}_\mathrm{E})$ and $(\hat{\mathbf{Q}}_\mathrm{I}, \hat{\mathbf{Q}}_\mathrm{E})$ denote the optimal solutions of the problems of (60) and (13), respectively, and C_I^* is the optimal value of the objective in (60). One can easily verify that $(\bar{\mathbf{Q}}_\mathrm{I}, \bar{\mathbf{Q}}_\mathrm{E})$ is a feasible solution of (13). Hence, it follows that

$$\mathrm{tr}(\hat{\mathbf{Q}}_\mathrm{I}+\hat{\mathbf{Q}}_\mathrm{E}) \leq \mathrm{tr}(\bar{\mathbf{Q}}_\mathrm{I}+\bar{\mathbf{Q}}_\mathrm{E}) \leq P_\mathrm{T}, \tag{61}$$

where the first inequality is due to the fact that $(\hat{\mathbf{Q}}_\mathrm{I}, \hat{\mathbf{Q}}_\mathrm{E})$ minimizes $\mathrm{tr}(\hat{\mathbf{Q}}_\mathrm{I}+\hat{\mathbf{Q}}_\mathrm{E})$ in (13) for given C_I^*; and the second inequality follows from the feasibility of $(\bar{\mathbf{Q}}_\mathrm{I}, \bar{\mathbf{Q}}_\mathrm{E})$ w.r.t. (60c). The inequality (61), together with (13d) and (13e), imply that $(\hat{\mathbf{Q}}_\mathrm{I}, \hat{\mathbf{Q}}_\mathrm{E})$ is a feasible solution of (60), i.e.,

$$\log\left(\frac{1+\mathbf{h}_\mathrm{I}^H(\hat{\mathbf{Q}}_\mathrm{I}+\hat{\mathbf{Q}}_\mathrm{E})\mathbf{h}_\mathrm{I}}{\gamma\,(1+\mathbf{h}_\mathrm{I}^H\hat{\mathbf{Q}}_\mathrm{E}\mathbf{h}_\mathrm{I})}\right) \leq C_\mathrm{I}^*. \tag{62}$$

Combining (62) with (13b) yields

$$\log\left(\frac{1+\mathbf{h}_\mathrm{I}^H(\hat{\mathbf{Q}}_\mathrm{I}+\hat{\mathbf{Q}}_\mathrm{E})\mathbf{h}_\mathrm{I}}{\gamma\,(1+\mathbf{h}_\mathrm{I}^H\hat{\mathbf{Q}}_\mathrm{E}\mathbf{h}_\mathrm{I})}\right) = C_\mathrm{I}^*. \tag{63}$$

A solution correspondence between (13) and the inner maximization problem of (12) is thus established. □

D. Proof of Theorem 3

Since (14) is convex and satisfies the Slater's condition, its duality gap is zero. The Lagrangian of the problem (14) can be expressed as

$$\mathcal{L}_{\text{src}} \triangleq \text{tr}\Bigg(\Bigg(\mathbf{I}_{N_\text{T}} - \lambda_\text{r}\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H + \mathbf{H}_\text{E}\mathbf{\Phi}\mathbf{H}_\text{E}^H - \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{\text{E},k} + \sum_{l=1}^{L}\lambda_{a,l}\mathbf{\Upsilon}_l\Bigg)\mathbf{Q}_\text{I}\Bigg) + \text{tr}\Bigg(\Bigg(\mathbf{I}_{N_\text{T}} - \lambda_\text{r}\beta\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H + (1-\gamma)\,\mathbf{H}_\text{E} \times\mathbf{\Phi}\mathbf{H}_\text{E}^H - \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{\text{E},k} + \sum_{l=1}^{L}\lambda_{a,l}\mathbf{\Upsilon}_l\Bigg)\mathbf{Q}_\text{E}\Bigg) - \beta\lambda_\text{r} + (1-\gamma)\,\text{tr}(\mathbf{\Phi}) - \sum_{l=1}^{L}\lambda_{a,l}p_l + \sum_{k=1}^{K}\lambda_{e,k}\eta_k, \quad (64)$$

where $\lambda_\text{r} \geq 0, \mathbf{\Phi} \succeq \mathbf{0}, \lambda_{a,l} \geq 0, \forall l, \lambda_{e,k} \geq 0, \forall k$, are the dual variables associated with the constraints (14b)–(14e), respectively. We find it useful to define

$$\mathbf{B} \triangleq \mathbf{I}_{N_\text{T}} + \mathbf{H}_\text{E}\mathbf{\Phi}\mathbf{H}_\text{E}^H - \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{\text{E},k} + \lambda_{a,l}\mathbf{\Upsilon}_l, \quad (65)$$

$$\mathbf{D} \triangleq \mathbf{I}_{N_\text{T}} - \lambda_r\beta\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H + (1-\gamma)\,\mathbf{H}_\text{E}\mathbf{\Phi}\mathbf{H}_\text{E}^H - \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{\text{E},k} + \lambda_{a,l}\mathbf{\Upsilon}_l, \quad (66)$$

$$\mathbf{A} \triangleq \mathbf{B} - \lambda_\text{r}\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H, \quad (67)$$

and let $r_\text{B} \triangleq \text{rank}\,(\mathbf{B})$ denote the rank of $\mathbf{B}$.

The KKT conditions of the problem (14) that are relevant to the proof can be defined as

$$\mathbf{A}\mathbf{Q}_\text{I} = \mathbf{0}, \text{ and } \mathbf{D}\mathbf{Q}_\text{E} = \mathbf{0}. \quad (68)$$

Lemma 5: Let $\mathbf{X}$ and $\mathbf{Y}$ be two matrices of the same dimension. Then it holds true that $\text{rank}\,(\mathbf{X}-\mathbf{Y}) \geq \text{rank}(\mathbf{X}) - \text{rank}(\mathbf{Y})$.

Proof: We know that $\text{rank}(\mathbf{X}) + \text{rank}(\mathbf{Y}) \geq \text{rank}\,(\mathbf{X}+\mathbf{Y})$. Therefore, we have $\text{rank}\,(\mathbf{X}+\mathbf{Y}) + \text{rank}(-\mathbf{Y}) \geq \text{rank}(\mathbf{X})$. Since $\text{rank}(\mathbf{Y}) = \text{rank}(-\mathbf{Y})$, we can conclude that $\text{rank}\,(\mathbf{X}+\mathbf{Y}) \geq \text{rank}(\mathbf{X}) - \text{rank}(\mathbf{Y})$. Since $\text{rank}\,(\mathbf{X}-\mathbf{Y}) = \text{rank}\,(\mathbf{X}+\mathbf{Y})$, *Lemma 5* is thus proved. □

Using *Lemma 5*, we have from (67) that

$$\text{rank}(\mathbf{A}) \geq r_\text{B} - 1. \quad (69)$$

If $\mathbf{B}$ in (65) is positive-definite, $r_\text{B} = N_\text{T}$ and $\text{rank}(\mathbf{A}) \geq N_\text{T} - 1$. However, if $\text{rank}(\mathbf{A}) = N_\text{T}$, i.e., $\mathbf{A}$ is of full-rank, then it follows from (68) that $\mathbf{Q}_\text{I} = \mathbf{0}$, which cannot be an optimal solution to (14). Therefore, we have $\text{rank}(\mathbf{A}) = N_\text{T} - 1$. According to (68), we have $\text{rank}(\mathbf{Q}_\text{I}) = 1$. That is, $\mathbf{Q}_\text{I} = b\boldsymbol{v}\boldsymbol{v}^H$ such that $\boldsymbol{v}$ spans the null space of $\mathbf{A}$ and $b > 0$. Now the key is to show that $\mathbf{B} \succ \mathbf{0}$.

To guarantee that the Lagrangian of the problem (14) is bounded from below such that the dual function exists, we have $\mathbf{A} \succeq \mathbf{0}$. Since $\mathbf{A} \succeq \mathbf{0}$ and $-\lambda_r\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H \preceq \mathbf{0}$ for $\lambda_r \geq 0$, $\mathbf{B} \succeq \mathbf{0}$. Let us now construct a vector $\mathbf{x} = \mathbf{\Pi}_\text{E}\mathbf{h}_\text{I}$, where $\mathbf{\Pi}_\text{E} = \mathbf{I}_{N_\text{T}} - \mathbf{H}_\text{E}\left(\mathbf{H}_\text{E}^H\mathbf{H}_\text{E}\right)^{\dagger}\mathbf{H}_\text{E}^H$. Since $\mathbf{h}_\text{I} \notin \text{range}(\mathbf{H}_\text{E})$, we have $\mathbf{x} \neq 0$. Moreover, $\mathbf{x}^H\mathbf{h}_{\text{E},k} = 0, \forall k$, and $\mathbf{x}^H\mathbf{h}_\text{I} > 0$. Thus, we have

$$\mathbf{x}^H\mathbf{B}\mathbf{x} = \mathbf{x}^H\Bigg(\mathbf{I}_{N_\text{T}} + \mathbf{H}_\text{E}\mathbf{\Phi}\mathbf{H}_\text{E}^H - \sum_{k=1}^{K}\lambda_{e,k}\mathbf{H}_{\text{E},k} + \lambda_{a,l}\mathbf{\Upsilon}_l\Bigg)\mathbf{x} = \mathbf{x}^H\left(\mathbf{I}_{N_\text{T}} + \lambda_{a,l}\mathbf{\Upsilon}_l\right)\mathbf{x} > 0, \quad (70)$$

i.e. $\mathbf{B} \succ \mathbf{0}$. Since $\mathbf{A} \succeq \mathbf{0}$ and $\mathbf{B} \succ \mathbf{0}$, $\lambda_r > 0$ must hold in (67). Otherwise, $\mathbf{A}$ becomes full-rank, and it follows from (68) that $\mathbf{Q}_\text{I} = \mathbf{0}$. Next, we prove that $\mathbf{B} \succ \mathbf{0}$ must always hold by contradiction.

Suppose the minimum eigenvalue of $\mathbf{B}$ is zero. Then, there exists at least a vector $\mathbf{z} \neq \mathbf{0}$ such that $\mathbf{z}^H\mathbf{B}\mathbf{z} = 0$. According to (67), it follows that

$$\mathbf{z}^H\mathbf{A}\mathbf{z} = -\lambda_r\mathbf{z}^H\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H\mathbf{z} = -\lambda_r|\mathbf{z}^H\mathbf{h}_\text{I}|^2. \quad (71)$$

Since $\lambda_r > 0$, it follows that $\mathbf{z}^H\mathbf{A}\mathbf{z} \leq 0$. This implies that $\mathbf{A}$ is not positive semidefinite, which in turn violates the KKT condition in (68). Hence, we conclude that $\mathbf{B} \succ \mathbf{0}$ must hold. □

E. Proof of Proposition 1

In this proof, we use the identities already developed in the proof of *Theorem 3*. The KKT conditions of the problem (18) relevant to this proof can be defined as

$$\mathbf{A}\mathbf{Q}_\text{I} = \mathbf{0}, \text{ and } \mathbf{D}\mathbf{Q}_\text{E} = \mathbf{0}. \quad (72)$$

From the proof of *Lemma 5*, we notice that $\text{rank}\,(\mathbf{X}+\mathbf{Y}) \geq \text{rank}(\mathbf{X}) - \text{rank}(\mathbf{Y})$. Thus we obtain from (22) that $\text{rank}(\mathbf{A}) \geq r_\text{B} - 1$. If $\mathbf{B}$ in (20) is of full-rank, i.e., $r_\text{B} = N_\text{T}$, then following similar arguments as that in the proof of *Theorem 3*, we can conclude that $\text{rank}(\mathbf{A}) = N_\text{T} - 1$. Accordingly, $\text{rank}(\mathbf{Q}_\text{I}) = 1$. That is, $\mathbf{Q}_\text{I} = b\boldsymbol{v}\boldsymbol{v}^H$ such that $\boldsymbol{v}$ spans the null space of $\mathbf{A}$ and $b > 0$.

For the case when $r_\text{B} < N_\text{T}$, let $\mathbf{\Psi} = \left[\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \ldots, \boldsymbol{\psi}_{N_\text{T}-r_\text{B}}\right]$ with $\mathbf{\Psi}^H\mathbf{\Psi} = \mathbf{I}_{N_\text{T}-r_\text{B}}$ denote the orthogonal basis for the null space of $\mathbf{B}$, i.e., $\mathbf{B}\mathbf{\Psi} = \mathbf{0}$. Then we have

$$\boldsymbol{\psi}_i^H\mathbf{A}\boldsymbol{\psi}_i = \boldsymbol{\psi}_i^H\left(\mathbf{B} + \mathbf{h}_\text{I}\mathbf{h}_\text{I}^H\right)\boldsymbol{\psi}_i = |\mathbf{h}_\text{I}^H\boldsymbol{\psi}_i|^2 \geq 0, \quad \text{for } i = 1, \ldots, N_\text{T} - r_\text{B}. \quad (73)$$

To guarantee that the Lagrangian of the problem (18) is bounded from above such that the dual function exists, it follows that $\mathbf{A} \preceq \mathbf{0}$. Since $\mathbf{A} \preceq \mathbf{0}$, it follows from (73) that $\mathbf{h}_\text{I}^H\boldsymbol{\psi}_i = 0, \forall i$. That is,

$$\mathbf{h}_\text{I}\mathbf{h}_\text{I}^H\mathbf{\Psi} = \mathbf{0}. \quad (74)$$

As a result, we have

$$\mathbf{A}\mathbf{\Psi} = \mathbf{0}. \quad (75)$$

Let $\mathbf{W}$ denote the orthogonal basis for the null space of $\mathbf{A}$. Then it follows that

$$\text{rank}(\mathbf{W}) = N_\text{T} - \text{rank}(\mathbf{A}) \leq N_\text{T} - r_\text{B} + 1. \quad (76)$$

According to (75), $\mathbf{\Psi}$ spans $N_\text{T} - r_\text{B}$ orthogonal dimensions of the null space of $\mathbf{A}$, i.e., $\text{rank}(\mathbf{W}) \geq N_\text{T} - r_\text{B}$. If $\text{rank}(\mathbf{W}) = N_\text{T} - r_\text{B}$, then we have $\mathbf{W} = \mathbf{\Psi}$. Since $\mathbf{A} \preceq \mathbf{0}$, $\mathbf{Q}_\text{I}$ can be expressed as $\mathbf{Q}_\mathrm{I} = \sum_{i=1}^{N_\mathrm{T}-r_\mathrm{B}} a_i \boldsymbol{\psi}_i \boldsymbol{\psi}_i^H$, for $a_i \geq 0$, satisfying the KKT conditions in (72). However, no information will be transferred to the IR since all $\boldsymbol{\psi}_i$'s lie in the null space of $\mathbf{h}_\mathrm{I}\mathbf{h}_\mathrm{I}^H$ according to (74). As a consequence, $\mathrm{rank}(\mathbf{W}) \neq N_\mathrm{T} - r_\mathrm{B}$. According to (76), there exists only a single subspace spanned by $\boldsymbol{v}$ of unit norm such that $\mathbf{A}\boldsymbol{v} = \mathbf{0}$, and is orthogonal to the span of $\boldsymbol{\Psi}$, i.e., $\boldsymbol{\Psi}^H \boldsymbol{v} = \mathbf{0}$. Therefore, we can define $\mathbf{W}$ as $\mathbf{W} = [\boldsymbol{\Psi}, \boldsymbol{v}]$ with $\mathrm{rank}(\mathbf{W}) = N_\mathrm{T} - r_\mathrm{B} + 1$. Finally, according to (72) and (76), the optimal $\mathbf{Q}_\mathrm{I}$ can be expressed as $\mathbf{Q}_\mathrm{I} = \sum_{i=1}^{N_\mathrm{T}-r_\mathrm{B}} a_i \boldsymbol{\psi}_i \boldsymbol{\psi}_i^H + b\boldsymbol{v}\boldsymbol{v}^H$, where $a_i \geq 0, \forall i, b > 0$.

The first part of *Proposition 1* is thus proved. The proof of the second part of *Proposition 1* is identical to that of [12, Proposition 4.1(3)] and is thus omitted for brevity. □

F. Proof of Proposition 3

Any feasible point $(\check{\mathbf{Q}}_\mathrm{I}, \check{\mathbf{Q}}_\mathrm{E})$ of the design (30) satisfies $\mathrm{SINR}_k \leq \gamma$ for all k. The corresponding constraints (30b) can be expressed as

$$\mathrm{tr}\left(\check{\mathbf{Q}}_\mathrm{I}\left(\frac{1}{\sigma_k^2}\mathbf{R}_{\mathrm{E},k}\right)\right) \leq \gamma\left(\mathrm{tr}\left(\check{\mathbf{Q}}_\mathrm{E}\left(\frac{1}{\sigma_k^2}\mathbf{R}_{\mathrm{E},k}\right)\right) + 1\right), \quad \text{for } k = 1, \ldots, K. \tag{77}$$

Summing (77) over all k and using the notation in (34), we obtain

$$\mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{I})\boldsymbol{\Sigma}^{-1}\right) \leq \gamma\left(\mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1}\right) + K\right). \tag{78}$$

Note that the newly added constraint (36) is equivalent to (see [20])

$$\lambda_{\min}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1}\right) \geq \frac{1}{K}\mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1}\right), \tag{79}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of its argument.

Using the basic matrix properties $\mathbf{x}^H\mathbf{A}\mathbf{x} \geq \lambda_{\min}(\mathbf{A})\|\mathbf{x}\|^2$ and $\mathbf{x}^H\mathbf{A}\mathbf{x} \leq \mathrm{tr}(\mathbf{A})\|\mathbf{x}\|^2$ for $\mathbf{A} \succeq \mathbf{0}$, we have from (35) that

$$\begin{aligned}\mathrm{SINR}_\mathrm{ce} &= \frac{(\boldsymbol{\Sigma}\mathbf{w})^H\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{I})\boldsymbol{\Sigma}^{-1}\right)(\boldsymbol{\Sigma}\mathbf{w})}{(\boldsymbol{\Sigma}\mathbf{w})^H\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1} + \mathbf{I}_K\right)(\boldsymbol{\Sigma}\mathbf{w})} \\ &\leq \frac{(\boldsymbol{\Sigma}\mathbf{w})^H\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{I})\boldsymbol{\Sigma}^{-1}\right)(\boldsymbol{\Sigma}\mathbf{w})}{\left(\lambda_{\min}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1}\right) + 1\right)\|\boldsymbol{\Sigma}\mathbf{w}\|^2} \\ &\leq \frac{\mathrm{tr}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{I})\boldsymbol{\Sigma}^{-1}\right)}{\lambda_{\min}\left(\boldsymbol{\Sigma}^{-1}\mathcal{M}(\check{\mathbf{Q}}_\mathrm{E})\boldsymbol{\Sigma}^{-1}\right) + 1}. \end{aligned} \tag{80}$$

By putting (79) and then (78) into (80), we obtain the end result $\mathrm{SINR}_\mathrm{ce} \leq K\gamma$. □

G. Proof of Proposition 4

Following *Lemma 1*, it suffices to show that (42b) is equivalent to (46b). Let us now substitute $\mathbf{H}_\mathrm{E} = \hat{\mathbf{H}}_\mathrm{E} + \boldsymbol{\Delta}$ into (42b) and then set $\mathbf{X} = \boldsymbol{\Delta}$, $\mathbf{A} = (\gamma - 1)\mathbf{Q}_\mathrm{E} - \mathbf{Q}_\mathrm{I}$, $\mathbf{B} = ((\gamma - 1)\mathbf{Q}_\mathrm{E} - \mathbf{Q}_\mathrm{I})\hat{\mathbf{H}}_\mathrm{E}$, $\mathbf{C} = (\gamma - 1)\mathbf{I} + \hat{\mathbf{H}}_\mathrm{E}^H((\gamma - 1)\mathbf{Q}_\mathrm{E} - \mathbf{Q}_\mathrm{I})\hat{\mathbf{H}}_\mathrm{E}$, and $\mathbf{D} = \varepsilon^{-2}\mathbf{I}$. Thus, (42b) can be represented by the left-hand side of (45). Applying the implication in *Lemma 3*, we obtain (46b) as an equivalent form of (42b). □

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